STUDIES IN THE HISTORY OF THE EXACT SCIENCES IN HONOUR OF DAVID PINGREE

EDITED BY

CHARLES BURNETT, JAN P. HOGENDIJK, KIM PLOFKER AND MICHIO YANO

STUDIES IN THE HISTORY OF THE EXACT SCIENCES IN HONOUR OF DAVID PINGREE

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Preface

This collection of essays is an expression of respect and gratitude from a group of scholars who have worked closely with David Pingree, either as doctoral students or as colleagues, in the Department of the History of Mathematics at Brown University and elsewhere. A much larger number of scholars could have been invited to participate if we had included all those who have benefited from his written work and his generously given advice. The geographical spread of the contributors shows how the influence of his scholarship has taken root not only in North America, but also in Europe, India and Japan. The breadth of the subject matter indicates how wide David's own expertise extends: from ancient Mesopotamia, through Greece, India and Persia, into the Islamic and medieval European worlds. The emphasis at the Department of the History of Mathematics, ever since Otto Neugebauer was invited to Brown in 1939, has been on the interpretation of pre-modern mathematical texts (in the widest sense), and David has followed this admirable tradition in editing texts in Akkadian, Arabic, Greek, Latin, Persian and Sanskrit. Thus, most of the articles here are also concerned with texts, and several of them include editions. David has always believed that pre-modern astronomy and astrology form a single science of the stars; the theoretical paradigms for the movements of the Sun, Moon and planets generate the astronomical tables which are used for making astrological predictions for everyday personal matters and for affairs of the nation. The articles on astrology in this collection all deal with texts that describe or imply some mathematical basis for their calculations.

Those who have worked with David in Wilbour Hall in the shadow of the University Library will remember how he would turn up every morning with his dog—for many years a gentle but intellectually challenged black mongrel called Junior—whom Neugebauer would reward with a titbit. Gerald Toomer would

be working in the basement with his two corgis; Abe Sachs was quietly unravelling the secrets of his cuneiform tablets in a neighbouring office. Throughout the day one would read with him a text in one of David's many languages, pulling books off the shelves which contained the most comprehensive collection of works on the history of mathematics that has ever been assembled in one place. Besides the books one might plunge one's hand into a sea of microfilms of manuscripts, or consult one of the many immaculate transcriptions of unpublished texts that David made, or his card-file of datable horoscopes. Only rarely did one have to go to the Department's big brother next door to supplement the resources of the unique library. Shortly before mid-day (incredibly early for most Europeans) one would accompany David and Neugebauer to the university cafeteria where one would pile a mixture of salads and sauces into one bowl and wash it down with juice or beer. The afternoon stint would continue until five or six o'clock, when David walked back home with his dog. But one knew that more work waited there for him: perhaps an edition of a Sanskrit text, or a set of astronomical tables.

The scholars associated with the Department of the History of Mathematics referred to one another by various nicknames. The founder, Neugebauer, was 'the Elephant'. His colleague Ted Kennedy, the expert on Arabic mathematics, naturally acquired the name of the ninth-century Arabic philosopher and scientist, al-Kindī. Gerald Toomer's Oxford origin earned him the title *Homo oxoniensis* ('Home-Ox' for short). Abe Sachs was 'the Owl', whose office walls were adorned with numerous postcards and other pictures of his namesake. Pingree's nickname, 'Abū Kayd', was inspired by the similarity of his family name to that of Alexandre Pingré, the great eighteenth-century authority on comets (*Cométographie*, 2 vols, Paris, 1783–4). The Arabic 'kayd' in the sense of 'comet' is derived from the Sanskrit 'ketu', which originally meant 'brightness, rays of light', but came to be applied both to a comet (called the 'tailed star' in Arabic and medieval Latin) and the descending lunar node, which was regarded as being the tail of a dragon. The idea of brightness and of swishing tails persists in another meaning of 'ketu', denoting 'ensign' or 'banner'; moreover, the word also means an 'eminent person'. So we are happy to honour David not only in his own name, but also

as Abū Kayd, 'the father (or epitome) of the eminent scholar', for whom the banners can be unfurled.

In addition to the twenty-nine articles edited here, we have attempted to put together a complete bibliography of David's publications until the end of July 2003. We fear that we have not entirely succeeded: he has been so prolific, and published in such a wide range of journals, that we may have inadvertently left out a few of his articles, let alone several judicious reviews of other people's books. Nonetheless, we hope that the *curriculum operis* presented here will serve as a boon to all scholars who rely on David's work, as well as a testimony to his vast contributions.

Acknowledgements

We are deeply indebted to the current and recent doctoral students in the Brown History of Mathematics Department who generously added to their heavy workloads the task of helping prepare this tribute to their teacher. Setsuro Ikeyama, Toke Knudsen, and Micah Ross compiled and arranged most of the Pingree bibliography, and Clemency Williams helped edit the papers in the Assyriology section. Additionally, a graduate student at the University of Utrecht, Sybren Botma, indexed the volume. Two colleagues, Taro Mimura of Tokyo and David Juste of the Warburg Institute, were equally generous with their time and effort. Finally, we owe much to the skilful editorial support of Trudy Kamperveen at Brill. The able assistance of each of them is greatly appreciated.

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Mesopotamia

Constellation into Planet

ERICA REINER

It was more than twenty years ago that David Pingree identified many of the terms used in astral omens to describe the ominous phenomena, both the astronomical and the atmospheric ones.[1] During those twenty years I have learned much from him, but I am still unable to solve some of the simplest philological problems that have arisen in the course of my work on Enūma Anu Enlil (henceforth EAE), Pingree's comments and patient instruction notwithstanding. Here I will offer, as a small tribute, a sample of what I consider my ignorance, not on the astronomical level, which should be obvious, but on the basic philological level.

Among the words used to describe the appearance of a celestial body the terms denoting brightness span the scale from dim or faint to various degrees of brightness, using a vocabulary the exact nuances of which we cannot establish, and for which we use conventional translations that can hardly capture the nuance attributed to them in antiquity. Maximum brightness is designated by *baʾil* for a single star or planet, *buʾulu* for a plurality, such as several stars of a constellation. David Pingree used the term 'brilliant'. Another term, also indicating brightness, *nebû* or *nabû*, normally qualifies the word for star, *kakkabu*, and according to the Chicago Assyrian Dictionary (CAD) denotes the brightest star of a constellation. This interpretation of the CAD is based on the references which speak of the 'bright star' (*kakkabu nebû*) of one or another constellation. Significantly, its Sumerian equivalent is d i l. b a t, also read d e l e. b a t, the very name of the planet Venus which is often qualified with this epithet *nebû*.

Similar is the case in regard to the terms for 'dark' and 'faint'. The most common ones are *adir* and *duʾum*, for which Pingree proposed 'obscured' and 'dark', but *ekil* 'dim'[2] and *ukkul* (for the

[1] [Reiner & Pingree 1981, 16–21].

[2] David Brown has pointed out that the translations given in [Reiner &

plural) are also attested. Of the terms used to denote faintness, the most common is *unnut* 'faint'.

The predicate may not only be an adjective (or stative of a verb), but an inflected verb form too; to the descriptive terms I have just listed belong the verb forms *iba›il* and *inambuṭ* (the latter also occurs in the ingressive *ittanambiṭ*) 'shines brightly' and, to indicate faintness, *ūtannat* (Rep. 114 [Hunger 1992, 72–3]).

Moreover, several terms for scintillation—or change in intensity—occur in these texts: *ittananbiṭu* 'they shine brightly repeatedly' and *ittananpahu* 'they flare up again and again'; this latter term is explained in the commented text as 'the planets become very bright' (^{d}UDU.BAD.MEŠ *iba›iluma*).[3] Two difficult terms, *ummulu* and *mulluh*, are, as Pingree put it,[4] not convincing candidates for the meaning scintillation. To make matters even more complicated, *ummul* is given as explanation, or variant, to *apil* 'late' referring to the rising of Entenamašhum.[5] The term *ṣirhu* was translated provisionally in [Hunger 1992] as 'flare'. The similarly elusive terms *mišhu* and *šarūru* were taken by Pingree to mean mirage, meteorites, or fireballs. The use of color terms to describe celestial objects are the same as those used of various other ominous manifestations in house and fold: MI (*ṣalmu* or *tarku*) 'black', SIG_7 (*arqu*) 'green', SA_5 (*sāmu*) 'red', BABBAR (*peṣû*) 'white'. Some terms are used metaphorically, thus *lummun* 'in a bad condition' (said of Lyra, Taurus, and other constellations) probably denotes poor quality in brightness; why *lummun* is used instead of, e.g., *adir* or *unnut* is not known.

Stars and constellations can also be provided with a LAL or *išpalurtu*, presumably a light phenomenon. Whether LAL is the logogram for *išpalurtu* is not certain; some texts use one, some the other term, which seems to refer to a cross-shaped object, while in K.4571 the two appear, though in broken context, in the consecutive lines 2 and 3.

We could look up these terms in the dictionaries—we have

Pingree 1998] differ from those in [Reiner & Pingree 1981]; see [Brown 2000, 153 n. 364].

3 [Reiner & Pingree 1981, 42].

4 [Reiner & Pingree 1981, 19].

5 *Entenamašhum ina* È-*šú apil*: [Virolleaud 1908–12, Part 1, 24] (Sin 19:17); *apil* explained as 'rises in month V' and *ummul* (*ibid.* and K.3579).

now one complete dictionary, W. von Soden's *Akkadisches Handwörterbuch* (AHw.); another, the CAD, almost complete (it lacks only the letters Ṭ (*ṭet*), U, and W); and we even have a Concise Dictionary (CD). Unfortunately, when we try to look up these terms, it turns out that they are restricted—with very few exceptions—to precisely the phenomena for which we seek a definition.

If philology deserts us, we may as well turn to the explanations that the astrologers and astronomers of the first millennium resorted to. They were, as we sometimes are, trying to be clever, and sometimes too clever. Not content with the literal meaning of such terms as 'bright' or 'dark', 'green' or 'red', they resorted to explanations in terms of the presence—and presumably the benefic or malefic influence—of a planet. Such explanations are common procedure in commented texts: the description of or reason for a constellation's appearance as dark or bright, etc., is often given not in terms of its actual appearance, but in reference to the planet which is seen with it or in it.

Thus, darkness is explained as the presence of Mars or Saturn, but also of Mercury: for example, *adir* (said of the western Fish, called SIM.MAH 'Swallow') is explained as 'Mercury stands in SIM.MAH' (Rep. 253);[6] MUL.ŠU.GI *adir* 'The Old Man star is obscured' is explained as ^{d}GENNA(TUR.DIŠ) *ina* ŠÀ IGI 'Saturn is seen in it' (TCL 6 18 + r. 10). MUL.UDU.BAD *adir* 'a planet is obscured' is explained as (Mars) *ūtanna*[*t*] 'Mars becomes faint' (K.2329 r. 9); ^{d}*Ea adir* is explained as Mercury *ina* MUL.SUHUR.MÁŠ.KU$_6$ *ūtannatma* 'Mercury becomes faint in Capricorn' (K.2064:9f.).

When the sides of the Scorpion are said to be dark (*ukkula*), this is explained as 'Mars stands in Scorpius'. If the front star of Enmešarra is very dark (*mādiš ekil*), that means that Mercury is seen in the constellation Old Man (ŠU.GI). Note: MUL.KU$_6$ *a-dir*... MUL.KU$_6$ ^{d}*Ṣalbatānu a-dir* ^{d}SAG.UŠ 'If the Fish is obscured—the Fish is Mars; (if it is) obscured, it is Saturn' (Rm.192:7f.).

Elsewhere too, Saturn is associated with blackness and darkness: for example, the description *zīmūšu uṣṣanallamu* '(if the Crook star's) appearance turns quite black', is explained

[6] [Hunger 1992, 141].

as Saturn having approached Jupiter in the Old Man star;[7] similarly in the omen ¶ MUL.UR.GU.LA GI_6... ¶ MUL GI_6 ^{d}UDU.[BAD.SAG.UŠ] 'If the Lion is black—the Black Star is Saturn' (Rep. 180 r. 3′–4′)[8] and the same is said of the constellation Dead Man (UG_5.GA), although the name of the constellation is a variant writing of UGA 'Raven'.

When the stars of Orion *ittananbiṭu* 'gain radiance', this results from the fact that Venus precedes it, or stands in front of it (¶ MUL.SIPA.ZI.AN.NA MUL.MEŠ-*šú ittananbiṭu*... MUL *Dilbat ina pa-an* MUL.SIPA.ZI.AN.NA GUB-*ma* (Rep. 255 r. 9).[9] Not only the derivative *nibṭu* from *nabāṭu* is usually translated with the term 'radiance'; so also is *šarūru*. A star or planet can both be bright and carry radiance: Mars *baʾil u šarūru naši* (Rep. 491 r. 7f.),[10] the opposite being *unnut u šarūrūšu maqtu* (said of Saturn) (ibid. r. 10).

When the predicate referring to the light of a planet is *iltappû* (for *ištappû*), and even written as *ultappû*, so that it remains unclear whether the verb is *(w)apû* or *šapû*, Hermann Hunger very wisely leaves the verb untranslated,[11] while the CD boldly attributes to *šapû* the meaning, in the Gtn stem, 'to repeatedly flare up', and to *(w)apû* D 'to make visible' (from AHw.'s 'sichtbar machen'). Still, if *iltappû* is predicated of the stars of Leo, it is explained as Jupiter standing in Leo; that is, it refers to a great brightness (Rep. 54).[12]

Redness, of course, is associated with the red planet, Mars; thus the protasis MUL NN *maʾdis* SA_5 'If the constellation NN is very red' is explained as 'Mars stands in the constellation NN'. E.g., 'If the star of the Kidney is very red—Mars [stands?] in Aquarius' (K.6519:11′f.). Redness, however, can also refer to Mercury, as in the omen 'If one star in the Fish is very red' which is explained as 'Mercury is very bright in Capricorn' (K.7945:3′ff.).

Why this should be so, I can only speculate as a philologist, without knowledge of astronomy. It seems to me that by the seventh century the ominous significance of superficial features

[7] TCL 6 no. 18 line 23, cited in [Weidner 1925, 356].
[8] [Hunger 1992, 104].
[9] [Hunger 1992, 143].
[10] [Hunger 1992, 271].
[11] Rep. 437 [Hunger 1992, 248].
[12] [Hunger 1992, 32].

and events regarding celestial bodies was dismissed as irrelevant. Early omen literature considered any visible aspect or disturbance of a star or planet—such as its color, its brightness, or its passing through the lunar halo—as ominous, just like the appearances and events of other non-provoked omens (e.g., those of the *šumma ālu* type). But the astronomers of the Sargonid period realized that these were atmospheric phenomena, not computable as the astronomical phenomena were. In order to make these 'atmospheric' phenomena relevant, they therefore interpreted them in astronomical terms.

Similar explanations were applied to various other phenomena too, whether possible, plausible, or impossible. Thus, apart from simple adjectival predicates, descriptions may also include the behavior of the star or constellation in relation to other celestial bodies or celestial directions, for example, whether it faces East or West (*ana* ^{d}UTU.È or *ana* ^{d}UTU.ŠÚ.A *panūšu šaknu*), or a particular direction (*ana* IM.1—also IM.2, IM.3, IM.4—*panūšu šaknu*); its position (*harrān Šamaš ikšud* 'reaches the path of the Sun'), and its relation to the Moon; scintillation and other light phenomena (*ṣirhu iškun, mišhu imšuh*), and such idiosyncratic behavior as being visible by day or not being visible by night. Omens are also derived from whether a star or constellation is high or low, bright or dim, faces East or West at heliacal rising.

Certain constellations that are in animal shape—the Scorpion, the Fish, the Worm—have phenomena predicted of them in terms of these animals; that is, in reference to their pincers, tails, horns. When a constellation is considered as consisting of several stars, these stars take on a life independent of the constellation and are described in relationship to one another. For example, the two Fish can come closer together or recede one from the other (*ibēš*, *iqrib*, *issanqu*), and the stars of the Pleiades can recede (*nehsu*) or become elongated (*šathu*). The upper or the lower stars of the constellation Field, i.e., the square of Pegasus,[13] are connected (*ritkusu*) or meet (*nenmudu*). The latter is also said of the stars of the constellation Old Man, and a commentator explains this phenomenon by attributing to them brilliance (MUL.MEŠ-*šu ba-*ˀ*lu* K.2894:16f.). These phenomena are therefore most likely due to atmospheric conditions.

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[13] [Pingree & Reiner 1981, 64] (Text XII 45).

Phenomena predicated of planets require, of course, different terminology. These phenomena are: first and last visibility; visibility in each of the twelve (sometimes thirteen) lunar months; and visibility in each of the three 'paths' or segments on the horizon. It may also be stated whether a planet is invisible longer than its expected period, e.g. *ina šamê uhharamma* 'it lingers in the sky', or (Venus) *manzassa urrik* 'prolongs its position', or that it becomes visible when it is not expected, e.g., *ihrumma innamir* 'became visible early' (Rep. 27).[14] A planet may also be obscured (*adir*) in each of the twelve months. Several times attested is the protasis: *Nergal ina šubtišu zi-ir* 'Nergal (i.e., Mars) is crooked? in its location?' (AHw sub *zâru*), a phrase which occurs only in this collocation, and only in reference to Nergal.

Since the planets move relative to the fixed stars, the predicates describing their behavior are the regular verbs of motion: a planet reaches (*ikšud*), passes (*ītiq*), approaches, comes close (*iṭhi*, *uqerrib*, *isniq*), leaves behind (*īzib*) another planet or a constellation; similarly, a planet may enter into (*īrub*) another. Two such bodies—two planets or a planet and a star—may face? one another (*imdahharu*), meet (*ittenmidu*), follow closely (*ittentû*, *innetû*), stand above (*elēnu... izziz*) or inside (*ina libbi... izziz*) another.

More rare and unusual are predications such as that a constellation 'inseminates' (*irhi*) another,[15] or 'bends down' toward the apsû (*ana apsî šūr*) or toward the sky; the former is explained as 'its stars are very bright', and the latter as 'its stars are very dim'. A constellation may 'lean against' (*īmid*) a fixed star or constellation.

In addition to the above and similar phenomena predicated of planets that, within certain boundaries, can be assumed to occur, the texts also describe movements of individual stars of a constellation and movements of constellations or their parts relative to other constellations. For these motions the texts use the same terminology that they use when they refer to planets.

The astronomical impossibility of such statements can be explained in two different ways. One explanation has a long his-

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[14] [Hunger 1992, 16].

[15] *šumma* MUL.UD.KA.DU$_8$.A [GÌ]R 15 MUL *Lu-lim ir-hi* K.3780, ii, 17 in [Virolleaud 1908–12, 2nd Suppl., 95, 104].

tory and claims that the names of the stars and constellations are *Decknamen* (that is, substitutes for names) of planets, and thus the motions ascribed to them actually refer to motions of planets; obviously, there is nothing impossible in the movements attributed to them.[16] This explanation has a confirmation in the comments appended to such omens by the ancient scribes, both in the astral omens and in the Reports. The other possibility is that the verbs used to describe phenomena affecting fixed stars may have had a meaning different from the usual meanings describing motions, and designated some other phenomenon that could be predicated of fixed stars and planets alike.[17] Examples:

> ¶ MUL.KU$_6$ *ana* MUL UG$_5$.GA *īmid*... MUL.UDU.BAD.SAG.UŠ *ina libbi* MUL.SIM.MAH *lu* MUL *Anunītu* GU[B-*ma*] 'If the Fish stands against the Dead Man—Saturn stands inside the Swallow or Anunītu', i.e., between the western and the eastern Fish (Sm.1154:6f.). But the same omen is explained as referring to UDU.BAD.GUD.UD = Mercury (Rep. 73 r. 1–4).[18]

Omens about the Pleiades are explained as referring to Mars, as in

> [¶ MUL.MUL MUL.KA].MUŠ.Ì.KÚ.E KUR-*ud* [MUL *Ṣal*]-*bat-a-nu* ^{d}SAG.UŠ KUR-*ma* (Rep. 491r. 3f.,[19] quoting K.3558:6);

> ¶ MUL.MUL MUL.KA.MUŠ.Ì.KÚ.E KUR-*ud* ^{d}*Ṣalbatānu* ^{d}UDU.BAD.SAG.UŠ KUR-*ma* 'If the Pleiades reach β Andromedae—Mars reaches Saturn'; (K.3558:6, also Rm.191:13f., both EAE 53 commentary);

> ¶ MUL.MUL *ana* MUL.AŠ.GÁN TE 'If the Pleiades come close to the Field' (K.3558:8), with comm. *Ṣalbatānu ana* MUL.AB.SÍN TE-*ma* 'Mars comes close to Virgo' (K.5713:20), or ^{d}*Ṣalbatānu ana* ^{d}GUD.UD *lu ana* MUL.AB.SÍN TE-*ma* 'Mars comes close to Mercury or to Virgo' (K.3558:8);

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[16] Unless the planet is said to reach or enter, etc., a constellation too far from the ecliptic.

[17] For a résumé of suggestions, see, e.g., [Hunger 1992, xvi], and [Brown 2000, 54 n. 161].

[18] [Hunger 1992, 43].

[19] [Hunger 1992, 271].

¶ MUL.MUL *ana Sin ithûma* (wr. TE-*ma* with gloss) 'If the Pleiades come close to the Moon—[MUL.MUL MUL] *Ṣal-bat-a-nu* 'the Pleiades are Mars' (Rep. 50:1–7);[20]

¶ MUL.MUL *ana* IGI *Sin* TE-*ma* GU[B-*iz*]... MUL.MUL MUL *Ṣalbatānu* (Rep. 72:1 and r. 2);[21]

but note the omen

¶ MUL.Á.MUŠEN *ana* MUL.MUL TE... MUL *A-hu-ú Ṣalbatānu* 'If the Eagle (perhaps read phonetically as Á-*hu*, not Á.MUŠEN) comes close to the Pleiades—the Strange? star is Mars' (K.7129:14).

Still, some Pleiades omens are said to refer either to Mercury or to Saturn, as in

¶ MUL.MUL MUL.MUŠ KUR-*ud* dGUD.UD *lu* SAG.UŠ d*Ṣalbatānu* KUR-*ma* 'If the Pleiades reach the Snake—Mercury or Saturn reach Mars' (K.3558:7, EAE 53 commentary).

In the Jupiter Tablet (EAE 64) we have the omen

¶ MUL.ŠU.GI MUL.GÀM (= *Gamlum*) *īzib* 'If the Old Man leaves the Crook behind', with explanation... MUL.GÀM d*Marduk* MUL.SAG.ME.GAR KI MUL [...] 'the Crook is Marduk, Jupiter (break)'

and another commented text has the explanation

MUL *Marduk ana* dGUD.UD *iqabbi šanîs mā* MUL d*Marduk ana* MUL.GÀM *iqabbi* 'the Marduk star is said with reference to Mercury, or alternatively, the Marduk star is said with reference to the Crook'.

Mercury is also adduced as the planet to be understood in the omens ostensibly dealing with the Kidney star (MUL.BIR) in K.2064:6f. and in VAT 7830.

Sometimes one, sometimes another of the constellations is equated with a planet. Note, e.g., the Eagle omens

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20 [Hunger 1992, 30]; cf. Rep. 376:9 (comm. on line 6) [Hunger 1992, 213].
21 [Hunger 1992, 42].

¶ *ina* ITI.ŠU MUL.KAK.SI.SÁ MUL Á.MUŠEN KUR-*ud*...^{d}GUD.UD *ina* ITI.ŠU MUL *Ṣalbatānu* KUR-*ma* 'If in month IV the Arrow (= Sirius) reaches the Eagle—Mercury in month IV reaches Mars' (K.2346+ r. 17),[22]

and

¶ MUL.PAN *ana* MUL Á.MUŠEN *ikšud*... *Ṣalbatānu* MUL *Zappa* KUR-*ma* 'If the Bow reaches the Eagle—Mars reaches the Pleiades' [Borger 1973, 41:r. 12],

versus

¶ MUL.PAN *ana* MUL.KAK.SI.SÁ *ikšud*...MUL.UDU. BAD.[GUD.UD *ina* ŠÀ MUL.AB.SÍN DU-*ma*] 'If the Bow reaches the Arrow—Mercury stands in Virgo' (ibid. 13; also K.7129:19)

and

¶ MUL.PAN *ana* MUL *Šulpae* DIM$_4$... MUL.SAG.ME.GAR *ina* ŠÀ MUL.AB.SÍN [DU]-*ma* 'If the Bow approaches Šulpae (i.e., Jupiter)—Jupiter stands in Virgo' [Borger 1973, 41:r. 14];[23]

The next lines in [Borger 1973], dealing with MUL.ÙZ 'the Goat', all refer to Venus, as do Goat-star omens elsewhere (e.g., Rep. 175, 247).[24] The Erua star (A.EDIN) is also identified with Venus, as in

¶ MUL.A.EDIN *ana* MUL.MUL KUR-*ud* 'If Erua reaches the Pleiades' (Rep. 55 r. 9 [Hunger 1992, 33]), also with explanation MUL *Dilbat ina* ŠÀ MUL.MUL [GUB-*ma*] 'Venus stands in the Pleiades' (Rep. 536:3f.)[25] and MUL *Dilbat* MUL *Zappu* KUR-*ma* 'Venus reaches the Pleiades' (K.5713+:11).[26]

MUL.APIN is often equated with Mars, e.g.,

[22] See [Reiner & Pingree 1998, 246:46].
[23] Restored from K.5713+:8′ (EAE 53 commentary).
[24] [Hunger 1992, 102–3, 137–8].
[25] [Hunger 1992, 294].
[26] But note that the following line has the explanation 'Saturn reaches Virgo, variant: Venus', because in this omen Erua is said to reach the 'Dead Man', which, as we have seen, is associated with Saturn; whereas in the dupl. K.1494a r. 3′, and in the following line describing the Stag (*Lu-lim*), the comment only refers to Saturn reaching the Pleiades.

¶ MUL.APIN *ana* MUL.GÍR.[TAB TE]... [MU]L *Ṣal-bat-a-nu* *ana* MUL.[GÍR.TAB TE-*ma*] 'If the Plow comes close to the Scorpion—Mars comes close to the Scorpion' (Rep. 219:1ff., also 502 r. 1ff.).[27]

¶ MUL.APIN *harrān Šamaš ikšud... Ṣalbatānu* MUL.UDU. BAD.SAG.UŠ *ikaššadma* 'If the Plow reaches the "Path of Šamaš"—Mars reaches Saturn' (Rep. 49 r. 4–6).[28]

¶ MUL.AŠ.GÁN *ana* MUL.APIN TE 'If the Field comes close to the Plow', with commentary ^{d}*Ṣalbatānu ana* MUL.AB.SÍN *ulu* *ana* MUL.AŠ.GÁN TE-*ma* 'Mars comes close to either Virgo or the Field' (K.1522 r. 1 and duplicates K.6415:19, Rm.487:2′–4′), and with commentary ^{d}*Ṣalbatānu ina* ŠÀ KUN [...] (K.2329:11).

Note also the substitution of the Sun for Saturn in

¶ MUL.UR.BAR.RA ^{d}UTU *ikšud*... MUL.UR.BAR.RA MUL *Ṣalbatānu* [MUL^{d}UTU] MUL.UDU.BAD.SAG.UŠ 'If the Wolf reaches the Sun—the Wolf is Mars, the Star of the Sun is Saturn' (this refers to a conjunction of Mars and Saturn) (Rep. 48:1–6),[29]

a substitution that also occurs in

¶ ^{d}UTU *ana* ŠÀ *Sin* TU 'if the Sun enters into the Moon' (Rep. 166:1),[30] explained as ^{d}UDU.BAD.SAG.UŠ *ina* ŠÀ *Sin* *ētarab* 'Saturn entered into the Moon' (ibid. line 7).

It must be noted, however, that the Reports equate fixed stars and constellations with planets even when nothing unusual is predicated of them, such as the omen stating that the Yoke star stands in the lunar halo, which is interpreted to refer to Mars (Rep. 383)[31] or, as mentioned, that the Lion is black, which is commented upon as 'the Black star is Saturn' (Rep. 180).[32]

In closing, even though I have to admit that I am no closer to understanding what the Babylonian astronomers meant by these terms describing celestial phenomena, I would like to point to

[27] [Hunger 1992, 121–2, 279].
[28] [Hunger 1992, 29].
[29] [Hunger 1992, 28].
[30] [Hunger 1992, 98].
[31] [Hunger 1992, 219–20].
[32] [Hunger 1992, 104].

a characteristic feature of the commented texts, a feature that deserves to be followed up further. This is that the explanations, and the way they are presented, seem to me to be attempts to make use of the process of inference.[33] Examples are:

1. *Zibanītu manzassa kēni*... *Zibanītu* [MUL.UDU.BAD. SAG.UŠ], *manzassa kēni: ina libbi* MUL.AL.LUL [...], (=) MUL.UDU.BAD.SAG.UŠ *ina* [*libbi* MUL.AL.LUL] *izzazma* 'If the Scales' position is steady...the Scales are [Saturn], its position is steady (means) [...] inside the Crab, (that is), Saturn stands inside the Crab' (Rep. 39:1ff.).[34]

2. MUL.UR.BAR.RA MUL.UR.MAH [*ikšud*]...MUL.UR. BAR.RA MUL [*Ṣalbatānu*] MUL.UR.MAH MUL.[UR.GU.LA] *Ṣalbatānu ina* ŠÀ M[UL.UR.GU.LA GUB-*ma*] 'The Wolf reaches the Lion...the Wolf is Mars, the Lion is Leo, Mars stands inside Leo' (Rep. 45:1–6, also similar 48:1–5).[35]

3. Another way to arrive at the statement that Mars is in Cancer, which is an omen predicting evil, is to cite first the omen ¶ MUL.AL.[LUL *ana* MUL.APIN TE] 'If the Crab comes close to the Plow' (with an unfavorable apodosis), then the omen ¶ MUL MAN-*ma ana* MUL.AL.[LUL TE] 'If the Strange star (i.e., Mars) comes close to the Crab (with another unfavorable apodosis), and finally the omen ¶ MUL *Ṣalbatānu ana* MUL.AL.[LUL TE] 'If Mars comes close to the Crab' (with another unfavorable apodosis) (Rep. 452).[36]

4. A similarly ill-portending omen, evening first visibility of Mercury in Leo, is cited in Rep. 337[37] again as two omens: a) 'if a planet (UDU.BAD) rises in Month IV', apodosis; b) 'if Leo is black', apodosis. The same protasis, 'if a planet (UDU.BAD) rises in Month VI', now predicting a favorable omen, is attested in Rep. 340,[38] but does not have to be interpreted.

Oppenheim, in his article on the scribes of Enūma Anu Enlil,[39] noted this practice:

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[33] I am grateful to my friend and colleague, Professor Leonard Linsky, of the Department of Philosophy at the University of Chicago, for discussing this possibility with me.

[34] [Hunger 1992, 24].

[35] [Hunger 1992, 27, 28].

[36] 'Mars in Cancer' [Hunger 1992, 255].

[37] [Hunger 1992, 193].

[38] [Hunger 1992, 194].

[39] [Oppenheim 1969, 98].

> As a rule, a string of such quotations is given, each corresponding to a special feature of the event such as timing, accompanying circumstances, etc. Occasionally these quotations permit us to reconstruct the event in some detail.

I would like to assume that if no exact parallel could be found in the corpus available to the writers of the Reports, they sought to approximate it by enumerating the features that were pertinent to the event observed. Thus, it seems to me, they practiced what would qualify in terms of classical logic as inference. While their statements are not syllogisms, they seem to be attempts at propositional logic, and perhaps constitute a forerunner to them.

Abbreviations

AHw.	*Akkadisches Handwörterbuch* by W. von Soden
CAD	*The Assyrian Dictionary of the Oriental Institute of the University of Chicago*
CD	*A Concise Dictionary of Akkadian* by J. Black et al.
EAE	Enūma Anu Enlil
K	Texts in the British Museum
Rep	Reports in [Hunger 1992]
Rm	Texts in the British Museum
Sm	Texts in the British Museum
TCL	Textes cunéiformes du Louvre
VAT	Texts in the Vorderasiatisches Museum, Berlin

Bibliography

R. Borger, 'Keilschrifttexte verschiedenen Inhalts', in *Symbolae Biblicae et Mesopotamicae: F. M. T. de Liagre Böhl Dedicatae*, ed. M. A. Beek et al., Leiden, 1973, pp. 38–55.

David Brown, *Mesopotamian Planetary Astronomy-Astrology* (Cuneiform Monographs 18), Groningen, 2000.

Hermann Hunger, *Astrological Reports to Assyrian Kings* (State Archives of Assyria, vol. 8), Helsinki, 1992.

Erica Reiner & David Pingree, *Babylonian Planetary Omens. Part 1: The Venus Tablet of Ammiṣaduqa* (Bibliotheca Mesopotamica 2/1), Malibu CA, 1975.

——. *Babylonian Planetary Omens. Part 2: Enūma Anu Enlil Tablets 50–51* (Bibliotheca Mesopotamica 2/2), Malibu CA, 1981.

——. *Babylonian Planetary Omens. Part 3* (Cuneiform Monographs 11), Groningen, 1998.

Charles Virolleaud, *L'astrologie chaldéenne: le livre intitulé 'Enuma (Anu) Bêl'*, fasc. 1-14, Paris, 1908–12.

Ernst F. Weidner, 'Ein astrologischer Kommentar aus Uruk', *Studia Orientalia*, 1, 1925, pp. 347–58.

Stars, Cities, and Predictions

HERMANN HUNGER

The tablet BM 47494 (= 81-11-3, 199) is part of the 81-11-3 collection of the BM and was acquired by purchase.[1] According to the circumstances, it most likely comes from Babylon. This is supported by the name of the scribe, Šemaʾja, descendant of Eṭiru. He is mentioned in other tablets of the same collection and lived in the Achaemenid period, during the reign of an Artaxerxes.[2] Unfortunately, the date once written at the end of the tablet is now broken.

The tablet is damaged in several places, but its contents can be established to a large extent. There is ruling (done before writing) all over the tablet, but between rev. 16 and 17, 18 and 19, 20 and 21, and 22 and 23, lines seem to separate sections; there is also a line before the colophon.

1 *Transliteration*

1 DIŠ MUL.⸢MUL x⸣ ŠÁR$^{?}$⸢^{ki}⸣

2 DIŠ ^{mul}GU$_4$.AN.NA EN.LÍ[L]⸢^{ki} x⸣ ŠEŠ.UNUG^{ki} nin da la ta de-e-ri u ⸢x x⸣ $^{ki?}$

3 DIŠ ^{mul}SIPA.ZI.AN.NA UD.[KI]B.NUN^{ki} u La-ár-si$^{?ki}$

4 DIŠ ^{múl}ŠU.GI EN^{ki} u NUN^{ki}

5 DIŠ ^{múl}[MAŠ].TAB.BA.GAL.GAL GÚ.DU$_8$.A^{ki} u ŠEŠ.UNUG^{ki}

6 DIŠ ^{mú}[^{l}ALL]A UD.KIB.NUN^{ki} Dil-bat^{ki} u Gír-su^{ki} MURUB$_4$^{meš} UD.KIB.NUN^{ki} IGI^{meš} MIN MIN

[1] I thank the Trustees of the British Museum for permission to publish this tablet.

[2] [Finkel 1988, 154].

7 DIŠ SAG.DU $^{\text{múl}}$UR.A UNUG$^{\text{ki}}$ GABA-su TIN.TIR$^{\text{ki}}$ GÌR EN.LÍL$^{\text{ki}}$

8 DIŠ $^{\text{múl}}$AB.SÍN ⸢KUR⸣ NIM.MA$^{\text{k[i]}}$

9 DIŠ $^{\text{múl}}$RÍN KUR URI$^{\text{ki}}$

10 DIŠ $^{\text{múl}}$GÍR.TAB Dil-mun u Bar-⸢sipa$^{\text{ki}}$⸣

11 DIŠ $^{\text{múl}}$Pa-bil-sag TIN.TIR$^{\text{ki}}$ AMAR.DA$^{\text{ki}}$ u ⸢KUR NIM.

MA⸣$^{\text{ki}}$

12 DIŠ $^{\text{múl}}$SUHUR MÁŠ KUR Su-bar-tú$^{\text{ki}}$

13 DIŠ MÚL.GU.LA E EN UŠ ⸢x x⸣ A RA ⸢x⸣ [x x x x] ba$^{?}$ az zu

14 DIŠ KUN$^{\text{meš}}$ TIN.TIR$^{\text{ki}}$ [x] ⸢x⸣ ni$^{?}$ KUR

15 DIŠ $^{\text{múl}}$ $^{\text{lú}}$HUN.GÁ ⸢x⸣$^{\text{ki}}$

16 DIŠ šá in-nam-ma-ru šá-ru-ru na-šú-ú u NI BAD ⸢x⸣ [x N]E RU bu-ú

17 ana ZI KÚR ina ŠÀ $^{\text{mul}}$AL.LUL $^{\text{mul}}$UR.GU.LA u $^{\text{[mu]l}}$GÍR.TAB

18 ana SU.KÚ ina ŠÀ $^{\text{mul}}$AB.SÍN $^{\text{[mu]l}}$AŠ.IKU $^{\text{mul}}$ $^{\text{giš}}$R[ÍN u MU]L.GU.LA

19 ana BE$^{\text{meš}}$ ina ŠÀ ⸢MUL⸣.MUL $^{\text{mul}}$GU$_4$.AN.NA u $^{\text{mu[l}}$SI]PA.ZI.AN.NA

20 ana BURU$_{14}$ ina ŠÀ MUL.GU.LA $^{\text{mul}}$AB.SÍN $^{\text{mul}}$AŠ.IKU [KU]N$^{\text{meš}}$

21 ana MÍ$^{\text{meš}}$ Ù.TU ina ŠÀ $^{\text{mul}}$GÍR.TAB $^{\text{mul}}$SUHUR MÁŠ $^{\text{mu[l}}$x x] u $^{\text{mul}}$ $^{\text{d}}$A-⸢nu-ni⸣-t[um]

22 ana bu-lim ina ŠÀ $^{\text{mul}}$SUHUR MÁŠ $^{\text{mul}}$AL.LUL u $^{\text{mu[l}}$GU$_4$.A]N.⸢NA⸣

23 ana KI.LAM ina ŠÀ $^{\text{mul}}$AL.LUL $^{\text{mul}}$ $^{\text{giš}}$RÍN $^{\text{mul}}$SUHUR MÁŠ $^{\text{mul}}$AŠ.IKU

24 ana še-im ina ŠÀ ^{mul}AŠ.IKU ^{mul}AB.SÍN u MUL.GU.LA

25 ana ZÚ.LUM.MA ina ŠÀ ^{mul} ^{giš}RÍN ^{mul} ^{lú}HUN.GÁ [u mu]l ŠU.GI

26 ana ŠE.GIŠ.Ì ina ŠÀ ^{mul}GÍR.TAB u [mu]l⸢Á?⸣$^{mušen?}$

27 ana SÍG$^{ḫi-a}$ ina ŠÀ ^{mul} ^{lú}HUN.GÁ ^{mul} ^{giš}RÍN u [mu]l SIPA.ZI.AN.NA

28 ana A.AN A.KAL ina ŠÀ ^{mul}KU$_6$ MUL.GU.LA u MUL.MUL

29 ana ZI IM ina ŠÀ ^{mul}MAŠ.MAŠ ^{mul}Pa-b[il-sag ^{mul}AL.L]UL ^{mul}UR.GU.LA

30 ^{mul}⸢SIM.MAH⸣ ^{mul}A-nu-ni-tum MUL.MUL [u] ^{mul} ^{lú}HUN. GÁ

31 ana ZI me-ḫe-[e UD.D]È?.RA.RA ri-iḫ-ṣu ina ŠÀ MUL.MUL u ^{mul} ^{lú}HUN.GÁ

32 ana GIG^{meš} ina ŠÀ ^{mul}SUHUR MÁŠ ^{mul}A-nu-ni-tum u [m]ul ^{lú}HUN.GÁ

33 [ana x] ⸢x⸣ [x ina ŠÀ mu]l SIPA.ZI.AN.NA ^{mul}ŠU.GI [u m]ul AŠ.IKU

34 [ ] LUGAL

35 [ana x (x)] AN? ina Š[À is? l]e?-e

36 [ana x x] NIM ina ŠÀ ⸢mul⸣[ ] ⸢x⸣

37 [ana x x x] ⸢x⸣ ina ŠÁ ^{mul}SUHUR [MÁŠ ...]

38 [ana ...] ina ŠÀ ^{mul}Pa-bil-sa[g ...]

Rev.

1 [^{mul} ^{lú}HUN.GÁ?] ^{mul}UR.GU.LA u ^{mul}[Pa-bi]l-sag

2 [...] KI.LAM šá K[UR UR]I^{ki}

3 [BE-ma ^{mul}Sag-me]-gar ina ŠÀ GUB-ma KUR$_4$ KI.LAM ina-pu-⸢uš⸣ BE-ma SIG LAL-ṭi

4 [BE-ma ^{mul}Dele-bat$^{?}$] ina ŠÀ KUR$_4$ KI.LAM ina-pu-uš [BE]-ma SIG KI.LAM LAL-ṭi

5 [BE-ma ^{d}S]in ina ŠÀ ÍR KI.LAM LAL-ṭi

6 [BE-ma ^{d}]⸢UDU⸣ .[TIL.SA]G.UŠ ú-lu ^{d}Ṣal-bat-a-nu ina ŠÀ KUR$_4$ [KI.]LAM LAL-ṭi

7 ⸢BE⸣-m[a] ina$^{?}$ [^{itu}B]ÁR ^{itu}IZI ^{itu}GAN UD-11+[x-KAM x] ⸢x⸣ GAR-an

8 ⸢^{d}Sin⸣ ina IGI ^{d}UTU GUB^{meš}-zu [KI.LAM] LAL-ṭi

9 BE-ma UD-13-KAM.TA.ÀM GUB^{meš}-zu [KI.LA]M LAL-ṭi

10 BE-ma UD-14-KAM.TA.ÀM GUB^{meš}-z[u KI.LA]M ina-pu-uš

11 BE-ma ^{mul}KAK.SI.SÁ ina ŠÀ KUR$_4$ [KI.]LAM ina-pu-uš$^{!}$: ZI KÚR

12 BE-ma ^{mul}Sag-me-gar un-nu-tu KI.LAM [ina$^{?}$ MU$^{?}$] BI LAL-al

13 BE-ma ^{mul}Dele-bat un-nu-tu KI.LAM LAL-al

14 BE-ma ^{mul}Ṣal-bat-a-nu ú-lu ^{d}SAG.U[Š] ⸢x⸣ [(x) K]I.LAM LAL-al

15 ⸢BE-ma⸣ ^{d}Sin ina IGI.LAL-šú un-nu-tu [K]I.LAM LAL-al

16 BE-ma ^{d}Sin ina IGI ^{d}UTU NA-su NU IGI ⸢KI⸣.[LAM] LAL-al

17 DIŠ ^{mul}GU$_4$.AN.NA ^{mul}AB.SÍN u ^{mu}[^{l}SUHUR MÁŠ]

18 3 KI^{meš} a-n[a KUR] ⸢NIM $^{?}$⸣.[MA^{ki}]

19 DIŠ ^{mul}MAŠ.TAB.BA.GAL$^{!?}$.GAL ^{mul}Zi-ba-an-n[a u M]UL.GU.⸢LA⸣

20 3 KI^{meš} a-na [K]UR Mar-tu^{ki}

21 DIŠ ^{mul}AL.LUL ⸢^{mul}GÍR.TAB⸣ u ^{mul}AŠ.IKU

22 3 KI^{meš} [a-na KU]R SU^{ki}

23 DIŠ ina KI šá <...> ^{d}Sag-me-gar ⸢KUR$_4$$^?$ x⸣ LU[GAL$^?$] ⸢x x x⸣ [x x x] ⸢x⸣ SUMUN-bar BALA-šú GÍD.DA

24 DIŠ ina KI šá KUR NIM.MA^{ki} KI.MIN ina KI šá KUR Ma[r-tu^{ki} KI.MIN ina KI šá KUR] URI^{ki} KI.MIN

25 DIŠ ^{d}Ṣal-bat-a-nu ina KI šá KUR URI^{ki} ⸢x x⸣ [x x x] AŠ.TE DIB-bat

26 DIŠ ^{mul}KAK.SI.SÁ ina KI šá KUR URI^{ki} ⸢x⸣ [x] ⸢LUGAL$^?$⸣ HI.GAR-ma AŠ.TE DIB-bat

27 DIŠ ^{mul}Dele-bat ina KI šá KUR URI^{ki} ⸢x x⸣ ú$^?$-lu ⸢DUMU$^?$ LUGAL$^?$⸣ AŠ.TE DIB-bat

28 DIŠ ^{d}UDU.TIL.SAG.UŠ ina KI šá KUR URI^{ki} AŠ.TE MAN KUR ZÀH

29 DIŠ ^{d}UTU AN.KU$_{10}$ GAR ERÍN^{meš} LUGAL ÚŠ^{meš}

30 DIŠ ^{d}Sin AN.K[U$_{10}$ GAR] KUR BI uš-tál-pat

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31 GIM SUMUN-šú SAR-ma IGI.TAB pa-liḫ ^{d}AG li-šá-qir

32 ṭu[p]-pi ^{I}Še-ma-ʾ-ia A ^{I}E$_4$-ṭi-rù ^{itu}IZI UD.28$^?$.KAM

(rest of colophon on edge broken)

2 *Translation*

1 |[3] The Stars: Keš$^?$

2 | The Bull of Heaven: Nippur, ..., Ur, ..., Der$^?$ and ...

3 | The True Shepherd of Anu: Sippar and Larsa

4 | The Old Man: ... and Eridu

[3] In the translation, I represent the vertical wedge (transliterated DIŠ) by | when it introduces a new item, but translate it by 'if' in protases of omens. The translations of star names are the literal ones used in [Hunger & Pingree 1989].

5 | The Great Twins: Kutha and Ur

6 | The Crab: Sippar, Dilbat and Girsu; the middle: Sippar; the front part: dto. (i.e. Dilbat), dto. (i.e. Girsu)

7 | The Head of the Lion: Uruk; its breast: Babylon; the foot: Nippur

8 | The Furrow: Elam

9 | The Scales: the land of Akkad

10 | The Scorpion: Dilmun and Borsippa

11 | Pabilsag: Babylon, Marad and Elam

12 | The Goat-Fish: Subartu

13 | The Great One: ...

14 | The Tails: Babylon ... of the land[?].

15 | The Hired Man: ...

16 | which is seen, carries radiance and keeps entering[?] ...

17 For an attack of an enemy: within the Crab, the Lion and the Scorpion

18 For famine: within the Furrow, the Field, the Scales, and the Great One

19 For deaths: within the Stars, the Bull of Heaven, and the True Shepherd of Anu

20 For harvest: within the Great One, the Furrow, the Field, the Tails

21 For women giving birth: within the Scorpion, the Goat-Fish, [...], and Anunitu

22 For cattle: within the Goat-Fish, the Crab, and the Bull of Heaven

23 For business: within the Crab, the Scales, the Goat-Fish, the Field

24 For barley: within the Field, the Furrow, and the Great One

25 For dates: within the Scales, the Hired Man, [and] the Old Man

26 For sesame: within the Scorpion and the Eagle$^{?}$

27 For wool: within the Hired Man, the Scales, and the True Shepherd of Anu

28 For rain and flood: within the Fish, the Great One, and the Stars

29 For rising of wind: within the Twins, Pabilsag, the Crab, the Lion,

30 the Swallow, Anunitu, the Stars, [and] the Hired Man

31 For rising of storm, [devast]ation by Adad$^{?}$, destruction: within the Stars and the Hired Man

32 For sick people: within the Goat-Fish, Anunitu, and the Hired Man

33 [For ...: with]in the True Shepherd of Anu, the Old Man, [and] the Field

34 [...] the King

35 [For ...] ...: within [...] ...

36 [For ...] ...: within [... the Jaw of the] Bull$^{?}$

37 [For ...] within the Goat-[Fish ...]

38 [For ...] within Pabilsag [...]

Rev.

1 [The Hired Man$^{?}$,] the Lion, and Pabilsag

2 [*concern*] the business (lit., equivalent) of the land of Akkad.

3 [If Jupi]ter stands therein and is bright: business will prosper; if it is faint, (business) will decrease.

4 [If Venus] is bright therein: business will prosper; if it is faint: business will decrease.

5 [If the M]oon is eclipsed therein: business will decrease.

6 [If Sat]urn or Mars is bright therein: business will decrease.

7 If [in] month I, month V, month IX, day 1[2$^{?}$, there] is [an eclipse, (and)]

8 the Moon$^{?}$ stands each time in front of the Sun: [business] will decrease.

9 If it stands there on each 13th day: [busi]ness will decrease.

10 If it stands there on each 14th day: [busi]ness will prosper.

11 If the 'Arrow' is bright therein: business will prosper; attack of an enemy.

12 If Jupiter is faint: business in that [year$^{?}$] will be rare.

13 If Venus is faint: business will be rare.

14 If Mars or Saturn is [...:] business will be rare.

15 If the Moon at its appearance is faint: business will be rare.

16 If the Moon's NA in front of the Sun is not visible: business will be rare.

17 | The Bull of Heaven, the Furrow, and the [Goat-Fish:]

18 3 areas for [Elam.]

19 | The Great Twins, the Scales, [and] the Great One:

20 3 areas for the Westland.

21 | The Crab, the Scorpion, and the Field:

22 3 areas [for] Subartu.

23 If in the area of <...>, Jupiter is bright$^{?}$ ...: the king ... [...] will live to old age, his reign will become long.

24 If in the area of Elam: dto.; in the area of the Westland: [dto.; in the area of the land of] Akkad: dto.

25 If Mars ... in the area of the land of Akkad: [...] will seize the throne.

26 If the 'Arrow' (...) in the area of the land of Akkad: ... will revolt against the king? and seize the throne.

27 If Venus in the area of the land of Akkad: ... or? the king's son will seize the throne.

28 If Saturn in the area of the land of Akkad: the throne will change, the land will disappear.

29 If the Sun makes an eclipse: the troops of the king will die.

30 If the Moon [makes] an eclipse: this land will be overthrown.

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31 According to its original written and collated. Whoever reveres Nabû shall hold (the tablet) in esteem.

32 Tablet of Šemaʾja, descendant of Eṭiru. Month V, day 28?,

(rest broken)

3 *Notes to the text*

1: Unfortunately, I am uncertain how to read the first city name. The tablet MNB 1849[4] lists constellations along the ecliptic (not strictly in their sequence in the sky), combining them with cities (and rivers) which are affected by omens from a lunar eclipse occurring in these constellations. For MUL.MUL (and ŠU.GI), it lists Dēr, Nippur (written *Dur-an-ki*), and Ur. None of them can I find in the signs of this name. R. Biggs suggests that the signs resemble Kèš. This city is long abandoned at the time of the tablet, but so are others listed here, like Girsu in line 6. The scribe may have had difficulty in writing the unusual city name.

[4] Partly published in [Weidner 1963, 118].

2: Not surprisingly, the Bull of Heaven is correlated to the same cities Ur, Dēr and Nippur as are the Pleiades in MNB 1849:39. — *de-e-ri* could be the city of Dēr (this syllabic writing is attested in Neo-Babylonian); but the determinative KI is lacking. Separated by *u* 'and', there probably follows another city name, now illegible. The signs *nin da la ta* before *de-e-ri* may be a part of the Bull of Heaven; compare the parts of the Crab in line 6, or those of the Lion in line 7. I do not understand *nin da la ta*, however.

3: The True Shepherd of Anu is correlated to Sippar and Larsa also in MNB 1849:40.

4: I could not find EN^{ki} anywhere else, and can only suggest that it is an error for $EN.LÍL^{ki}$; Nippur, although written *Dur-an-ki*, happens to be connected with the constellation ŠU.GI in MNB 1849:38.

5: Cutha occurs with the Twins in MNB 1849:41.

6: For the Crab, MNB 1849:49 lists Sippar.

10: MNB 1849:42 agrees with the present tablet in assigning Tilmun and Borsippa to the Scorpion.

11: Babylon is combined with Pabilsag in MNB 1849:43.

13: Since I do not understand this line, I am not sure whether there is a geographical name in it at all.

14: The end of the line could be an epithet of Babylon rather than another city name since it does not end in KI.

16: This line may be a kind of summary of the first section. Something not mentioned, probably the constellations listed before, is said to become visible (*innammaru*) and become bright (*šarūru našû*). The last part of the line should therefore also mention an observable phenomenon. A possible restoration ⌜*i*⌝ [*te-n*]*e-ru-bu-ú* (from *erēbu* 'to enter') would presuppose an unusual writing with long *-u* at the end. Also, I do not know how to understand NI BAD before it. The last four signs of the line could also be read *izi-šub-bu-ú* 'stroke of lightning', but that does not fit the first part of the line.

17ff.: I could not detect any pattern in the distribution of the constellations. Most of them are near the ecliptic, but the Eagle (line 26) and the Fish (line 28) are not.

29: The Crab is listed 'for the rising of wind' also in [Reiner & Pingree 1981, 40] (III 7).

31: If the restoration [UD.D]È.RA.RA is correct, it is to be read *riḫiṣti Adad* 'devastation by Adad' (see CAD s.v. *riḫiṣtu*); this is somewhat strange immediately before *riḫṣu*.
35: A possible restoration of the fragmentary signs at the end of the line could be [*is l*]*ê*, the Jaw of the Bull (= Aldebaran), although this does not occur elsewhere in this section.

Rev.
1: Restoration of the Hired Man (or an equivalent for the first sign in the zodiac) is based on the assumption that the three constellations listed form a triplicity, i.e. they are four zodiacal signs apart. The months I, V, and IX, corresponding to the constellations of line 1, are mentioned in line r. 7, and the remaining three triplicities of zodiacal constellations in lines r. 17–22. The correlations between months (or the zodiacal constellations corresponding to them) and countries are the same as those in the 'Great Star List';[5] they are also stated in [Virolleaud 1908–12, 2nd. Suppl., 118:19–20].
2: in the beginning of the line, words have to be restored to express that astronomical events in the three constellations listed in line r. 1 give omens for prices, and thereby for business (see CAD s.v. *maḫīru*) in Babylonia.
3: Restoration in the beginning of the line according to the writing of Jupiter in line r. 12.
4: I restore Venus because the apodosis is favorable.
5: 'Weeping' is an expression for an eclipse of the moon: the eclipse reports use ÍR for the duration of the maximal phase of an eclipse.[6]
6: The beginning is barely legible and could be restored slightly differently; Saturn is written both ^{d}SAG.UŠ and ^{d}UDU.TIL.SAG.UŠ on this tablet.
7: In view of lines r. 9 and 10, the day number is probably to be restored as 12. At the end of the line, the traces of signs before GAR-*an* do not look like AN.KU$_{10}$, but an eclipse is most likely meant here.
8: GUB^{meš} can be understood not only as plural, but also as expressing the iterative stem; the interpretation as an iterative is

[5] [Koch-Westenholz 1995, 202:274–7].
[6] For a summary, see [Sachs & Hunger 1988, 24]; an edition of the eclipse reports by P. J. Huber is in preparation.

justified by the occurrence of the phenomenon in three different months. The complement -*zu*, however, indicates a plural; the singular would end in -*iz*. I nevertheless assume a singular and take -*zu* to be an error; a similar confusion between singular and plural occurs in r. 12f., see below.

11: Here and in line r. 26, KAK.SI.SÁ 'Arrow' must mean Mercury, because this planet would otherwise be missing from the planets mentioned here; also, the omen presupposes that the 'Arrow' can be seen in different constellations, and therefore cannot be a fixed star. For other instances of this use of the star name, see CAD s.v. *šukūdu* mng. 2a.

12f.: *unnutu* here (and in line r. 15) looks like a plural; but the context requires a singular so that it is necessary to consider the ending -*u* as erroneous. Similar laxness can be found (looking at astronomical texts only) in [Hunger 1976], no. 94:23–25.

14: One expects the verb referring to Mars and Saturn to mean something like 'to be bright', since the apodosis is unfavorable; this suggests that the two malefic planets were exerting their influence.

16: NA is here the time between sunrise and moonset after full moon, when the moon sets for the first time after sunrise. For this and other time intervals observed by the Babylonians, see [Sachs 1948, 273].

23–28: Damages in the middle of these lines cause many uncertainties. After KI *šá* in line 23, the name of a country seems to be missing: in the following lines, the expression KI *šá* is always followed by a country name. — In line 23, the verb of the protasis, to be expected after Sag-me-gar, is probably *ba*ʾ*il* 'is bright'; but I do not understand the sign following it. In line 25 and line 26, the traces after KUR URI$^{\text{ki}}$ are different. Although I cannot read them, it seems to me more likely that they contained not any more the verb of the protasis but already the subject of the apodosis. In line 27, there seems to be no verb expressly written in the protasis, if in the apodosis *ūlu* 'or' is read correctly; a noun has to precede *ūlu*, and there is not enough space for both such a noun beginning the apodosis, and a verb ending the protasis. There is certainly no verb in the protasis of line 28; of course, it may have been omitted inadvertently by the scribe.

Note that the section of lines 23–30 considers all of the seven

planets (Moon and Sun included). Their sequence is interrupted by line 24, which states that what was said in the preceding line is valid for other countries too. One would expect that the text started with Akkad, and in line 24 listed Elam, Amurru and Subartu, as it did in the preceding section of the reverse (see lines 2, 18, 20, and 22). Unfortunately, line 24 has, on the contrary, the countries Elam, Amurru and Akkad. The country missing from line 23 would therefore be Subartu. It is however very unlikely that a Babylonian text would begin with Subartu, which is equivalent to Assyria. The choice of countries in line 24 remains therefore unexplained.

4 *Discussion*

This tablet contains a first part about the correlation of constellations with geographical units (mostly cities), and then several sections concerning the use of constellations for purposes of prediction.

Section 1: The geographical part extends from line 1 to 16. For fifteen constellations situated more or less along the ecliptic, and in some cases for parts of these constellations, a city or country is listed. Similar correlations can be found in MNB 1849, a text published in [Weidner 1963, 118]; parallels are given in the Notes above. In Weidner's text, a lunar eclipse occurring in the constellations listed gives an omen for a particular city. Line 16 of the tablet edited here may contain a similar statement, but I do not understand it completely.

There is no obvious regularity in the choice of cities and countries.[7] Several of them occur more than once: Nippur in 2 and 7, Ur in 2 and 5, Sippar in 3 and 6, Elam in 8 and 11, Babylon in 7, 11 and 14.

Section 2: Lines 17 to 38 list a number of topics frequent in predictions, and give for each topic the constellations which are pertinent to predictions on this topic. The text literally says 'inside of' a constellation; it means that ominous events, like first and last visibilities of planets, that occur within the constellation will concern the topic 'for' which the constellation is listed. This section is reminiscent of 'Tablet 50' of Enūma Anu Enlil, where

[7] The same was observed by E. Weidner for MNB 1849.

constellations are said to be 'for' (*ana*) a certain event on earth.[8] However, there is almost no parallel to be found between the two texts.

E. Reiner pointed out to me that some of the constellations in this section could be connected by association to the events listed. For example, for 'harvest' (and for 'barley' and 'famine'), the Field and the Furrow are given; for 'business', the Scales are among the constellations. Similarly, Crab, Lion, and Scorpion can be seen as 'enemies', and so they are associated with an 'attack of an enemy'.

Section 3: Rev. 1 to 16 concern business possibilities in the land of Akkad (= Babylonia), for which the three constellations listed in line 1 are particularly relevant. They are arranged in triplicity, i.e. four zodiacal signs apart from each other. Depending on which celestial body is bright or dim in these constellations, the equivalent (of what can be bought with one shekel of silver) is predicted to be high or low.[9]

Section 4: Rev. 17 to 22 assign again three constellations each to a particular country, again grouped in triplicities. While for Akkad explicit omens were listed in the preceding section, the other countries are treated here in a summary fashion.

Section 5: Rev. 23 to 30 derive predictions of a general type, in formulaic expressions identical to those of the traditional omen series, from a phenomenon of the planets in the (celestial) area associated with Babylonia. Unfortunately, the reading of this phenomenon shown by the planets is uncertain.

The arrangement of constellation names in triplicities (r. 1 and 17–22) presupposes the existence of 12 constellations of equal extension, as in the zodiac; elsewhere in the tablet, more than 12 names are used. Section 1, while proceeding in order of what would be rising longitude in the sky, has 15 names, sometimes mentioning parts of constellations. Section 2 uses the same names, but also additional ones which did not occur in Section 1. Section 4, which contains the triplicities, implies 12 zodiacal constellations of more or less equal size. These differences probably follow from the different sources used in compilation of the tablet.

[8] [Reiner & Pingree 1981, 28–51].

[9] Similar predictions of equivalents, depending on the brightness and position of planets, are given in [Hunger 1976] no. 94. That text does not arrange the constellations in triplicities, however.

This tablet provides one more instance of an ad-hoc compilation of astral lore by a particular scribe of the Late period;[10] as such, it may be of interest to our friend who pays attention also to seemingly lowly texts.

Bibliography

I. L. Finkel, 'Adad-apla-iddina, Esagil-kin-apli, and the Series SA.GIG', in *A Scientific Humanist: Studies in Memory of Abraham Sachs*, ed. Erle Leichty et al., Philadelphia, 1988, pp. 143–59.

H. Hunger, *Spätbabylonische Texte aus Uruk*, vol. 1, Berlin, 1976.

H. Hunger & D. Pingree, *MUL.APIN, An Astronomical Compendium in Cuneiform* (*Archiv für Orientforschung* Beiheft 24), Horn, 1989.

U. Koch-Westenholz, *Mesopotamian Astrology*, Copenhagen, 1995.

E. Reiner & D. Pingree, *Babylonian Planetary Omens. Part 2: Enūma Anu Enlil Tablets 50–51*, Malibu 1981.

A. Sachs, 'A Classification of the Babylonian Astronomical Tablets of the Seleucid Period', *Journal of Cuneiform Studies*, 2, 1948, pp. 271–90.

A. Sachs & H. Hunger, *Astronomical Diaries and Related Texts from Babylonia*, vol. 1, Vienna, 1988.

F. Thureau-Dangin, *Tablettes d'Uruk* (Textes Cunéiformes du Louvre, 6), Paris, 1922.

Charles Virolleaud, *L'astrologie chaldéenne: le livre intitulé 'Enuma (Anu) Bêl'*, fasc. 1-14, Paris, 1908–12.

E. Weidner, 'Astrologische Geographie im Alten Orient', *Archiv für Orientforschung*, 20, 1963, pp. 117–21.

[10] A similar tablet is [Thureau-Dangin 1922] no. 11. Note however that Šemaʾja is probably not the one who composed this tablet, because he states in the colophon that he copied the text from another tablet.

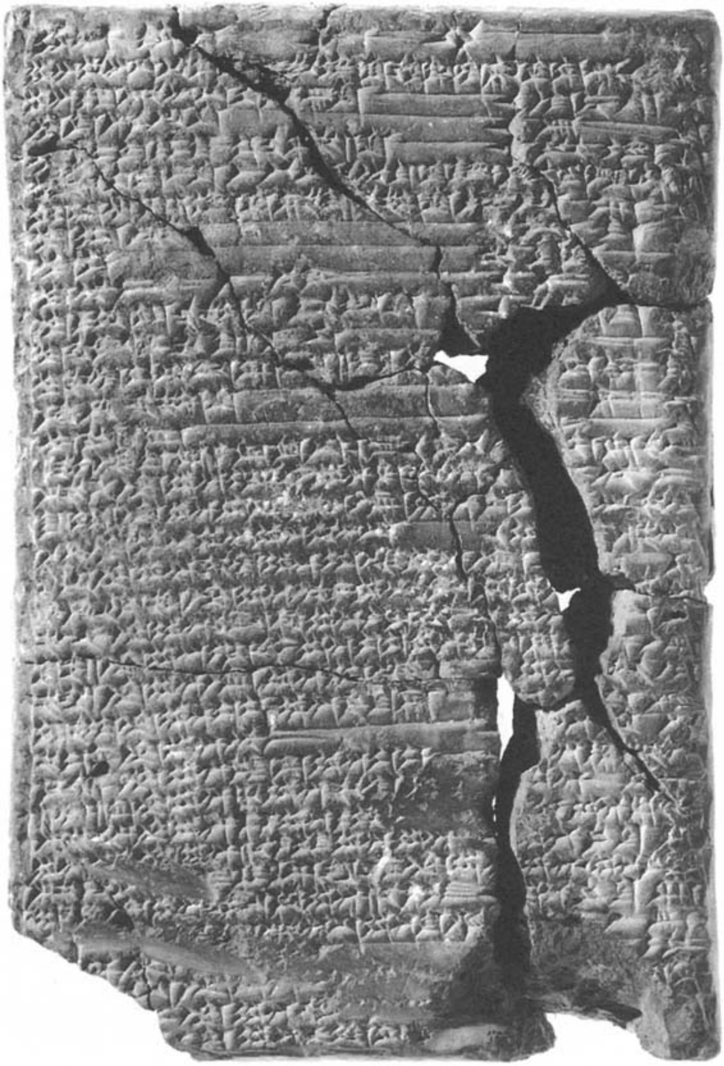


Plate 1: BM 47494 Obverse (courtesy of the Trustees of the British Museum)

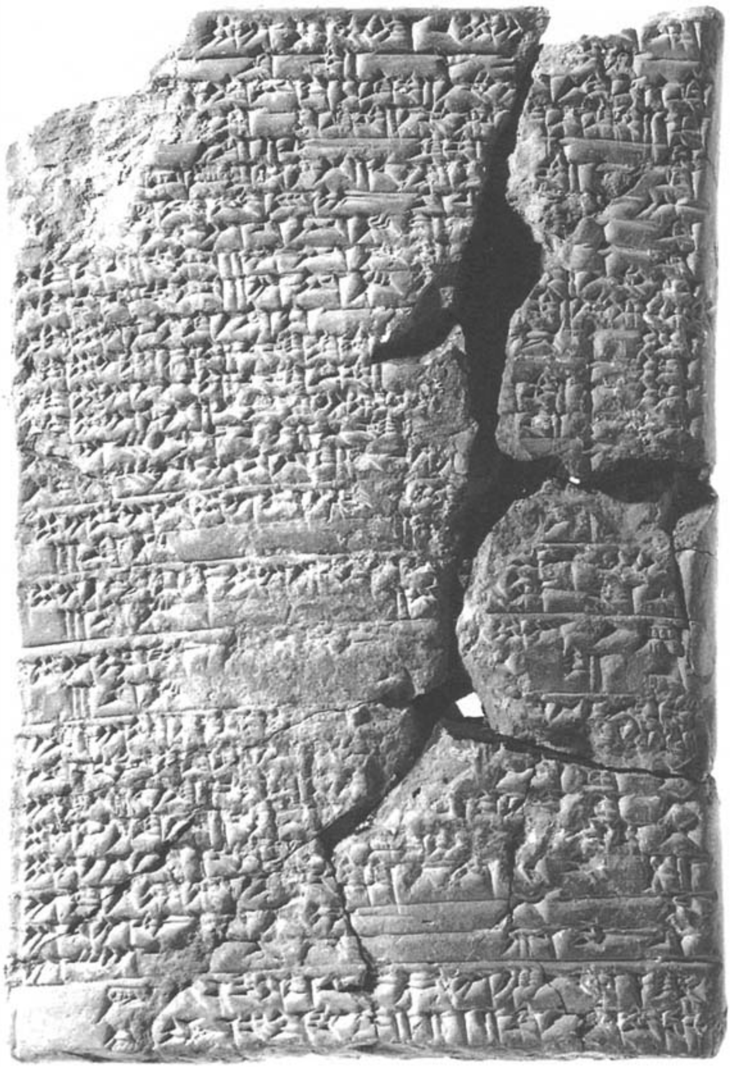


Plate 2: BM 47494 Reverse (courtesy of the Trustees of the British Museum)

An Early Observation Text for Mars: HSM 1899.2.112 (=HSM 1490)

JOHN P. BRITTON

This tablet, acquired by Harvard in 1899 and probably from Babylon, is a fragment of a text which recorded observations and calculated phenomena of Mars, year by year, from around the beginning of the reign of Esarhaddon (−679) through the end of the reign of Nebuchadnezzar (−561). The astronomical nature of the text was recognized by Claudine Vincente in 1994. It is published with the kind permission of Prof. Piotr Steinkeller, Curator of the Collection of Cuneiform Tablets at the Harvard Semitic Museum. I am indebted to Prof. Paul-Alain Beaulieu of Harvard for his generous assistance and suggestions with difficult readings, and for the copy of the text presented here as Figure 1. I also wish to thank Prof. Hermann Hunger for his helpful comments on the challenging reverse of the tablet. Remaining errors of reading, interpretation, or over-ambitious reconstruction are, of course, my own.

Physical Description

The surviving fragment, 8cm (h) × 4.5cm (w), appears to comprise between 1/2 and 1/3 of the original tablet in height. Left and bottom edges are preserved on the obverse (left and top on the reverse). The tablet turns over the bottom edge (O/R). Columns, separated by vertical rulings, run left to right on obverse; right to left on reverse, so that the last column on obverse continues over onto reverse. The fragment thus preserves the bottom of column (i) with traces of column (ii) on the obverse and part of the top of the last column on the reverse. The tablet is well written in a Neo-Babylonian scholarly hand,[1] and uses the

[1] Description courtesy of Paul-Alain Beaulieu.

cursive form of '9', but possibly the older form, *mul*, instead of *múl*.

Column (i) preserves statements of the dates of appearances and disappearances of Mars together with one positional statement for years 0–10 of Šamaš-šuma-ukin (−667 to −656). Horizontal rulings separate successive years, and the ten preserved years take up 13 lines. By the end of the text more information is recorded for each year, and in the last column data for years 35 to 38 Nebuchadnezzar (−569 to −566) requires 15 lines. Thus by the end of the text there is nearly three times as much information per year as at the beginning.

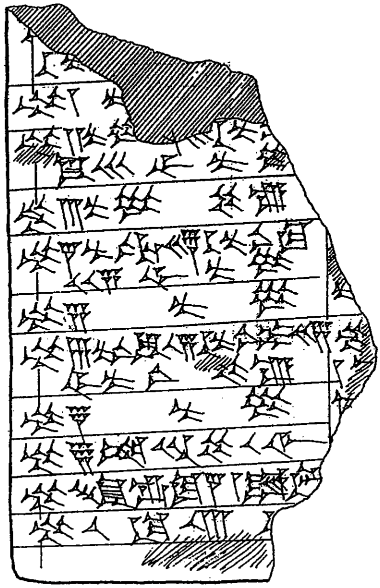
If the unbroken tablet was 20cm in height, it would have had room for ca. 33 lines on the obverse and a few more on the more tightly written reverse, allowing room for up to 16 years at the beginning of column 1 and up to 6 years at the end of the reverse. Thus the text almost certainly included information from the beginning of Esarhaddon's reign (ca. −680), and perhaps a few years earlier, and it probably continued through the last (43rd) year of Nebuchadnezzar's reign (−561/0) or perhaps one year later.[2]

Column (i) appears to have covered roughly 25 years, whereas the last column covered only 9 or 10. In between there are 86 years of information distributed over an even number of columns. Two columns of 25 years (implying no change in information density) on the obverse and two averaging 18 years (e.g. 24 and 12 as in Figure 2) on the reverse would accommodate the missing years, and imply a marked increase in information density between ca. −605 and −569. The alternative, three missing columns per side, would imply a steady increase in information density until roughly −600, followed by a level density at 9 to 10 years per column on the reverse. On balance it seems more likely that the text contained three columns to a side, with a substantial increase in informational density occurring in the first half of the 6th century. However, in the absence of additional evidence this remains a tentative assumption.

[2] It is perhaps coincidental, but noteworthy, that BM 36731, a compilation of computed dates of solstices, equinoxes and Sirius visibility phenomena from 0 or 1 Nabopolassar, also appears to conclude with the last year of Nebuchadnezzar's reign.

Text

Obverse:
1′
2′
3′
4′
5′
6′
7′
8′
9′
10′
11′
12′
13′
14′
15′



Reverse:
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15



Figure 1 (courtesy of Paul-Alain Beaulieu)

Transliteration

Obverse:

Column (i) Column (ii)

1′ || M[U SAG ^{md}GIŠ.NU$_{11}$-MU-GI.NA (mn)(d)
2′ || ŠÚ NU [ŠEŠ (mn) (d) IGI NU ŠEŠ
3′ || MU 1 NU [ŠEŠ KIN DIRI
4′ || MU 2 NU ⌜ŠEŠ GU$_4$ 20⌝ ŠÚ NU ⌜x⌝ [o
5′ || KIN 30 IGI NU ŠE[Š o
6′ || MU 3 NU ŠEŠ ŠE DIRI [
7′ || MU 4 NU PAP SIG 27 ŠÚ NU PAP DU$_6$ [
8′ || 25 IGI NU ŠEŠ | [
9′ || MU 5 NU ŠEŠ | M$^?$[U$_x$ 1′
10′ || MU 6 NU PAP KIN 15 ŠÚ NU PAP GAN 25 | MU[2′
11′ || IGI NU PAP ŠE DIRI | *ana* ⌜x⌝ 3′
12′ || MU 7 NU ŠEŠ | ⌜x⌝ [4′
13′ || MU 8 NE 20 ŠÚ ŠE 10 IGI [
14′ || MU 9 KIN DIRI DU$_6$ 4 *ana* MURUB$_4$ ⌜ALLA⌝ [
15′ || MU 10 DU$_6$ 13 Š[Ú

{lower edge}

Reverse:

{upper edge}

1 || { blank }
2 || M[U 35]⌜SAG SIG⌝ [Í]M *ina* GÌR G[U$_4$ o]
3 || Š[Ú KIN] ⌜1 IGI⌝ { blank$^?$ }
4 || MU 36 NE 12 $\frac{1}{2}$ KÙ[Š *ina* IGI MUL IGI]
5 || šá GÌR$^!$ LU 2$\frac{1}{2}$ KÙŠ [*ana* ULU SIG UŠ]
6 || ⌜KIN⌝ 23 ⌜SIG$^?$⌝ [MUL KUR šá DUR *nu-nu*]
7 || E ⌜DU$_6$⌝ 22 4 KÙŠ ⌜*meš-hat*⌝ [*ana* ULU]
8 || *ana* ⌜ŠÚ DIB⌝ UŠ 30 *ana* NIM LAL ŠE DIR
9 || MU 37 SIG 22⌜$\frac{1}{2}$⌝ KÙŠ ⌜ár MUL⌝
10 || ⌜TUR⌝ [šá] ⌜4⌝ KÙŠ ár [o A] ⌜ŠÚ⌝
11 || [DU$_6$ o o *n*]*a*-⌜*su*⌝ *ina* 4 IG[I x o o]
12 || [MU 38] ⌜KIN⌝ 22 ⌜E$^?$ MUL⌝ [ŠUR GIGIR]
13 || [šá ULU U]Š ⌜APIN⌝ 4 ⌜ár⌝ [IS DA E]
14 || [GAN o x+]1 *ina* IGI [IS DA UŠ]
15 || [MU 39]⌜NE$^?$⌝ [o o o o SÚ]
16 || [o o o] x [o o o o o IGI]

Translation

Obverse:

	Column (i)	Column (ii)	
1′	‖ Accession year of Šamaš-šuma-ukin, (month) (day)		
2′	‖ Ω, not [observed, (month) (day) Γ, not observed		
3′	‖ Year 1, not(hing) [observed, VI_2		
4′	‖ Year 2, not(hing) observed, II 20 Ω, not [observed		
5′	‖ VI 30 Γ, not obser[ved		
6′	‖ Year 3, not(hing) observed, XII_2		
7′	‖ Year 4, no watch, III 27 Ω, not watched for, VII		
8′	‖ 25 Γ, not observed		
9′	‖ Year 5, not(hing) observed	| Y[ear? x	1′
10′	‖ Year 6, no watch, VI 15 Ω, not watched for, IX	| Year [	2′
11′	‖ Γ, not watched for, XII2	| towards [x	3′
12′	‖ Year 7, not(hing) observed	| ⌜x⌝	4′
13′	‖ Year 8, V 20 Ω, XII 10 Γ		
14′	‖ Year 9, VI_2, VII 4 towards the middle of the Crab		
15′	‖ Year 10, VII 13 Ω		

Reverse:

Column (vi)

1 ‖ { blank }

2 ‖ Ye[ar 35]⌜beginning of III o⌝r end of I[I,]

3 ‖ ⌜Ω⌝. [VI] ⌜1, Γ⌝. { blank? }

4 ‖ Year 36 V 12, $\frac{1}{2}$ cub[it in front of]

5 ‖ β-ari, $2\frac{1}{2}$ cubits [below to the south, Φ;]

6 ‖ ⌜VI⌝ 23, ⌜below?⌝ [η-psc]

7 ‖ Θ; VII 22, 4 cubits ⌜measured⌝ [to the south (having)]

8 ‖ ⌜passed⌝ to ⌜the west⌝, Ψ; the 30th back towards the east; XII_2.

9 ‖ Year 37 III 22, ⌜$\frac{1}{2}$⌝ cubit ⌜behind⌝

10 ‖ ⌜ρ-leo, Ω;⌝

11 ‖ [VII x y° (h]igh); around the 4th Γ, [...].

12 ‖ [Year 38] VI 22 ⌜above? MUL⌝

13 ‖ [ζ-tau,] Φ; ⌜VIII⌝ 4 ⌜behind⌝ [α-tau, Θ;]

14 ‖ [IX x+]1, in front of [α-tau, Ψ.]

15 ‖ [Year 39]⌜V⌝ [... Ω;]

16 ‖ [......x ... Γ]

Critical Apparatus

Obverse:

col. (i)

1′	The location of ŠÚ on the following line argues against MU 13 (Esarhaddon).
2′	There is enough space in the missing column for a second NU ŠEŠ.
4′	Possibly 30 in place of 20.
7′ff.	NU PAP/NU ŠEŠ (∼*naṣaru*, 'to watch for'), here translated 'not watched for' (PAP) and 'not(hing) observed' (ŠEŠ), but no distinction may be warranted.
14′	Cursive (3-wedge) 9.

col. (ii)

1′	N[U possible for M[U.
1′/2′	Rows not aligned precisely with col. (i); traces of possible NU between 1′ and 2′.

Reverse (generally, very poorly preserved):

2	GÌR, or possibly GAN (HÉ), appears in BM 37361 in the context SAG, $MURUB_4$, GÌR, which evidently denote 'beginning, middle and end'. It seems to be an archaic form which was replaced in later Diaries and observation texts by the more widely used TIL.
5	GÌR, scribal error for SAG or different description of β-ari? Note LU(DIB), not LÚ, for Aries. $2\frac{1}{2}$, possible scribal error for $5\frac{1}{2}$.
13	Scoring should be under the following line.

Commentary[3]

Dates

Four intercalary months are mentioned: in column (i) a XII_2 in years 3 and 6 and a VI_2 in year 9; in column (vi) a XII_2 in year 36*. From the 8th century onwards, the last could only refer to

[3] Notation here follows the conventions in ACT. Years with intercalary months XII_2 and VI_2 are designated Y* and Y** respectively. Synodic phenomena are designated by Greek letters as follows: Ω (ŠÚ∼disappearance); Γ (IGI∼first appearance); Φ (UŠ∼first station); Θ (E∼opposition); Ψ (2-UŠ∼second station).

36 Nebuchadnezzar (−568/7), which thus establishes the dates in that column. In theory (although improbably from the contents) this might be the obverse of the tablet, but no subsequent reign of at least 10 years reflects the intercalations in column (i).[4] Before Nebuchadnezzar, attested intercalations and reign lengths rule out all but Šamaš-šuma-ukin (SSU), confirmation of which is provided by the positional remark in year 9. The text thus provides evidence of 3 hitherto unconfirmed intercalary years, namely SSU 3*, 6*, and 9**.

Contents

The planet's name is not mentioned, but the synodic intervals of roughly 26 months for šú(Ω) and igi(Γ) in column (i) and the close agreement between reported and calculated data for all five synodic phenomena recorded in column (vi) show that the text concerns Mars exclusively.

Data is recorded for every year, whether or not there is anything to report. In column (i), except in year 9 (line 13′) which contains a unique statement of position, only dates of appearances, igi(Γ), and disappearances, šú(Ω), are reported together with intercalary months. Until year 8 these are invariably preceded and followed by either 'nu šeš' or 'nu pap', both essentially meaning 'not observed', (∼*naṣaru*) and thus by implication computed in some fashion, but with what difference of nuance and whether predicted or computed back is not at all clear. Nor is any consistent pattern of usage evident: in year 2 we find three instances of 'nu šeš'; in year 6 three of 'nu pap', and in year 4 two 'nu pap's followed by a 'nu šeš'. In contrast, years 3, 5, and 7 in which neither phenomenon occurred are marked by a single 'nu šeš'. In years 8–10 (−659/8) these remarks no longer appear, suggesting that for these years the text records actual observations.

[4] These intercalations are attested for the reign of Cyrus which, however, lasted only 9 years with a (late) intercalary VI_2 in year 2, which is not mentioned in the text.

line	(i) −680	(ii) −656	(iii) −631
1	Heading	SSU 11 II 26 igi	KAN 16 IV 22 šú
2	SA2 8 III 25 šú	SSU 12 X 19 šú	KAN 16 VII 29 igi
3	SA2 8 VI 28 igi	SSU 13 III 22 igi	17
4	1	14	KAN 18 V 12 šú
5	ASR 2 IV 17 šú	SSU 15 XII 20 šú	KAN 18 IX 9 igi
7	3	16	KAN 20 VI 6 šú
8	ASR 4 V 8 šú	SSU 17 II 29 šú	KAN 20 XII 26 igi
9	ASR 4 IX 9 igi	SSU 17 VI 3 igi	21
10	5	18	KAN 22 VII 25 šú
11	ASR 6 VI 4 šú	SSU 19 III 26 šú	NBL 1 II 29 igi
12	ASR 7 I 9 igi	SSU 19 VI 28 igi	NBL 2 XII 2 šú
13	ASR 8 IX 2 šú	20	NBL 3 III 23 igi
14	ASR 9 II 25 igi	KAN 1 IV 17 šú	4
15	ASR 10 XII 6 šú	KAN 1 VIII 1 igi	NBL 5 I 26 šú
16	ASR 11 IV 17 igi	2	NBL 5 V 13 igi
17	12	KAN 3 V 8 šú	6
18	ASR 13 I 25 šú	KAN 3 IX 19 igi	NBL 7 III 2 šú
19	ASR 13 V 7 igi	4	NBL 7 VI 5 igi
20	1	KAN 5 VI_2 6 šú	8
21	SSU 2 II 28 šú	KAN 6 I 21 igi	NBL 9 III 27 šú
22	SSU 2 V 30 igi	KAN 7 IX 22 šú	NBL 9 VII 1 igi
23	3	KAN 8 III 26 igi	10
24	SSU 4 III 22 šú	KAN 9 XII 13 šú	NBL 11 IV 17 šú
25	SSU 4 vi 28 igi	KAN 10 iv 18 igi	NBL 11 VIII 4 igi
26	5	11	12
27	SSU 6 V 12 šú	KAN 12 I 28 šú	NBL 13 V 9 šú
28	SSU 6 IX 3 igi	KAN 12 V 7 igi	NBL 13 X 3 igi
29	7	13	14
30	SSU 8 VI 4 šú	KAN 14 II 30 šú	NBL 15 VI_2 13 šú
30	SSU 8 XI 18 igi	KAN 14 VI 1 igi	NBL 16 I 27 igi
31	9	15	NBL 17 X 14 šú
32	SSU 10 VII 12 šú		

Figure 2: Schematic potential arrangement of HSM 1490 showing calculated dates of igi(Γ) and šú(Ω). In practice the empty space in each column would have been filled with additional data.

line	(iv) −606	(v) −582	(vi) −569
1	NBL 18 III 27 igi	NBK 22 IV 17 šú	NBK 35 III 2 šú
2	NBL 19 XII 21 šú	NBK 22 VIII 10 igi	NBK 35 VI 4 igi
3	NBL 20 IV 17 igi	23	36
4	NBK 1 II 3 šú	NBK 24 V 10 šú	NBK 37 III 22 šú
5	NBK 1 V 9 igi	NBK 24 XI 2 igi	NBK 37 VII 7 igi
6	2	25	38
7	NBK 3 III 1 šú	NBK 26 VII 19 šú	NBK 39 IX 27 šú
8	NBK 3 VI 4 igi	NBK 27 II 3 igi	NBK 39 V 13 igi
9	4	NBK 28 X 28 šú	40
10	NBK 5 IV 22 šú	NBK 29 II 28 igi	NBK 41 VI_2 13 šú
11	NBK 5 VII 3 igi	NBK 30 XII 27 šú	NBK 42 I 29 igi
12	6	NBK 31 IV 17 igi	NBK 43 IX 4 šú
13	NBK 7 V 12 šú	32	
14	NBK 7 VIII 15 igi	NBK 33 II 6 šú	
15	8	NBK 33 V 9 igi	
16	NBK 9 VI 9 šú	34	
17	NBK 9 XII 17 igi		
18	10		
19	NBK 11 VIII 12 šú		
20	NBK 12 II 1 igi		
21	NBK 13 XI 12 šú		
22	NBK 14 III 21 igi		
23	15		
24	NBK 16 I 1 šú		
25	NBK 16 IV 12 igi		
26	17		
27	NBK 18 II 4 šú		
28	NBK 18 V 6 igi		
29	19		
30	NBK 20 III 27 šú		
30	NBK 20 VII 4 igi		
31	21		
32			

Figure 2 (cont'd.): For columns (i)–(iii) and possibly column (iv) this could have been remarks on the location of visibility phenomena. By column (v) at the latest, however, data for stations and oppositions would have been included.

The preserved traces of column (ii) concern 2 or 3 years, perhaps years 11–13 Kandalanu, and contain at least one positional report implied by the *ana* in line 3′. This would be consistent with the practice of recording the approximate location of visibility phenomena with less frequent 'nu šeš/pap' remarks, reflected in the compilation of appearances and disappearances of Saturn recorded in BM 76738[5] for the reign of Kandalanu. Most of this information could be incorporated in place of the frequent 'nu šeš/pap's in column (i) without materially increasing the number of lines per year as long as mainly visibility phenomena are reported.

By the end of the text, however, the information recorded has changed dramatically in substance, precision, and—as we shall see—accuracy. Here we find both dates and positions recorded for each of the five characteristic synodic phenomena: disappearance,[6] šú(Ω); first appearance, igi(Γ); first station, uš(Φ); opposition, e(Θ); and second station, 2-uš(Ψ). The dates often omit the months—evidently the intervals between successive synodic phenomena were well enough known by this time that the omission was not thought important—while positions, often in two coordinates, are recorded in cubits relative to stars which appear consistent with Normal Stars preserved in later texts. These are accompanied by occasional comments suggesting greater or less reliability and precision, as well as an observation of when the planet resumed perceptible direct motion after its second station (Ψ). Thus by its end, the text reflects systematic and precise observations of virtually all planetary phenomena recorded in the later Diaries,[7] including detail—e.g., the precise locations of šú and igi and the dates of perceptible resumption of direct motion—which at some later date ceased to be part of the regular observational regimen.[8]

Figure 2 shows the possible approximate beginning of each

[5] [Walker 1999, 61ff.]

[6] 'Last visibility' in ACT, but shown for Venus by [Huber 1982] to be the day following last visibility.

[7] The most conspicuous omission is the absence of data for the time interval between planet-rise and sunrise (*nasu*) at first appearance found in later Diaries and observation texts.

[8] It is also not impossible that the apparent reference to 'me' (day) between šú and igi in year 37 is part of some comment about conjunction.

column together with the dates of šú and igi, arranged to show how the text might have appeared if one line were accorded each of these phenomena as at the beginning of column (i). The blank space at the bottom of each column suggests the expansion of information recorded, although probably understating this process in column (ii), where additional information would have replaced the 'nu šeš's and 'nu pap's in column (i). As shown here the pattern is consistent with a sharp increase in informational density around −580, although the increase could have occurred more gradually. Obviously this is only a schematic rendering of a process whose precise details remain obscure, but which appears to have taken place early in the sixth century.

Accuracy

Occurrences: Figure 3a compares the dates and negative solar altitudes at planet-rise/set ($h*$) for šú(Ω) and igi(Γ) recorded in column (i) with those calculated from modern theory[9] using Schoch's *arcus visionis* values[10] as visibility criteria. Dates in the text accompanied by 'nu šeš/nu pap' are underlined. These are significantly less accurate than the last three dates, lending support to the assumption that they were calculated in some fashion. On balance they are systematically late, especially the igi(Γ)'s, and reflect a collective scatter (standard deviation) in $h*$ of $\pm 6°$. In contrast, the last three dates, unqualified by 'nu šeš/nu pap' and presumably observed, reflect an average $h*$ that is slightly higher (by 1.3°) than assumed in the calculations, but with a much lower scatter of only $\pm 1.3°$.

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[9] The calculations employ the program PLANETS.EXE by Peter Huber to calculate the daily positions of the planet and sun and the altitude of the sun at planet-rise and set for the years in question, whence calculated dates for visibility phenomena are derived from assumed visibility criteria. Sidereal longitudes, denoted by $\lambda*$, are calculated from tropical longitudes to be consistent with the mean Babylonian sidereal zodiac (of later date) by the formula $\lambda* = \lambda_{\text{trop}} + 9.91° - (Y + 500) \times 1.3825°/100$, where Y is the Julian year in astronomical notation.

[10] [Schoch 1928, 103]. For Mars these are 13.2° at šú(Ω) and 14.5° at igi(Γ).

Mars

				Calculated				
	King(JYr)	Yr	Phen	JD	M	D	$h*$	$\lambda*$
1	SSU	0	šú(Ω)	1477538	I	25	13.0	40.6
2	(−667)	0	igi(Γ)	1477639	V	7	14.7	107.0
3	(−665)	2	šú(Ω)	1478309	II	28	13.1	80.1
4	(−665)	2	igi(Γ)	1478399	V	30	14.5	137.8
5	(−663)	4	šú(Ω)	1479071	III	22	13.0	112.2
6	(−663)	4	igi(Γ)	1479165	VI	28	14.7	173.4
7	(−661)	6	šú(Ω)	1479829	V	12	13.0	141.7
8	(−661)	6	igi(Γ)	1479938	IX	3	14.6	216.3
9	(−659)	8	šú(Ω)	1480589	VI	4	13.1	173.8
10	(−658)	8	igi(Γ)	1480750	XI	18	14.6	293.6
11	(−657)	10	šú(Ω)	1481365	VII	13	13.2	219.8
12	SSU	11	igi(Γ)	1481585	II	26	14.5	28.3
13					η-cnc			100.6
14					θ-cnc			100.9
15	SSU	9	(−658)	1481002	VII	4		101.5
16					γ-cnc			102.8
17					δ-cnc			103.9

Figure 3a: HSM 1490: Comparisons of calculated and recorded data for Mars from column (i), obverse. Underlinings indicate record accompanied by 'not observed' ('nu šeš/nu pap'). According to Huber's Cresdat algorithm, month VI in SSU 10 should have been 31 days long, here treated as 30 days long.

				Calculated				
	King(JYr)	Yr	Phen	JD	M	D	$h*$	$\lambda*$
1	NBK	35	šú(Ω)	1513395	III	2	13.0	101.7
2	(−569)	35	igi(Γ)	1513486	VI	4	14.5	160.4
3	(−568)	36	uš(Φ)	1513821	V	15		8.1
4	(−568)	36	e(Θ)	1513856	VI	20		0.2
5	(−568)	36	uš(Ψ)	1513889	VII	24		353.8
6	(−567)	37	šú(Ω)	1514153	III	22	13.2	131.1
7	(−567)	37	igi(Γ)	1514256	VII	7	14.6	200.0
8	(−566)	38	uš(Φ)	1514597	VI	23		59.9
9	(−566)	38	e(Θ)	1514636	VIII	3		50.2
10	(−566)	38	uš(Ψ)	1514673	IX	10		41.8
11	NBK	39	šú(Ω)	1514912	V	13	13.1	161.9

Figure 3b: HSM 1490: Comparisons of calculated and recorded dates for synodic phenomena of Mars from column (vi), reverse.

Mars

Text				Differences (Text−Calc)		
JD	M	D	$h*$	δD	$\delta h*$	
	[I ff]	na				1
	na	na				2
1478301	II	20	15.5	−8	2.4	3
1478429	VI	30	24.9	30	10.4	4
1479076	III	27	11.6	5	−1.4	5
1479192	VII	25	23.5	27	8.8	6
1479861	VI	15	6.1	32	−7.0	7
1479960	IX	25	20.4	22	5.8	8
1480575	V	20	15.9	−14	2.7	9
1480772	XII	10	15.8	22	1.3	10
1481365	VII	13	13.2	0	0.0	11
1481585	column	(ii)				12
	igi šá alla šá si					13
	igi šá alla šá ulú					14
1481002	VII	4	'middle of the Crab'			15
	ár šá alla šá si					16
	ár šá alla šá ulú					17

Figure 3a (cont'd.): Visibility criteria used in calculations are from [Schoch 1928]: 14.5 for igi(Γ); 13.2 for šú(Ω). $\lambda*$ = sidereal longitude calculated from $\lambda_{\text{trop}} + \delta\lambda$, where $\delta\lambda = 9.91 - 1.3825 \times (Y + 500)/100$, where Y is the Julian year in astronomical notation. $h*$ = negative altitude of sun at moment when planet sets.

Text				Differences (Text−Calc)		
JD	M	D	$h*$	δD	$\delta h*$	
1513393	II/III	0±	13.6	∼ −2	∼ 0.6	1
1513483	VI	1	13.4	−3	−1.1	2
1513818	V	12		−3		3
1513859	VI	23		3		4
1513887	VII	22		−2		5
1514153	III	22	13.2	0	0.0	6
1514253	[VII]	4	13.5	−3	−1.1	7
1514596	VI	22		−1		8
1514637	VIII	4		1		9
1514680	[IX]	x+1				10
	[V]					11

Figure 3b (cont'd.): $h*$ and $\lambda*$ are calculated as in Figure 3a.

Saturn

				Calculated $h* \geq 12.0$				
	King(JYr)	Yr	Phen	JD	M	D	$h*$	$\lambda*$
1	KAN	1	igi(Γ)*	1485303	IV	23	11.9	102.92
2	(−645)	2	igi(Γ)*	1485681	V	17	12.0	116.43
3	(−644)	3	igi(Γ)*	1486059	V	11	12.1	129.63
4	(−643)	4	igi(Γ)*	1486437	VI	5	12.2	142.52
5	(−642)	5	igi(Γ)*	1486815	VI	29	12.4	155.07
6	(−641)	6	igi(Γ)*	1487192	VI	22	12.1	167.21
7	(−640)	7	igi(Γ)*	1487569	VII	15	12.0	179.10
8	(−639)	8	igi(Γ)*	1487946	VII	8	12.2	190.75
9	(−638)	9	igi(Γ)*	1488322	VII	30	11.7	202.10
10	(−637)	10	igi(Γ)*	1488699	VIII	23	12.3	213.43
11	(−636)	11	igi(Γ)*	1489075	VIII	15	12.0	224.57
12	(−635)	12	igi(Γ)*	1489451	IX	7	11.8	235.67
13	(−634)	13	igi(Γ)	1489828	IX	30	12.2	246.89
14	KAN	14	igi(Γ)	1490205	IX	23	12.4	258.16

				Calculated $h* \geq 12.0$				
	King(JYr)	Yr	Phen	JD	M	D	$h*$	$\lambda*$
15	KAN	1	šú(Ω)	1485269	III	19	11.0	98.52
16	(−645)	2	šú(Ω)	1485648	IV	14	10.9	112.20
17	(−644)	3	šú(Ω)	1486026	IV	8	11.1	125.46
18	(−643)	4	šú(Ω)	1486404	V	2	11.1	138.40
19	(−642)	5	šú(Ω)	1486782	V	25	11.0	151.01
20	(−641)	6	šú(Ω)	1487159	V	19	11.2	163.20
21	(−640)	7	šú(Ω)*	1487536	VI	12	11.3	175.13
22	(−639)	8	šú(Ω)	1487913	VI_2	5	11.3	186.81
23	(−638)	9	šú(Ω)*	1488290	VI	28	11.3	198.31
24	(−637)	10	šú(Ω)	1488667	VII	21	11.1	209.65
25	(−636)	11	šú(Ω)	1489043	VII	13	11.5	220.80
26	(−635)	12	šú(Ω)	1489420	VIII	6	11.3	232.02
27	(−634)	13	šú(Ω)	1489797	VIII	28	11.1	243.22
28	KAN	14	šú(Ω)	1490174	VIII	21	10.9	254.49

Figure 4: Comparison of reported and calculated dates for first appearance, igi(Γ), and disappearance, šú(Ω), of Saturn from BM 76738+. Calculated dates use *arcus visionis* values of 12.0 degrees (igi) and 11.5 degrees (šú); corresponding values from [Schoch 1928] are 13.0 and 10.0 respectively.

Saturn

Text				Differences (Text−Calc)		
JD	M	D	$h*$	δD	$\delta h*$	
1485304	IV	24	12.6	1	0.7	1
	V					2
1486059	V	16				3
	VI					4
	VI	end		~0	0.0	5
1487192	VI	22	12.1	0	0.0	6
1487569	VII	15	12.0	0	0.0	7
1487943	VII	5	9.5	−3	−2.6	8
1488323	VIII	x1	12.6	1	0.9	9
1488699	VIII	23	12.3	0	0.0	10
1489075	VIII	15	12.0	0	0.0	11
1489449	IX	5	10.0	−2	−1.7	12
1489829	X	1	13.1	1	0.8	13
	IX	2x				14
	avg		11.8	−0.2	−0.2	
	std		1.2	1.4	1.1	

Text				Differences (Text−Calc)		
JD	M	D	$h*$	δD	$\delta h*$	
	III					15
	IV					16
1486024	IV	7	11.7	−1	0.6	17
	IV	end		~ −3		18
1486780	V	23	12.0	−2	1.0	19
1487160	V	20	10.7	1	−0.5	20
	VI	1x				21
1487913	VI_2	5	11.3	0	0.0	22
1488290	VI	28?	11.3	0	0.0	23
1488666	VII	20	11.6	−1	0.5	24
1489043	VII	13	11.5	0	0.0	25
1489419	VIII	5	11.8	−1	0.5	26
1489795	VIII	26	12.2	−2	1.1	27
1490173	VIII	20	11.5	−1	0.6	28
	avg		11.6	−0.7	0.4	
	std		0.4	0.9	0.5	

Figure 4 (cont'd.): Phenomena designated with * include reported positions; those underlined are described as not observed. $h*$ = negative altitude of sun at planetset; $\lambda*$ = sidereal longitude, calculated as described in Figure 3a.

It is tempting to assume that the month name 'kin' for šú(Ω) in year 6 is a gross error, but the deviation in $h*$ of (−)7.1° is less than those for igi(Γ) in years 2 and 4. Conversely, the deviation of 22 days in date for igi(Γ) in year 8, seemingly simply in keeping with the error evidenced by the three preceding igi(Γ)'s, reflects only a modest deviation in $h*$ (1.3°), which is not inconsistent with an actual observation.

In short, the apparently computed dates of visibility phenomena comprising the first 6 preserved entries in column (i) reflect distinctively large deviations from assumed visibility criteria leading to errors of a month or more in the dates of visibilities. In contrast, the last three entries, beginning in −659, reflect smaller deviations in $h*$ consistent with fairly careful actual observations. Similar care is reflected in the one positional remark in column (i), which placed Mars 'in the middle of the Crab' on VII 4 of SSU 9. On the evening of that date Mars was indeed located midway between η-cnc/θ-cnc and γ-cnc/δ-cnc, the reference stars (later) associated with Cancer, with a tropical longitude of 89.4°, curiously close to the solstitial point.

By the end of the text, a century later, the accuracy of observation has clearly improved as may be seen from Figure 3b which shows the same data, but for column (vi). Here the text's dates are a day early on average, with a scatter or (2 days and maximal differences from calculation which do not exceed 3 days, including the hard-to-observe dates of stations and oppositions. For the small number of visibility phenomena deviations of $h*$ average −0.4° with a scatter of ±0.8°, again reflecting improvement over the earlier period.

It is instructive to compare the accuracy of these records with that of the dates of visibility phenomena for Saturn preserved for years 1–14 of Kandalanu in BM 76738+. These are shown in Figure 4, separated by phenomenon. Here neither phenomenon exhibits a sensible systematic error,[11] while the scatter in $h*$ is ±1.1° for igi and half as much for šú. Evidently, for visibility phenomena at least, a level of observational accuracy consistent with that reflected in the records at the end of our text (ca −570)

[11] The calculation utilizes *arcus visionis* values, which best fit these observations ($h*$[igi] = 12.0°, $h*$[šú] = 11.5°) and differ slightly from those of [Schoch 1928] ($h*$[igi] = 13.0°, $h*$[šú] = 10.0°).

was achieved by the reign of Kandalanu (−646 ff).

Measurements: Column (i) contains only one statement of position which locates Mars, accurately but imprecisely 'in the middle of the Crab'. A century later, column (vi) preserves four measurements of position—two of longitude and two of latitude, supplemented by an imprecise (before/behind:above/below) directional remark. All measurements are in cubits, relative to reference stars which seem to be identical with those found in the Diaries and later observation texts.[12] Longitudinal measurements are preserved for first station and disappearance—uš(Φ) and šú(Ω)—but omitted for opposition, e(Θ), whose position is only described by 'under' in line 6 and 'behind' in line 13. No positional data is preserved for first appearance, igi(Γ), but in light of the greater attention paid in earlier Saturn observations to the location of igi(Γ) compared to šú(Ω), it seems likely that longitudinal measurements were often included for it as well. Latitude measurements are preserved for both stations—uš(Φ) and uš(Ψ)—together with a longitude interval in the case of uš(Φ). Thus it seems that measurements of both coordinates were frequently, but not systematically, recorded for the four unambiguous synodic phenomena [igi(Γ), uš(Φ), uš(Ψ), and šú(Ω)], while for opposition, e(Θ), only the date and approximate position were recorded.[13]

Figure 5 compares computed and reported positions for synodic phenomena recorded in column (vi) of HSM 1490. With one exception the interval measurements appear to be accurate within approximately a cubit with no sensible average error. The exception could easily be a scribal error in recording the latitude for uš(Φ) in year 36, as '$2\frac{1}{2}$ cubits [below] β-ari' instead of $5\frac{1}{2}$ cubits.

Finally, the text remarks on the date, following second station, when the planet [was perceived to have] resumed direct motion, approximately a week after the planet became stationary. The date is consistent with a direct motion from Mars's stationary position of a quarter of a degree, which is a reasonable magnitude for the smallest positional difference which could be distinguished.

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[12] No star names are preserved, but the agreement with modern calculation is too good to be otherwise.

[13] Cf. Diary −567 [Sachs & Hunger 1988–96], which records an opposition of Jupiter on I 11 or 12 with no positional remark.

				Text			Mars (calc)	
	King	Yr	Phen	JD	M	D	λ*	β
1	NBK	35	šú(Ω)	1513390	II/III	0±	100.47	1.12
2	NBK	35	igi(Γ)	1513483	VI	1	158.16	1.12
3	NBK	36	uš(Φ)	1513818	V	12	8.08	−4.61
4	NBK	36	e(Θ)	1513859	[VI]	23	359.51	−3.73
5	NBK	36	uš(Ψ)	1513887	[VII]	22	353.81	−1.90
6	NBK	37	šú(Ω)	1514153	III	22	131.15	1.18
7	NBK	37	igi(Γ)	1514253	VII	4	197.67	0.60
8	NBK	38	uš(Φ)	1514596	VI	22	59.92	−0.80
9	NBK	38	e(Θ)	1514637	[VIII]	4	50.13	0.90
10	NBK	38	uš(Ψ)	1514680	[IX]	~12	41.84	1.94
11	NBK	39	šú(Ω)		na			

12	NBK	36	2uš(Ψ)	1513887	[VII]	22	353.80	Text
13				1513889	[VII]	24	353.76	Calc
14				1513895	[VII]	30	353.99	Direct
15							0.23	Direct−Calc

Figure 5: Comparison of calculated and reported positions for Mars from HSM 1490, column (vi). $\lambda*$ = sidereal longitudes calculated as described in Figure 3a.

Reference Star			Planet–Star (cubits)				Error (cubits)		
			Calc		Text		(Text–Calc)		
Star	$\lambda*$	β	$\delta\lambda*$	$\delta\beta$	$\delta\lambda*$	$\delta\beta$	$\delta\lambda*$	$\delta\beta$	
[θ-cnc]	100.94	−1.00	−0.2	0.9					1
[γ-vir]	165.67	3.00	−3.1	−0.8					2
β-ari	9.16	8.40	−0.4	−5.4	−0.5	−2.5	−0.1	**2.9**	3
η-psc	2.01	5.24	−1.0	−3.7		under		under	4
η-psc	2.01	5.24	−3.4	−3.0		−4.0		−1.0	5
ρ-leo	131.58	0.01	−0.2	0.5	0.5		0.7		6
[α-lib]	200.29	0.66	−1.1	0.0					7
ζ-tau	59.97	−2.52	0.0	0.7		above		above	8
[α-tau]	44.94	−5.65	2.2	2.7		behind		behind	9
[α-tau]	44.94	−5.65	−1.3	3.2		before		before	10
									11
			1 c. = 2.4 degrees						
									12
									13
									14
									15

Transformations to cubits assume 1 cubit = 2.40 degrees [Grasshoff 1999, 138]. Apparent scribal error is highlighted in bold.

Conclusion

Apart from the Venus Tablet of Amizaduga, the text contains the earliest observations of planetary phenomena, systematically compiled, that we possess. It is a pity that more of it is not preserved, for the complete text would provide an invaluable overview of the emergence and development of observational practice in the seventh and sixth centuries (B.C.). Nevertheless, several features bear comment. First, it is clear that the text represents a systematic compilation of observations of Mars's synodic phenomena, with only occasional interjections of other positional reports. At the outset and still by the end of the first column (−656) only appearances (igi) and disappearances (šú) are recorded, and before −658 even these are reported to have been 'not observed', It seems likely, however, that at least some actual observations were recorded in the missing beginning of column (i), and that these would have comprised the earliest ones available to the text's author. Thus it would seem that at least occasional observations of Mars's appearances and disappearance began to be recorded around the beginning of Esarhaddon's reign in −679.

Only 10 years after the end of column (i) observations of appearances and (some) disappearances of Saturn record the approximate location of the planet at the time of the event, information which is missing in column (i) of this text, but which could have been included in column (ii) without much disturbance, replacing the ubiquitous 'nu šeš/pap's of column (i). Thus it would appear that regular observations of the dates of appearances and disappearances began during the reign of Šamaš-suma-ukin, a practice expanded by the reign of Kandalanu to include the approximate location of the planet at the time of first appearance and, occasionally, disappearance.

During the subsequent lacuna of roughly 60 years (−633 to −569) systematic observations of the stations and oppositions were introduced,[14] and positions of the distinctive synodic phenomena (i.e. excepting oppositions) began to be recorded as measured intervals from Normal (reference) Stars in two coordinates. These innovations were accompanied by an increase in the accu-

[14] But note that the Diary for −651 records a station of Mars.

racy of the dates of recorded phenomena, which by the end of the text reflected average errors of roughly 1 day and maximal errors of less than 3 days. Positional measurements, recorded with a precision of half a cubit, seem to have had a generally accuracy (scribal errors excepted) consistent with that precision, suggesting an ongoing program of careful, systematic observations.

In short, by the end of the reign of Nebuchadnezzar, regular planetary observations included nearly all of the elements reflected in the later Diaries and observation texts, including a record of the interval between planet-rise/set and sunrise/set near first appearance (line 11). In addition, the text records the date after second station when the planet is seen to resume direct motion, which drops out of later observational practice and which demonstrates that motion of $\frac{1}{4}^{\circ}$ was perceptible to sixth-century observers.

Abbreviations

ACT	[Neugebauer 1955]
ASR	Esarhaddon
KAN	Kandalanu
NBK	Nebuchadnezzar
NBL	Nabopolassar
SA2	Sennacherib
SSU	Šamaš-šuma-ukin

Bibliography

G. Grasshoff, 'Normal Star Observations in Late Babylonian Astronomical Diaries', in *Ancient Astronomy and Celestial Divination*, ed. N. M. Swerdlow, Cambridge MA, 1999, pp. 97–147.

P. J. Huber, *Astronomical Dating of Babylon I and Ur III* (*Occasional Papers on the Near East*, vol. 1, no. 4), Malibu CA, 1982.

O. Neugebauer, *Astronomical Cuneiform Texts*, 3 vols, London, 1955.

A. J. Sachs & H. Hunger, *Astronomical Diaries and Related Texts from Babylon*, 3 vols, Vienna, 1988–96.

C. Schoch, 'Astronomical and Calendarical Tables', in S. Langdon, J. Fotheringham, and C. Schoch, *The Venus Tablets of Ammizaduga*, London, 1928, pp. 94–109.

C. B. F. Walker, 'Babylonian Observations of Saturn during the Reign of Kandalanu', in *Ancient Astronomy and Celestial Divination*, ed. N. M. Swerdlow, Cambridge MA, 1999, pp. 61–76.



Plate 1: HSM 1899.2.112 (+HSM 1490) Obverse (courtesy Harvard University)



Plate 2: HSM 1899.2.112 (+HSM 1490) Reverse (courtesy Harvard University)

A Babylonian Rising-Times Scheme in Non-Tabular Astronomical Texts

FRANCESCA ROCHBERG

1 *Introduction*

One of the elements of Babylonian astronomy adopted in Greco-Roman astronomy and astrology before the first century A.D. was the concept of the rising times of the twelve consecutive 30° signs of the zodiac, the Greek ἀναφοραί. Neugebauer indicated that

> the historical significance of the Babylonian schemes for the rising times reaches far beyond their applications in the solar and lunar theory. Since Greek mathematical geography characterized the latitude of a locality by its maximum daylight M the Babylonian method of finding the function $C(\lambda)$ of daylight depending on the solar longitude was properly modified, but under preservation of the arithmetical types A or B for the rising times. The geographical system of the 'seven climata' preserved vestiges of the Babylonian oblique ascensions until deep into the Middle Ages. On the other hand one finds the unaltered set of Babylonian rising times of System A in Indian astronomy of the sixth century A.D. without any consideration for India's far more southern position. Rising times and related patterns have thus become an excellent indicator of cultural contacts, ultimately originating in Mesopotamia.[1]

A rising time (α) is the time required for one zodiacal sign to cross the eastern horizon. Since both horizon and ecliptic are great circles on the celestial sphere, at any moment, one-half of the ecliptic (6 zodiacal signs) is above the horizon and the other half is below. During the interval of sunrise to sunset, 180° of the ecliptic will have crossed the horizon. As Neugebauer showed, evidence for the rising times of the zodiac in Babylonian astronomy is embedded in the ephemerides, in the column for generating length of

[1] [Neugebauer *History*, 371].

daylight (Column C).[2] The assumption that if the rising time of each individual zodiacal sign is known, the length of daylight for any day of the year is also known, underlies the computation of daylight length in column C, which derives the length of daylight from the sum of the rising times for the appropriate half of the zodiac that rises on the day in question, beginning with the position of the sun (C_1 [daylight length for a given solar position] $= \alpha_1+\alpha_2+\alpha_3+\ldots+\alpha_6$, $C_2 = \alpha_2+\alpha_3+\alpha_4+\ldots+\alpha_7$, and so on).[3] Cognizance of the connection between the position of the sun in the ecliptic and the length of daylight is expressed in this scheme. Column C in fact presupposes the lunar longitudes of Column B, from which solar longitudes are easily substituted, being either the same at conjunction, or 180° apart at full moon. This notion of the variation of daylight as an astronomical phenomenon is quite different from earlier attested calendaric schemes, such as we find in the Astrolabe texts[4] and MUL.APIN,[5] which account for the change in the length of the day throughout the year strictly as a function of the calendar month. The Babylonian values (α) for the rising times are only implicit in the computed daylight lengths of Column C of the ephemerides, as the values themselves are not found in those texts.

In a group of non-tabular late Babylonian astronomical texts (sources are given below sub 2), rising times of twelve micro-zodiac 'portions' (HA.LA = *zittu*), each representing $2\frac{1}{2}°$ of the ecliptic, are given, as are totals (PAP) for the sign as a whole in a number of instances.[6] That such totals in fact represent values of α is clear, although complete agreement with the System A rising times is not found. The reason for the discrepancy between the rising-times scheme underlying System A and that of the 'micro-zodiac' texts is clarified below. Suffice it to say here that these

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[2] [Neugebauer 'Jahreszeiten'], especially pp. 530–50. See also [Neugebauer 'Rising Times', 100 n. 4], citing his earlier [Neugebauer 'Astronomical Papyri'].

[3] Ibid., and see also [Neugebauer *History*, 368–71].

[4] A new edition of the 'Astrolabes' is being prepared by W. Horowitz. For now, see [Weidner *Handbuch*, 65–6], [Walker & Hunger 'Zwölfmaldrei'], [Donbaz & Koch 'Astrolab'], and [Horowitz *Geography*, 154–66].

[5] [Hunger & Pingree *MUL.APIN*].

[6] The rising time for the sign itself is given for Aries (Text B: rev. 27 and 29), Scorpius (Text A: 14 and 16), and Pisces (Text C: 11). See Section 2 below.

texts provide the only direct evidence thus far for values of the rising times of the zodiac in cuneiform sources.

The micro-zodiac texts attest to an awareness of the problem of the oblique ascensions of the zodiacal signs in that a determination, however crude, of values for the rising times is developed. Of further interest is the implication of the micro-zodiac rising times scheme for the understanding of the variation in daylight as a function of the position of the sun in the ecliptic, in the manner of late Babylonian mathematical astronomy, but with far simpler parameters. Section 2 below presents the transliterations and translations of three micro-zodiac texts (A–C, see below). In Section 3, the particular values for the rising times of the zodiac from the intervals between meridian crossings of culminating (*ziqpu*) stars is discussed. Further implications of this rising times scheme for a daylight scheme that seems to be a hybrid of the early calendaric method of the Astrolabes and the later astronomical method of the ephemerides' Column C will be drawn.

The micro-zodiac texts have much to contribute to our continuing efforts to understand Babylonian astronomy of the non-ACT type. Here, however, only the analysis of the rising-times scheme will be undertaken. Discussion of other elements of these sources, e.g., the lunar KUR and the *mešhu* stars that flare in each month, must be left for a separate study. Neugebauer's above quoted comment as to the broad significance of the rising times, not only as a feature of Babylonian but also ancient Greek and Indian astronomy, and as an indicator of the transmission of astronomical science within the *oikoumene*, shows how especially appropriate this material is for a volume dedicated to David Pingree. On a more immediate level, however, the present paper would have been impossible to produce without everything I have learned and gained from David's good guidance and the permanence of his work.

2 *Sources*

Text A: A 3427. Previous publication: [Schaumberger 'Anaphora', 238–41], without translation.

Text B: LBAT 1499 rev. 10ff. Previous publication: [Schaumberger 'Anaphora', 245–47], transliterates lines 10–30 but omits 31–34 , and does not include a translation.

Text C: LBAT 1503

Text D: U 196. Previous publication: [Schaumberger 'Anaphora', 242–43], without translation.

Text E: BM 77242. See [Horowitz 'Ziqpu-Star Texts', 97–8]. This text is far too fragmentary to add to our understanding of the scheme utilized for the rising times, and will therefore not be included below.

Text A

A 3427 Transcription
obv. upper edge: ina a-mat ^{d}60 u ^{d}An-tum liš-lim

1 [T]A 5 UŠ ár 2 MÙL.ME šá SAG MÚL.A EN 5 UŠ ár MÚL.DELE šá KUN-šú MÚL.[GÍR.TAB TA SAG-šú EN TIL-šú KUR-ma]

2 8 UŠ 20 NINDA ár 2 MÚL.ME šá SAG A KIMIN HA.LA reš-tú šá MÚL.GÍR.TAB MÚL.GÍR.TAB [šá MÚL.GÍR.TAB ITI.APIN KUR ina ITI.APIN ina še-rim UŠ.28.KAM]

3 MÚL.UR.IDIM meš-ha im-šúh ZI 1 UŠ 40 NINDA ár 4 šá GABA-šú KIMIN 2-tú H[A.LA šá MÚL.GÍR.TAB MÚL.PA šá MÚL.GÍR.TAB ITI.GAN KUR]

4 ina ITI.GAN ina še-rim UD.28 ^{d}Ṣal-bat-a-nu KIMIN 5 UŠ ár 4 šá GABA-šú [KIMIN 3-tú HA.LA šá MÚL.GÍR.TAB MÚL.MAŠ šá MÚL.GÍR.TAB]

5 ITI.AB KUR ina ITI.AB ina še-rim UD.28 MÚL.ALLA KIMIN 8 UŠ 20 NINDA ár 4 šá ⌜GABA⌝-[šú KIMIN(?) 4-tú HA.LA šá MÚL.GÍR.TAB MÚL.GU šá MÚL.GÍR.TAB]

6 ITI.ZÍZ KUR ina ITI.ZÍZ ina še-rim UD.28 MÚL.Nu-muš-da KIMIN 11 UŠ 40 NINDA ár 4 [šá GABA-šú KIMIN 5-tú HA.LA šá MÚL.GÍR.TAB MÚL.AŠ.GÁN šá MÚL.GÍR.TAB]

7 ITI.ŠE KUR ina ITI.ŠE ina še-rim UD.28 MÚL.KU$_6$ KIMIN $\frac{1}{2}$ DANNA ár 4 šá GABA-š[ú KIMIN 6-tú HA.LA šá MÚL.GÍR.TAB LU šá MÚL.GÍR.TAB]

8 ITI.BAR KUR ina ITI.BAR ina še-rim UD.28 <MÚL>.AŠ.GÁN KIMIN 18 UŠ 20 NINDA ár 4 šá GA[BA-šú KIMIN 7-tú HA.LA šá MÚL.GÍR.TAB MÚL.MÚL šá MÚL.GÍR.TAB]

9 ITI.GU$_4$KUR ina ITI.GU$_4$ ina še-rim UD.28 MÚL.MÚL KIMIN 1 UŠ 40 NINDA ár 2 šá GIŠ.K[UN-šú KIMIN 8-tú HA.LA šá MÚL.GÍR.TAB MAŠ.MAŠ šá MÚL.GÍR.TAB]

10 ITI.SIG KUR ina ITI.SIG ina še-rim UD.28 SIPA KIMIN 5 UŠ ár 2 šá GIŠ.KUN-[šú KIMIN 9-tú HA.LA śa MÚL.GÍR.TAB MÚL.ALLA šá MÚL.GÍR.TAB]

11 ITI.ŠU KUR ina ITI.ŠU ina še-rim UD.28 MUL.KAK.SI.SÁ KIMIN 8 UŠ 20 NINDA ár 2 šá GIŠ.[KUN-šú KIMIN 10-tú HA.LA šá MÚL.GÍR.TAB MÚL.A šá MÚL.GÍR.TAB]

12 ITI.NE KUR ina ITI.NE ina še-rim UD.28 MÚL.BAN KIMIN 1 UŠ 40 NINDA ár MÚL.DIL šá KU[N-šú KIMIN 11-tú HA.LA šá MÚL.GÍR.TAB MÚL.ABSIN šá MÚL.GÍR.TAB]

13 ITI.KIN KUR ina ITI.KIN ina še-rim UD.28 MÚL.BIR KIMIN 5 UŠ ár MÚL.DIL šá KUN[-šú KIMIN 12-tú HA.LA šá MÚL.GÍR.TAB MÚL.RÍN šá MÚL.GÍR.TAB]

14 ITI.DU$_6$ KUR ina ITI.DU$_6$ ina še-rim UD.28 MÚL.NIN.MAH KIMIN PAP 1 DANNA 10 UŠ [TA 5 UŠ ár 2 MÚL.ME šá SAG MÚL.A EN 5 UŠ]

15 ár MÚL.DELE šá KUN-šú MÚL.GÍR.TAB TA SAG-šú EN TIL-šú KUR-ha 1-et HA.LA

[ ] 16 2 UŠ 30 NINDA 1-et HA.LA šá MÚL.GÍR.TAB KUR-ha ina 12 HA.LA 1 DANNA 10 UŠ [MÚL.GÍR.TAB]

17 TA SAG-šú EN TIL-šú KUR-ha PAP 2 DANNA ina ziq-pi i-lak-ma MÚL.[GÍR.TAB TA SAG-šú]

18 EN TIL-šú 1 $\frac{1}{2}$ DANNA KUR NIM.MA SAR

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19 TA 5(?) UŠ ár MÚL.DELE šá KUN-šú EN MÚL na-at-tul-lum MÚL.PA TA SAG-šú [EN TIL-šú KUR-ma ZI $\underline{n}$ UŠ ár MÚL.DELE šá KUN-šú]

20 ana ziq-pi DU-ma ^{d}UTU KIMIN HA.LA reš-tu šá SAG MÚL.PA šá SAG MÚL.PA šá MÚL.PA [KUR ina ITI.GAN ina AN.NE UD.29]

21 MÚL.GÍR.TAB meš-hi im-šuh ZI 8 UŠ ár MÚL e_4-ru_6 KIMIN 2-tú HA.L[A šá MÚL.PA MÚL.MAŠ]

22 [šá MÚL.PA ITI.AB KUR ina] ITI.AB ina AN.NE UD.29 MÚL.UD.KA.DU_8.A KIMIN 8 UŠ ár MÚL.e_4-ru_6 [KIMIN 3-tú HA.LA MÚL.PA MÚL.GU]

23 [šá MÚL.PA ITI.ZÍZ KUR ina] ITI.ZÍZ ina AN.NE UD.29 MÚL.ALLA KIMIN 8 UŠ 20 NINDA ár e_4-ru_6 [KIMIN 4-tú HA.LA MÚL.PA MÚL.AŠ.GÁN]

24 [šá MÚL.PA ITI.ŠE KUR ina] ITI.ŠE ina AN.NE UD.29 MÚL.NIN.MAH KIMIN 11 UŠ 40 NINDA ár MÚL.e_4-ru_6 [KIMIN 5-tú HA.LA MÚL.PA MÚL.LU]

25 [šá MÚL.PA ITI.BAR KUR ina] ITI.BAR ina AN.NE UD.29 MÚL.KA_5.A KIMIN $\frac{1}{2}$ DANNA ár MÚL.e_4-ru_6 [KIMIN...]

Remainder broken
Reverse badly damaged, but there appear to be the remains of a four-line colophon following a ruling

A 3427 Translation
upper edge: Upon the command of Anu and Antum, may it (the tablet) remain intact.

1 From 5° east of the 2 stars of the head of Leo, to 5° east of the single star of his tail, Scor[pius rises from its beginning to its end and]

2 8;20° after the 2 stars of the head of Leo ditto. The first portion of Scorpius (is called) Scorpius [of Scorpius. Arahsamna (Month VIII): KUR in Arahsamna, morning of the 28th.]

3 Wolf produced a flare. The distance 1;40° east of the 4 stars of his chest ditto. The second po[rtion of Scorpius (is called) Sagittarius of Scorpius. Kislīmu (Month IX): KUR]

4 in Kislīmu morning of the 28th. Mars flared. (The distance) 5° east of the 4 stars of his chest [ditto. The third portion of Scorpius (is called) Capricorn of Scorpius.]

5 Ṭebētu (Month X): KUR in Ṭebētu morning of the 28th. Great One produced a flare. (The distance) 8;20° east of the 4 stars of [his chest ditto. The fourth portion of Scorpius (is called) Aquarius of Scorpius.] Šabaṭu (Month XI):

6 Šabaṭu (Month XI): KUR in Šabaṭu morning of the 28th. Numušda produced a flare. (The distance) 11;40° east of the 4 stars [of his chest ditto. The fifth portion of Scorpius (is called) Pisces of Scorpius.] Addaru (Month XII):

7 Addaru (Month XII): KUR in Month XII morning of the 28th. Fish produced a flare. (The distance) 15° east of the 4 stars of his chest [ditto. The sixth portion of Scorpius (is called) Aries of Scorpius.]

8 Nisannu (Month I): KUR in Nisannu morning of the 28th. The Field produced a flare. (The distance) 18;20° east of the 4 stars of his che[st ditto. The seventh portion of Scorpius (is called) Taurus of Scorpius.]

9 Ajjaru (Month II): KUR in Ajjaru morning of the 28th. Stars produced a flare. (The distance) <2>1;40° east of the two stars from the ru[mp ditto. The eighth portion of Scorpius (is called) Gemini of Scorpius.]

10 Simānu (Month III): KUR in Simanu morning of the 28th. The True Shepherd of Anu produced a flare. (The distance) <2>5° east of the two stars of [his] rump [ditto. The ninth portion of Scorpius (is called) Cancer of Scorpius.]

11 Du'ūzu (Month IV): KUR in Du'ūzu morning of the 28th. Arrow produced a flare. (The distance) <2>8;20° east of the two stars of his [rump ditto. The tenth portion of Scorpius (is called) Leo of Scorpius.]

12 Abu (Month V): KUR in Abu morning of the 28th. Bow produced a flare. (The distance) <3>1;40° east of the single star of his tai[l ditto. The eleventh portion of Scorpius (is called) Virgo of Scorpius.]

13 Ulūlu (Month VI): KUR in Ulūlu morning of the 28th. Kidney produced a flare. (The distance) <3>5° east of the single star of his tai[l ditto. The twelfth portion of Scorpius (is called) Libra of Scorpius.]

14 Tašrītu (Month VII): KUR in Tašrītu morning of the 28th. Ninmah produced a flare. The total (distance) 40° [from 5° east of the 2 stars of the head of Leo to 5°]

15 east of the single star of his tail, Scorpius from its beginning to its end rises. The first portion [...]

16 2;30° (which is) the first portion of Scorpius rises. In twelve portions 40° [Scorpius,]

17 from its beginning to its end, rises. Total (distance) 60° culminates, and Scor[pius from its beginning]

18 to its end 45° the rising in the east becomes visible.

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19 From 5° east of the single star of its tail to the Rear Harness, Sagittarius from its beginning [to its end rises and the distance n° east of the Single star of its tail]

20 culminates, and ditto the sun. The first portion of Sagittarius (is called) 'the beginning of Sagittarius' of Sagittarius. [Kislīmu (Month IX): KUR in Kislīmu midday of the 29th.]

21 Scorpion produced a flare. The distance 8° east of the Frond of Eru ditto.The second porti[on of Sagittarius (is called) Capricorn]

22 [of Sagittarius. Ṭebētu (Month X): KUR in] Ṭebētu midday of the 29th. Panther ditto (= flared). (The distance) 8° east of the Frond of Eru [ditto. The third portion of Sagittarius (is called) Aquarius]

23 [of Sagittarius. Šabaṭu (Month XI): KUR in] Šabaṭu midday of the 29th. Cancer ditto. (The distance) 8;20° east of the Frond of Eru [ditto. The fourth portion of Sagittarius (is called) Pisces]

24 [of Sagittarius. Addaru (Month XII): KUR in] Addaru midday of the 29th. Ninmah ditto. (The distance) 11;40° east of the Frond of Eru [ditto. The fifth portion of Sagittarius (is called) Aries]

25 [of Sagittarius. Nisannu (Month I): KUR in] Nisannu midday of the 29th. Fox ditto. (The distance) 15° east of the Frond of Eru [ditto ...]

A 3427 Philological Commentary

obv. 1 Transits of *ziqpu* stars are used here to measure time in degrees (UŠ), as in the 'GU-text', see [Pingree & Walker 'Star-Catalogue', 315]: 7, 12, and 19. The use of *ziqpu* star transits to express time at night is attested as early as the Neo-Assyrian period in letters from diviner-scholars to the king. Primarily, such times are given so as to note the time of a lunar eclipse occurrence, as in the following from a Neo-Assyrian letter, see [Parpola SAA X No.149 rev. 1–4]: *ina* KI MÚL.GÍR.TAB *a-dir* MUL *ku-ma-ru šá* MUL.UD.KA DU_8.A *ziq-pu* '(the moon) was eclipsed in the region of (the constellation) Scorpius. The Shoulder of the Panther culminated'. The Diaries provide more examples of the use of *ziqpu* stars to express time, often of a lunar eclipse, although few passages give degrees either 'before' (*ina* IGI, meaning to the west of) or 'after' (*ár*, meaning to the east of) the culmination of a *ziqpu* star. The following Diaries entries contain degrees with respect to a culminating star: No. -163:20′ 3 UŠ *ár* MUL *na-ad-dul ár ziq-pi*; No. -149:5′ 4 UŠ *ina* IGI MUL *kin-sa ziq-pi*; No. -122 D obv. 8 5 UŠ *ár* MUL DELE *ziq-pi*; and No. -95 F 4′ 5 UŠ *ár* SA_4 *šá* GABA-*šú ziq-pi*, see [Sachs & Hunger *Diaries*, vol. 3].

obv. 3 *mešhu imšuh*: *mešhu* has been defined as a luminous phenomenon produced by stars (CAD M s.v. *mišhu*). In a commentary to celestial omens, the *mešhu* is explained as the bright appearance of the star (or planet), see CAD M s.v. *mišhu* b)2′. Stellar *mešhu*'s are frequently associated with months. Here the

month associations made with stars producing *mešhu*'s are in accordance with the Astrolabe text, where heliacally rising stars are arranged month by month. In the case of Text B (LBAT 1499), the Astrolabe is written on the obverse of the same tablet. The choice of *mešhu* stars is determined by the month of a given micro-zodiac portion. In the Astrolabe section of Text B, the stars whose heliacal risings are (not always correctly) assigned to the months are indeed the *mešhu* stars in the months stated in the micro-zodiac texts (LBAT 1499 obv. 1–12). Table I shows the correlation between months and their *mešhu* stars in the micro-zodiac texts and the Astrolabe. It readily points up the error in the copy of the Astrolabe of LBAT 1499, namely, the second and third stars of each month are displaced one month ahead of the standard copies of the Astrolabe (i.e., MÚL.GÍR.TAB belongs to the middle 'ring' of Mo. VIII in Astrolabe B, but the middle 'ring' of Mo. IX in Text B, and so on). Nonetheless, the intended relation is clear, i.e., to match up the month of the micro-zodiac portion to a star rising in that month in a particular path of the sky (assigned to Ea, Anu, or Enlil) according to the Astrolabe tradition.[7]

[7] The astronomical sense of the *mešhu* stars' connection to the twelfths of zodiacal signs is difficult to see. Note that the obverse of LBAT 1499 (= Text B): 13–42 consists of three ruled sections in which the Astrolabe stars are said to produce a flare (*mešhu imšuh*) in the morning (*šērim*), midday (*muslālu*), and afternoon (*kinsigu*), and are assigned to the months and to the values of daylight length associated with those months. As in Text B rev., the *mešhu* stars are assigned to one of the three times of day in accordance with the 'ring' (outer, middle, or inner) of the Astrolabe with which they are identified in that text. See the comments of [Weidner *Gestirn-Darstellungen*, 19–20 n. 60].

Sign	HA.LA	HA.LA month	*mešhu*	Astrolabe # (ring/path)
♈	VI ♍ of ♈	VI morning	Kidney	VI out./Ea
	VII ♎ of ♈	VII morning	Ninmah	VII out./Ea
	VIII ♏ of ♈	VIII morning	Wolf	VIII out./Ea
	IX ♐ of ♈	IX morning	Mars	IX out./Ea
	X ♑ of ♈	X morning	Great One	X out./Ea
	XI ♒ of ♈	XI morning	Numušda	XI out./Ea
	XII ♓ of ♈	VII morning	Fish	XII out./Ea
♉	I ♉ of ♉	II morning	Stars	II out./Ea
	II ♊ of ♉	III morning	True Shep.	III out./Ea
♎	VII ♈ of ♎	I morning	Field	I out./Ea
	VIII ♉ of ♎	II morning	Stars	II out./Ea
	IX ♊ of ♎	III morning	True Shep.	III out./Ea
	X ♋ of ♎	IV morning	Arrow	IV out./Ea
	XI ♌ of ♎	V morning	Kidney	VI out./Ea[8]
♏	I ♏ of ♏	VIII morning	Wolf	VIII out./Ea
	II ♐ of ♏	IX morning	Mars	IX out./Ea
	III ♑ of ♏	X morning	Great One	X out./Ea
	IV ♒ of ♏	XI morning	Numušda	XI out./Ea
	V ♓ of ♏	XII morning	Fish	XII out./Ea
	VI ♈ of ♏	I morning	Field	I out./Ea
	VII ♉ of ♏	II morning	Stars	II out./Ea
	VIII ♊ of ♏	III morning	True Shep.	III out./Ea
	IX ♋ of ♏	IV morning	Arrow	IV out./Ea
	X ♌ of ♏	V morning	Bow	V out./Ea
	XI ♍ of ♏	VI morning	Kidney	VI out./Ea
	XII ♎ of ♏	VII morning	Ninmah	VII out./Ea
♐	I ♐ of ♐	IX midday	Scorpion	VIII mid./Anu
	II ♑ of ♐	X midday	Panther	IX mid./Anu
	III ♒ of ♐	XI midday	Crab	X mid./Anu
	IV ♓ of ♐	XII midday	Swallow	XI mid. ring
	V ♈ of ♐	I midday	Fox	VIII mid. ring
♓	[IX ♏ of ♓]	VIII afternoon	Mouselike	VII in./Enlil
	X ♐ of ♓	IX afternoon	King	VIII in./Enlil
	XI ♑ of ♓	X afternoon	She-Goat	IX in./Enlil
	XII ♒ of ♓	XI afternoon	Raven	X in./Enlil

Table I: Relation between *mešhu* stars of micro-zodiac texts and Astrolabe[9] (restored data is not indicated as such)

[8] The Astrolabe requires MUL.BAN (*Qaštu*) 'Bow' in the outer ring of Month V. Our text simply skipped Month V and entered MUL.BIR (*Kalītu*) 'Kidney' from Month VI.

[9] For the standard text of the Astrolabe see [Weidner *Handbuch*, 65–66],

obv. 3 ZI: ZI = *nishu* 'distance', from the verb *nasāhu* 'to move (forward)', here in terms of degrees per unit. For astronomical usage, see ACT glossary s.v. ZI.

obv. 5 KUR: Two possible interpretations of KUR in the present context may be entertained. The basic meaning of KUR = *niphu* as the rising of heavenly bodies can be taken to refer either to the heliacal rising of a fixed star or, alternatively, the last rising of the moon before sunrise on the last day of the month. The heliacal rising of a star may be argued on the basis of the assignment of the *mešhu* star to the month of the given microzodiac portion, especially if we take account of the relationship between the present texts and the Astrolabe, as indicated by Table I.

The time designations morning (*šêru*), midday (AN.NE), and afternoon (KIN.SIG), are easily correlated with the three paths of the *mešhu* stars in a purely schematic fashion. Weidner's suggestion[10] to take morning as south, midday as east, and afternoon as north would make some sense out of the correlation of the paths to the time designations.

The other possibility, to read KUR in reference to moonrise on the day of the last visibility of the moon, is supported by the dates given, which range from the 28th to the 30th of each month. Here, the time designations must also be otherwise explained, perhaps in terms of the three paths Ea, Anu, or Enlil, since the lunar KUR is by definition a dawn phenomenon.

obv. 7 The writing of Aries as LU, while not commonly used, is found in e.g., horoscope texts (see [Rochberg *Horoscopes*]) and in BRM 4 19 (see [Ungnad 'Besprechungskunst', 274-82]). See also Text B passim.

obv. 17 *ina ziqpi illakma* is the only occurrence in these texts constructed with the preposition *ina*. Elsewhere the expression 'to go toward the zenith', i.e., 'to culminate', is constructed with *ana*. See also line obv. 20, and Text B rev. 14, 16, 18, 20, 22, 24, 26, 28, 29, 32, 33, and Text C 3, 6, 8, 13, and rev. 3 and 6.

obv. 19 *nattulum*: For the Harness constellation, see CAD s.v. *nattulum* mng. 2, and [Schaumberger, ZA 50 218f., ZA 51 243]. The identification of the Harness as η Boötis (*ziqpu* star no. XXVI

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and see [Reiner & Pingree *BPO*, 4 (Table II)].

[10] [Weidner *Gestirn-Darstellungen*, 20 n. 60].

in the list AO 6478) follows [Hunger & Pingree *Astral Sciences*, 87].

Sagittarius is written here MÚL.PA, which is the standard writing of the ACT. Note that Text B:20 has MÚL.PA.BIL, a form which appears occasionally elsewhere, as in the early Diary -378:9' [Sachs & Hunger *Diaries*, vol. I, 92].

<u>obv. 21 and 22</u> The text has 8° for the ZI of the first portion of Sagittarius and again 8° for the second portion. From the scheme in which 40°/12 = 3,20° as the constant difference between the rising times of each portion, we expect 1,40° for portion 1 and 5° for portion 2. The value for the third portion is correct, i.e., 8;20°, and thereafter the values are separated by 3,20° as required by the reconstructed scheme.

<u>Reverse</u> The entire surface of the reverse is badly damaged, but there is a ruling followed by a four-line colophon containing the imprecation *pālih Ani Enlil u Ea la itabbalšu* 'whoever honors Anu, Enlil, and Ea, may he not remove it (the tablet)'. This particular form of the *pālih* DN formula is not so common; see [Hunger *Kolophone*, Nos. 97 and 98].

Text B

LBAT 1499 rev. 10–34 Transcription

10 [ki]-i ina ITI.BARA$_2$ ITI AN.KU$_{10}$ ^{d}UTU ina UGU ⌜MÚL⌝.[ku-mar šá] MÚL.UD.KA.DU$_8$.A KUR-ha

11 ina UGU-hi MÚL.ME ár-tú šá MÚL.ALLA ŠÚ-ma 6 DANNA U$_4$-mu 4 x x [(x)] ki(?) šá-aṣ-ma

12 HA.LA reš-tú ⌜2(?)⌝DANNA še-rim UD.28!.KAM 2-tú HA.LA 2 DANNA [AN.NE] ⌜UD.29.KAM⌝

13 3-tú HA.LA 2 DANNA EN.USAN UD.30.KAM ki-i ina ITI.BARA$_2$ KI KUR šá ^{d}UTU MÚL.ku-mar šá MÚL.UD.KA.DU$_8$

14 ana ziq-pi DU-ma ^{d}UTU ki-i GIŠ.KUN MÚL.LU AN.KU$_{10}$ TAB-ú 6-tú HA.LA šá MÚL.LU MÚL.ABSIN

15 šá MÚL.LU KIN KUR ina KIN ina še-rim UD.28.KAM MÚL.BIR meš-hu imšuh ZI 1 UŠ 40 NINDA

16 ár MUL.ku-mar šá MÚL.UD.KA.DU$_8$ ana ziq-pi DU-ma šamaš KI.MIN 7-tú HA.LA šá MÚL.LU

17 MÚL.RÍN šá MÚL.LU DU$_6$ KUR ina DU$_6$ ina še-rim UD.28 MÚL.NIN.MAH meš-hu im-šuh ZI

18 3 UŠ 20 NINDA ár ku!-mar MÚL.UD.KA.DU$_8$ ana ziq-pi DU-ma šamaš KI.MIN 8-tú HA.LA

19 šá MÚL.LU MÚL.GÍR.TAB šá MÚL.LU APIN KUR ina APIN ina še-rim 28 MÚL.UR.IDIM meš-hu imšuh ZI

20 5 UŠ ár ku-mar MÚL.UD.KA.DU$_8$ ana ziq-pi DU-ma šamaš KI.MIN 9-tú HA.LA šá MÚL.LU MÚL.PA.BIL

21 šá MÚL.LU GAN KUR ina GAN ina še-rim 2[8 MÚL.Ṣal]-bat-a-nu meš-hu im-šuh ZI 6 UŠ 40 NINDA

22 ár MÚL.ku-mar MÚL.UD.KA.D[U$_8$] ⌜ana ziq-pi DU⌝-ma šamaš KI.MIN 10-tú HA.LA šá MÚL.LU MÚL.MAŠ šá MÚL.LU

23 AB KUR ina AB ina še-rim 28 MÚL.GU.LA meš-hu im-šuh ZI 8 UŠ 20 NINDA ár MÚL.ku-mar

24 MÚL.UD.KA.DU$_8$ ana ziq-pi DU-ma šamaš KI.MIN 11-tú HA.LA šá MÚL.LU MÚL.GU.LA šá MÚL.LU

25 ZÍZ KUR ina ZÍZ ina še-rim 28 MÚL.Nu-muš-da meš-hu im-šuh ZI MÚL.SA$_4$ šá GABA-šú ana ziq-pi

26 DU-ma šamaš KI.MIN 12-tú HA.LA šá MÚL.LU MÚL.AŠ.GÁN šá MÚL.LU ŠE KUR ina ŠE ina še-rim 28 MÚL.KU$_6$

27 meš-ha im-šuh ZI PAP 10 UŠ TA MÚL.ku-mar šá MÚL.UD.KA.DU$_8$ EN SA$_4$ šá [G]ABA-šú MÚL.LU

28 [TA GIŠ].KUN-šú EN TIL KUR 1-et HA.LA 1 UŠ 40 NINDA ziq-pi i-lak-ma 2 UŠ 30 NINDA 1-et HA.LA

29 [šá MÚL.L]U KUR 6 HA.LA.MEŠ 10 UŠ ziq-pi i-lak-ma

30 [6 HA].LA šá MÚL.LU TA MAŠ-šú [E]N TIL-šú KUR

31 [TA n UŠ SA$_4$ šá] GABA EN 4 UŠ ina IGI MÚL.kin-ṣi MÚL.MÚL EN TIL(?) ⸢KUR(?)⸣11 UŠ 40 NINDA ár SA$_4$

32 [šá GABA-šú ana ziq-p]i DU-ma šamaš KI.MIN HA.LA reš-tú šá MÚL.[MÚL MÚL].MÚLšá MÚL.MÚL GU$_4$ KUR

33 [ina GU$_4$ ina še-rim 28 MÚL.MÚL(?) m]eš-ha im-šuh ZI 3 UŠ ⸢20(?)⸣[NINDA ár SA$_4$ šá GABA] ⸢ana⸣ziq-pi DU-ma

34 [šamaš KI.MIN 2-tú HA.LA š]á MÚL.MÚL MÚL.MAŠ.MAŠ šá MÚ[L.MÚL SIG KUR ina SIG ina še-rim 28 MÚL.SIPA.Z]I.AN.NA

Remainder broken

LBAT 1499 rev. 10–34 Translation

10–11 When in Nisannu the month of an eclipse, the sun rises before [the Shoulder of] the Panther, and sets before the Rear Stars of the Crab; 6 DANNA is the day (=12 hours, or 180°), 4 [...] ....

12 The first portion (is) 2(?) DANNA morning of the 28th; second portion (is) 2 DANNA [midday] of the 29th;

13 third portion (is) 2 DANNA evening of the 30th. When in Nisannu with the rising of the sun, the Shoulder of the Panther

14 culminates, and the sun .... the loins of Aries begins an eclipse. The 6th portion of Aries (is called) Virgo

15 of Aries. Ulūlu (month VI): KUR in Ulūlu morning of the 28th. Kidney produced a flare. The distance 1;40°

16 east of the Shoulder of the Panther culminates, and the sun ditto. The 7th portion of Aries (is called)

17 Libra of Aries. Tašrītu (month VII): KUR in Tašrītu morning of the 28th. Ninmah produced a flare. The distance

18 3;20° east of the Shoulder of the Panther culminates and ditto the sun. The 8th portion

19 of Aries (is called) Scorpius of Aries. Arahsamna (month VIII): KUR in Arahsamna morning of the 28th. Wolf produced a flare. The distance

20 5° east of the Shoulder of the Panther culminates and ditto the sun. The 9th portion of Aries (is called) Sagittarius

21 of Aries. Kislīmu (month IX): KUR in Kislīmu morning of [the 28]th. Mars flared. The distance 6;40°

22 east of the Shoulder of the Panther culminates and ditto the sun. The 10th portion of Aries (is called) Capricorn of Aries.

23 Ṭebētu (month X): KUR in Ṭebētu morning of the 28th. Great One produced a flare. The distance 8;20° east of the Shoulder

24 of the Panther culminates and ditto the sun. The 11th portion of Aries (is called) Aquarius of Aries.

25 Šabaṭu (month XI): KUR in Šabaṭu morning of the 28th. Numušda produced a flare. The distance (east of) the Bright star of its Chest culminates

26 and ditto the sun. The 12th portion of Aries (is called) Pisces of Aries. Addaru (month XII): KUR in Addaru morning of the 28th. Fish

27 produced a flare. The distance a total of 10° from the Shoulder of the Panther to the Bright star of its Chest; Aries

28 [from it]s loins to its end rises. The first portion (of this part of the equator) 1;40° culminates and 2;30° the first portion

29 [of Aries] rises. 6 portions (equal to) 10° culminate and

30 [6 por]tions of Aries from its midpoint to its end (equal to 15°) rise.

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31 From n° east of the Bright star of its] Chest to 4° west of the Knee, Taurus until its end rises(?). (The distance) 11;40° east of the Bright star

32 [of its Chest] culmin[ates] and ditto the sun. The first portion of Taur[us (is called) Tau]rus of Taurus. Ajjaru (month II): KUR in Ajjaru

33 [morning of the 28th. Stars] produced a flare. The distance 3;⌜20 (?)⌝ east of the Bright star of its Chest] culminates and

34 [ditto the sun. The 2nd portion o]f Taurus (is called) Gemini of Taur[us. Simanu (Month III): KUR in Simanu morning of the 28th. True] Shepherd of Anu

LBAT 1499 rev. 10–34 Philological Commentary

rev. 10 MÚL.ME *ár-tú šá* MÚL.ALLA (*kakkabānu arkâtu ša Alluttu*): The 'rear stars of the Crab' seem to function as *ziqpu* stars here, and in Text C rev. 1–3 and 8, although these are not included star list referred to above (AO 6478, [Schaumberger '*Ziqpu*-Gestirne', 214–29]), nor do they seem to be attested elsewhere as such. As a *ziqpu* star, Cancer is attested as star no. XX in AO 6478 and is also found in the 'GU-text', BM 78161:5; see [Pingree & Walker 'Star-Catalogue', 313]. Two Normal Stars, MÚL *ár šá* ALLA *šá* SI 'Rear Star of the Crab to the North' (γ Cancri) and MÚL *ár šá* ALLA *šá* ULÙ 'Rear Star of the Crab to the South' (δ Cancri), however, are known; see [Sachs & Hunger *Diaries*, I, 18]. The function of Normal Stars, to indicate the relative position of a planet and a star, where the star becomes the reference point 'above', 'below', 'in front of', or 'behind' which the planet is observed to be, is not the same as the use of a *ziqpu* star as an indication of the time of the appearance of a planet, or a phenomenon such as an eclipse, by reference to its culmination. In view of this, the modern identification of MÚL.ME *ár-tú šá* MÚL.ALLA is uncertain. It is the case, however, that when the sun is in Aries in Mo. I, at sunset, Cancer with a R.A. of about 90° sits on the meridian, being 90° from Libra on the eastern horizon and Aries on the western horizon with the setting sun. Note that the reverse situation is given in Text C rev. 1–3, where the sun is in Libra in Mo. VII and the stars of Cancer cross the meridian around sunrise.

rev.20 MÚL.PA.BIL is obviously an abbreviated form of PA.BIL.SAG, which is the spelling of Sagittarius in nearly all texts outside the tradition of ACT. Interestingly, the three texts

treated here do not agree on the spelling of this sign: Text A has MÚL.PA throughout, as in the late mathematical astronomical texts; Text C:4 has the unabbreviated form MÚL.PA.BIL.SAG as in earlier texts, such as MUL.APIN (where Sagittarius is of course a constellation rather than a zodiacal sign) and is followed in the texts BRM 4 19 and 20; see [Ungnad 'Besprechungskunst', 274–82].

rev. 24 The spelling of Aquarius MÚL.GU.LA is a reflection of an earlier tradition. Elsewhere in the micro-zodiac texts the abbreviated late form MÚL.GU is used.

rev. 26 The representation of Pisces MÚL.AŠ.GÁN is consistent with that found in BRM 4 19 (see note to rev. 20 above for reference). Other astrological and astronomical texts, including the horoscopes and ACT use KUN.(ME) or ZÍB.(ME), following the normal sequence of zodiacal constellation names of the early tradition of MUL.APIN, where Pisces is MUL.KUN.MEŠ.

rev. 28 *illakma*: In astronomy, DU means 'to move (toward or forward)', see ACT glossary s.v. DU. In relation to the zenith or meridian (*ziqpu*), *alāku* means 'to culminate'. The expression *ana ziqpi illak* is normally written with the logogram DU, but for other syllabic spellings, see Text B rev. 29 and Text C:13.

rev. 33 The value 3 UŠ followed by what appears to be ⌜30⌝ or possibly ⌜20⌝ [NINDA] is difficult to resolve because no other values are available in the section for Taurus. Without at least one other, we cannot be certain of the constant difference between the values between *ziqpu* transits, hence of the rising time for Taurus.

Text C

LBAT 1503 Transcription

1 ⌜x x x x⌝

2 [EN.TE.N]A.BAR.HUM meš-hu im-šuh ZI ⌜6⌝U[Š 40 NINDA]

3 [ár] MÚL.GAŠAN.TIN ana ziq-pi DU-ma šamáš KI.MIN 10-tú HA.LA šá MÚL.[AŠ.GÁN]

4 MÚL.PA.BIL.SAG šá MÚL.AŠ.GÁN GAN KUR ina GAN ina KIN.SIG U[D.30.KAM]

5 MÚL.LUGAL meš-hu im-šuh ZI 8 UŠ 20 NINDA ár MÚL.GA[ŠAN.TIN]

6 ana ziq-pi DU-ma šamáš KI.MIN 11-tú HA.LA šá MÚL.AŠ.GÁ[N]

7 MÚL.MAŠ šá MÚL.AŠ.GÁN AB KUR ina AB ina KIN.SIG UD.30.KAM MÚL.Ù[Z(?)]

8 meš-hu im-šuh ZI 10 UŠ ár MÚL.GAŠAN.TIN ana ziq-pi D[U-ma]

9 šamáš KI.MIN 12-tú HA.LA šá MÚL.AŠ.GÁN MÚL.GU šá [MÚL.AŠ.GÁN]

10 [ZÍ]Z KUR ina ZÍZ ina KIN.SIG UD!.30.KAM MÚL.UGA.MUŠEN me[š-hu]

11 im-šuh ZI PAP $\frac{2}{3}$ DANNA TA MÚL.AŠ.<GÁN> EN [...]

12 [á]r MÚL.GAŠAN.TIN MÚL.AŠ.GÁN TA SAG-šú EN TIL!-šú [...]

13 ⌜1⌝ UŠ 40 NINDA ziq-pi i-lak DIRI UŠ 30 NINDA x [...]

14 ⌜x x⌝ šá MÚL.AŠ.GÁN KUR-ha ina 12 HA.LA.MEŠ $\frac{2}{3}$ DAN[NA ...]

rev.

1 [ki-i ina I]TI.DU$_6$ UD.15 ina UGU MÚL.ME ár.ME [šá MÚL.ALLA šamáš KUR-ma(?)]

2 [ina UGU M]ÚL.ku-mar šá MÚL.UD.KA.DU$_8$ ŠU-ma [...]

3 [MÚL.ME ár].ME šá MÚL.ALLA ziq-pi DU šamáš K[I.MIN 7-tú HA.LA šá MÚL.RÍN]

4 [MÚL.L]U(?) šá MÚL.RÍN BAR KUR ina BAR ina še-rim U[D.28.KAM MÚL.AŠ.GÁN meš-hu]

5 [im-šu]h ZI erasure(?) 5 UŠ x [...]

6 [ana zi]q-pi DU-m[a šamáš KI.MIN 8-tú HA.LA šá MÚL.RÍN]

7 [MÚL.MÚL] šá MÚL.RÍN GU_4 KUR ina [GU_4 ina še-rim UD.28.KAM MÚL.MÚL]

8 [meš-hu im-šuh] ZI 10 UŠ ár MÚL.ME ár.ME [šá MÚL.ALLA ana ziq-pi DU-ma]

9 [š]amáš KI-MIN 9-tú HA.LA šá MÚL.R[ÍN MÚL.MÁŠ šá MÚL.RÍN]

10 [SIG KUR ina] SIG ina še-rim UD.28.KAM MÚL.SIPA.[ZI.AN.NA meš-hu im-šuh]

11 [Z]I 13 UŠ 20 NINDA ár MÚL.ALLA [ana ziq-pi DU-ma šamáš KI.MIN]

12 [10]-tú HA.LA šá MÚL.RÍN MÚL.ALLA [šá MÚL.RÍN ŠU KUR ina ŠU ina]

13 [še-rim U]D.28 MÚL.KAK.SI.SÁ meš-h[u im-šuh ZI 16 UŠ 40 NINDA]

14 [ár MÚL.II] MÚL.ME šá SAG.DU [MÚL.UR.GU.LA ana ziq-pi DU-ma šamáš KI.MIN]

15 [11-tú] ⌜HA.LA⌝šá MÚL.[RÍN ...]

16 [ ] x [...]

remainder broken

LBAT 1503 Translation

1 ⌜...⌝

2 [Habaṣ]ī rānu flared; the distance 6[,40°]

3 [east of] Lady of Life culminates and ditto the sun. The 10th portion of Pi[sces (is called)]

4 Sagittarius of Pisces. Kislīmu (Month IX): KUR in Kislīmu in the afternoon of the [30th.]

5 The King produced a flare. The distance 8,20° east of Lady of Life

6 culminates and ditto the sun. The 11th portion of Pisces (is called)

7 Capricorn of Pisces. Ṭebētu (Month X): KUR in Ṭebētu in the afternoon of the 30th. The She-[Goat]

8 produced a flare. The distance 10° east of Lady of Life culminates and

9 ditto the sun. The 12th portion of Pisces (is called) Aquarius of [Pisces].

10 Ṣaba[ṭu (Month XI]: KUR in Šabaṭu in the afternoon of the 30th. Raven [a flare]

11 produced. The distance total 20° from Pisces to [...]

12 [ea]st of Lady of Life Pisces from its beginning to its end

13 1,40° (of the equator) culminates extra from(?) 30 DANN[A ...]

14 ... of Pisces rises in 12 portions 20[° ...]

rev.

1 [When in Tašrītu (Month VII) the 15th day in front of the Rear stars of [Cancer(?) the sun rises(?)]

2 and [in front of(?) Shoulder of P]anther sets [...]

3 [the Rear stars of] Cancer(?) culminate and dit[to] the sun. [The 7th portion of Libra]

4 [(is called) Ari]es of Libra. Nisannu (Month I): KUR in Nisannu in the morning [of the 28th. The Field produced a flare.]

5 The distance 5° x [...]

6 [... cu]lminates an[d ditto the sun. The eighth portion of Libra (is called)

7 [Taurus] of Libra. Ajjaru (Month II): KUR in [Ajjaru in the morning of the 28th. Stars]

8 [produced a flare]. The distance 10° east of the rear stars [of Cancer culminates and]

9 ditto [the sun]. The ninth portion of Lib[ra (is called) Gemini of Libra.]

10 [Simanu (Month III): KUR in] Simanu in the morning of the 28th. True Sh[epherd of Anu produced a flare.]

11 [The di]stance 13;20° east of Cancer [culminates and ditto the sun.]

12 [The ten]th portion of Libra (is called) Cancer [of Libra. Du'ūzu (Month IV): KUR in Du'ūzu in]

13 [the morning of the] 28th. Arrow produc[ed a flare. The distance 16;40°

14 [east of the 2] stars of the head [of Leo culminates and ditto the sun.]

15 [... the eleventh] ⌜portion⌝ of Li[bra (is called) Leo of Libra ...

Remainder broken

LBAT 1503 Philological Commentary

obv. 4 MÚL.AŠ.GÁN is used here to write the zodiacal sign Pisces. The constellation MÚL.AŠ.GÁN (*Ikû*), which in Astrolabe B (KAV 218 B i 1–4) is the first to rise heliacally at the new year, is identified as α, β, γ Pegasi and α Andromedae; see [Hunger & Pingree *Astral Sciences*, 272]. Pisces does have a number of writings in late astrological texts, among them MUL.KUN.MEŠ and ZÍB.ME, or ZÍB. See [Rochberg 'New Evidence', 119 (Table 1)] for a comparison of the various spellings of all the signs of the zodiac (or, in the case of MUL.APIN, constellations of the ecliptic) in a number of texts.
rev. 1 On the *ziqpu* stars MÚL.ME *ár*.ME *šá* MÚL.ALLA (also rev. 2, 3, and 8), see commentary to Text B rev. 11.
rev. 5 Text has ZI 'the distance' followed by an obscured sign, then a clear 5 UŠ when we expect 6 UŠ 40 NINDA from the scheme.

3 *Analysis*

The micro-zodiac Texts A–C share a number of features. Of primary concern are the one-twelfths, or $2\frac{1}{2}°$ 'portions' (HA.LA = *zittu*) of the twelve zodiacal signs (*dodekatemoria*). Associated with these portions are 'distances' (ZI = *nishu*) or intervals between transits of the so-called *ziqpu*, or culminating, stars, measured in degrees of arc. The distances ZI give rise to the degree values for the rising times of the signs that cross the eastern horizon in equal times. Values for such distances were collected in texts such as the well-known *ziqpu* list (TCL 6 21 = AO 6478).[11] The ZI's in our texts, however, must correspond to the number of degrees equal to twelfths of a zodiacal sign or $2\frac{1}{2}°$. It will be seen that the individual ZI's have constant differences equal to $\frac{1}{12}$ of the value of the total ZI for the oblique ascensions. Nonetheless, in Texts A–C, the reckoning of the *ziqpu* star distances is in accordance with the values from such a list as the *ziqpu* text AO 6478. Not surprisingly, the derived rising times do not conform to those of System A (or B), but are far simpler. Despite the differences in values, the principles employed in the present texts, i.e., the concept of rising times and the implicit relation of the change of daylight length to the passage of the sun through the ecliptic, remain the same. As will be shown in the following discussion, the simple structure of the rising-times scheme of the micro-zodiac texts is derivative of the scheme for the variation in daylight known in the Astrolabe and MUL.APIN, as it preserves the 2:1 ratio of longest to shortest daylight. Given this, one would not expect the rising-time values of the micro-zodiac texts to be consistent with those of System A, whose daylight scheme presumes a 3:2 ratio for the daylight length extrema. Although Texts A–C do not concern the length of daylight per se, a daylight scheme is implicit in the rising times preserved (and reconstructed) from these texts, and can be discussed.

The main body of each of the three texts presents data concerning the micro-zodiac portions, the *ziqpu* star intervals corresponding to their risings, dates of a phenomenon KUR, and the appearances of stars that produce a luminous flare termed *mešhu*.

[11] For a recent discussion and bibliography, see [Horowitz *Geography*, 182–8].

The dates of KUR, here taken to represent the last visibility of the moon at the end of the month, are also given in schematic fashion. The data follows the same formulary in each text:

> The *n*th portion of sign_x (is called) sign_y of sign_x. Month *m* : KUR in Mo. *m* morning/midday/afternoon of the 28th/29th/30th day. Star *m* flared. The distance d° east of star *z* stood on the meridian, ditto the sun.

The number in the sequence of twelve $2\frac{1}{2}^\circ$ portions per zodiacal sign determines the name of that portion. Portion 1 is named for the sign itself, and the rest follow in order of the remaining signs of the zodiac.[12] For example, Aries' portions are named Aries of Aries, Taurus of Aries, etc. The nature of the correspondence between the number of the micro-zodiac portion and the month is determined by the particular sign whose name is given to the portion. For example, in Text B, the 6th portion of Aries is 'Virgo of Aries'. The month is then Month VI, corresponding to the position of Virgo as the sixth sign of the zodiac. For the sign Scorpius (see Text A), however, the 6th portion is 'Aries of Scorpius'. Month I is then correlated with the 6th portion, since Aries is the first sign of the zodiac.

The months are in turn associated with the phenomenon KUR and the flare of a star (*mešhu*). The pairing of a star said to produce a *mešhu* and associated with a particular month as the month of its supposed heliacal rising together with a *ziqpu* star is reminiscent of the association made in MUL.APIN between twelve *ziqpu*'s crossing the meridian before sunrise in mid-month and the heliacal risings of certain constellations.[13] As can be readily seen from this section of MUL.APIN, however, no relation can be made between pairs of *ziqpu* and rising stars of MUL.APIN and the stars mentioned in Texts A–C. Remarks on the *mešhu* stars and the phenomenon KUR have already been made in the commentary to Text A. I leave these questions as to the function of the *mešhu* stars and meaning of KUR unresolved and turn to the central question of the rising-times scheme.

[12] The method is seen as well in the texts published in [Weidner *Gestirn-Darstellungen*]. See the summary description in [Hunger & Pingree *Astral Sciences*, 29].

[13] MUL.APIN I iv 13–30, and see [Hunger & Pingree *MUL.APIN*, 142 (Table IV)].

The use of the *ziqpu* stars is key to understanding the scheme developed in Texts A–C. Table II summarizes the data on the *ziqpu* stars preserved in the micro-zodiac texts, and follows the identifications and right ascensions as given by Hunger and Pingree.[14]

No.	Star name	Mod. ID	R.A.$^{(\circ)}$	ΔR.A.	Sign
VII	MÚL.GAŠAN. TIN, *Bēlet balāṭi* 'Lady of life'	α Lyrae	256;42		♓
VIII	MÚL *kumar ša* UD.KA.TUH.A *Kumar ša Nimri* 'Shoulder of the Panther'	β Cygni	265;38		♈
IX	MÚL SA$_4$ *ša* GABA-*šu* *Nibî ša irtišu* 'Bright Star of his Chest'	α Cygni	287;32	41;54	
X	MÚL *kinṣu* Lower leg (of Panther)	α Lacertae	311;36	24;4	♉
XX	MÚL.AL.LUL *Alluttu* 'The Crab'	ϵ Cancri	89;55		♎
	MÚL.ME *ár*.ME *šá* MÚL.ALLA	γ Cancri(?)			
	kakkabānu arkâ- *tu ša Alluttu*	δ Cancri(?)			
XXI	MÚL 2 MÚL. MEŠ *ša* SAG.DU MÚL.UR.GU.LA 2 *Kakkabānu ša* *rēš* UR.GU.LA 'Two Stars of the Head of the Lion'	ϵ Leonis μ Leonis	105;30 106;50	 16;55	

[14] See the *ziqpu* table in [Hunger & Pingree *Astral Sciences*, 87].

XXII	MÚL 4 *ša* GABA-*šu* 4 *ša irtišu* 'The Four Stars of his Chest'	η Leonis ζ Leonis α Leonis γ Leonis	112;54 113;46 114;26 115;17		♏
XXIII	MÚL 2 *ša* KUN-*šu* 2 *ša zibbatišu* 'Two Stars of his Rump'	δ Leonis θ Leonis	130;1 131;4		
XXIV	MÚL DELE *ša* KUN-*šu* *Edu ša zibbatišu* 'The Single Star of his Rump'	β Leonis	141;2	35;32	
XXV	MÚL *e*$_4$-*ru*$_6$ (A.EDIN) *Erua* 'The Frond'	γ Coma Berenices	150;49		♐
XXVI	MÚL *na-at-tul-lum* *Nattullum* 'The Harness'	η Boötis	175;47	34;44	

Table II: *Ziqpu* stars in Texts A–C

As indicated by the Roman numerals, this table is arranged according to the order of the *ziqpu* stars in AO 6478 (see [Schaumberger '*Ziqpu*-Gestirne', 228–9]). Only the 'rear stars of the Crab' (MÚL.ME *ár*.ME *šá* MÚL.ALLA, possibly γ and δ Cancri), mentioned in both Texts B and C, do not appear in Schaumberger's table. The arrangement is in progressive sequence by right ascension. The ΔR.A.'s have been given to indicate the distance (ZI) between culminations of the *ziqpu* stars used to measure the rising of a given zodiacal 30° arc of the ecliptic. These R.A. values, however, are useful only for the three zodiacal signs for which complete data on *ziqpu* transits is preserved. This situation obtains for Aries, Scorpius, and Sagittarius. Here one can compare the modern ΔR.A. values against the schematized values adopted in the text. For example, for *ziqpu*'s correlated with Aries ΔR.A. $= 41;54°$, which rounds down to 40° and gives a rising-time value for Aries of 40°. For Scorpius, measured from ϵ Leonis + μ Leonis to β Leonis, ΔR.A. $= 35;32°$, but this value is also taken as 40° and the rising time adopted for Scorpius is 40°. For Sagittarius as well, which is measured from β Leonis to

η Boötis, ΔR.A. $= 34;44°$, but is found to be 40° in the text, and assigned a rising time of 40° for the sign. It is interesting to see how close the ΔR.A.'s for Scorpius and Sagittarius in fact are to the System A rising-time values for these signs, for Scorpius 36° and Sagittarius 32°. Our texts, as we shall see, present a far simpler scheme. The other three zodiacal signs give us only partial information. For Pisces, only the last *ziqpu* is preserved, so no estimate of the distance between such culmination stars can be supplied. The same applies to Libra. For Taurus, the beginning is preserved, but we lack the end. As shown in Table III, however, the 'degrees of arc' (UŠ *ina qaqqari*)[15] between *ziqpu* stars, as given in the star list AO 6478, do not in every instance correspond to the distances between culminations of *ziqpu* stars presumed in our texts.

Sign	No.	*Ziqpu star*	UŠ *ina qaqqari*	Total ZI	ZI in micro-zodiac texts
♓	VI	[μ Herculis]	10		
	VII	α Lyrae	10	20	20
♈	VIII	β Cygni	20		
	IX	α Cygni	10	30	20
♉	X	α Lacertae	20	20	[20]
♎	XX	ϵ Cancri	20		
	XXI	$\epsilon + \mu$ Leonis	20	40	[40]
♏	XXII	$\alpha + \gamma + \zeta + \eta$ Leonis	10		
	XXIII	$\delta + \theta$ Leonis	20		
	XXIV	β Leonis	10	40	40
♐	XXIV	β Leonis	10		
	XXV	γ Coma Berenices	10	20	40

Table III: Degrees of arc between *ziqpu* stars

This table shows that the numerical data for the intervals between *ziqpu* star transits as stated in the micro-zodiac texts do not with any certainty follow from the UŠ *ina qaqqari* of the *ziqpu* star list. From Texts A–C's numerical values, however, a scheme can be reconstructed for the rising times of the zodiac whose method resembles System A, but whose numbers are clearly cruder, as

[15] See the discussion in [Horowitz *Geography*, 183–5].

seen in the above Table III. Before considering this scheme in more detail, the method already well known from ACT should be briefly described.

Hitherto, two rising-times schemes were known for Babylonian astronomy only as derived from the two Babylonian schemes for computing the length of daylight from a solar (or a lunar) longitude in Column C of the late astronomical ephemerides. That schemes for the rising times of the zodiac underpinned this column was demonstrated by Neugebauer.[16] He showed that the length of a day for a given position of the sun in the ecliptic was indeed the sum of the rising times of the 180° of the ecliptic beginning with the sun's position: length of daylight (C) = the rising time of $\frac{1}{2}$ of the ecliptic, viz., the half from the longitude of the sun (λ_{sun}) to $\lambda_{\text{sun}} + 180°$.[17] Necessarily, the rising times were constrained by the 3:2 ratio of longest to shortest daylight adopted in the ephemerides.

Underlying the computation of daylight in Systems A and B were rising-times schemes of simple linear sequences in which the rising-times values of System A have a constant difference *d* of 4° and System B of 3° but for one middle difference of 6°, as seen below (Table IV).

Zod. sign	$\alpha^{(\circ)}$ System A	*d*	$\alpha^{(\circ)}$ System B	*d*
♈ / ♓	20		21	
♉ / ♒	24	4	24	3
♊ / ♑	28	4	27	3
♋ / ♐	32	4	33	6
♌ / ♏	36	4	36	3
♍ / ♎	40	4	39	3

Table IV

System A, which normed the zodiac at Aries 10°, derived values for daylight lengths for every 10th degree of a sign, System B for every 8th degree. Consequently, the cardinal points of the year were set at 10° and 8° of their respective signs.[18] It then becomes clear that the lengths of daylight for the 10th or 8th degrees of

[16] [Neugebauer 'Rising Times', 100 n. 4], citing his earlier [Neugebauer 'Astronomical Papyri'].

[17] See [Neugebauer *History*, 368].

[18] As Neugebauer has explained, 'When finally the irregular configurations

the zodiacal signs computed in Systems A and B respectively are in fact the sums of the rising times for the appropriate half of the ecliptic that rises on the day in question. For example, and using the values of α from System A, when the sun is in Aries 10°, the value C is a result of the sum of the rising times of signs 1–6:

$$20 + 24 + 28 + 32 + 36 + 40 = 3,0(= 180)^\circ = 12\,\text{hours}.$$

For the sun in Taurus 10°

$$24 + 28 + 32 + 36 + 40 + 40 = 3,20(= 200)^\circ = 13\,\text{hours}\ \ 20\,\text{min}.$$

The following Table V gives the daylight scheme for λ_{sun} according to System A.

Solar λ	C		(System A)	
♈ 10°	3;0^H	=	12;0	hours
♉ 10°	3;20	=	13;20	
♊ 10°	3;32	=	14;8	
♋ 10°	3;36	=	14;24	(M)
♌ 10°	3;32	=	14;8	
♍ 10°	3;20	=	13;20	
♎ 10°	3;0	=	12;0	
♏ 10°	2;40	=	10;40	
♐ 10°	2;28	=	9;52	
♑ 10°	2;24	=	9;36	(m)
♒ 10°	2;28	=	9;52	
♓ 10°	2;40	=	10;40	

Table V

For all other solar positions, values of daylight were interpolated by factors derived from the differences of column C divided by 30°, which then represent the increase or decrease in daylight length per degree of solar longitude,[19] as shown in Table VI for System A.

[constellations] were replaced by real ecliptic coordinates in signs of equal 30° length the sign "Aries" obtained by some accidental compromise such a position within the constellation Aries that the vernal equinox took place when the sun was at the 10th, respectively 8th, degree of the sign.' He did not find any chronological significance to the two norms, and certainly dismissed any connection with a knowledge of precession. For details, see [Neugebauer *History*, 368–9].

19 ACT 200 sect. 2; ACT 200b sect. 2, both for System A.

Zodiacal sign	C	d	interpolation factor		
♈	3,0	+20	20°/30°	=	+0;0,40
♉	3,20	+20	20°/30°	=	+0;0,40
♊	3,32	+12	12°/30°	=	+0;0,24
♋	3,36	+4	4°/30°	=	+0;0,8
♌	3,32	−4	4°/30°	=	−0;0,8
♍	3,20	−12	12°/30°	=	−0;0,24
♎	3,0	−20	20°/30°	=	−0;0,40
♏	2,40	−20	20°/30°	=	−0;0,40
♐	2,28	−12	12°/30°	=	−0;0,24
♑	2,24	−4	4°/30°	=	−0;0,8
♒	2,28	+4	4°/30°	=	+0;0,8
♓	2,40	+12	12°/30°	=	+0;0,24

Table VI

The method of computation for the daylight length when the sun is at any longitude other than the 10th degree of a sign requires that the daylight length for the sign of the sun (or opposite the sun when the longitudes are based on full moons) be modified (increased) by the number of degrees by which the sun exceeds 10° of the sign multiplied by the interpolation factor, as shown in Table VI. In this way, the rising times are implied by column C of the ACT ephemerides and indeed can be derived from them, but it should be noted that the values themselves do not appear in the ACT material.

It is only in the group of micro-zodiac texts presented here that several of the actual values of the zodiacal rising times are given to us directly, as noted by Schaumberger in his publication of three of these texts, which Schaumberger designated as *ziqpu*-texts.[20] Schaumberger identified the rising-time values as those connected with System A. He pointed out that these same values also appear in the Greco-Roman treatises of Manilius (A.D. 15),[21] Vettius Valens (ca. A.D. 150),[22] and Firmicus Maternus (ca. A.D. 350),[23] attesting to the adoption of Babylonian astronomical parameters in Hellenistic Greek astronomy. On the basis of the

[20] [Schaumberger 'Anaphora'].

[21] *Astronomica* III, 275ff. [Breiter *Astronomica*, 74, 88]; [Housman *Astronomicon*, 24, xiii].

[22] I, 7 and 14 [Kroll *Anthologiae*, 23, 28].

[23] *Mathesis* II, 11 [Kroll & Skutsch *Mathesis*, I, 53–5], see [Neugebauer *History*, 719].

identification between rising-time values in the micro-zodiac texts and some of those of System A, Schaumberger proceeded on the assumption that the micro-zodiac texts' rising-times scheme was identical to that of System A.[24] Although he did not address the question of the related scheme for length of daylight in his article, he would have to have inferred that it too was identical to that of System A. As already indicated above, a different and simpler scheme than that of System A is discernible in Texts A–C.

As mentioned above, two of the main features of the micro-zodiac texts are the transits of *ziqpu* stars and the twelfth portions of zodiacal signs (*dodekatemoria*) that define the ecliptic. Because the texts reckon the time required for *dodekatemoria* to rise (i.e., rising times of twelfths of zodiacal signs) in terms of the distance (expressed in time) since meridian crossings of *ziqpu* stars, twelve 'distances' called ZI (*nishu*) are given for each zodiacal sign. The rising time of each sign, therefore, is given, as it should be, in time degrees with reference to the equator, and the constant differences of the ZI's are therefore twelfths of rising times to correspond to the twelve 'portions' or *dodekatemoria*. The sum of the twelve ZI differences should equal the rising time for the sign. The four extant micro-zodiac texts of this type preserve data only for the signs Aries, part of Taurus, part of Libra, Scorpius, part of Sagittarius, and part of Pisces.[25] Because of the symmetry of the rising times, according to which

$$\begin{array}{rcl} \alpha_1 & = & \ulcorner\alpha_{12}\urcorner \\ \ulcorner\alpha_2\urcorner & = & [\alpha_{11}] \\ [\alpha_3] & = & \ulcorner\alpha_{10}\urcorner \\ [\alpha_4] & = & \ulcorner\alpha_9\urcorner \\ [\alpha_5] & = & \alpha_8 \\ [\alpha_6] & = & \ulcorner\alpha_7\urcorner \end{array}$$

the rising-times values for nearly all twelve signs may be reconstructed.

[24] His analysis of U 196, however, entertains the possibility of correspondence with either System A or B; see [Schaumberger 'Anaphora', 242–3].

[25] U 196:10 may provide a value for Capricorn, but is utterly fragmentary and needs collation. The value given in U 196:10 for the zodiacal sign Capricorn (Schaumberger put two question marks by his reading MÁŠ) is $\frac{2}{3}$ DANNA 7 UŠ, '27°', which belongs to System B. If the reading is correct, the text reflects a different scheme than that of the others, and so U 196 is left out of consideration for now.

If the rising-times values of System A truly underlie this scheme, we should expect the following values of the difference of ZI (Table VII), derived by dividing the rising times of System A by 12, to obtain 12 constant increments in ZI, which together constitute a total distance crossed by the *ziqpu* stars equal to the time taken for the 30° of the sign to rise.

Zodiacal sign	rising time/12	*d* of ZI's
♈ / ♓	20°/12	1,40°
♉ / ♒	24°/12	2°
♊ / ♑	28°/12	2,20°
♋ / ♐	32°/12	2,40°
♌ / ♏	36°/12	3°
♍ / ♎	40°/12	3,20°

Table VII

In fact, only 1,40° and 3,20° appear as differences in the ZI's given in our texts, implying that the rising-times scheme underlying these texts is limited to the values 20° and 40° for rising times of signs. The best preserved sections illustrate:

HA.LA (of ♈)	degrees of ZI	*d* of ZI
6	1,40	
7	3,20	1,40
8	5	1,40
9	6,40	1,40
10	8,20	1,40
11	<>	
12	TOTAL 10	1,40

Table VIII

Only the second half of the Aries section is given,[26] so the total of 10° is the rising time for 15° of Aries. For the entire sign, the rising time will be $2 \times 10° = 20°$, and $20° \div 12 = 1,40$. The value for Aries is the same as that of System A. The derivation of the rising time for Scorpius is shown in Table IX below.

[26] See Text B (LBAT 1499) rev. 14–30, referring to the half of the sign Aries TA MAŠ-*šú* [E]N TIL-*šú* 'from its middle to its end' (line 30).

HA.LA (of ♏)[27]	degrees of ZI	*d* of ZI
1	1,40	
2	5	3,20
3	8,20	3,20
4	11,40	3,20
5	15	3,20
6	18,20	3,20
7	<2>1,40	3,20
8	<2>5	3,20
9	<2>8,20	3,20
10	<2>1,40	3,20
11	<2>5	3,20
12	TOTAL 40	

Table IX

Note that the difference of ZI from portion 11 to 12 is 5°. If, however, the ZI of the first portion is subtracted, the correct difference is obtained. The total 40°, the value of the rising time of Scorpius, is not in agreement with System A, where Scorpius has the rising time of 36°. It is the only possible value in light of the constant difference of 3,20. There is enough textual evidence preserved to establish that one-half the ecliptic consisted of zodiacal signs having rising times of 40° each and in the other half, two signs, Aries and its symmetrical Pisces, had rising times of 20°. Although the texts do not preserve values for the remaining four signs, the nature of the scheme which emerges from the data that are preserved is sufficient to argue for restoring the values 20° for these too.[28]

The scheme divides the ecliptic into equal halves. From Cancer to Capricorn the signs are assigned the rising time 40° and from Capricorn to Cancer, the signs are assigned the value 20°.

[27] See Text A (A 3427) obv. 1–18.

[28] The one seemingly contradictory text, the fragmentary U 196, has the value 27 for Capricorn(?). This value, however, if the reading is correct (text needs collation), corresponds to System B, and so would not be relevant to the other texts. In order for U 196 to be considered evidence that this entire text group corresponds to System A rising-times scheme (as Schaumberger assumed), the value 27 must be emended to 28. That still would not mitigate the fact that for those data which are preserved or restorable in the other three extant texts, the value 40 is repeated from Cancer to Sagittarius, a fact which cannot be reconciled with System A. A wholly different, and much cruder, rising-times scheme must be seriously considered.

If we compare these rising times against those of System A, we note that two of the six rising times are found in the micro-zodiac texts. Indeed these texts seem to divide the ecliptic into only two kinds of signs, those with a fast rising time (20°) and those with a slow (40°), as in Table X. It is interesting to see that the same symmetry is followed as in the fully developed scheme, i.e., $\alpha_1 = \alpha_{12}$, $\alpha_2 = \alpha_{11}$, $\alpha_3 = \alpha_{10}$, etc. The rule that the sum of the rising times equals 360 is also obeyed by the cruder scheme: $20° \times 6 + 40° \times 6 = 360°$.

Zodiacal sign	rising time
♈ / ♓	20
♉ / ♒	[20]
♊ / ♑	[20]
♋ / ♐	40
♌ / ♏	40
♍ / ♎	40

Table X

If this reconstructed scheme is a precursor to System A, the resulting model of two equal 'zones' of the ecliptic, one in which the signs rise 'fast' and the other in which the signs rise 'slow' is only reminiscent of System A's model of solar progress around the zodiac. In System A, the ecliptic is also divided into two parts, but not equal halves, and the sun moves with one rate of progress in a 'fast' zone and another in a 'slow' zone. The sum of the lengths of each zone must be 360°. The points where the sun's progress in longitude jumps from fast to slow and back to fast are placed roughly near the solsticial points, at Gemini 25° and Scorpius 30° respectively.

In terms of daylight length, the scheme implied in the micro-zodiac text tradition divides the year into symmetrical halves at the solstices: From Cancer to Capricorn the days become progressively shorter, and from Capricorn to Cancer they become progressively longer. Stressing again the hypothetical nature of the daylight scheme that follows from the rising times reconstructed here for the micro-zodiac texts, the values C which would be obtained by taking the sum of the rising times of the six signs that cross the horizon from sunrise to sunset, are given in Table XI.

Zodiacal sign	α		C		
♈	20	3	=	12 h	Vernal Equinox
♉	20	3,20	=	13 h 20′	
♊	20	3,40	=	14 h 40′	
♋	40	4	=	16 h	Summer Solstice
♌	40	3,40	=	14 h 40′	
♍	40	3,20	=	13 h 20′	
♎	40	3	=	12 h	Autumnal Equinox
♏	40	2,40	=	10 h 40′	
♐	40	2,20	=	9 h 20′	
♑	20	2	=	8 h	Winter Solstice
♒	20	2,20	=	9 h 20′	
♓	20	2,40	=	10 h 40′	

Table XI

As can readily be seen, C is obtainable from the sum of the six rising times of the zodiacal signs crossing the horizon over the course of a day, from sunrise to sunset (λ_{sun} to $\lambda_{\text{sun}} + 180°$, in exactly the same manner as System A. For Aries, the length of daylight is found by adding the rising times from Aries to Virgo: $20+20+20+40+40+40 = 3,0 = 12$ hours. For Taurus, the sum of the rising times of Taurus to Libra: $20+20+40+40+40+40 = 3,20 = 13$ hours 20 minutes. Although the method of deriving the values C from the sums of rising times is identical with that of System A, the values of the daylight scheme are identical to those of earlier Babylonian astronomical texts, such as the Astrolabe, *Enūma Anu Enlil* XIV, and MUL.APIN. Whereas, however, In the Astrolabe tradition, the cardinal points of the year are placed in months XII (VE), III (SS), VI (AE), and IX (WS), MUL.APIN II i 9–21 divides the year around months I (VE), IV (SS), VII (AE), and X (WS), as shown in Table XII. It is clear that Texts A–C's assignment of daylight lengths to the month in which the sun is in a given zodiacal sign accords with MUL.APIN rather than the Astrolabe/*Enūma Anu Enlil* scheme, and remains consistent with System A as well (see the assignment of the cardinal points above in Table V).

Month	C (in *mana*)	C in UŠ[29]			
I	3;0	3	=	12 h	Vernal Equinox
II	3;20	3,20	=	13 h 20′	
III	3;40	3,40	=	14 h 40′	
IV	4;0	4	=	16 h	Summer Solstice
V	3;40	3,40	=	14 h 40′	
VI	3;20	3,20	=	13 h 20′	
VII	3;0	3	=	12 h	Autumnal Equinox
VIII	2;40	2,40	=	10 h 40′	
IX	2;20	2,20	=	9 h 20′	
X	2;0	2	=	8 h	Winter Solstice
XI	2;20	2,20	=	9 h 20′	
XII	2;40	2,40	=	10 h 40′	

Table XII

While many aspects, both philological and astronomical, of the micro-zodiac Texts A–C must here be left unresolved, these sources may be viewed as evidence for the development of the characteristically Babylonian method of solving the problem of determining oblique ascensions of arcs of the ecliptic, a problem which would continue to be of the highest importance in later ancient spherical astronomy. Of course, we may be dealing with an archaized rising-times scheme as opposed to evidence of a precursor to the methods of the late tabular texts. Regardless of the date of its invention, the hybrid daylight scheme that follows from the rising-times values in Texts A–C, i.e., a cross between that of Column C of the System A ephemerides on one hand, and the early astronomical tradition of the Astrolabes, *Enūma Anu Enlil*, and MUL.APIN on the other, certainly adds a new dimension to our picture of late Babylonian non-tabular astronomical texts.

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[29] The relation between these measures is 1 *mana* = 1,0 UŠ.

Abbreviations

ACT [Neugebauer *ACT*]
CAD *The Assyrian Dictionary of the Oriental Institute of the University of Chicago*

Bibliography

T. Breiter (ed.), *Astronomica* of Manilius, 2 vols, Leipzig, 1907–08.

V. Donbaz & J. Koch, 'Ein Astrolab der dritten Generation: NV.10', *Journal of Cuneiform Studies*, 47, 1995, pp. 63–84.

W. Horowitz, 'Two New Ziqpu-Star Texts and Stellar Circles', *Journal of Cuneiform Studies*, 46, 1994, pp. 89–98.

——. *Mesopotamian Cosmic Geography*, Winona Lake IN, 1998.

A. E. Housman (ed.), *Astronomicon* of Manilius, 5 vols, Cambridge, 1937, 2nd ed.

H. Hunger, *Babylonische und assyrische Kolophone*, Neukirchen-Vluyn, 1968.

H. Hunger & D. Pingree, *MUL.APIN, An Astronomical Compendium in Cuneiform* (*Archiv für Orientforschung* Beiheft 24), Horn, 1989.

——. *Astral Sciences in Mesopotamia*, Leiden, 1999.

W. Kroll (ed.), *Anthologiae* of Vettius Valens, Berlin, 1908.

W. Kroll & F. Skutsch (eds), *Mathesis* of Firmicus Maternus, Leipzig, 1897–1913.

O. Neugebauer, 'Jahreszeiten und Tageslangen in der babylonischen Astronomie', *Osiris*, 2, 1936, pp. 517–50.

——. 'On some Astronomical Papyri and Related Problems of Ancient Geography', *Transactions of the American Philosophical Society*, New Series 32, 1942, pp. 251–63.

——. 'The Rising Times in Babylonian Astronomy', *Journal of Cuneiform Studies*, 7, 1953, pp. 100–102.

——. *Astronomical Cuneiform Texts*, 3 vols, London, 1955.

——. *A History of Ancient Mathematical Astronomy*, 3 vols, New York, 1975.

J. Oelsner & W. Horowitz, 'The 30-Star Catalogue HS 1897 and the Late Parallel BM 55502', *Archiv für Orientforschung*, 44–45, 1997/98, pp. 176–85.

S. Parpola, *Letters from Assyrian and Babylonian Scholars* (State Archives of Assyria, Vol. X), Helsinki, 1993.

D. Pingree & C. B. F. Walker, 'A Babylonian Star Catalogue: BM 78161', in *A Scientific Humanist: Studies in Memory of Abraham Sachs*, ed. Erle Leichty et al., Philadelphia, 1988, pp. 313–21.

E. Reiner & D. Pingree, *Babylonian Planetary Omens. Part 2: Enūma Anu Enlil Tablets 50–51*, Malibu 1981.

F. Rochberg, 'New Evidence for the History of Astrology', *Journal of Near Eastern Studies*, 43, 1984, pp. 115–40.

——. *Babylonian Horoscopes* (American Philosophical Society Transactions 88), Philadelphia, 1998.

A. J. Sachs (with J. Schaumberger), *Late Babylonian Astronomical and Related Texts*, Providence RI, 1955.

A. J. Sachs & H. Hunger, *Astronomical Diaries and Related Texts from Babylon*, 3 vols, Vienna, 1988–96.

J. Schaumberger, 'Die *Ziqpu*-Gestirne nach neuen Keilschrifttexten', *Zeitschrift für Assyriologie*, 50, 1952, pp. 214–29.

——. 'Anaphora und Aufgangskalender in neuen Ziqpu-Texten', *Zeitschrift für Assyriologie*, 51, 1955, pp. 237–51.

A. Ungnad, 'Besprechungskunst und Astrologie in Babylonien', *Archiv für Orientforschung*, 14, 1941–44, pp. 274–82.

C. B. F. Walker & H. Hunger, 'Zwölfmaldrei', *Mitteilungen der Deutschen Orientgesellschaft zu Berlin*, 109, 1977, pp. 27–34.

E. Weidner, *Handbuch der babylonischen Astronomie*, Band I, Leipzig, 1915 (repr. 1976).

——. *Gestirn-Darstellungen auf babylonischen Tontafeln*, Graz, 1967.

Babylonian Mathemagics: Two Mathematical Astronomical-Astrological Texts

LIS BRACK-BERNSEN, JOHN M. STEELE

The two tablets published here combine aspects of mathematics, astronomy and astrology and so they were an obvious choice when we came to decide upon the subject of this article in honour of David Pingree. It is with the greatest of respect that we offer this small contribution towards the study of the astral sciences to a scholar who has done so much to elucidate their history throughout ancient and medieval Europe and Asia.

BM 96258 and BM 96293 were brought to our attention by Christopher Walker during a visit to the British Museum in September 2000. He kindly supplied a preliminary transliteration from which we were able to identify the numerical scheme contained on the tablets. We thank the Trustees of the British Museum for granting permission to publish the tablets.

1 *The texts*

BM 96258 (1902-4-12, 370) and BM 96293 (1902-4-12, 405) come from a purchased collection which contains a substantial number of Neo-Babylonian tablets from Babylon and (predominantly) Borsippa covering the time range from Nebuchadnezzar to Artaxerxes.[1] The tablets are similar in shape and size; when complete each tablet would have been about 4 cm wide by 7 cm high.

The two tablets were almost certainly written by the same scribe, as is evident from the fact that he writes the number nine using both the old (9 vertical wedges) and new (3 slanting wedges) forms, the use of light vertical guide lines for each column of numbers, the horizontal rulings after each 5 lines, and the general similarity of script and style. From their contents (see

[1] C. B. F. Walker, private communication.

below), it is likely that these were the first two tablets in a series of twelve.

BM 96258 (1902-4-12, 370)
Size: 4 × 4.5 cm.
Top, left and right edges are preserved.

Obverse:

	I	II	III	IV
#1	10	7	⌜1⌝	[1]
#2	7	14	1	2
#3	4	21	1	⌜3⌝
#4	1	28	1	⌜4⌝
#5	11	5	1	⌜5⌝
#6	8	12	1	6
#7	5	19	1	7
#8	2	29	1	8
#9	12	3	1	9
#10	[9]	[10]	[1]	⌜10⌝

Reverse:

	I	II	III	IV
#1′	[12]	⌜4⌝	⌜1⌝	22
#2′	9	11	[1]	⌜23⌝
#3′	6	⌜18⌝	[1]	⌜24⌝
#4′	⌜3⌝	25	[1]	[25]
#5′	⌜1⌝	⌜2⌝	⌜1⌝	[26]
#6′	⌜10⌝	[9]	⌜1⌝	[27]
#7′	7	⌜16⌝	[1]	[28]
#8′	4	[23]	[1]	[29]
#9′	1	[30]	[1]	[30]

Critical Apparatus:

Obv. 7: 19 written with new style 9.
Obv. 8: 29 written with new style 9. Error for 26.
Obv. 9: 9 written old style.
Rev. 2′: 9 written old style.

BM 96293 (1902-4-12, 405)
Size: 4 x 5.25 cm.
Bottom, left and right edges are preserved.

Obverse:

	I	II	III	IV
#1′	3	26	⌜2⌝	[8]
#2′	1	3	2	[9]
#3′	10	10	2	⌜10⌝
#4′	⌜7⌝	17	2	11
#5′	4	⌜24⌝	2	12
#6′	[1]	1	2	13
#7′	[11]	⌜8⌝	2	14
#8′	[8]	⌜15⌝	2	15

Reverse:

	I	II	III	IV
#1	5	22	2	⌜16⌝
#2	2	29	2	⌜17⌝
#3	12	6	2	18
#4	9	13	2	19
#5	6	20	2	⌜20⌝
#6	3	27	[2]	21
#7	1	4	[2]	⌜22⌝
#8	10	11	[2]	23
#9	7	18	2	24
#10	4	⌜25⌝	⌜2⌝	[25]

Critical Apparatus:

Rev. 2: 29 written with old style 9.

Rev. 4: 9 written new style, 19 written with new style 9.

2 *Commentary*

Both tablets are divided into 4 columns. Column I contains numbers between 1 and 12, and column II numbers between 1 and 30. Normally, column I decreases line by line by 3 (mod. 12), and column II increases by 7 (mod. 30). However, if the increase in column II would take the value beyond 30, then the number in column I is only reduced by 2. This implies that columns I and II are linked such that 30 units of column II are equal to one unit of column I. For simplicity, let us call column I 'signs' since there are twelve of them and column II 'degrees' since there are thirty. Moving on one line, therefore, we move back three signs and forward seven degrees (i.e., -83°), or alternatively we move forward nine signs and seven degrees (i.e., +277°), since -83 $\equiv$ 277 (mod. 360). Column III contains a constant number: 1 on BM 96258, 2 on BM 96293. Finally column IV contains a number that increases line by line by 1 from 1 to 30. Once again, for convenience we can call column III 'signs' and column IV 'degrees'. The question of whether these actually represent signs and degrees will be discussed below. Mathematically we can formulate this scheme as follows: If we have in columns III and IV a number n degrees within sign s, then the associated position in columns I and II is equal to $277n$ from the beginning of the same sign s.

The mathematical scheme presented on BM 96258 and BM 96293 is identical to that found on a group of texts called by Weidner '*Kalendertexte*'.[2] These texts associate entries from a numerical *Kalendertext* scheme with trees, stones, animal parts and various other items probably used in rituals. Six *Kalendertexte* have previously been published:[3]

VAT 7816 and VAT 7815, both from Uruk, were edited in [Weidner *GDBT*, 41-49], with an explanation of the numerical scheme by van der Waerden on pp. 50-52. VAT 7815 was owned

[2] [Weidner *GDBT*, 41].

[3] In addition to the published *Kalendertexte*, we can add the unpublished tablet W.20030/133, a *Kalendertext* for month V cited by [Hunger 'Kalendertext', 43], and a small fragment in the British Museum, BM 36995, edited below, which contains *Kalendertexte* for months II and VIII:

by Anu-Belšunu and written by his son, Anu-abi-utterri,[4] on the 14th of Month X, year 120. The colophon of VAT 7816 is mostly broken away, but it is quite possible that both tablets were part of the same archive. Beside each numerical entry in the texts are two or three lines which give the name of a tree, a plant, a stone, a place name, and some prescribed action. After the 30th entry in the *Kalendertext* scheme, VAT 7816, Rev. 17′ begins LAL 1 TA UD-1-KAM EN UD-30-KAM...'Subtract (?) 1. From the 1st day to the 30th day...' Similarly VAT 7815, Rev. 9′ begins ⌜LAL⌝ [...][5] TA UD-1-KAM EN UD-30-KAM. On these two texts, therefore, we should understand the fourth number in an entry as a day number, and so presumably the third number as the month. It is not immediately clear whether we should read the first two numbers in an entry as month and day as well.

LBAT 1586+1587,[6] from Babylon, is a *Kalendertext* for the first 15 days of Month III. This is clear from Rev. 11 which reads *šal-šú* [*ár*]-*ḫu* 'third month'. Once more, there is no indication of whether we should also read the first two numbers in an entry as month and day. Accompanying each numerical entry in the *Kalendertext* scheme there is often a statement that the moon is 'in' (*ina*) a star or constellation, a statement concerning the star MÚL.SIPA, and various names. In his edition of the text, Hunger remarked that the first of these was 'astronomische Angaben über die Stellung des Mondes in verschiedenen Sternbildern'.[7] Not all

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Obv.

[...] x	ŠID$^{?}$	BE x	[...]
11	7	2	1
⌜8⌝	14	2	⌜2⌝
[5]	21	2	⌜3⌝
[2	2]8	2	[4]
Rev.			
[7	3]	8	[9]
[4]	⌜10⌝	8	10
[1]	17	8	1[1]
[10]	24	8	1⌜2⌝
[8]	1	⌜8⌝	[13]

4 Both Anu-Belšunu and Anu-abi-utterri are well known from astronomical and other texts. See [Steele 'A3405', 131].

5 [Weidner *GDBT*, 46] restores [30] here. Why?

6 [Hunger 'Kalendertext'].

7 [Hunger 'Kalendertext', 41].

of the entries are preserved. From what does remain, however, we can see that these cannot represent the (approximate) position of the moon on days during the month. Instead, the stars and constellations relate to the zodiacal signs given by the *Kalendertext* scheme. This is made clear by the following table:[8]

Kalendertext entry	*Sin ina...*	Identification	Zodiacal sign
4 26 3 8	M[Ú]L.NAGAR	Cancer	Cancer
2 3 3 9	[*is l*]*e-e*	α Tau + Hyades	Taurus
11 10 3 10	MÚL.GU.LA	Aquarius	Aquarius
5 24 3 12	MÚL.GU.LA	Aquarius	Aquarius
3 1 3 13	MÚL.MAŠ	Gemini	Gemini
12 8 3 14	MÚL.SÌM.MAḪ	Western fish of Pisc. with some of the W. part of Pegasus	Pisces
9 15 3 15	MÚL.PA	Sagittarius	Sagittarius

For example, on days 13, 14 and 15, the moon is said to be in Gemini, Pisces and Sagittarius respectively. Astronomically this makes no sense since the moon moves approximately 13 degrees per day. As a consequence, many of the lunar positions are impossible. For instance on day 13 of month 3, the moon is said to be in sign 3. This might have been the case on day 1 where the moon was near the sun (which in month 3 approximately will be in sign 3). On day 13 (shortly before opposition), however, the moon must be near sign 8 or 9. It can never be in sign 3 on day 13 of month 3. Thus we must conclude that what is written here is astronomically impossible.

However, if we compare the zodiacal signs with the first number of the *Kalendertext* scheme, we find that in all but one case they agree. The exception is for day 12 where we should have Leo instead of Aquarius, but since Leo may be written as MÚL.UR.GU.LA, it is not hard to imagine that this is just a simple scribal error. The entry for day 11 is damaged. Hunger read MÚL.PA? (Sagittarius?) and indeed this looks quite possible from the copy in *LBAT*. However, if we are correct to link

[8] The identifications of stars and constellations are generally taken from [Reiner & Pingree *BPO 2*].

these stars and constellations with the entries in the *Kalendertext* scheme, we would expect Scorpio.

We do not know whether to read the statement concerning MÚL.SIPA as part of the clause refering to the moon, or as a separate statment. MÚL.SIPA is generally used as the name for Orion, although it is also a name for Saturn.[9] We might possibly translate entries such as that for day fourteen: *Sin ina* MÚL.SIM.MAḪ KI MÚL.SIPA GUB as 'The moon (which was) in Pisces stood with Orion/Saturn', but this is by no means certain. Nevertheless, it does seem clear that this text is associating the moon with its position according to the *Kalendertext* scheme, rather than even approximating its actual motion.

W.20030/127,[10] from Uruk, gives no indication as to whether we should interpret the numbers as months and days or signs and degrees. The 'month' number is 2.

W.22704 and W.22619/6+22554/2b,[11] from Uruk, are *Kalendertexte* for Months IV and VIII respectively. The colophons to both texts indicate that they belonged to Iqīša, who lived around the end of the 4th century BC. In contrast to all of the above texts, however, month names and the names of the zodiacal signs replace some of the numbers in the *Kalendertext* scheme. Furthermore, the order of the entries in the scheme is different on these two texts. First we have a month name (or the repetition sign MIN after line 1) and the day number from 1 to 30, and then a sign of the zodiac and what we must interpret as the degrees within that sign. These are not always the normal signs of the zodiac, however.[12] For example, BAR[13] (Month I) is used in both texts for the sign Aries, and ŠU (Month IV) is used for Cancer as well as the month name. In the first line of W.22704, NE (Month V) is used for Leo, although UR.A is used consistently elsewhere. Nevertheless, it is likely that the zodiacal signs, and not the months, were meant since the associated text for each entry lists ingredients that are usually linked to the corresponding zodiacal sign.[14] For example, when the sign is Cancer, these are

9 [Hunger 'Kalendertext', 42].
10 [van Dijk *Texte*, no. 79].
11 [von Weiher *SpTU*, nos. 104–5].
12 [Reiner *Astral Magic*, 115].
13 BAR, not MAŠ as read by von Weiher.
14 [Reiner *Astral Magic*, 116–17]. Interestingly, these associations can only

the blood and fat of a crab.

From these texts, therefore, it seems most likely that we should understand the *Kalendertext* scheme as relating a position in the ecliptic to a date in the year. However, since the months in the *Kalendertext* are always 30 days long, we are not dealing here with real calendar years, but instead the schematic or 'ideal' calendar of twelve 30-day months. This schematic calendar probably has its origins in the administrative calendar of the Uruk III period or earlier,[15] and was used in texts such as MUL.APIN. Traces of it are still evident in the mathematical astronomical texts of the Late Babylonian period with the use of 1/30 of a synodic month as a unit.[16] This schematic calendar also, of course, provided the means by which the ecliptic could be divided up. Just as the schematic year had 12 months, so there were twelve signs of the zodiac. Likewise, the division of each zodiacal sign into 30 degrees mirrored the 30 days of a month. Furthermore, in a schematic year of 360 days, the sun makes one full revolution of the zodiac, and so, assuming simple mean motion, 1 day corresponds to 1° of solar longitude. Thus a date in the schematic calendar corresponds directly to a position in the zodiac. No doubt this was the justification for the substitution of month names for some of the zodiacal signs in W.22704 and W.22619/6+22554/2b. Thus the *Kalendertext* scheme can equally be seen as relating a day in the year to a position in the ecliptic (or vice versa), a day to another day, or a position to another position. Most probably it was used in all of these ways by the Babylonian scribes.

Further evidence for the equivalence between the schematic year and the ecliptic is provided by BM 47851.[17] This text lists side by side the *Kalendertext* scheme and a similar scheme known as the *Dodekatemoria*. This latter scheme is similar to the *Kalendertext* scheme except that one moves on 13 instead of 277 each line. Instead of zodiacal signs, this text always gives the names of the months.

There is undoubtedly a close relationship between the *Kalen-*

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be understood in terms of the classical zodiac.

[15] Described in [Englund 'Administrative Timekeeping'].

[16] Conventionally called a *tithi* after the analogous unit in Indian astronomy. In cuneiform it was simply written with the sign UD meaning 'day'.

[17] [Hunger 'Zahlenschema'].

dertext and the *Dodekatemoria*. Their mathematical similarities will be discussed below, but first we can note that they seem to have had similar astrological uses. The unpublished tablet BM 36326 contains numbers from the *Dodekatemoria* scheme written in the same format as many of the *Kalendertexte*; i.e., four columns of numbers which we can interpret as sign-degree-month-day. The 'month' number is 1. Before each numerical entry is the name of one of the five planets. As in the two texts published here there is a ruling after every fifth line, and each planet is named once within each group of five. The order of the planets varies between the five groups, but we are unable to see any patern in the ordering. The tablet is broken away to the right of the *Dodekatemoria* numbers, but the other side of the tablet contains, *inter alia*, the names of various cities, so it seems that the text was probably similar to the *Kalendertexte*.

The text Clay, *BRM IV*, no. 19 associates incantations with positions in the ecliptic.[18] For each position the related *Dodekatemoria* position is also given. By associating one position on the ecliptic with another, one immediately doubles the amount of material that can be interpreted astrologically.

A pictoral representation of the *Dodekatemoria* is found on the tablets VAT 7847 and AO 6448.[19] The zodiacal sign Leo is divided into 12 micro signs, starting at (0°) with Leo and ending at (30°) with Cancer. Similarly, under sign Virgo, depicted on the reverse side of the same tablet, the 12 micro signs Virgo up to Libra are drawn. It is interesting to note that Leo is drawn standing on the back of a long snake (both looking toward left), the tail of which continues on the reverse of the tablet. After the tail of the snake (= Hydra), Virgo is drawn, looking toward the left, in direction of Hydra and Leo. This representation on a Babylonian tablet is very peculiar. In the zodiac, Virgo comes correctly after Leo, but the direction of the consecutive signs as seen from the earth is from right to left, and not the opposite way around as on these pictures. When the stars are drawn on the outside of a sphere, representing the celestial sphere, then the figures of the stellar constellations are mirror pictures of the

[18] [Ungnad 'Besprechungskunst']; [Koch-Westenholz *Mesopotamian Astrology*, 169–70].
[19] [Weidner *GDBT*].

constellations as seen on the sky from below (this might explain the name 'mirror' of Eudoxos's first book on the stars and constellations). The Babylonian drawings mentioned here are similarly mirror pictures of what one can see. This might testify to another example of Greek reimport into Babylonian astronomy such as the one claimed by Rawlins.[20]

Ungnad suggested that the *Dodekatemoria* was a crude scheme for lunar motion.[21] If the sun moves forward one degree per day, then after 30 days, it will have moved on by precisely one sign. After 30 days, however, the *Dodekatemoria* has moved on by 390 degrees, or one complete revolution plus one sign. Thus the sun and moon will once again be in the same relationship. Neugebauer-Sachs criticised this explanation, arguing that 'the simplification of the solar and lunar motion is such that already one year would suffice to make predictions based on this scheme completely wrong. Furthermore, it would be hard to understand why a scheme of this type... would have been followed during the Seleucid period, when a detailed insight into the lunar motion (was known).'[22] To our minds, dismissing the lunar origin of this scheme on the grounds that it is too crude is wrong. Most likely it did start out as a simple lunar scheme, but one which was meant to be astrologically convenient rather than astronomically accurate. Perhaps at some stage in its use the scheme came to be applied to other heavenly bodies as well. This was how the *Dodekatemoria* was utilised after being assimilated into Hellenistic astrology.[23]

The second section of the late astrological text LBAT 1593 also refers to the *Dodekatemoria* and *Kalendertext* schemes.[24] In Obv. 18′ we read 'the animal(s) (*umāmu*) of 13 and 4,37 you take one with the other, you salve, feed, and fumigate the patient with the stone, herb, and wood (respectively)',[25] which must surely be

[20] [Rawlins 'Hipparchos'].

[21] [Ungnad 'Besprechungskunst'].

[22] [Neugebauer & Sachs 'Dodekatemoria'].

[23] [Bouché-Leclercq *L'Astrologie grecque*, 300–305.].

[24] [Reiner 'Early Zodialogia'].

[25] Translation of [Reiner 'Early Zodialogia', 424]. On p. 427 she 'put[s] forward the strange and surprising notion... that these *umāmu* "animals" are to be connected with the *zōa* that make up the Greek zodiac'. This notion is in fact not so strange. In W.22704 above we have seen that the ointment pertaining to a particular day is produced by means of ingredients from

a reference to the ingredients used in the preparation of medicine or other ointments used in ritual practice.[26] Indeed, it is tempting to interpret the beginning of the Obv. 19′, *bi-ib-lu šá* BAR TA 1 EN 30 'Almanac/handbook of Month I from (day) 1 to (day) 30', as referring to *Kalendertext* and *Dodekatemoria* texts such as the examples described above. The 'animals of 13' and 'animals of 4,37' are also mentioned in a similar context in Obv. 15′–17′, and in Obv. 15′ we find the numbers 1,1,1,13 (the tablet apparently reads 1,2,13, but this must surely be a simple scribal error) and 1,2,1,26. If we interpret these numbers as month-day-sign-degree, we have the first two entries from the *Dodekatemoria*.

The two texts published here differ from all of the *Kalendertexte* described above in that there is no text to accompany the mathematical scheme. Possibly the texts functioned as a template for the scribe when he was making a *Kalendertext*, or as an *aide-mémoire*; the shape of the tablets might even indicate that they were a scribal exercise, which would have interesting consequences for how we understand the role of astronomy and astrology in scribal education.[27] In any case, it seems practically certain that these two tablets were part of a series of twelve which covered each month in the schematic calendar.

Although we must be wary of dating the two tablets on the basis of other material in the same collection (especially since this collection was acquired by purchase), it is nevertheless interesting to note that a date of 5th century BC or before is significantly earlier than the other known *Kalendertexte*. Indeed, such a date would place the development of the *Kalendertext* scheme very soon after the first attested use of the zodiac.

3 *The mathematical structure of the* Kalendertext *scheme*

As mentioned above, the numerical schemes we are concerned with can be interpreted and used in different ways: as a date

the animal whose zodiac sign is connected to that day by the Kalendertext scheme.

[26] We remind the reader that 13 is the number of degrees from day to day in the *Dodekatemoria* scheme, and $4,37 = 277$ is the daily change in position in the *Kalendertext* scheme.

[27] For a scribal exercise containing parts of *Enūma Anu Enlil*, see [Mauer 'Ein Schülerexcerpt'].

(month, day) associated with a date (month, day), as a date (month, day) associated with a position in the zodiac (sign, degree), as a position associated with a position, or finally as a position associated with a date. We shall now investigate the *Kalendertext* scheme more closely, and for this purpose we will first consider a concrete example, namely the version where positions are associated to dates.[28]

month	day	zodiacal		sign	degree
I	1	10	=	♑	7
I	2	7	=	♎	14
I	3	4	=	♋	21
I	4	1	=		28
I	5	11	=	♒	5
I	6	8	=	♏	12
I	7	5	=	♌	19
I	8	2	=	♉	26
I	9	12	=	♓	3
I	10	9	=	♐	10
I	11	6	=	♍	17
I	12	3	=	♊	24
I	13	1	=		1
I	14	10	=	♑	8
I	15	7	=	♎	15
I	16	4	=	♋	22
I	17	1	=		29
I	18	11	=	♒	6
I	19	8	=	♏	13
I	20	5	=	♌	20
I	21	2	=	♉	27
I	22	12	=	♓	4
I	23	9	=	♐	11
I	24	6	=	♍	18
I	25	3	=	♊	25
I	26	1	=		2
I	27	10	=	♑	9
I	28	7	=	♎	16
I	29	4	=	♋	23
I	30	1	=		30
II	1	11	=	♒	7

Table 1: The *Kalendertext* scheme for month I.

[28] The well preserved tablets W.22704 and W.22619/6+22554/2b are examples of this type of *Kalendertext*.

For our example we have chosen month I. The basic structure is the same for all months and thus what follows also applies to the other months as well. Because of the ideal mean motion of the sun, there is an obvious connection between month M and sign M. This is shown by the substitution of the name of month M for sign M in some texts. We will therefore call sign M the 'special sign' of month M.

We remind the reader how the *Kalendertext* scheme is constructed: from one line to the next, the position in the zodiac is shifted by 277 degrees (which equals −83 degrees (modulo 360°)). The rule is then: go 9 zodiacal signs and 7 degrees forward or go 3 signs back and 7 degrees forward, applying the appropriate correction in the case where the degrees become larger than 30. Once the starting point is known the whole scheme is uniquely determined.

Day 1 of month I is associated with ♉ 7 degrees. We would however rather see month XII, day 30 ⇌ (associated with) the thirtieth degree of sign 12, ♓ 30 degrees, as the starting point of month I, and similarly for the other months. The last day of a month equals day zero of the next month, and degree 30 of a sign can be seen as degree 0 of the next sign:

So we would see month I, day 0 ⇌ 0 degrees
written as month XII, day 30 ⇌ ♓ 30 degrees

as the starting point of our scheme, which is reproduced in Table 1. Similarly, the last (30th) day of month I is associated with the thirtieth degree of sign 1, ( ) 30 degrees, which is then the starting point of the scheme for month II. The scheme for month II will therefore for each day associate the same number of degrees within a sign as for month I, and the sign number will simply be larger by one in each case (i.e., representing the following sign).

I	II	III	IV	V	VI
(1	2	3	4	5	6
10	**11**	**12**	**1**	**2**	**3**
7	**8**	**9**	**10**	**11**	12
4	**5**	**6**	**7**	**8**	9
1	**2**	**3**	**4**	**5**	6
11	**12**	**1**	**2**	**3**	4
8	**9**	**10**	**11**	**12**	1
5	**6**	**7**	**8**	**9**	10
2	**3**	**4**	**5**	**6**	7
12	**1**	**2**	**3**	**4**	5
9	**10**	**11**	**12**	**1**	2
6	**7**	**8**	**9**	**10**	11
3	**4**	**5**	**6**	**7**	8
1	**2**	**3**	**4**	**5**	6
10	**11**	**12**	**1**	**2**	3
7	**8**	**9**	**10**	**11**	12
4	**5**	6	**7**	**8**	9
1	**2**	3	**4**	**5**	6
11	**12**	1	**2**	**3**	4
8	**9**	10	**11**	**12**	1
5	**6**	7	**8**	**9**	10
2	**3**	4	**5**	**6**	7
12	**1**	2	**3**	**4**	5
9	**10**	11	**12**	**1**	2
6	**7**	8	**9**	**10**	11
3	**4**	5	**6**	**7**	8
1	2	3	**4**	**5**	6
10	11	12	**1**	**2**	3
7	8	9	**10**	**11**	12
4	5	6	**7**	**8**	9
1	2	3	**4**	**5**	6

VII	VIII	IX	X	XI	XI	degree	day
7	8	9	10	11	12	0	0)
4	**5**	**6**	**7**	8	9	**7**	1
1	**2**	**3**	4	5	6	**14**	2
10	**11**	**12**	1	2	3	**21**	3
7	**8**	**9**	10	11	12	**28**	4
5	**6**	**7**	8	9	10	**5**	5
2	**3**	**4**	5	6	7	**12**	6
11	**12**	**1**	2	3	4	**19**	7
8	**9**	**10**	11	12	1	**26**	8
6	**7**	**8**	9	10	11	**3**	9
3	**4**	**5**	6	7	8	**10**	10
12	**1**	**2**	3	4	5	**17**	11
9	**10**	**11**	12	1	2	**24**	12
7	**8**	**9**	10	11	12	**1**	13
4	**5**	**6**	7	8	9	**8**	14
1	**2**	**3**	4	5	6	**15**	15
10	**11**	**12**	1	2	3	**22**	16
7	**8**	**9**	10	11	12	**29**	17
5	**6**	**7**	8	9	10	**6**	18
2	**3**	**4**	5	6	7	**13**	19
11	**12**	**1**	2	3	4	**20**	20
8	**9**	**10**	11	12	1	**27**	21
6	**7**	**8**	9	10	11	**4**	22
3	**4**	**5**	6	7	8	**11**	23
12	**1**	**2**	3	4	5	**18**	24
9	**10**	**11**	12	1	2	**25**	25
7	**8**	**9**	10	11	12	**2**	26
4	**5**	**6**	7	8	9	**9**	27
1	**2**	**3**	4	5	6	**16**	28
10	**11**	**12**	1	2	3	**23**	29
7	8	**9**	10	11	12	**30**	30

Table 2: The complete *Kalendertext* scheme. For a each day in a year of 12 months with 30 days, the corresponding zodiacal positions are written: the first line gives the months, the last column the day number, the second last column gives the position in degrees within the signs, and columns 1 to 12 record the number of the zodiacal signs. The bold numbers indicate which parts of the scheme have been found in cuneiform texts.

One can ask which number X of degrees (other than $277° \equiv -83°(\text{mod. } 360)$) will result in a similar scheme. If we start at 0 degrees for day 0 of month I, and go X degrees forward each day, then after 30 days, the position will be shifted by $30 \times X$ degrees. We want the position on day 30 to be 30, i.e. 30° further and n complete circuits through the zodiac. This will be the case if $30X$ equals $n \times 360 + 30$. Therefore, for X we must claim that

$$30 \cdot X = 30 + 360n, \quad n \in N_0, \quad \text{hence} \quad X = 1 + 12n, \quad n \in N_0.$$

We see X must be of the form $X = 1 + 12n$ where n is a natural number (or 0). Van der Waerden has already remarked that only numbers of this form will result in the position shifting by one sign each month.[29] This means that all the (29) numbers $X = 1, 13, 25, 37, 49, \ldots, 277, \ldots, 337, 349$ would be possible candidates for constructing a *Kalendertext* type scheme. We will therefore go a little further and ask why $X = 277° \equiv -83°$, corresponding to $n = 23$, was chosen? What is special about the distribution of positions resulting from this choice of X?

A glance at Table 1 shows us that all possible degree numbers, 1 to 30, occur during the 30 days of our month, and also that all 12 zodiacal signs are represented, and are nicely distributed. Thus the *Kalendertext* scheme for the whole ideal year (see Table 2) gives a one-to-one correspondence between the 360 days of the ideal year and the 360 degrees (positions) of the zodiac.

[29] van der Waerden *apud* [Weidner *GDBT*, 50–52].

Kalendertext scheme							*Dodekatemoria* scheme			
month	day	zodiac		sign	°		sign	°	month	day
(I)	(0)	(1)	=	( )	(0)		( )	(0)	(I)	(0)
I	1	10	=	♑	7			13	I	1
I	2	7	=	♎	14			26	I	2
I	3	4	=	♋	21		♉	9	I	3
I	4	1	=		28		♉	22	I	4
I	5	11	=	♒	5		♊	5	I	5
I	6	8	=	♏	12		♊	18	I	6
I	7	5	=	♌	19		♋	1	I	7
I	8	2	=	♉	26		♋	14	I	8
I	9	12	=	♓	3		♋	27	I	9
I	10	9	=	♐	10		♌	10	I	10
I	11	6	=	♍	17		♌	23	I	11
I	12	3	=	♊	24		♍	6	I	12
I	13	1	=		1		♍	19	I	13
I	14	10	=	♑	8		♎	2	I	14
I	15	7	=	♎	15		♎	15	I	15
I	16	4	=	♋	22		♎	28	I	16
I	17	1	=		29		♏	11	I	17
I	18	11	=	♒	6		♏	24	I	18
I	19	8	=	♏	13		♐	7	I	19
I	20	5	=	♌	20		♐	20	I	20
I	21	2	=	♉	27		♑	3	I	21
I	22	12	=	♓	4		♑	16	I	22
I	23	9	=	♐	11		♑	29	I	23
I	24	6	=	♍	18		♒	12	I	24
I	25	3	=	♊	25		♒	25	I	25
I	26	1	=		2		♓	8	I	26
I	27	10	=	♑	9		♓	21	I	27
I	28	7	=	♎	16			4	I	28
I	29	4	=	♋	23			17	I	29
I	30	1	=		30			30	I	30
=		=					=		=	
(II)	(0)	(0)	=	(♉)	(0)		(♉)	(0)	(II)	(0)

Table 3: The *Kalendertext* and *Dodekatemoria* schemes for month I.

Let us imagine the schemes which would be produced by a different choice of X. Choosing 25, 85, 145, etc. would result in a distribution of positions with only 5, 10, 15, 20, 25, and 30 degrees within the signs. This is not a one-to-one correspondence between the 360 days and 360 degrees, so we understand why these numbers were not chosen. The choice $X = 1$ results in the (boring) distribution Month M day d $\rightleftharpoons$ Sign M degree d, which represents the ideal movement of the sun during the ideal year. However, the next possible choice, $X = 13$, does result in a distribution with a one-to-one correspondence (see table 3). This is the *Dodekatemoria* described above.

From Table 3 we see that on day 0 of a month M the position (sign M, 0 degrees) given by the *Kalendertext* scheme is the same as that given by the *Dodekatemoria* scheme, i.e., the ideal position of sun and moon at conjunction, and that the position corresponding to day 15 of month M in both schemes is the ideal position of the moon at opposition, namely 15 degrees in sign $M + 6$. This must be one of the reasons why the number $X = 277$ was chosen for the *Kalendertext* scheme. We can therefore ask the question: in what other ways is the scheme with $X = 277$ special? Does it in some way results in a 'better" distribution of positions than other possible numbers? Some of the numbers $X = 12n + 1$ can be excluded because they result in a scheme in which not all zodiacal signs are represented in a month, or in which the distribution of signs is rather boring.[30]

Another remarkable feature of the *Kalendertext* scheme concerns the zodiacal signs. A special structure becomes apparent when we consider how often and on which days the zodiacal signs occur. In each month M, its special sign M occurs 5 times: on the days 4, 13, 17, 26, and 30 (i.e., at intervals of 9, 4, 9, and 4 days).[31] Each other sign occurs at intervals of 13 days. The 3

[30] We remind the reader that the purpose of the *Kalendertexte* W.22704 and W.22619/6+22554/2b was to determine the ingredients of substances to be used on the 30 days of month IV and VIII, respectively. If a day corresponds to a position in Aries, then the medical substance for that day should contain blood, fat, and hair of a sheep. The greater the diversity of signs, the more substances could be used. There are also several examples of Greek texts in which connections between astral and meterological phenomena and medical care are apparent. See [Bowen and Goldstein 1988].

[31] Just as the astrological significance of a planet was apparently increased when it was in its 'secret house' (*bīt niṣirti*), one might speculate that the

signs which together with the special sign constitute a quadrangle occur 3 times, whilst the remaining 8 signs occur only twice. This structure is demonstrated in Table 4: the zodiacal signs are reproduced together with the days within month I to which the signs are associated. Aries, being the first sign of the zodiac, is the special sign of month I. This is shown more clearly in Table 5.

Month I

sign	day		sign	day		sign	day
	4, 13, 17, 26, 30		♉	8, 21		♊	12, 25
♋	3, 16, 29		♌	7, 20		♍	11, 24
♎	2, 15, 28		♏	6, 19		♐	10, 23
♑	1, 14, 27		♒	5, 18		♓	9, 22

Table 4: Zodiacal signs and days on which they occur in month I.

M	d	sign	°	M	d	sign	°	M	d	sign	°
(I)	(0)	(	0)	I	13		1	I	26		2
I	1	♑	7	I	14	♑	8	I	27	♑	9
I	2	♎	14	I	15	♎	15	I	28	♎	16
I	3	♋	21	I	16	♋	22	I	29	♋	23
I	4		28	I	17		29	I	30		30
I	5	♒	5	I	18	♒	6				
I	6	♏	12	I	19	♏	13				
I	7	♌	19	I	20	♌	20				
I	8	♉	26	I	21	♉	27				
I	9	♓	3	I	22	♓	4				
I	10	♐	10	I	23	♐	11				
I	11	♍	17	I	24	♍	18				
I	12	♊	24	I	25	♊	25				
I	30		30								
=		=									
(II)	(0)	(♉)	(0)	II	13	♉	1	II	26	♉	2
II	1	♒	7	II	14	♒	8	I I	27	♒	9
.	.	.	.	.	.	.	.				

Table 5: Here the structure of the *Kalendertext* scheme becomes a little more transparent. Each sign will occur at intervals of 13 days, the position within the sign being enlarged by 1 degree. Only the special sign of the month occurs more often, namely at intervals of 4 and 9 days (which adds up to 13 days).

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ingredients etc. relating to sign M would be especially effective when they were applied in month M. If this was the case, it is an advantage of the *Kalendertext* scheme that in each month M its special sign M is given 5 times.

We once more ask the question, is the value $X = 277$ especially chosen, or would other choices be as good? I.e., would other X values induce schemes with the special features of the *Kalendertext* scheme? In Table 6, for all the possible numbers X, we have analyzed the resulting scheme: are all signs and all degrees represented in each month, does the position on day 15 give the ideal moon position, and does its special sign occur frequently within each month? If a scheme fails one of these requirements it is rejected.

X	$\bar{X}$	all signs each month	all degrees each month	'full moon position' on day 15	special sign occurring n times
1	− 359	−	+	−	30
13	− 347	+	+	+	5
25	− 335		−		
37	− 323	+	+	+	3
49	− 311	+	+	−	2
61	− 299	−			
73	− 287	−	+	−	6
85	− 275		−		
97	− 263	+	+	−	4
109	− 251	+	+	−	2
121	− 239	−			
133	− 227	+	+	+	3
145	− 215		−		
157	− 203	+	+	+	3
169	− 191	+	+	−	1
181	− 179	−			
193	− 167	+	+	−	3
205	− 155		−		
217	− 143	−			
229	− 131	−	+	+	0
241	− 119	−			
253	− 107	+	+	+	3
265	− 95		−		
277	− 83	+	+	+	5
289	− 71	−	+	−	6
301	− 59	−			
313	− 47	+	+	−	2
325	− 35		−		
337	− 23	+	+	−	2
349	− 11	−	+	+	0

Table 6: The suitability of other schemes.

Except for the numbers 13 (*Dodekatemoria*) and 277 (*Kalendertext*) only the numbers 37, 133, 157, and 253 will produce schemes which have all of these fine features. But in all these imagined schemes the special sign occurs only 3 times within the month. The numbers $X = 73$ and 289 result in schemes in which the special sign occurs more often, namely 6 times, but here not all signs are represented within each month.

These results have demonstrated the very special and fine structure of the *Kalendertext* and *Dodekatemoria* schemes.

4 *An intimate connection between the* Kalendertext *and* Dodekatemoria *schemes*

We return to Table 2 and remember that some of the cuneiform schemes only have numbers and do not differentiate between dates and positions. That we have identified some numbers with signs or months and others with degrees or days is our choice and interpretation (although it is based on the two tablets W.22704 and W.22619/6+22554/2b.) We shall now demonstrate a very special feature of the two schemes.

If in the *Kalendertext* scheme (Table 2) we exchange all the roman month numbers for arabic numbers to be understood as signs, and exchange the signs for month numbers, and likewise swap the degrees and days, we end up with table 7.[32] What we have in this new table is simply a rearrangement of the *Dodekatemotria* scheme. We can enter the table with a position in the zodiac and we find a corresponding date. This date is the date in the ideal year on which the ideal moon would be at that position according to the *Dodekatemoria* scheme.

[32] This idea was suggested by the curious fact that the number sequences $1, 1, 1, 13$ and $1, 2, 1, 26$ (from the text LBAT 1593) could also be found in the *Kalendertext* scheme.

1	2	3	4	5	6
(I	II	III	IV	V	VI
X	XI	XII	I	II	III
VII	VIII	IX	X	XI	XII
IV	V	VI	VII	VIII	IX
I	II	III	IV	V	VI
XI	XII	I	II	III	IV
VIII	IX	X	XI	XII	I
V	VI	VII	VIII	IX	X
II	III	IV	V	VI	VII
XII	I	II	III	IV	V
IX	X	XI	XII	I	II
VI	VII	VIII	IX	X	XI
III	IV	V	VI	VII	VIII
I	II	III	IV	V	VI
X	XI	XII	I	II	III
VII	VIII	IX	X	XI	XII
IV	V	VI	VII	VIII	IX
I	II	III	IV	V	VI
XI	XII	I	II	III	IV
VIII	IX	X	XI	XII	I
V	VI	VII	VIII	IX	X
II	III	IV	V	VI	VII
XII	I	II	III	IV	V
IX	X	XI	XII	I	II
VI	VII	VIII	IX	X	XI
III	IV	V	VI	VII	VIII
I	II	III	IV	V	VI
X	XI	XII	I	II	III
VII	VIII	IX	X	XI	XII
IV	V	VI	VII	VIII	IX
I	II	III	IV	V	VI

7	8	9	10	11	12	day	degree
VII	VIII	IX	X	XI	XII	0	0)
IV	V	VI	VII	VIII	IX	7	1
I	II	III	IV	V	VI	14	2
X	XI	XII	I	II	III	21	3
VII	VIII	IX	X	XI	XII	28	4
V	VI	VII	VIII	IX	X	5	5
II	III	IV	V	VI	VII	12	6
XI	XII	I	II	III	IV	19	7
VIII	IX	X	XI	XII	I	26	8
VI	VII	VIII	IX	X	XI	3	9
III	IV	V	VI	VII	VIII	10	10
XII	I	II	III	IV	V	17	11
IX	X	XI	XII	I	II	24	12
VII	VIII	IX	X	XI	XII	1	13
IV	V	VI	VII	VIII	IX	8	14
I	II	III	IV	V	VI	15	15
X	XI	XII	I	II	III	22	16
VII	VIII	IX	X	XI	XII	29	17
V	VI	VII	VIII	IX	X	6	18
II	III	IV	V	VI	VII	13	19
XI	XII	I	II	III	IV	20	20
VIII	IX	X	XI	XII	I	27	21
VI	VII	VIII	IX	X	XI	4	22
III	IV	V	VI	VII	VIII	11	23
XII	I	II	III	IV	V	18	24
IX	X	XI	XII	I	II	25	25
VII	VIII	IX	X	XI	XII	2	26
IV	V	VI	VII	VIII	IX	9	27
I	II	III	IV	V	VI	16	28
X	XI	XII	I	II	III	23	29
VII	VIII	IX	X	XI	XII	30	30

Table 7: This table is obtained by swapping dates and positions in the *Kalendertext* scheme (Table 2). By entering the table with a position, we find the date on which the moon was at that position according to the *Dodekatemoria* scheme.

We can now construct Table 8 by rearranging Table 7 so that we enter the table with a date and we find the associated position according to the *Dodekatemoria* scheme.

We have therefore been able to derive the *Dodekatemoria* scheme by simply swapping the dates and positions in the *Kalen-*

dertext scheme.[33] Of course, we could also work the other way, and obtain the *Kalendertext* scheme from the *Dodekatemoria*. Indeed, we believe this is how the *Kalendertext* scheme was in fact constructed. This investigation has once again illustrated how careful one has to be in interpreting ancient texts: having found all the special and nice features of the *Kalendertext* scheme, we initially concluded that the number of 277 degrees was the result of a purposeful search for a very special one to one correspondence between (360) dates and (360) positions. Now it is evident that such a conclusion is totaly wrong. The very special distribution of positions in the *Kalendertext* scheme is a direct consequence of the way it was constructed.

The *Kalendertext* scheme has no inherent astronomical or other significance (such as the duration of pregnancy). Instead it comes from playing around with the numbers in another astrological scheme, the *Dodekatemoria*.[34] The *Dodekatemoria*, however, does have astronomical significance: it represents the mean motion of the moon during the ideal year of 360 days.

This shows an interest on the part of the Babylonian astrologers in increasing the astrological potential of a date or position in the zodiac by mathematical manipulation — a procedure which we term 'mathemagics'.

[33] The reason why 13 and 277 give inverse schemes comes from the fact that $13 \times 277 = 3601 = 1\text{SAR} + 1$ $(= 10 \times 360 + 1 \equiv 1 \text{ (modulo 360)})$.

[34] Learned numerical and philological arguments linking one idea with another are common in cuneiform. For example, numerical games are played with the cunieform signs of dates in the explanatory work i.NAM.giš.ḫur.an.ki.a. K2164 + 2195 + 3510, Obv. 16–17 reads 'The 22nd day: the 14th day. You multiply 14 and 10. $14 \times 10 = 140$. It becomes 22 reversed.' (translation of [Livingstone *Explanatory Works*, p. 23]). The author of this text uses the following argument to relate the 22nd and the 14th day: Separate 14 (which is written with one Winkelhaken followed by 4 stacked vertical wedges) into 10 and 4. Then multiply 10 by 14 to give 140, which is written sexagesimally 2,20. In cuneiform 2,20 is written with two vertical wedges followed by two Winkelhaken ($\| \ll$). Reversing their order to get 20,2, we have two Winkelhaken followed by 2 vertical wedges ($\ll \|$) which can be read together as 22. Thus, we have been able to transform the number 14 into the number 22 simply by rearranging the constituent signs, and performing some simple arithmetic. See also [Brown 2000, pp. 127–8].

I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	°	d
(1	2	3	4	5	6	7	8	9	10	11	12	0	0)
1	2	3	4	5	6	7	8	9	10	11	12	13	1
1	2	3	4	5	6	7	8	9	10	11	12	26	2
2	3	4	5	6	7	8	9	10	11	12	1	9	3
2	3	4	5	6	7	8	9	10	11	12	1	22	4
3	4	5	6	7	8	9	10	11	12	1	2	5	5
3	4	5	6	7	8	9	10	11	12	1	2	18	6
4	5	6	7	8	9	10	11	12	1	2	3	1	7
4	5	6	7	8	9	10	11	12	1	2	3	14	8
4	5	6	7	8	9	10	11	12	1	2	3	27	9
5	6	7	8	9	10	11	12	1	2	3	4	10	10
5	6	7	8	9	10	11	12	1	2	3	4	23	11
6	7	8	9	10	11	12	1	2	3	4	5	6	12
6	7	8	9	10	11	12	1	2	3	4	5	19	13
7	8	9	10	11	12	1	2	3	4	5	6	2	14
7	8	9	10	11	12	1	2	3	4	5	6	15	15
7	8	9	10	11	12	1	2	3	4	5	6	28	16
8	9	10	11	12	1	2	3	4	5	6	7	11	17
8	9	10	11	12	1	2	3	4	5	6	7	24	18
9	10	11	12	1	2	3	4	5	6	7	8	7	19
9	10	11	12	1	2	3	4	5	6	7	8	20	20
10	11	12	1	2	3	4	5	6	7	8	9	3	21
10	11	12	1	2	3	4	5	6	7	8	9	16	22
10	11	12	1	2	3	4	5	6	7	8	9	29	23
11	12	1	2	3	4	5	6	7	8	9	10	12	24
11	12	1	2	3	4	5	6	7	8	9	10	25	25
12	1	2	3	4	5	6	7	8	9	10	11	8	26
12	1	2	3	4	5	6	7	8	9	10	11	21	27
1	2	3	4	5	6	7	8	9	10	11	12	4	28
1	2	3	4	5	6	7	8	9	10	11	12	17	29
1	2	3	4	5	6	7	8	9	10	11	12	30	30

Table 8: The *Dodekatemoria* scheme, constructed by rearranging Table 7.

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Abbreviations

AO Tablets in the Antiquités orientales collection, Musée du Louvre
BM Tablets in the collection of the British Museum, London
LBAT [Sachs LBAT]
VAT Tablets in the Vorderasiatisches Abteilung Tontafeln collection, Berlin
W Tablets excavated at Warka by the Deutsche Orient-Gesselschaft

Bibliography

A. Bouché-Leclercq, *L'Astrologie grecque*, Paris, 1899.

A. C. Bowen and B. R. Goldstein, 'Meton of Athens and Astronomy in the Late Fifth Century B.C.', in E. Leichty et al. (eds.), *A Scientific Humanist: Studies in Memory of Abraham Sachs* (Occasional Papers of the Samuel Noah Kramer Fund 9), Philadelphia, 1988, pp. 39–81.

D. Brown, *Mesopotamian Planetary Astronomy-Astrology*, Groningen, 2000.

A. T. Clay, *Babylonian Records in the Library of J. Pierpont Morgan, Volume IV*, New Haven, 1923.

J. van Dijk, *Texte aus dem Rēš-Heiligtum in Uruk-Warka* (Baghdader Mitteilungen Beiheft 2), Berlin, 1980.

R. Englund, 'Administrative Timekeeping in Ancient Mesopotamia', *Journal of Economic and Social History of the Orient*, 31, 1988, pp. 121–85.

H. Hunger, 'Noch ein "Kalendertext"', *Zeitschrift für Assyriologie*, 64, 1975, pp. 40–43.

——. 'Ein astrologisches Zahlenschema', *Wiener Zeitschrift für die Kunde des Morgenlandes*, 86, 1996, pp. 191–7.

U. Koch-Westenholz, *Mesopotamian Astrology: An Introduction to Babylonian and Assyrian Celestial Divination*, Copenhagen, 1995.

A. Livingstone, *Mystical and Mythological Explanatory Works of Assyrian and Babylonian Scholars*, Oxford, 1986.

G. Mauer, 'W.20044,62, Ein Schülerexzerpt aus Enūma Anu Enlil', *Baghdader Mitterlungen*, 18, 1987, pp. 239–42.

O. Neugebauer and A. Sachs, 'The "Dodekatemoria" in Babylonian Astrology', *Archiv für Orientforschung*, 16, 1953, pp. 65–6.

D. Rawlins, 'Hipparchos' Ultimate Solar Orbit and the Babylonian Tropical Year', *Dio*, 1.1, pp. 49–66.

E. Reiner, *Astral Magic in Babylonia*, Philadelphia, 1995.

——. 'Early Zodiologia and Related Matters', in A. R. George and I. L. Finkel (eds.), *Wisdom, Gods and Literature: Studies in Assyriology in Honour of W. G. Lambert*, Winona Lake IA, 2000, pp. 421–8.

E. Reiner and D. Pingree, *Babylonian Planetary Omens. Part 2: Enūma Anu Enlil Tablets 50-51*, Malibu CA, 1981.

A. J. Sachs, *Late Babylonian Astronomical and Related Texts from Babylonia*, Providence RI, 1955.

J. M. Steele, 'A3405: An Unusual Astronomical Text from Uruk', *Archive for History of Exact Sciences*, 55, 2000, pp. 103–35.

A. Ungnad, 'Besprechungskunst und Astrologie in Babylonien', *Archiv für Orientforschung*, 14, 1941–1944, pp. 251–84.

E. F. Weidner, *Gestirn-Darstellungen auf babylonischen Tontafeln*, Vienna, 1967.

E. von Weiher, *Spätbabylonische Texte aus Uruk*, vol. 3, Berlin, 1988.



Plate 1: BM 96258 Obverse (courtesy of the Trustees of the British Museum)

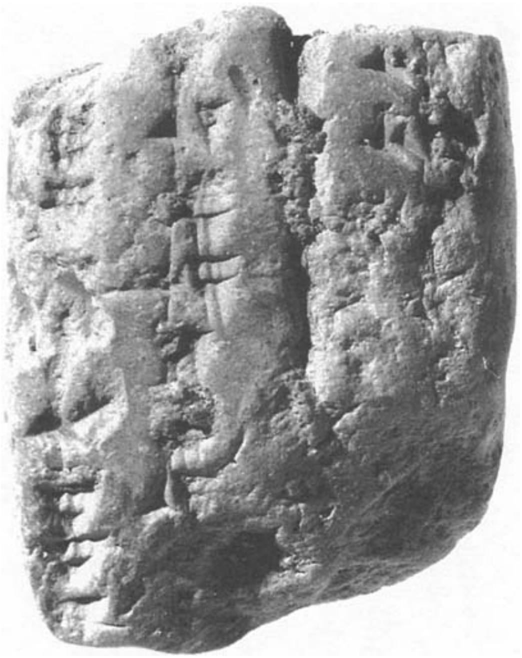


Plate 2: BM 96258 Reverse (courtesy of the Trustees of the British Museum)



Plate 3: BM 96293 Obverse (courtesy of the Trustees of the British Museum)



Plate 4: BM 96293 Reverse (courtesy of the Trustees of the British Museum)

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Classical and Medieval Europe

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An '*Almagest*' Before Ptolemy's?

ALEXANDER JONES

Among the remains of astronomy found in Greco-Roman period papyri from Egypt, truly theoretical writings are far outnumbered by tables and instructional texts.[1] On the basis of the subject matter so far as it can be determined in each fragment, I would identify the following papyri as clearly belonging under the heading of theory:

(1) *P. Aberd.* 12, a description of constellations, 2nd/3rd century A.D. (Parts of 10 lines of text.)

(2) *P. Iand.* V 84, spherical astronomy, 2nd century A.D. (Parts of 10 lines from the bottom of a column of text.)

(3) *P. Oslo* III 73, on measuring the apparent diameter of the Sun's disk, 1st/2nd century A.D. (27 lines from the top of a column of text.)

(4) *P. Oxy.* II 303, anomalistic motion of Moon with reference to a kinematic model, 1st century A.D. (Parts of 9 lines.)

(5) *P. Oxy.* LXI 4133 [Jones, *APO*, 1.69-80 and 2.2-5], analysis of dated observations of Jupiter, 2nd century A.D. (22 lines from the top of a column of text, and the left edge of the subsequent column.)

(6) *P. Oxy.* LXI 4138a [Jones, *APO*, 1.95-7 and 2.20-21], on eclipse intervals, 2nd century A.D. (Parts of 23 lines, the tops of two consecutive columns.)

(7) *P. Oxy.* LXI 4139 [Jones, *APO*, 1.97-9 and 2.22-3], on periods of lunar anomaly, 2nd century A.D. (Ends of eleven lines.)

[1] Most of the roughly two hundred published astronomical papyri are either included in Jones, *APO* or listed with bibliographical references on vol. 1, pp. 301–307 of that work.

(8) *P. Oxy.* LXI 4144 [Jones, *APO*, 1.108-109 and 2.40-41], discussion of kinematic model, 2nd/3rd century A.D. (Parts of 14 lines and part of a geometrical diagram.)

(9) *P. Par.* 1 [Blass, 1887], the 'Eudoxus Papyrus', 2nd century B.C. (Substantially complete manuscript, with 37 columns of text.)

(10) *PSI* XV 1490 [Manfredi, 1966], construction of tables for solar longitude with reference to a kinematic model, 1st/2nd century A.D. (Parts of 46 lines from a column of text, extending from the top margin to the bottom.)

Out of this list, items (1), (2), (4), and (8) are too poorly preserved to contribute significant historical information, while the Hellenistic papyrus (9) does not pertain to the kind of astronomy represented by the Roman period texts. Other fragments, not listed here, containing 'procedure text' material such as worked examples of computations may of course turn out to be from theoretical writings.

To reduce the census of this small body of texts by one will perhaps not appear to be a task meriting gratitude. In the present instance, however, nothing is lost and much is gained. We will see that two of the papyri listed above are parts of the same manuscript, a fact that one would scarcely have guessed from their contents, and that forces us to reconsider the character of the treatise to which they belonged.

Ptolemy's *Almagest* is the sole example from Greco-Roman antiquity of a book devoted to the exposition of advanced astronomical theory that the medieval manuscript has preserved for us. Its choice of subject matter and plan were emulated in a number of later Islamic and early modern European astronomical treatises. Whether there existed books comparable to the *Almagest* before Ptolemy is less easy to establish. Ptolemy himself makes reference to several books by Hipparchus concerning topics dealt with in the *Almagest*. From Ptolemy's reports it appears that several of Hipparchus' works shared characteristics with the *Almagest*, in particular the attempt to apply rigorous argument, mathematical deduction, and specific observational evidence (including dated observations) to the establishment of quantitative kinematic models for the heavenly bodies. On the other hand,

Hipparchus' work along these lines was limited, so far as Ptolemy knew, to models for the Sun and Moon, and even the various elements of these models were determined, sometimes more than once and with varying results, in a series of separate publications. The large-scale deductive structure of the *Almagest*, in which the solar model provides an indispensible foundation for the lunar model, which in turn must be worked out before the theory of precession and the star catalogue, upon which finally the planetary models depend, can have had no counterpart in Hipparchus' works.

As for the contributions of the astronomers who lived during the three centuries between Hipparchus and himself, Ptolemy makes only a brief and disparaging comment (*Almagest* 9.2) about unnamed authors who attempted to exhibit the behaviour of kinematic planetary models by means of 'Eternal Tables'. Their performances, he writes, were faulty and 'lacked proofs', which seems to mean that their determination of the models was not founded upon a logically cogent analysis of phenomena and observations.[2] That observations were made and recorded is, however, shown by the presence of three records of observations from the first century A.D. (by Menelaus and Agrippa) in *Almagest* 7.3.[3] But Ptolemy gives us no clue how these observations were applied in their original context.

Up to the present no fragments of ancient manuscripts of the *Almagest* have come to light. If one did, it would probably be no more extensive than the scraps and pieces listed above, and the text written on it would represent only one of the several 'textures' of the *Almagest*. An instructive exercise is to open Heiberg's edition of the *Almagest* at random, and imagine what we might conjecture about the nature of the whole work if all that we had was a half page, as it might be of historical review, mathematical argument, analysis of observations, or de-

[2] The 'Eternal Tables' are mentioned also by Vettius Valens (6.2, ed. Pingree, p. 232) and in the horoscope *P. Lond*, I 130 (= Neugebauer & Van Hoesen, *GH*, no. 81, lines 1–26) cast by Titus Pitenius for a person born in A.D. 81. Both indicate that these tables yielded precise numerical positions in degrees and minutes.

[3] Ptolemy also cites a few observations by Theon 'the mathematician' in *Almagest* 9.9, 10.1, and 10.2; but this Theon seems to have communicated the observations to Ptolemy directly.

scription of the layout of a table. As it happens, these four textures of prose are recognizable respectively in papyri (7), (2), (5), and (11) in the list above. That does not necessarily mean that the works to which these fragments belonged were all proto-*Almagests*, of course. What it does show is that the kinds of thing that Ptolemy does in the *Almagest* were not exclusive to Hipparchus and Ptolemy, but typical of the astronomical literature of Ptolemy's time.

Continuing our experiment of randomly dipping in Heiberg's edition, we might try what could be guessed from the top halves of *two* pages, with their page numbers intact. If the pages are close, we likely will learn little more about the broad scope of the work than from either single fragment. A bit further apart, and we will observe more than one variety of argument concerning the same heavenly body—say, the discussion of early astronomers' period relations for the Moon, and the instructions for use of the Moon's anomaly table. Still further apart, and we discover that the treatise dealt with more than one of the heavenly bodies.

In the Spring of 2000, through the kindness of Guido Bastianini (Istituto Papirologico G. Vitelli, Florence) and Rosario Pintaudi (Biblioteca Medicea-Laurenziana), I obtained photographs of several astronomical and astrological papyri destined for publication in the long-delayed fifteenth volume of the *Papiri della Societé Italiana (PSI)* series. I was astonished to recognize in *PSI* XV 1490, item (10) in our list, the same distinctive hand that wrote both *P. Oxy.* LXI 4133, item (5) in our list, and *P. Oxy.* LXI 4134. Further examination showed that the Florence papyrus, like *P. Oxy.* LXI 4133, has a column number above the text in the upper margin, and that the margins of both papyri contain jottings—apparently nothing to do with astronomy—in the same hand, which is different from the hand that wrote the text. It is practically certain that *PSI* XV 1490 and *P. Oxy.* LXI 4133 belonged to one and the same papyrus roll, and almost as certain that their contents are parts of a single treatise. (It is conceivable, but I think improbable, that a single roll contained the end of one text and the beginning of an unrelated one; if this was the case, then the inferences in the remainder of this article are false.)

The texts of *P. Oxy.* LXI 4133 and *PSI* XV 1490 have been

published (in the latter case in a professedly provisional, but accurate, transcription), and I have discussed both papyri in other articles in ignorance of their common provenance (Jones, 1999 and 2000). A summary will therefore suffice here.

P. Oxy. LXI 4133 begins with the tail end—actually just the last half on the final word—of a report of an observation of Jupiter near opposition on December 30/31, 241 B.C. The author reduces to an ecliptic frame of reference Jupiter's reported position relative to one or more nearby stars. Then he states the time interval—344 Egyptian years plus 87 days—from this observation to one of Jupiter near opposition that the author made himself on December 31, A.D. 104/January 1, A.D. 105. Again the reported position relative to nearby stars is reduced to the ecliptic. From this point the text becomes very broken, about twenty-four lines are lost, and then we have bits of the beginnings of a series of lines that evidently included citation and discussion of further observations.

Appealing to the date close to the beginning of the second century A.D., the manner of the observation, and the use of the Roman calendar, I conjectured in my original publication of *P. Oxy.* LXI 4133 that the author was the mathematician and astronomer Menelaus of Alexandria, who was active in Rome about A.D. 100. The new insights on the text offered below do not, I believe, significantly strengthen or weaken the case for this tentative attribution. However, it is enough to know that this is a treatise written less than half a century before the *Almagest.*

There are two plausible explanations of what the author is doing with the observations in *P. Oxy.* LXI 4133. Either he is trying to establish a period relation for Jupiter's anomalies, or he is investigating the long-term behaviour of some aspect of Jupiter's motion for which observations at opposition are useful. The specific technique of comparing pairs of widely separated observations of a heavenly body at the same phase and as nearly as possible the same longitude plays no part in Ptolemy's planetary theory, although analogous methods turn up in both Hipparchus' and Ptolemy's treatment of solar and lunar theory.

PSI XV 1490 preserves a longer stretch of text, but in a more broken condition so that almost every line has a gap that cannot be securely restored. The first lines prescribe how to lay out a ta-

ble in which the rows represent days in a calendrical scheme that groups days in thirties and also involves a four-year cycle—pretty clearly the Alexandrian (reformed Egyptian) calendar. We are given numbers representing mean motions in longitude, 'depth' (anomaly), and latitude to record in the second line (meaning the first line below the headings). Apparently rules are then set out for filling the remainder of the table, presumably through the repeated addition of constant increments for daily progress. The heavenly body to which these mean motions belongs is not named in the preserved parts of these lines. The text's editor, Manfredi, suggested the Moon; I believe that it is the Sun, according to a model in which the Sun has a latitudinal deviation from the ecliptic and a shifting apsidal line.

Now the text turns to the topic of solar anomaly, asserting that according to either an eccentric or an epicyclic model the Sun 'increases and decreases its motion by' (i.e. has a maximum equation of) 2;24=B0. The author refers us here to a discussion in a previous section on the Sun's anomaly. The rest of the preserved text appears to describe how to construct an anomaly table for the Sun, it is not clear on what mathematical basis.

And now we may raise the question: what kind of book would contain both the kind of discussion found in *P. Oxy.* LXI 4133 and the kind found in *PSI* XV 1490? It dealt with more than one heavenly body: Jupiter as well as the Sun. And once we have gone beyond one, the most likely remaining hypothesis is that all the planets as well as the Sun and Moon were the subjects of the complete work. It was about the motions of the heavenly bodies, explained by means of kinematic models of the epicyclic and eccentric varieties. Dated observation reports fully comparable to those cited in Ptolemy's *Almagest* were adduced, and the components of the models were assigned numerical values. Tables based on the principle of analysis of motion into mean motions and corrections for anomaly were derived from the quantitative models.

So far, the resemblances to the *Almagest* are obvious. But there are important contrasts too: the reliance on planetary observations separated by a periodic restitution; the very unptolemaic solar model with its three independent mean motions; the cumbersome-sounding calendrically structured mean motion ta-

ble. There are also differences in plan and scale. *P. Oxy.* LXI 4133 contains the fourteenth (and traces of the fifteenth) column of the papyrus roll; the damaged numeral in the upper margin of *PSI* XV 1490 seems to be 51, and is definitely a number in the fifties. Unless the fragments come from different rolls (which is improbable because of the marginal scrawls), the construction of the solar tables came *after* the analysis of observations of Jupiter. Perhaps this means that all the theoretical work was carried out in the first part of the treatise, and the tables reserved for the end.

One column of text would have contained a little more than an average page of Heiberg's *Almagest* edition: about two hundred forty words compared to about two hundred. Hence *P. Oxy.* LXI 4133 was the equivalent of about eleven or twelve Heiberg pages from the beginning of the roll, and *PSI* XV 1490 was about thirty Heiberg pages further along. Obviously there was not space for treatment of most of the other heavenly bodies in the thirteen lost columns of the beginning of the roll, so we must assume that this was a treatise in more than one 'book'. Even so, the scale of treatment has to have been much smaller than in the *Almagest*, each book of which averages nearly a hundred pages. The comparative concision is in fact apparent in the extant fragments, especially in *PSI* XV 1490, where the author turns from mean motion to anomaly table with a briskness unimaginable in Ptolemy.

What makes the *Almagest* so long is primarily the space it devotes to mathematical analyses. Our hypothetical treatise must have had much less of these; and we may recall Ptolemy's complaint that the presentations of his more immediate predecessors 'lacked proofs'. I have argued in [Jones, 1999] that Ptolemy probably saw this treatise, and plundered it for observation reports. From the point of view of methodology he would probably have professed to find little to learn from it.

Bibliography and Bibliographical Abbreviations

Blass, 1887: F. Blass, 'Eudoxi ars astronomica qualis in charta aegyptiaca superest denuo edita', Kieler Universitätsprogramm, Sommersemester, 1887. [Reprinted in *Zeitschrift für Papyrologie und Epigraphik*, 115, 1997, pp. 79–101.]

Jones, *APO*: A. Jones, *Astronomical Papyri from Oxyrhynchus*, 2 vols in 1, Memoirs of the American Philosophical Society, 233, Philadelphia, 1999.

——. 1999: A. Jones, 'A Likely Source of an Observation Report in Ptolemy's *Almagest*', *Archive for History of Exact Sciences*, 54, 1999, pp. 255–8.

——. 2000: A. Jones, 'Studies in the Astronomy of the Roman Period IV: Solar Tables Based on a Non-Hipparchian Model', *Centaurus*, 42, 2000, pp. 77–88.

Manfredi, 1966: M. Manfredi, 'Presentazione di un testo astronomico e discussione di un documento di Antinoe', *Atti dell' XI Congresso Internazionale di Papirologia, Milano 208 Settembre 1965*, Milan, 1966, pp. 237–43.

Neugebauer & Van Hoesen, *GH*: O. Neugebauer and H. B. van Hoesen, *Greek Horoscopes*, Memoirs of the American Philosophical Society, 48, Philadelphia, 1959.

P. Aberd.: *Catalogue of Greek and Latin Papyri and Ostraca in the Possession of the University of Aberdeen*, ed. E. G. Turner, Aberdeen University Studies, 116, Aberdeen, 1939.

P. Lond.: *Greek Papyri in the British Museum*, eds F. G. Kenyon *et. al.*, 7 vols, London, 1893–1974.

P. Oslo: *Papyri Osloenses*, eds S. Eitrem and L. Amundsen, 3 vols, Oslo, 1925–36.

P. Oxy.: *The Oxyrhynchus Papyri*, eds B. P. Grenfell, A. S. Hunt, *et al.*, 66 vols, Egypt Exploration Society, Graeco-Roman Memoirs, London, 1898–1999.

PSI: *Papiri greci e latini*, various editors, 14 vols, Pubblicazioni della Societ=E0 Italiana per la ricerca dei papiri greci e latini in Egitto, Florence, 1912–57.

Ptolemy's *Harmonics* and the 'Tones of the Universe' in the *Canobic Inscription*

N.M. Swerdlow

One of the oldest ideas of a relation of art and science in nature is that the heavens are formed according to the principles of music, now usually called the 'music of the spheres'. The earliest form, attributed to the Pythagoreans, was known by the late fifth century BC and perhaps earlier. Aristotle explains in *De caelo* 2.9 that they believe the speeds of the stars are in ratios of musical concords, and that the stars, which are large and move rapidly, produce such concords, but we hear no sound since it is always going on and sound is only perceived in contrast to silence. Aristotle dismisses the whole idea as elegant and ingenious nonsense, and refutes the explanation of why nothing is heard, but he hardly succeeded in killing it off, for any number of later sources contain lists of musical tones or intervals for the planets, presumed to be related to their speeds, their distances or to nothing in particular. These lists appear mostly in authors not known for the highest scientific acumen so they need not be taken all that seriously, but one of them appears in Ptolemy's *Canobic Inscription*, and since Ptolemy is known for scientific acumen this one ought to be taken seriously. And Ptolemy's interest in the relation of music to the heavens went beyond a list of tones, for half of Book 3 of his *Harmonics* is devoted to the most extensive surviving account from Antiquity, the most important before Kepler's *Harmonice mundi* (1619), in which he considered the relation of music to motions and configurations in the heavens of significance to both astronomy and astrology, as indeed Kepler did fifteen hundred years later. In fact, the *Harmonics* as we have it, missing most of the last three chapters of Book 3, does not even contain the tones assigned to the planets in the *Canobic Inscription*, but there is internal evidence that it once did and this shows that the tones are authentic and not some later, foolish addition to the manuscript

version of the inscription.

This paper was originally undertaken following the discovery, reported in Hamilton *et al.* (1987), that the *Canobic Inscription* is an early work of Ptolemy's, prior to the *Almagest*. Its intention was to investigate whether the *Harmonics*, perhaps also an early work, could make sense of the tones, could show what astronomical or astrological significance they may have. The answer turned out to be, not a lot, but in the course of returning to the paper from time to time, it became a fairly thorough exposition of the astronomical chapters of the *Harmonics*, which can hardly be accused of clarity, and of the music theory, mostly also from the *Harmonics*, necessary to understand those chapters. In this way, a former musicologist turned historian of astronomy could combine two subjects that had occupied his curiosity for years. At an early stage in this work, Andrew Barker's annotated translation of the *Harmonics* appeared in *Greek Musical Writings II: Harmonic and Acoustic Theory* (1989), which was not only of invaluable aid to this paper but is a landmark in the study of Greek music theory. I am greatly indebted to Barker's translation as well as to Ingemar Düring's edition (1930) and annotated translation (1934) of the *Harmonics* and, although I only saw it after the paper was mostly written, to Barker's recent *Scientific Method in Ptolemy's* Harmonics (2000). Beyond these I have made no attempt to consult secondary literature on the *Harmonics*, with the exception of Kepler's annotated translation of 3.3-16, which is very ingenious, even if much of it goes beyond what Ptolemy had in mind. Some of the colossal literature on Greek music and music theory was helpful, and several works are noted in the references at the end of the paper.

The purpose of this paper is modest. To set out an explanation, or a clarification, of the astronomical chapters of the *Harmonics*, which can best be done through drawing figures, worth many words, with a selective exposition of the music theory required to understand them, and to investigate the relation between the list of tones in the *Canobic Inscription* and the *Harmonics*. This relation in turn provides evidence that the *Harmonics* is also an early work of Ptolemy's, perhaps prior to the *Canobic Inscription*, which is interesting because the *Harmonics* contains Ptolemy's most detailed statement of what he considers

correct method in the applied mathematical sciences, the method he followed in his other works, and it is of interest if this statement is early. In no way is the exposition here exhaustive; the reader is referred to the sources and studies just mentioned and to other accounts of Greek music theory, and I leave more profound reflections on the harmonics of the heavens to heads wiser than my own. I assume that the reader knows something about music and elementary music theory, and I will not explain what an octave or a fifth is.

Elements of Greek Harmonic Theory

Harmonics (*harmonika*) is an applied mathematical science concerned with the analysis of the relations of the pitches, tunings, and scale forms of music. It certainly originated as a study of the practice of music, vocal but more so instrumental, plucked strings, such as the lyre and kithara, and winds, such as the aulos and syrinx, and however abstract harmonics became, in both its language and goals it remained grounded in musical practice. The treatises of the principal writers, Aristoxenus of Tarentum of the fourth century BC, Ptolemy of the second century AD, Porphyry of the third century, who wrote a commentary on the *Harmonics* as far as 2.7, and Aristides Quintilianus, probably of the third or fourth century, are among the most remarkable of Greek scientific works, although only Ptolemy's is truly mathematical. Boethius's *De institutione musica* of the sixth century is the closest to the *Harmonics* and is in part based upon it. While not as highly developed as pure mathematics or astronomy, each in its own way shows attention to the precise description of complex phenomena that do not on the surface seem amenable to such rational analysis. For the fundamental question addressed in these treatises is nothing less than why music of various kinds has its particular character and affect, called *ethos*. The answer agreed upon by the theorists is that the cause lies in the specific tuning (*harmonia*) of the notes (*phthongoi*) forming melodies, and harmonics is above all an analysis of those tunings. This answer is neither so naive nor so idealized as it may appear; for who would deny that the tight chromatic of Isolde's 'Den hab ich wohl vernommen' and the broad diatonic of Sachs's 'Verachtet mir die Meister nicht' provide at least the foundation of the character

and affect of those great scenes? And as Greek music was entirely monodic, and utilized for its melodies more numerous and subtle intervals than the twelve semitones into which our octave is divided, and more diverse orderings of notes than our own major and minor tonalities, it may not have seemed at all naive to derive the *ethos* of melody from the tunings of its successive notes. One thing of which we may be sure is that the writers of these treatises had something in their heads that we can never know, Greek music, for their analyses and reflections were based upon what they could hear and feel as surely as we can hear and feel *Tristan* and *Meistersinger*. And their extensions of harmonics to the soul and the heavens, whether through the simple principle of sympathetic magic, of like influencing like, or the refined philosophy of substantial forms, has left a legacy in the arts and even in the sciences that endured for some two thousand years and is perhaps still with us today. I say this by way of preface to a brief account of Greek harmonics, for the most part following Ptolemy, in which I refer to chapters of the *Harmonics*.

Ptolemy's exposition of harmonics is, like his astronomy, rigorously mathematical and rigorously empirical. In this way, he set the example of the method that has been followed in the applied mathematical sciences when practiced properly ever since, and the *Harmonics* even contains his most extensive reflections on such method (1.1-2 and elsewhere on specific points). His principal experimental instrument for demonstrating the ratios of pitches is the single string *kanon* (1.8), a monochord with a movable bridge and a *kanonion*, a graduated ruler, for measuring lengths, and he also uses *kanones* of eight and fifteen strings of equal length tuned in unison with movable bridges to divide respectively one and two octaves (1.11, 3.1-2). He is aware that pitch depends not only upon the length of strings, but also upon thickness and density, which he considers equivalent to tension, and explains how to eliminate the effects of thickness and density by tuning in unison so that pitch depends only upon length. The relations of pitches are thought of, not as chords, but as a series of discrete notes, although the simultaneous sounding of two notes is certainly a part of determining their relation. Two words of broad meaning assumed in what follows are *phthongos*: sound, musical sound or tone, musical note; *tonos*: pitch, interval of one

tone, species of octave. The fundamental relations demonstrated with the *kanon* are the unison, with lengths in the ratio of 1/1, the octave (*dia pason*) 2/1, the fifth (*dia pente*) 3/2, the fourth (*dia tessaron*) 4/3, and of those greater than an octave, the octave plus fourth (*dia pason* and *dia tessaron*) 8/3, the octave plus fifth (*dia pason* and *dia pente*) 3/1, and the double octave (*dis dia pason*) 4/1. The octave and double octave are called homophonic (*homophonos*) or homophones because they sound together as one note, and the fourth, fifth, and their octave additions are called concordant (*symphonos*) or concords, which are closest to homophones, presumably also when sounded together. Notes in other permissible relations, e.g. semitone, tone, ditone (*hemitonion*, *tonos*, *ditonon*), are called melodic (*emmeles*), as they are used in melody (but presumably cannot be sounded together), and impermissible relations which are not so used are unmelodic (*ekmeles*). It appears that the *kanones* could be used to distinguish and test even small intervals of less than a semitone.

The homophones and concords are invariable—what we would call pure tunings of 2/1, 3/2, 4/3, and their octave additions—but the melodic notes are quite variable in different tunings, and how to determine their ratios is not so straightforward. The 'Aristoxenians', followers of Aristoxenus, proceeded empirically, and either did not express the relations of notes as ratios or did so inconsistently, which Ptolemy shows led to errors such as taking the fourth to be exactly two and a half tones and the octave exactly six tones (1.9-10). The 'Pythagoreans', as Ptolemy calls them, such as Archytas, proceeded mathematically, deriving the notes by computation (1.5-6, 13). The only surviving work of this kind, the *Sectio canonis* attributed to Euclid, is limited to 'Pythagorean' ratios with prime factors 2 and 3 and seems primitive compared to the examples of Archytas's ratios given by Ptolemy (1.13). In general, the accepted ratios are either epimoric (*epimorios*, superparticular), of the form $(n + 1)/n$, or multiple (*pollaplasios*, multiplex), $(m \cdot n)/n$. Ptolemy criticizes the Pythagoreans (1.5-6) for excluding the concordant ratio 8/3 of the octave plus fourth on the ground that it is not epimoric, and for inconsistently accepting or rejecting different epimoric ratios as concordant by subtracting one from both terms of each ratio, summing the remaining terms, and then taking those with

the least sums as the most concordant. For example, if we take 1 from each term of 2/1, 3/2, 4/3, and sum the remainders, we shall have $1 + 0 = 1$, $2 + 1 = 3$, $3 + 2 = 5$, showing that the order of concord is octave, fifth, fourth. But suppose, Ptolemy says, that we express these ratios as 12/6, 9/6, 8/6; then we must subtract 6 from each, giving $6 + 0 = 6$, $3 + 0 = 3$, $2 + 0 = 2$, and the order of concord is reversed to fourth, fifth, octave. But here we must defend the Pythagoreans, for taking 6 from each of these terms is not the same as taking 1 from the ratios in lowest terms. Correctly, if 6 is the unit, the ratios are 12/6, 18/12, 24/18, and if we subtract 6 and add the remaining terms we shall have $6 + 0 = 6$, $12 + 6 = 18$, $18 + 12 = 30$, and the order of concord is, as before, octave, fifth, fourth. So the Pythagoreans may have known what they were doing.

Ptolemy's own determination of the ratios of melodic notes is in his treatment of the genus (*genos*) of tetrachords, four notes containing three intervals (1.15). In every tetrachord (*tetrachordon*) the outer two notes are fixed (*hestos*) at the interval of a fourth in the ratio 4/3 while the inner two are movable (*kinoumenos*) to form the three genera, diatonic, chromatic, enharmonic, and their subspecies or color (*chroma*). In all three genera the lowest interval is the smallest, while in the chromatic and enharmonic the two lower intervals together are close (*pyknon*), less than the highest interval, and in the diatonic the two lower intervals together are not close (*apyknon*), not less than the highest interval. Taking the notes in descending order, the intervals of the diatonic genus are tone/tone/semitone, as in modern notation a-g, g-f, f-e, but the equivalence is only approximate, the tones are not equal, and of course a semitone is not really half of a tone. Examples given in modern letter notation are only for illustration and imply no specific tuning of each note. In the chromatic and enharmonic genus, there is even less relation to modern notes and intervals. We may approximate the chromatic genus in the descending order sesquitone (minor third)/larger semitone/smaller semitone, as a-f#, f#-f, f-e, and the enharmonic genus ditone (major third)/larger microtone/smaller microtone, as a-f, f-e^+, e^+-e. I use microtone, an interval less than some, but not all, semitones, as preferable to the terms quarter tone and diesis.

The diatonic genus is said to be high or tense (*syntonon*)

and the enharmonic low or soft (*malakon*) with the chromatic in between. The same classification is used for the colors. Since the two lower intervals of the enharmonic are too small for audible alteration, it is already the softest and has no further forms, but the chromatic receives two colors and the wide intervals of the diatonic yet a third, called 'tonic' (*tonaion*). Ptolemy's method of finding the proper ratios is first to divide the fourth into two epimoric ratios to find the highest interval locating the higher movable note, and then to divide the lower part into two further epimoric ratios to find the two lower intervals locating the lower movable note. For example, in the tense chromatic, the fourth $4/3$ is first divided as $4/3 = 7/6 + 8/7$ to form the sesquitone of $7/6$. The remaining tone $8/7$ is then divided as $8/7 = 12/11 + 22/21$ to form the larger semitone $12/11$ and the smaller semitone $22/21$. Note that addition of ratios, called compounding or composition of ratios, is done by multiplication, e.g. $4/3 = 7/6 \cdot 8/7$, and subtraction by division, which is multiplication by the reciprocal, e.g. $7/6 = 4/3 \cdot 7/8$. The resulting notes are then tested on the eight-string kanon, and Ptolemy says that the octave, by which I assume he means all eight tones of two disjunct tetrachords, will be found so true that not even the most musical would alter it even a little. The six tetrachords in the three genera and colors with their ratios are as follows:

Diatonic	tense	tonic	soft	Chromatic	tense	soft	Enharmonic	
tone	10/9	9/8	8/7	sesquitone	7/6	6/5	ditone	5/4
tone	9/8	8/7	10/9	semitone	12/11	15/14	microtone	24/23
semitone	16/15	28/27	21/20	semitone	22/21	28/27	microtone	46/45

That the semitone 28/27 is smaller than what we are calling the microtone 24/23 is an idiosyncrasy of our terminology and because what are called semitones vary over a large range.

Ptolemy remarks that our ears are accustomed to the three colors of diatonic genera and the tense chromatic, but not to the soft chromatic and enharmonic, for they do not take delight in genera of such loose, that is, licentious, ethos. Perhaps we should imagine something like the Paris version of the Venusberg or the Zaubermädchen. He mentions two further colors of diatonic (1.16), an even (*homalon*), apparently of his own invention, with nearly equal ratios that he says may first appear strange and

rustic, but very pleasant when you get used to it, and a ditonic (*ditonaion*) with two equal tones that is in use as an instrumental tuning to accompany singing in the tense diatonic, to which it is very close:

Diatonic	even	ditonic
tone	10/9	9/8
tone	11/10	9/8
semitone	12/11	256/243

The latter is the familiar 'Pythagorean' or 'Platonic' tuning (*Timaeus* 36B), based purely upon computation from the prime factors 2 and 3, i.e. $4/3 \cdot 8/9 \cdot 8/9 = 256/243 = 2^8/3^5$. The tense diatonic, with prime factors 2, 3, 5, became a standard tuning, at least theoretically, in the sixteenth century, and was later called 'just intonation'. Ptolemy later (2.14) mentions that a tetrachord in the order 9/8, 10/9, 16/15 was used by Didymus 'the musician', a writer on music of the first century, and in the sixteenth century tunings with prime factors 2, 3, 5 were referred to as both 'tense diatonic' and 'according to Didymus'. Ptolemy's tunings are not based upon specific prime factors, which in the ratios given run as high as 23, but upon divisions of the tetrachord that are at once mathematical and empirical, mathematical or according to reason in that the ratios are all epimoric and the least numbers and closest series of ratios that will properly form the intervals and fill out the tetrachord, empirical in that they are tested with the eight-string kanon and found to agree with the phenomena, with perception.

The tetrachords are then arranged in two systems (*systemata*), which Ptolemy calls a concord of concords (2.4-6), the disjunct, perfect, or immutable system (*diezeugmenon, teleion, ametabolon systema*) of two octaves, and the conjunct or mutable system (*synemmenon, metabolikon systema*) of an octave and a fourth. The former is usually called the Greater Perfect System, and the latter the Lesser Perfect System, although Ptolemy says that only the disjunct system of two octaves, containing all the concords and the seven species of the octave, to be explained below, is properly complete or perfect (*teleion*). Both systems are shown in Table 1 with translations of terms, which are names of

Disjunct System			Conjunct System		
Eq	Note	Tetrachord	Eq	Note	Tetrachord
a'	NETE (lowest)				
g'	Paranete (next to lowest)	Hyperbolaion			
f'	Trite (third)	(Additional)			
e'	NETE				
d'	Paranete	Diezeugmenon	d'	NETE	
c'	Trite	(Disjunct)	c'	Paranete	Synemmenon
b	PARAMESE (next to middle)		b^b	Trite	(Conjunct)
a	MESE (middle)	→	a	MESE	
g	Lichanos (forefinger)	Meson			
f	Parhypate (next to highest)	(Middle)			
e	HYPATE (highest)				
d	Lichanos	Hypaton			
c	Parhypate	(Highest)			
B	HYPATE				
A	PROSLAMBANOMENOS (added note)				

Table 1: Disjunct and Conjunct Systems

notes (*phthongoi*), and the conventional modern equivalents (Eq) *in the diatonic genus* in which the lowest note (A) is a minor tenth below middle C (c′) and the highest note (a′) a major sixth above middle C. It must be understood that there are no absolute pitches in Greek music—although some musicians must have possessed perfect pitch and there must have been some convention for untunable wind instruments—so only the intervals between the notes are significant, and, as was true of our examples of genera, modern equivalents of notes and intervals are only approximate. As before, the two outer notes of each tetrachord, shown in upper case, are fixed at the interval of a fourth, while the two inner notes are movable and would have different modern equivalents in the chromatic or enharmonic genus. The terms lowest (*nete*) and highest (*hypate*) refer, not the pitches in the order low (*barytonon*) to high (*oxytonon*), in which the Greek convention is the same as our own, but to the position of the strings (*chordai*) of a lyre or kithara as it is held when played, just as two of the notes are called 'forefinger' (*lichanos*). Both systems are identical in the octave from proslambanomenos (A) to mese (a), following which the disjunct system has a tetrachord disjunct to mese, paramese (b) to nete diezeugmenon (e′), and an additional conjunct tetrachord, nete diezeugmenon (e′) to nete hyperbolaion (a′), forming a complete octave (a to a′), while the conjunct system has a single tetrachord conjunct to mese, mese (a) to nete

synemmenon (d′), forming a fourth (a to d′). The function of the conjunct tetrachord, Ptolemy tells us, is some kind of unexpected modulation, reaching to the fourth from mese (a) at nete synemmenon (d′) rather than the fifth at nete diezeugmenon (e′), that is appropriate when the transition is fitting and melodic but inappropriate when not. Although he does not mention it, there is probably also an effect from the semitone within the tetrachord from mese falling one step lower, in modern notation a, b-flat, c′, d′, instead of a, b, c′, d′, in the way that lowering the third and sixth distinguishes major and minor.

Mixolydian	b a g f e d c B	t t t s t t s
Lydian	c′ b a g f e d c	s t t t s t t
Phrygian	d′ c′ b a g f e d	t s t t t s t
Dorian	e′ d′ c′ b a g f e	t t s t t t s
Hypolydian	f′ e′ d′ c′ b a g f	s t t s t t t
Hypophrygian	g′ f′ e′ d′ c′ b a g	t s t t s t t
Hypodorian	a′ g′ f′ e′ d′ c′ b a	t t s t t s t

Table 2: Formation of the Seven Tonoi in the Diatonic Genus

The disjunct system is the basis of Ptolemy's analysis of the seven species of octave, called *tonoi*, as Dorian, Phrygian and so on, their names adapted from tunings (*harmoniai*) of the lyre or kithara thought to express their ethnic origin and character or, says Ptolemy (2.10), for whatever reason you wish! The tonoi may be formed simply as the order of intervals in seven species of descending octave beginning at the seven notes of the disjunct system from paramese to nete hyperbolaion, in the diatonic the descending order of tones and semitones beginning at b, c′, d′, e′, f′, g′, a′, but quite different in the chromatic and enharmonic, since the two internal notes of each tetrachord are movable. In the diatonic, this would give the notes and intervals shown in Table 2 in which *t* is a tone and *s* a semitone.

This derivation, however, gives the misleading impression that the Mixolydian is the lowest in pitch and the Hypodorian highest, when in fact it is the other way around and all tonoi may also be applied to the same octave. Ptolemy's approach, which is to alter the intervals between the notes of an octave of the immutable system, is more complex, but superior (2.11). He first distinguishes between notes according to position (*thesis*), thetic or positional notes, which are the notes of the immutable system

as on an instrument or kanon, what the notes would be called, and notes according to function (*dynamis*), dynamic or functional notes (2.5). These are also notes of the immutable system, but are movable along side the positional notes, like a template, such that a functional mese is assigned to a positional note and then the intervals between the functional notes above and below mese within the template determine the intervals between the positional notes. Each shift of the functional notes, of the template, next to an octave of positional notes imposes, in the diatonic genus, one of the series of tones and semitones in Table 2. In the chromatic or enharmonic genus, the intervals of the functional notes would be different, but the template could be shifted along the positional notes in the same way. Here, as an example is the central octave, nete diezeugmenon (e′) to hypate hypaton (e), of the diatonic Lydian, in which trite diezeugmenon receives functional mese, so the functional octave, the template, contains two notes above mese and six notes below mese:

Positional Notes		Eq	Functional Notes
Nete diezeugmenon		e′	Trite diezeugmenon
Paranete "		d#′	Paramese
Trite "	→	c#′	Mese
Paramese		b	Lichanos meson
Mese		a	Parhypate "
Lichanos meson		g#	Hypate "
Parhypate "		f#	Lichanos hypaton
Hypate "		e	Parhypate "

Note that the descending intervals, s t t t s t t, are those of the Lydian in Table 2 and are the intervals between trite diezeugmenon and parhypate hypaton in Table 1, taken as diatonic, between c′ b a g f e d c. Were the functional notes shifted one line higher and the mese assigned to paranete diezeugmenon, the functional octave would be paramese to hypate hypaton in Table 1, b to B, and the Mixolydian tonos would be formed; were the functional notes shifted one line lower and the mese assigned to paramese, the functional octave would be paranete diezeugmenon to lichanos hypaton in Table 1, d′ to d, and the tonos would be Phrygian. Were the genus other than diatonic, the intervals of the tetrachords would be different and the equivalent modern notes in Table 1 would not apply, as even here they do so only roughly.

The notes of the entire system are altered by this operation, but Ptolemy concentrates on two characteristic octaves between fixed notes, the central octave from nete diezeugmenon to hypate meson, e′ to e, and a lower octave from mese to proslambanomenos, a to A, or an equivalent higher octave from nete hyperbolaion to mese, a′ to a (2.15). For these he sets out fourteen tables of five columns, seven for each of the two octaves—six are duplicated so only eight are given—containing for each octave the notes of the seven tonoi in the five genera and colors to which the ears are accustomed. One column is tonic diatonic in both tetrachords, but in four columns the genera and colors are mixed with tonic diatonic such that, allowing for continuation of the octave, the tetrachord below functional mese is softer and the disjunct tetrachord above functional paramese tenser, which appears to reflect the actual musical practice of those subtle lyrodists and kitharodists. The order of the columns is: (1) tense chromatic and tonic diatonic; (2) soft and tonic diatonic; (3) tonic diatonic in both tetrachords; (4) tonic and ditonic diatonic; (5) tonic and tense diatonic. The numbers are computed in a single sexagesimal series, rounded to one fractional place, for all the tables so that for each of the octaves, central or lower or equivalent higher, the same number corresponds to exactly the same pitch, relative to some standard for, say, 60 or 120, regardless of genus, tonos, and the specific note for each number. Ptolemy does not explain how or even that this is done. His procedure is to raise the number for, it appears, functional paramese in the central octave, e′ to e, and functional hypate meson in the lower octave, a to A, through a tone or semitone in successive tables. Raising the number lowers the pitch of this fundamental note. If we take the highest notes of the seven tonoi in Table 2: b, c′, d′, e′, f′, g′, a′, the series is s t t s t t, and it is by these intervals, with $t = 9/8$ and $s = 256/243$ of the ditonic diatonic, that the numbers increase in the tables of the tonoi. For example, to raise functional paramese from 60 in Mixolydian to its value in Lydian, he computes $(256/243) \cdot 60 = 63;12,35 \approx 63;13$. That he uses and why he uses these ratios of the ditonic diatonic for this operation is not explained. The numbers for the fundamental note, paramese in the first seven tables and, in principle, hypate hypaton in the second seven tables, are as follows:

Tonos	Int	Ratio	Number
Mixolydian			60
	s	256/243	
Lydian			63;13
	t	9/8	
Phrygian			71; 7
	t	9/8	
Dorian			80
	s	256/243	
Hypolydian			84;17
	t	9/8	
Hypophrygian			94;49
	t	9/8	
Hypodorian			106;40

Having established the number for a single fundamental note in each table in this way, the remaining notes of the octave in each column are computed both above and below by the ratios for the intervals of the tetrachords. As a result of the adjustment of the fundamental note and the computation of the other seven notes by their ratios within the tetrachords, the range of the columns is not, as one might expect, 60 to 120, but somewhat wider, although the limits of each are always in the ratio 2/1. In the seven tables for the central octave, the lowest range of the columns is 59;16 to 118;31 and the highest 62;13 to 124;27; the single table for the lower octave has the range 56;11 to 112;22 in all columns and the other tables would duplicate tables for the central octave. To give an example, in Table 3 that follows, the lower limit, functional trite diezeugmenon, is computed from functional paramese by $(27/28) \cdot 63;12,35 = 60;57,8 \approx 60;57$. The remaining numbers are computed upward from paramese in the same way, e.g. for mese, $(9/8) \cdot 63;13 = 71;7,7 \approx 71;7$. Unfortunately, in these tables and elsewhere, the translations have replaced Ptolemy's sexagesimal fractions with a bewildering variety of common fractions, even different common fractions for the same sexagesimal fraction, which is not correct and makes the structure and computation of the tables far less clear. The sexagesimals may be found in the Greek text, and the common fractions are best ignored. Ptolemy follows these tables with another table listing

every number that is taken by each note of every octave. These tables of the tonoi are surely the tour de force of the *Harmonics*, for in principle they contain a complete set of tunings for all forms of the tonoi in common use, that is, he has reduced common musical practice to a precise mathematical description, the principal purpose of the *Harmonics*.

Positional Notes	Eq	Functional Notes	Int	Ratio	Number	Genus
NETE diezeugmenon	e'	Trite diezeugmenon			60;57	Tonic Diatonic
			s	28/27		
Paratete "	d#'	PARAMESE			63;13	
			t	9/8		
Trite "	c#'	MESE			71; 7	
			t+s	7/6		
PARAMESE	b^b	Lichanos meson			82;58	Tense Chromatic
			s	12/11		
MESE	a	Parhypate "			90;30	
			s	22/21		
Lichanos meson	g#	HYPATE "			94;49	
			t	9/8		
Parhypate "	f#	Lichanos hypaton			106;40	Tonic Diatonic
			t	8/7		
HYPATE "	e	Parhypate "			121;54	

Table 3: Lydian Tonos in Tense Chromatic and Tonic Diatonic Genera

Their computation, in the way just explained, is laborious but mechanical, and there is a great deal of duplication. We give as an example in Table 3 the first column of the Lydian tonos from nete diezeugmenon to hypate meson in the tense chromatic and tonic diatonic. Of the seven columns in our table, the Greek text gives only the sexagesimal numbers, and we have added the other columns to show the principle of the computation by using the ratios of the intervals between the notes of the tetrachords of the functional octave on either side of the fundamental note, here paramese, to alter the intervals of the notes of the positional octave. Each column of the tables in the text is based upon the principle shown in Table 3.

We shall see that every subject we have treated, genera of tetrachords, disjunct and conjunct systems, and tonoi, appear in Ptolemy's astronomical applications of music theory, to which we now turn.

Astronomy and Astrology in Harmonics *3.3-13*

Ptolemy begins with a new introduction in 3.3-4. Having shown that the power or function (*dynamis*) of *harmonia*, tuning and music in various tunings, can be reduced to proper ratios, that music itself is a rational science, a science of ratios, it remains to show the same power of *harmonia*, of tuning, in the world, where it is the cause which gives the proper form to underlying matter. This power lies in that kind of reason, concerned with movement, that causes and discovers what is ordered, good, and beautiful. It utilizes the highest senses, sight and hearing, which judge their objects not only as agreeable or disagreeable, as do the other senses, but according to their beauty, and these senses assist each other, as when speech is enhanced by diagrams or sight by poetic description. The application of this reason to sight and to the movements in position of things only seen, namely, the heavenly bodies, is astronomy, and to hearing and the movements in position of things only heard, namely, sounds, is harmonics. Both astronomy and harmonics make use of arithmetic and geometry as irrefutable instruments to determine the quantity and quality of the movements; both are, as it were, cousins, daughters of the sisters sight and hearing, nourished as of the closest kinship by arithmetic and geometry. The power of *harmonia* is present in all things that have their own source of movement, especially in those of a more perfect and rational nature, not however in their matter because of its inconstancy, but in their forms. These more perfect and rational natures are found, among the divine, in the movements of the heavenly bodies, and among the mortal, in the movements of the human soul, both of which are rational and have movement in position. There is far more to this introduction, which we have summarized briefly and not adequately to its complexity and subtlety.

3.5-7 are concerned with *harmonia* in the human soul, with ethics and virtues, and with the effects of music on the soul. The homophones and concords, with the intervals formed by the notes within each, correspond to the parts of the soul, rational, sensitive, animate, each with several further divisions and particular virtues. The genera are associated with the divisions of theoretical knowledge—physics, mathematics, theology—and practical knowledge—ethics, economics, politics—and the modulation of

tonoi to the changing condition of the soul in the course of the activities of life and the effect on the soul of the character (*ethos*) of different tonoi. Pythagoras advised calming music on awakening and even the gods are invoked with music. All of this is interesting and carefully considered, but lies beyond the limits of our study. I must confess to finding the analogies rather strained, although that probably reflects my own lack of competence in this subject. What I do note is that these chapters correspond to Boethius's *musica humana*, and are to my knowledge the most extensive exposition of it, as the remaining chapters of Book 3 correspond to *musica mundana* (*De Institutione musica*, 1.2). Boethius used parts of Ptolemy's *Harmonics*, but not for these distinctions, which were frequently referred to in medieval music theory, even if the writers did not have the faintest idea of what Boethius was talking about.

The purpose of applying the theory of music to astronomy, beginning in 3.8, is to describe in accordance with harmonic ratios the 'hypotheses' of the heavens, a technical term meaning the principles or models underlying the apparent motions of the heavenly bodies, and also the effects of the motions of the heavens. Compared to the *Almagest*, the astronomy is highly simplified; only the most fundamental apparent motions are considered and these independently of specific hypotheses to account for them. Throughout, Ptolemy's concern is as much astrological, the effects of the motions of the heavens, as astronomical, even when only astronomical subjects are considered. Both the effect of sound and the influence of the heavens depend upon the interval of motion alone without changes of being, meaning that all effects are due only to motion, not to turning into something else. The recurring revolutions of the aether are circular and orderly, just as the recurrences of order and pitch of the notes of the systems which, although apparently in a straight line, may begin at any point; that is, the tonoi may begin at any note, and if passing one end of the system, continue at the other end, and the same is true of motions in the heavens. Thus, concerning the 'likeness' or 'resemblance' (*homoiotes*) between the perfect system and the zodiac, as shown in Figure 1, if one cuts the circle through the middle of the zodiacal signs at one equinox and opens it up to fit equal lengths of the perfect system of two octaves, the uncut

equinox will correspond to mese (a) and the two ends of the cut equinox to proslambanomenos (A) and nete hyperbolaion (a′).

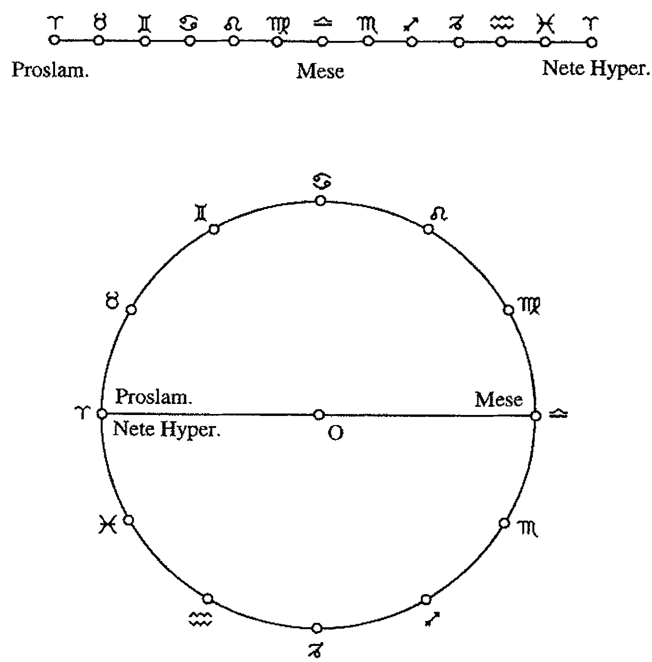


Figure 1

We are apparently to imagine the system superimposed upon the twelve zodiacal signs although which tones correspond to which signs is not specified aside from the equinoxes at the beginning, middle, and end. If one now rejoins the two ends of the double octave into a circle, and again forms the zodiac into a circle, joining proslambanomenos and nete hyperbolaion of the cut equinox, they will lie diametrically opposite mese at the homophone of an octave, for the diameter of the circle divides it as 2/1. Hence the aspects of stars opposite each other in the zodiac are, like the notes of the octave, the most effective, and this applies to any opposite points in the zodiac. In this way the effect of opposition is explained through harmonics.

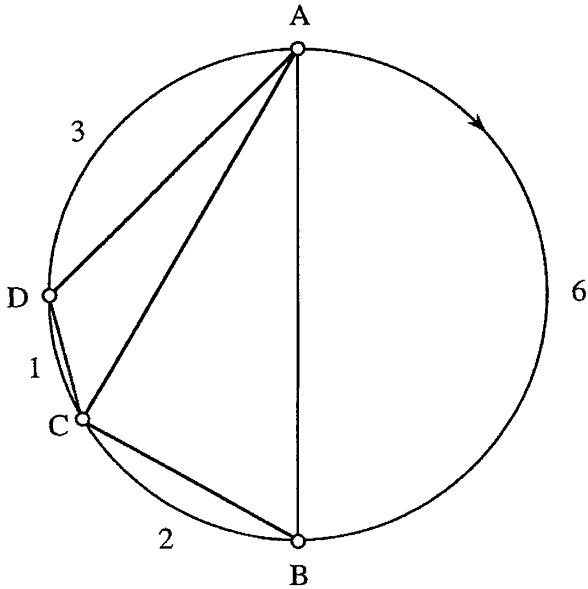


Figure 2

To show that concords (*symphona*) and discords (*diaphona*) in tuning are like those in divisions of the zodiac, a circle is further divided into four unequal parts (3.9) to produce all the concordant and effective aspects. If we imagine the circle of the zodiac divided into 12 parts, in Figure 2 the diameter AB will subtend half the circle or 6 parts, the side of an equilateral triangle AC will subtend one-third or 4 parts, the side of a square AD one-fourth or 3 parts, the side of a hexagon BC one-sixth or 2 parts, and CD, the difference between the triangle and square, one-twelfth or 1 part. These arcs and their sums divide the zodiac into homophones and concords in various ways, summarized in the following table in which we have modified Ptolemy's lettering so that all arcs are measured in the same direction around the circle, shown by the arrow, which is less confusing, and we have added a final A to those arcs that are the entire circle:

Octave 2/1	ABCDA/AB = 12/6	ABC/CA = 8/4	BA/DA = 6/3
Fifth 3/2	ABCDA/ABC = 12/8	ABD/AB = 9/6	BA/CA = 6/4
Fourth 4/3	ABCDA/ABD = 12/9	ABC/AB = 8/6	CA/DA = 4/3
Octave + fifth 3/1	ABCDA/CA = 12/4	ABD/DA = 9/3	
Double octave 4/1	ABCDA/DA = 12/3		
Octave + fourth 8/3	ABC/DA = 8/3		
Tone 9/8	ABD/ABC = 9/8		

These are intended to explain the efficacy of the aspects, opposition, trine, quartile, sextile, although Ptolemy does not say how each aspect is derived from these ratios, for there are various ways, and the number of divisions seems to go beyond the usual aspects. Conjunction is not an aspect since bodies in conjunction do not 'look upon' each other. He does say that it is reasonable that nature divided the zodiac into twelve parts since the perfect system of two octaves contains very nearly twelve tones, and an interval of a tone was fitted to one-twelfth of the circle, that is, $CD = ABD - ABC = 9 - 8 = 1$ and $ABD/ABC = 9/8$, a tone. This interval is not a concord, but melodic, as is 12/11, a semitone in Ptolemy's even diatonic and tense chromatic, and ratios of 12/5 or 12/7, made by a line cutting five-twelfths of the circle, would be unmelodic, which is why none of these aspects is effective.

In *Tetrabiblos* 1.13, something like these divisions is used to explain the aspects, although more briefly. If we take the two moric ratios (*morioi*), unit fractions $1/n$, 1/2 and 1/3, and epimoric ratios (*epimorioi*), $(n+1)/n$, 3/2 and 4/3, of the most importance for concords, then 1/2 of opposition makes quartile, and 1/3 of opposition sextile (text: 'and trine', probably an error), while of the epimorics, 2/3 of quartile makes sextile and 4/3 of quartile makes trine. What this appears to mean is that in Figure 2:

$\frac{1}{2}\text{arc}AB = \text{arc}AD$ quartile

$\frac{1}{3}\text{arc}AB = \text{arc}BC$ sextile $(\text{arc}AB - \text{arc}BC = \text{arc}AC$ trine)

$\frac{2}{3}\text{arc}AD = \text{arc}BC$ sextile

$\frac{4}{3}\text{arc}AD = \text{arc}AC$ trine

Trine and sextile, Ptolemy says, are called concordant (*symphonoi*) because they are of the same sex, masculine or feminine (masculine and feminine signs alternate beginning with Aries as masculine), while quartile and opposition are discordant (*asymphonoi*) because they are of opposite sex (although opposition is in fact of the same sex).

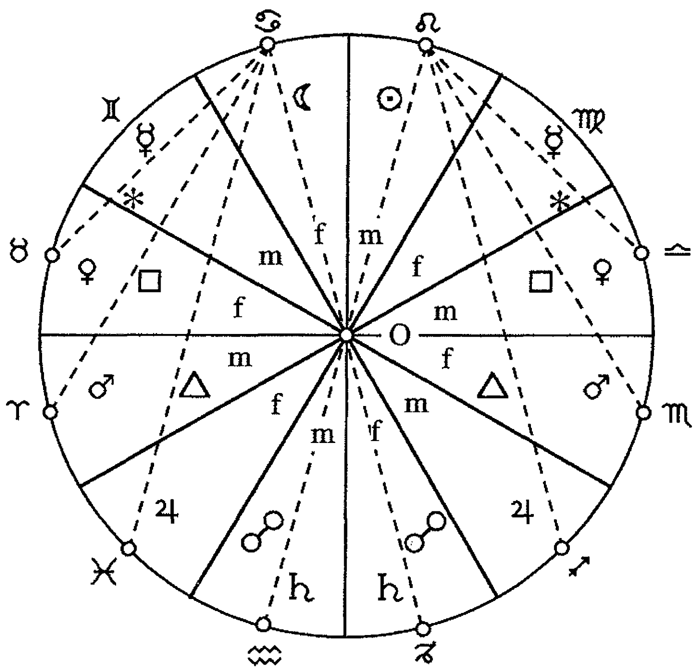


Figure 3

In *Tetrabiblos* 1.17, on the domiciles or houses (*oikoi*) of the planets, their relation to particular zodiacal signs to which they have an affinity (*synoikeiosis*), the Sun is assigned to Leo, which is masculine, and the Moon to Cancer, which is feminine, both of which are warm signs. The semicircle from Leo to Capricorn is of the Sun and from Cancer to Aquarius of the Moon, and the planets receive one sign in each. Thus, Saturn, which is highest and farthest from the Sun and Moon, is assigned to Aquarius and Capricorn, at the unfavorable aspect of opposition to each one as befits its cold nature, and Jupiter, which is below Saturn, to Pisces and Sagittarius, at the concordant and beneficent trine aspect to each as befits its temperate nature. Then to Mars, which is dry, are assigned Aries and Scorpio, at quartile to the Sun and Moon, and thus discordant and destructive, while Taurus and Libra, given to Venus, which is temperate, are in the fortunate and concordant sextile aspect to the Sun and Moon. Mercury, which is never farther than one sign from the Sun, is assigned to Gemini and Virgo, which are not at an aspect to the Sun

and Moon in the same semicircle—although they are at sextile in opposite semicircles—and nothing is said of its nature. Notice the terminology of concord and discord; although no ratios are specified, they are presumably those of the aspects described in 1.13. The domiciles of the planets and their aspects, opposition, trine, quartile, sextile, to the Sun and Moon are shown in Figure 3, in which the earth is at *O* and masculine and feminine signs are marked *m* and *f*. In this way we can see an application of *Harmonics* 3.9, which provides the foundation of *Tetrabiblos* 1.13 and, less directly, part of 1.17. The affinities of the planets among themselves are considered in *Harmonics* 3.16.

The next three chapters (3.10-12), form a group concerned with the relation of harmonics to the diurnal motion—rising, culmination, setting—and two further motions that may be interpreted as additions to the diurnal motion. Ptolemy treats these as comparisons or correspondences; he does not explicitly say that the harmonics are essential to the motions and never suggests that real sounds are produced.

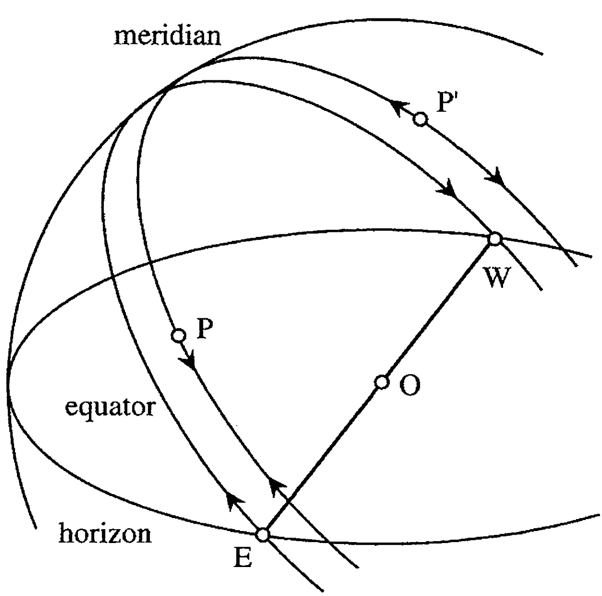


Figure 4

The initial idea, shown in Figure 4, is that a body, the Sun, Moon or planet, rises, as *P*, and sets, as *P'*, in a circle parallel to the

celestial equator, which intersects the horizon at its rising point E and setting point W. Below the horizon the body is silent. It rises with a low tone which becomes high as it approaches its culmination, where it crosses the meridian, which Ptolemy calls the midheaven, strictly the intersection of the meridian and the ecliptic, and then again becomes low as it moves toward setting. No specific notes of the systems are given. Ptolemy calls this motion in 'length' (*mekos*), a term that can refer to longitude in the ecliptic or to any motion in the general direction of the equator or ecliptic, but here it can mean only motion of rising and setting, parallel to the equator, and should not be called motion in longitude. Nothing is said of the body's proper motion from west to east in the ecliptic, which we call motion in longitude, although one may think of this as a subtraction from the diurnal motion parallel to the equator, shown by the arrow opposite the diurnal motion in the figure, very small for all bodies except the Moon. Nevertheless, all that may be intended here is the diurnal motion of rising, culmination, and setting without regard to proper motion, which is the way Kepler interprets it.

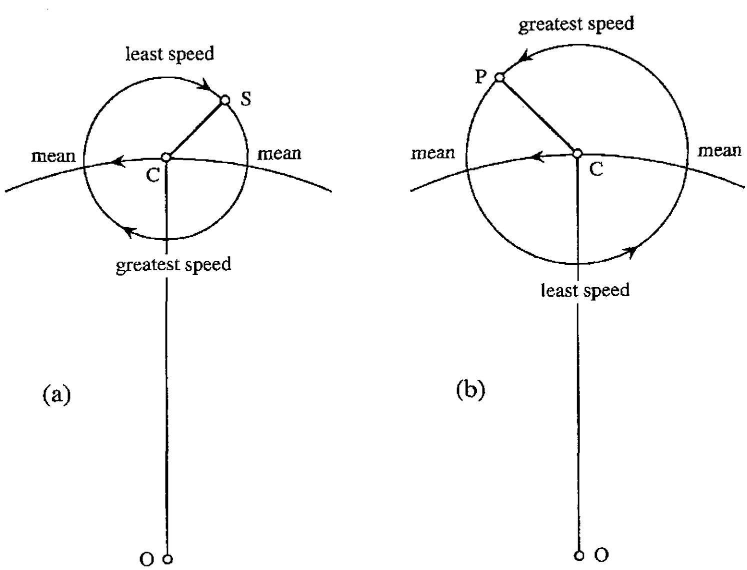


Figure 5

The second motion (3.11), in 'depth' (*bathos*), change of distance

and corresponding change of speed, is compared to the three genera, diatonic, chromatic, and enharmonic. Ptolemy does not specify any model to which the relation of distance and speed applies, but we shall illustrate it using an epicyclic model shown in Figure 5—an eccentric model is also possible—in which the relation is determined by the direction of motion on the epicycle. Now the Sun and Moon move slowest at greatest distance and fastest at least distance, so in 5a the motion of the Sun S when at apogee as seen from the earth O is opposite to the motion of the centre of the epicycle C and at perigee is in the same direction; hence the slowest motion is at apogee and fastest at perigee. For the planets it is the other way round, fastest at greatest distance, when in conjunction with the Sun, and slowest, in fact retrograde, at least distance, in opposition for superior planets and inferior conjunction for inferior. Thus in 5b the motion of the planet P when at apogee as seen from O is in the same direction as C and when at perigee in the opposite direction, so P is fastest at apogee and slowest at perigee. This is the second inequality of the planets. Ptolemy does not here consider the first inequality, which, like the Sun and Moon, has the least speed at greatest distance. The relation of speed and distance of the two inequalities is described in *Almagest* 9.5. In both cases, the speed is mean at about a quadrant from apogee and perigee. The Sun completes a revolution on its epicycle in one year, the Moon in an anomalistic month, and the planets in their synodic periods between conjunctions with the Sun, different for each planet. The change of distance and speed is compared to the genera of tetrachords. Intermediate speeds, at intermediate distances, are comparable to the chromatic genus in which the movable notes make intermediate divisions of the tetrachord. Least speeds, whether at greatest or least distance, are comparable to the enharmonic genus in which the two smaller intervals are close (*pyknon*), and greatest speeds, whether at greatest or least distance, are comparable to the diatonic genus in which the intervals are not close (*apyknon*). Perhaps these effects are to be applied to the range of tones of the diurnal motion of rising and setting described in 3.10 to determine their genus, which would imply that the proper motion of each planet is also to be considered there, for otherwise there would be no speed to vary. But that is only a guess, for the mo-

tions and their relations to harmonics may also be independent. After all, they are only analogies.

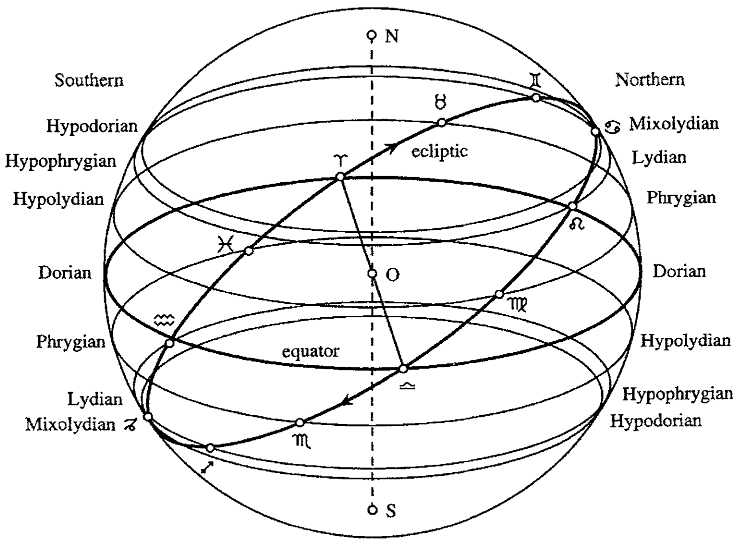


Figure 6

Now the planets do not move in the equator or parallel to the equator but nearly in the ecliptic which is inclined to the equator. The third motion (3.12), is in 'breadth' (*platos*), which can mean latitude from the ecliptic, but here means declination from the equator toward the two tropics, so even the Sun moving in the ecliptic has motion in breadth, while the specific latitudinal motions of the Moon and planets are not considered. The period for this motion is each body's zodiacal period, the longest for Saturn, the shortest for the Moon, with the Sun, Venus, and Mercury one year. Imagine a series of circles parallel to the equator drawn through the beginnings of the zodiacal signs, seven circles in all, as shown in Figure 6. Each circle corresponds to a tonos, the equator to the Dorian because it is in the middle, but the others differ depending upon whether the observer is north or south of the equator, the higher tonoi being in the direction of the visible pole above the horizon, the lower in the direction of the invisible pole below the horizon. The whole arrangement should be clear from the figure in which the earth is O, the north pole N and the south pole S. As the body moves through the zodiac, it crosses

the parallels of the different tonoi. Since the diurnal rising and setting arc is likewise parallel to the equator (see Figure 4), as the body reaches each parallel, its series of tones, from rising to culmination to setting, will be in a different tonos, and in the course of its motion around the zodiac it will modulate through all seven tonoi twice since it goes through each set of four twice above and below the equator. Again, we are assuming here that the motion in breadth is to be combined with the motion of rising and setting to give its tonoi, but they could be independent and nothing more than analogies.

Let us now summarize the motions and tones of these three chapters. As a planet or luminary passes through its diurnal motion from rising to culmination to setting it moves from a low tone to a high tone and back to a low tone. Let us assume for what is to follow that its range is at least an octave so it can cover all the notes of a tonos. Its motion in length (*mekos*), its diurnal motion of rising and setting, possibly modified by its proper motion, determines the rising and falling pitch of the tones. Its motion in depth (*bathos*), which causes its speed to vary with its distance from the earth, determines the genus of its tetrachords: diatonic when fastest, enharmonic when slowest, and chromatic at intermediate speeds. Finally, through its motion in breadth (*platos*) it passes through a series of parallels to the equator in accordance with which it modulates through the seven tonoi, Dorian when in the equator, the higher tonoi when on the side of the visible pole, the lower when on the side of the invisible pole. Of course, these could all be independent and nothing more than analogies. But if they are put together, something interesting happens. Since the synodic periods for motion in depth are not the same as the zodiacal periods for motion in breadth, all the tonoi will appear in all the different genera as the planet's speed changes, a complete cycle taking approximately: Saturn 59 years, Jupiter 71 years, Mars 79 Years, Venus 8 years, Mercury 46 years, here using periods, originally Babylonian, given by Ptolemy in *Almagest* 9.3 in which the planet completes, with residuals of only a few days or degrees, nearly integral numbers of synodic and zodiacal revolutions. When applied to all the planets, when their periods are combined, the complexity of this system is almost beyond comprehension, for it will not repeat as a whole

until they have all together completed integral numbers of synodic and zodiacal periods, which takes many, many thousands of years. If one can imagine a nearly eternal variety in the music of the spheres, Ptolemy surely provides it in these three seemingly innocent chapters. But, as noted, they could be independent, nothing more than analogies, and this grand music of the spheres not intended.

In 3.13 the motions of the Moon and planets are associated with notes in yet another way by relating tetrachords of the disjunct system to their phases with the Sun. The system is again formed into a circle as for the zodiac in 3.8, but this time without proslambanomenos, so the highest note nete hyperbolaion is disjunct to the lowest hypate hypaton. What happens is different for superior and inferior planets and for the Moon, and Ptolemy's explanation is so condensed, with all three run together, that it is not easy to distinguish the behavior of each. Our understanding of what he intends is illustrated in Figure 7a-c for superior planets, the Moon, and inferior planets, in which the arcs between notes are spaced for the diatonic genus and fixed notes are in upper case letters.

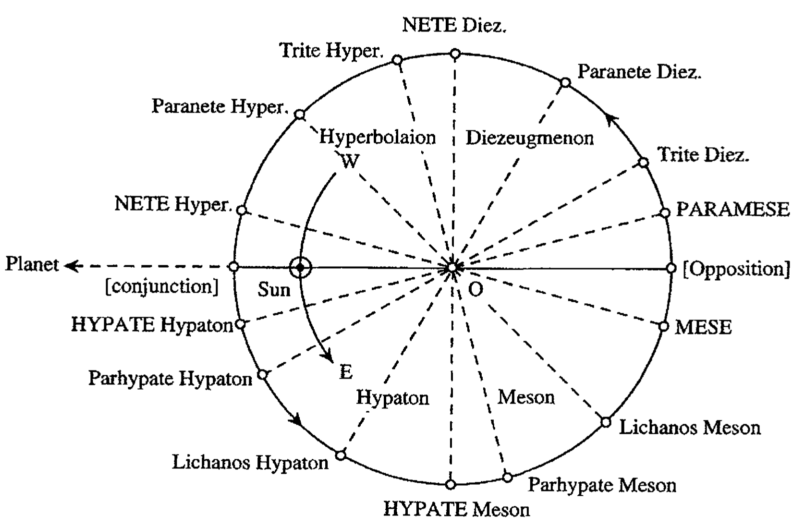


Figure 7a

Consider first a superior planet, Saturn, Jupiter or Mars, in 7a, in which, since most of the synodic motion belongs to the Sun,

the planet is shown in a fixed position and the Sun, moving from west to east, is shown initially in conjunction with the planet seen from the earth O. The arc containing conjunction, between heliacal setting at nete hyperbolaion and heliacal rising at hypate hypaton, during which the planet is invisible, according to Ptolemy, is about 1/12 of the circle, a rough estimate of a complex, variable quantity, corresponding to the disjunction of one tone (less two octaves, from a′ to B). From heliacal rising in the east just before sunrise, the Sun moves through the hypaton tetrachord until it reaches quadrature to the planet at hypate meson. The Sun then moves through the conjunct meson tetrachord until it reaches mese, following which there is a disjunction of one tone, 1/12 of the circle, to paramese, the beginning of the diezeugmenon tetrachord, during which opposition to the planet is reached. In the diezeugmenon tetrachord, Ptolemy says, evidently at paramese, the planet has its acronychal rising, in the east just after sunset. This, however, is not correct since acronychal rising takes place before, not after, opposition. But he later says that the interval of 1/12 the circle, from mese to paramese, is from acronychal rising to cosmical setting, in the west just before sunrise, with opposition thus falling in between, which would be qualitatively correct, although giving 1/12 of the circle, of the synodic motion, to this interval is very rough and too large. In any case, the Sun then moves through the diezeugmenon tetrachord until it again reaches quadrature to the planet at nete. Ptolemy points out that this tetrachord is diametrically opposite hypaton and makes an octave with it, which is true of all of its notes. Finally, the Sun moves through the conjunct hyperbolaion tetrachord to heliacal setting in the west just after sunset at nete, and here too each note is diametrically opposite and one octave from a note of meson. The distance between each rising or setting and quadrature is about $2\frac{1}{2}$ twelfths of the circle, just as each tetrachord is about $2\frac{1}{2}$ tones, and each disjunction, around conjunction and opposition, is about 1/12 of the circle, taken as one tone, so the whole circle nicely adds up to twelve tones as in the disjunct system.

In the case of the Moon the sequence is from first visibility, in the evening after sunset, through full Moon to last visibility, in the morning before sunrise, and Ptolemy mentions that everywhere opposite phases add up to a full Moon just as notes of an

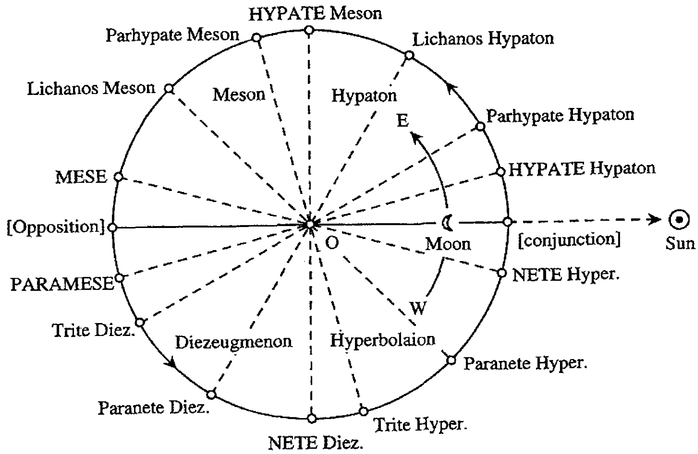


Figure 7b

octave make a unity. To illustrate this, in Figure 7b we have reversed figure 7a to show the Moon increasing its elongation from the Sun as seen from O, beginning at conjunction, and we now take the Sun as fixed. Hence, following conjunction, first visibility is at hypate hypaton, first quarter at hypate meson, and full Moon following mese at opposition. Ptolemy says that the Moon begins to wane in the diezeugmenon tetrachord, hence at paramese, reaches third quarter at nete diezeugmenon, and last visibility at nete hyperbolaion.

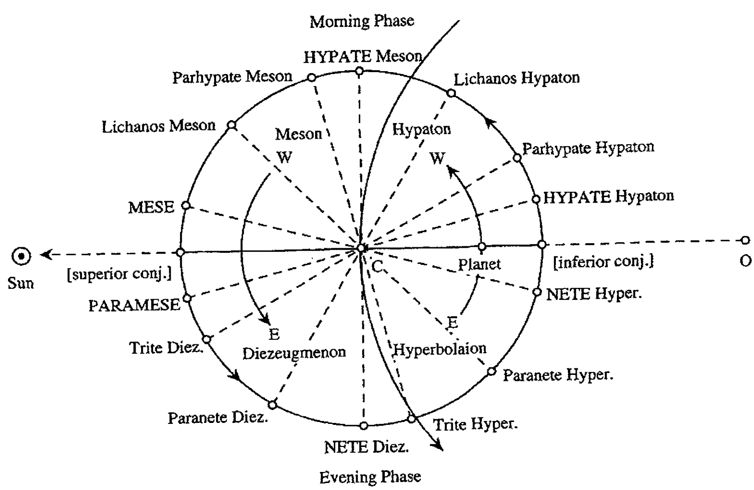


Figure 7c

For the inferior planets, Venus and Mercury, treated very briefly, each half of the circle corresponds to the morning or evening phase. This is most clearly illustrated, as in Figure 7c, by showing the planet on an epicycle with the earth at *O* and the centre of the epicycle *C* in the direction of the Sun. There is some ambiguity about just which phase goes with which tetrachord, but it appears that the morning phase is associated with the hypaton and meson tetrachords, as for heliacal rising of the superior planets. Hence, beginning from inferior conjunction the morning phase extends from morning rising at hypate hypaton through morning setting at mese, and following superior conjunction the evening phase from evening rising at paramese through evening setting at nete hyperbolaion. The planet is invisible through 1/12 of the circle near inferior conjunction and 1/12 near superior conjunction, two tones, and visible through the four tetrachords of ten tones, in all twelve tones of the disjunct system. This simple pattern hardly corresponds to the highly irregular periods of visibility and invisibility of Venus and, even more so, Mercury, but Ptolemy calls this entire relation between phases and tetrachords an analogy, so it need not be pressed closely. And the same probably applies to everything that has been described in 3.10-13 or even 3.8-13: a series of analogies.

Thus far, nothing in the *Harmonics* has any relation to the *Canobic Inscription*, but the surviving fragment of 3.16 does, and it appears as though the lost 3.14 did also. We shall therefore now set out the pertinent parts of the *Canobic Inscription* and then take up the remaining chapters of the *Harmonics*.

The Tones of the Universe in the Canobic Inscription *and* Harmonics *3.14-16*

The series of tones in the *Canobic Inscription* are headed 'Fixed tones (notes) of the universe of the system' (*systematos kosmikoi phthongoi hestotes*, which may contain a textual problem), presumably because it contains all the *fixed* notes of the disjunct and conjunct systems but no movable notes. In fact, it extends one note beyond the two octaves of the disjunct system.

Sphere of:	Note	Eq	No.	Ratio	Interval	Interval
Fixed Stars	[meta] hyperbolaion	b′	36			
				9/8	tone	
Saturn	nete hyperbolaion	a′	32			fifth
				4/3	fourth	
Jupiter	[nete] diezeugmenon	e′	24			
				9/8	tone	
Mars	nete synemmenon	d′	$21\frac{1}{3}$			fourth
				32/27	sesquitone	
Sun	paramese	b	18			fourth
				9/8	tone	
Venus & Mercury	mese	a	16			fifth
				4/3	fourth	
Moon	hypate meson	e	12			
				4/3	fourth	
Fire, Air	hypate hypaton	B	9			fifth
				9/8	tone	
Water, Earth	proslambanomenos	A	8			

Table 4: Fixed Tonos of the Universe in the *Canobic Inscription*

For the spheres (*sphairai*) of every body, beginning with the outermost sphere of the fixed stars (*aplanon*), Ptolemy gives the name of the body, the name of the note, and a number corresponding to the ratios of the intervals between all the notes in a continuous series from 36 to 8. We transcribe it here in Table 4 in which we have added the equivalent modern notes, the intervals between successive notes, and larger intervals that are significant in the formation of the series or in its application in the *Harmonics*. What is immediately noticeable about the list is the assignment of Venus and Mercury to the same note and the similar pairing of fire and air, water and earth. This is already an indication that the basis of the tones is not astronomical. While there are variants of these arrangements in different versions of the text, including an excerpt of this very system in manuscripts not containing the *Canobic Inscription*, the form we have given here appears to be the most secure and is in part supported by the fragment of *Harmonics* 3.16. The excerpt, which may come from *Harmonics* 3.14 rather than the *Canobic Inscription*, is translated in the Appendix. Its heading differs slightly and suggests that the heading in the *Canobic Inscription* may have read 'Fixed tones of the system of the universe' (*systematos kosmikou phthongoi hestotes*), a very small emendation, which

appears to make better sense. The name of the note assigned to the fixed stars is uncertain; the text appears to read 'mese hyperbolaion', which does not make sense. M. A. J. H. Vincent, in transcribing one of the excerpts, read the first word as *men* and emended it to *meta*, 'after', which is ingenious and could be correct, since *mese* could also be a misreading of *meta*. We have adopted this emendation because it correctly expresses the meaning 'beyond' the hyperbolaion tetrachord, which could also be given by 'hyper hyperbolaion', suggested by Düring, although we propose no specific emendation to the text. Whether the list originated with Ptolemy or was taken over from an earlier source is not known and there is no evidence for a decision.

The numbers, according to Ptolemy, encompass several occurrences of the three means:

arithmetic	$(a + c = 2b)$	5
geometric	$(a \cdot c = b^2)$	6
harmonic	$(\frac{1}{a} + \frac{1}{c} = \frac{2}{b})$	5

and of the six concords with their epimoric and multiple ratios:

fourth	4/3	5	octave + fifth	3/1	2
fifth	3/2	4	double octave	4/1	2
octave	2/1	5	tone	9/8	3

We note that the octave and double octave are not concordant, but homophonic, and the tone melodic. Lest the reader be puzzled about where to find the means and concords, a scholium lists them. First the means:

Arithmetic	36/24/12	32/24/16	24/18/12	24/16/8	16/12/8	
Geometric	36/24/16	36/18/9	32/24/18	32/16/18	18/12/8	16/12/9
Harmonic	36/24/18	$32/21\frac{1}{3}/16$	24/16/12	24/12/8	18/12/9	

Then the concords with their ratios:

Fourth 4/3	32/24	24/18	$21\frac{1}{3}/16$	16/12	12/9
Fifth 3/2	36/24	24/16	18/12	12/8	
[Octave 2/1	36/18	32/16	24/12	18/9	16/8]
Octave + fifth 3/1	36/12	24/8			
Double octave 4/1	36/9	32/8			
Tone 9/8	36/32	18/16	9/8		

Surely by oversight, the octave was omitted from the scholium, and we have added it. Two further intervals are also omitted from the scholium and, one must assume, from Ptolemy's enumeration, a fifth between Saturn and Mars, $32/21\frac{1}{3} = 3/2$, and a tone between Jupiter and Mars, $24/21\frac{1}{3} = 9/8$; but here, as we shall see, the omission is probably deliberate and the reason astrological. There are also two occurrences of the octave plus fourth 8/3, between Saturn and the Moon, 32/12, and between Jupiter and the fire and air, 24/9, not mentioned at all, presumably because it is neither an epimoric nor multiple ratio, and perhaps also for astrological reasons.

And what is the significance of all these notes, numbers, means, and concords? In order to answer this, we shall resume our review with *Harmonics* 3.14-16, referring to the *Canobic Inscription* wherever it is pertinent, and in our conclusion take up whatever further can be said of the relation of the *Harmonics* and the *Canobic Inscription.*

3.14 has the title 'By which least numbers the fixed tones (notes) of the perfect system may be compared to the primary spheres in the universe,' which certainly sounds promising. Note the similarity to the heading of the tones in the *Canobic Inscription*: 'Fixed tones of the universe of the system', or to the excerpt: 'Fixed tones of the system of the universe.' However, except for the first sentence and part of the second, the chapter is lost, and its completion by Nicephorus Gregoras—trivialities about dividing the circle according to the aspects and then assigning some notes to these divisions—has nothing to do with its title. Nevertheless, 3.14 is surely where Ptolemy assigned numbers and the fixed notes of the perfect system to the heavenly bodies, and that these were essentially or exactly those of the *Canobic Inscription* is confirmed by the fragment of 3.16 in which four notes are mentioned in agreement with the *Canobic Inscription*, as we shall see. The notes and concords referred to in 3.16 must have been given somewhere, and that could only have been in 3.14. I also believe that the arithmetic, geometric, and harmonic means, and the ratios of the concords in the scholium to the *Canobic Inscription* were contained in this chapter, which was in fact the scholiast's source, like the enumeration of concords and ratios of divisions of the zodiac in 3.9. It is also possible that 3.14 rather than the

Canobic Inscription was the source of the excerpt of the system given in the Appendix.

One would like to think that Ptolemy also said something concerning the astronomical or cosmological significance of the tones, numbers, intervals, means, and ratios, but there is little or no evidence for speculation. The most interesting possibility is that the numbers refer in some way to distance, the least known quantity in the heavens, since we see only direction, not distance. A variety of scales of the planets of one octave in other ancient sources are said to refer to distance or to speed. But this could be incautious here, as we shall show. For preceding the system in the *Canobic Inscription* Ptolemy gives the distances of the Moon and Sun as 64 and 729 terrestrial radii, remarking that these are the lowest numbers that are both cubes and squares, that is, 64 $= (2^3)^2$ and 729 $= (3^3)^2$. How these distances were computed is shown in Hamilton *et al.* Now the ratio of the distances is 729/64 $\approx 11\frac{2}{5}/1$, while the ratio of the Sun and Moon in the system is but $18/12 = 3/2$, not even close. Note, however, that if 3/2 is cubed and then squared, in accordance with the remark in the *Canobic Inscription*, we shall have

$$((3/2)^3)^2 = (3/2)^6 = 729/64,$$

exactly the distances in terrestrial radii just given. If this is not a coincidence, it shows a way of converting the numbers in the system to distances by cubing and squaring their ratios, taking their sixth power, to a body of known distance beginning with the Moon or Sun. Consider the sixth power of ratios for bodies up to the Sun to the distance of the Moon and for bodies above the Sun to the distance of the Sun:

Body	Ratio	Distance	Body	Ratio	Distance
Water, Earth	$(2/3)^6 \cdot 64$	5;37	Mars	$(32/27)^6 \cdot 729$	2020;26
Fire, Air	$(3/4)^6 \cdot 64$	11;23	Jupiter	$(4/3)^6 \cdot 729$	4096
Mercury, Venus	$(4/3)^6 \cdot 64$	359;35	Saturn	$(16/9)^6 \cdot 729$	23014
Sun	$(3/2)^6 \cdot 64$	729	Stars	$(2/1)^6 \cdot 729$	46656

But there is no evidence for confirming these numbers, some of which appear nonsensical—what would it mean to have water and earth five terrestrial radii from the centre of the earth?—or, for that matter, any numbers that may be computed either from sixth powers or other, presumably lower, powers of the ratios.

Hence, even if the ratio $729/64 = (3/2)^6$ for the Sun and Moon is not a coincidence, there seems to be no verifiable method of converting all the numbers into distances and thus nothing to support this interpretation of the ratios. Kepler comes up with a series of numbers remarkably close to those of the *Canobic Inscription*, which he did not know, and even computes 4096 for Jupiter, as in the preceding table, as a term of a ratio between Jupiter and Saturn, by a method somewhat like our own but squaring the ratios between adjacent planets. He then compares distances computed in this way to distances from al-Battānī, computed according to the theory of contiguous spheres of Ptolemy's *Planetary Hypotheses*, which he also did not know. But neither, we assume, did Ptolemy when he wrote the *Harmonics* and the *Canobic Inscription*, and it is most unlikely that such distances have anything to do with 3.14. Indeed, it appears that there is *nothing* of astronomical significance or with an astronomical basis in the tones and numbers in the *Canobic Inscription* and, we presume, in 3.14; they have no more basis in serious astronomy than 3.10-13. But we shall see in 3.16 that they do have astrological significance, and perhaps that was in part their motivation.

3.15 has the title 'How the ratios of the motions belonging (to each sphere) may be conceived by means of numbers.' Again the chapter is lost and Nicephorus's addition has nothing to do with the title, but simply considers ratios between the arcs of aspects, so 3.14-15 as we have them are nothing more than trivial commentary on 3.9. As for what Ptolemy wrote in this chapter, there is not a clue. Neither the periods nor mean motions of the planets in longitude or anomaly suggest harmonic ratios of any kind, and the same is true of the Babylonian period relations in *Almagest* 9.3. Numerical ratios of these motions can of course be given without reference to harmonics — e.g. Mars completes 37 synodic periods and about 42 zodiacal periods in about 79 years — but why would Ptolemy do so here? Kepler considers these relations as well as a variety of ratios between the periods or mean motions of the planets, and even ratios of their true daily motions, but few correspond to harmonic ratios, and we may be certain that these many comparisons did not occur to Ptolemy. A more specific, quantitative treatment of the three motions in 3.10-12 also seems unlikely, since the title suggests comparing motions

proper to each planet rather than these relations of pitch, genus, and tonos. Hence, for the contents of this chapter, we are, quite simply, at a total loss.

For 3.16, called 'How the affinities of the planets may be compared to those of the tones,' we are more fortunate. Nicephorus found a passage in the middle of 3.9 that he believed did not fit, and he guessed that it belonged to 3.16. Here he was correct. The subject is astrological, on the 'affinities' (*synoikeioseis*) of the planets to each other, and fits the title. Planets are said to have natures, powers, and affinities, all of which, with their applications, are treated in *Tetrabiblos* 1-2. For example, the nature of Jupiter is hot, moist, beneficent, masculine, diurnal, and so on, and its power over weather, nations, individuals and so on, is derived from its nature and its aspects with regard to signs, houses, and other planets with which it does or does not have affinities. The principle of 'affinity' (*oikeiosis or synoikeiosis*) is very large, taking in the relations between planets, stars, zodiacal signs, houses, geographical regions, nations and so on, in accordance with common or opposite attributes of their natures. We earlier considered masculine and feminine zodiacal signs and the affinities of planets for particular signs, their domiciles, in our comment on 3.9. In the fragment of 3.16 the affinities are of diurnal and nocturnal planets, called the 'sects' (*haireseis*) of the Sun and Moon, and of beneficent and malificent planets. In *Tetrabiblos* 1.5, we have the division:

Beneficent: Jupiter, Venus, Moon
Malificent: Saturn, Mars
Common: Sun, Mercury

and in 1.7 the sects:

Diurnal: Sun, Jupiter, Saturn, Mercury (as morning star)
Nocturnal: Moon, Venus, Mars, Mercury (as evening star)

Note that each sect has one malificent planet, the power of which, Ptolemy explains, is moderated by its assignment. Thus, Saturn, which is cold, is assigned to the warm diurnal sect of the Sun, and Mars, which is dry, to the moist nocturnal sect of the Moon. But in 3.16 Ptolemy gives a harmonic explanation, the reason being that the members of each sect are assigned a note and these notes are at intervals of a fourth, 4/3. In the following table, in which the four notes for Saturn, Jupiter, Mars, and Venus are identical

to those in the *Canobic Inscription*, I have filled in the two notes for the Sun and Moon and given the number for each note from the system in the *Canobic Inscription.*

Diurnal			Nocturnal		
Saturn	nete hyperbolaion	32	Mars	nete synemmenon	$21\frac{1}{3}$
Jupiter	nete diezuegmenon	24	Venus	mese	16
Sun	paramese	18	Moon	hypate meson	12

Just as Ptolemy says, the ratios of the notes for each planet in each sect are at the interval of a fourth, 4/3, which both confirms the authenticity of the system in the *Canobic Inscription*, lest there be any question about it, and shows its relation to the *Harmonics.* Unfortunately Mercury, common both as to sect and benefit, is not mentioned, so we have no further information about why it is assigned the same note as Venus in the *Canobic Inscription.*

Ptolemy goes on to say that between beneficent and malificent planets, the aspects are good or bad as follows, which we give along with ratios of the tones in the *Canobic Inscription:*

Favorable			Unfavorable		
Saturn & Jupiter	all	$32/24 = 4/3$	Saturn & Venus	all	$32/16 = 2/1$
Saturn & Sun	trine	$32/18 = 16/9$	Saturn & Moon	all	$32/12 = 8/3$
Mars & Venus	all	$21\frac{1}{3}/16 = 4/3$	Jupiter & Mars	all	$24/21\frac{1}{3} = 9/8$
Mars & Moon	trine	$21\frac{1}{3}/12 = 16/9$	Mars & Sun	all	$21\frac{1}{3}/18 = 32/27$

The reasoning here seems to depend upon harmonic relations for which I have added the ratios of the tones of the planets reduced to their lowest terms. Since Saturn and Jupiter, Mars and Venus are at the concord of a fourth, 4/3, they are favorable in all aspects. It is possible to translate the text as Mars and Venus are favorable only at trine, but the phrase is ambiguous and the concord supports the interpretation of all aspects. But Saturn and the Sun, Mars and the Moon are at 16/9, that is, one tone less than an octave, which is not even melodic; thus only their trine is favorable, as it is the most beneficent aspect, dividing the zodiac as 3/2 (see Figure 2 for 3.9), which, according to Ptolemy, is more concordant than the others. On the other side, since 2/1 between Saturn and Venus is an octave, the ratio for opposition, an unfavorable aspect, all aspects are unfavorable. Evidently the other intervals, with ratios 8/3, 9/8, and 32/27, correspond to

no beneficent aspect, and so are unfavorable in all aspects. We had noted that in the scholium giving ratios of intervals in the *Canobic Inscription*, a fifth between Saturn and Mars, $32/21\frac{1}{3} = 3/2$, was omitted, presumably because both planets are malificent, and a tone between Jupiter and Mars, $24/21\frac{1}{3} = 9/8$, which was omitted there is listed here as unfavorable in all aspects. There is nothing very profound in any of these explanations, but they do show again that *Harmonics* 3.16 is based upon the same notes and numbers as the system in the *Canobic Inscription*. It is also interesting that this chapter seems to be the source of Macrobius's explanation (Comm. on *Somnium Scipionis* 1.19.23-26), following a reference to the *Harmonics*, of why aspects of certain planets to the Sun and Moon are favorable or unfavorable. Although the reasoning about aspects and harmonic ratios is the same as in 3.16, the examples are not and may come from a different part of the chapter or be some sort of extension of it.

Concluding Remarks on the Harmonics *and* Canobic Inscription

Having reviewed the astronomical chapters of the *Harmonics* and the related tones of the universe in the *Canobic Inscription*, we may address the question of what it all means. If we are looking for science on the level of the *Almagest* or of the rest of the *Harmonics*, the answer is, not a lot. Ptolemy is searching for analogies, correspondences between harmonics, which he has already shown to explain the power of *harmonia* in music, and a highly simplified description of motions in the heavens, and he finds them. As is clear from his introduction to this section (3.3-4), for him this is profound and beautiful in a way difficult for us to appreciate, since the correspondences themselves seem so vague and, apparently, made up. I have tried to imagine something beautiful in planets' modulating from the Phrygian to the Lydian tonos as they pass from Taurus to Gemini, but I have not been able to do so. Perhaps it is because I do not know the sound of Greek music in those tonoi. For those who know Kepler's meticulous application of harmonics to the heavens in the fifth book of the *Harmonice mundi*, for the derivation of the eccentricities of the planets, which are really there, from harmonic ratios of extreme angular speeds, which are also really there, Ptolemy's correspondences appear without substance or empirical basis. And Kepler

himself was disappointed when, after years of anticipation, he read Book 3 of the *Harmonics* and found that it had little or nothing of what he expected, which, by the way, is clear from his commentary on the text even though he tries to put it in the best light. Yes, the planets do rise and set, vary in distance and speed, and move north and south of the equator as they move through the zodiac, but not one of Ptolemy's comparisons of these motions to rising and falling pitch, change of genus, and change of tonos is quantitative, not one is based on anything empirical or specific, all are merely fanciful. This is very far from the *Almagest.* The same must be true of the tones of the universe in the *Canobic Inscription*; they are mere fancies without empirical basis or astronomical significance.

In applying harmonics to astrology, Ptolemy is somewhat more successful. The aspects divide the zodiac into arcs in simple numerical ratios, and the same ratios are found in musical concords. In this way, the efficacy of the recognized aspects is explained on the basis of harmonics as well as why signs separated by arcs not in the ratios of concords do not have effective aspects. These harmonic causes of aspects are retained in the *Tetrabiblos*, which, however, omits the explanation in the *Harmonics*, without which they are barely intelligible. And it is not difficult to extend these same ratios of concords to the relation of aspects to the sex of signs, to the affinities of planets to their domiciles, and, using the ratios of tones in the *Canobic Inscription*, which may be of concords or discords, to the affinities of planets to diurnal and nocturnal sects, and to favorable and unfavorable aspects of beneficent and malificent planets. Thus, harmonics explains or gives the formal cause of some simple parts of astrology, and perhaps this is even the reason for the tones in the *Canobic Inscription.* But even here, compared to Kepler's investigation, in Book 4 of the *Harmonice mundi*, of the cause of aspects from the harmonic ratios of the sides of constructible regular polygons, including his discovery of additional aspects not previously recognized, Ptolemy's treatment of harmonics in astrology is (it must be said) superficial. None of this is to blame Ptolemy for not being on a level with Kepler; in the *Almagest* he most certainly is, but in truth his application of harmonics to astrology is as simple, as vague, as its application to astronomy. Perhaps he

knew this all along and thought it just as well.

There is one further point to be made, a more important one I believe. The relation of the *Harmonics* to the *Canobic Inscription* provides evidence that the *Harmonics* is an early work of Ptolemy's. The *Canobic Inscription*, a list of parameters of Ptolemy's mathematical astronomy, is dated to the tenth year of Antoninus, AD 146/47. It has been shown in Hamilton *et al.* that the parameters, differing slightly from the *Almagest*, show an earlier stage of Ptolemy's work, perhaps from an earlier draft of the *Almagest*, which he corrected in the published *Almagest*, where he noted some of the corrections. That the *Harmonics* makes use of the tones of the planets in the *Canobic Inscription* is evidence that it is also an early work, perhaps even preceding the *Canobic Inscription*, *at least in its contents*, although nothing is certain about when it was actually published. For no particular reason, it has been assumed that the *Harmonics* is a late work, perhaps because parts of it are philosophical, perhaps because it seems peripheral to Ptolemy's major works in astronomy, perhaps even, as has apparently been suggested, because Ptolemy died before completing its last three chapters, which can hardly be true. In fact, there is no evidence for assigning the work a late date, while the relation to the *Canobic Inscription* is at least *some* evidence for an early date. From internal references, corrections, and other pieces of information, it is known that the *Tetrabiblos*, *Handy Tables*, *Planetary Hypotheses*, *Geography*, and *Optics* are later than the *Almagest*. But the *Harmonics* contains no reference to earlier work and does not correct or modify anything in Ptolemy's other writings. And it appears more likely that the brief application of harmonics in *Tetrabiblos* 1.13 and 1.17 is based upon the detailed explanation in *Harmonics* 3.9 than the other way around, for the *Harmonics* gives the reasons for what is merely stated in the *Tetrabiblos*.

Since attempts to establish the order of writings of ancient authors, a common enough pastime for people with nothing better to do, are seldom of interest, why should we care when Ptolemy wrote the *Harmonics*? For one reason, that *Harmonics* 1.1-2 and some other parts of the work contain Ptolemy's most detailed account of what he considers proper method in the applied mathematical sciences. It is thus of interest if, or that, such statements

are part of his early, perhaps even earliest, surviving work, showing that he had recognized the importance of such method and knew how to apply it from the beginning. I had intended to write an account of Ptolemy's reflections on method, but the recent, excellent book by Andrew Barker, *Scientific Method in Ptolemy's Harmonics* (2000), makes anything I could say superfluous. I shall only mention, as I did earlier, that Ptolemy's method is rigorously mathematical and rigorously empirical, that it is exactly the method he follows in the *Almagest*, and that it is exactly the method that has been followed in the best work in the applied mathematical sciences ever since. Even if the astronomical and astrological parts of the *Harmonics*, which we have considered here, perhaps in greater detail than they are worth, are not on Ptolemy's highest level (as indeed they are not), his statements of method and his exposition of harmonics itself certainly are, and that is enough to make the *Harmonics*, perhaps Ptolemy's earliest work, of lasting importance to the history of the mathematical sciences.

Appendix

Excerpt from the Canobic Inscription *or from* Harmonics *3.14*

M. A. J. H. Vincent and Carl von Jan have published what appears to be an excerpt, from either the *Canobic Inscription* or *Harmonics* 3.14, containing the tones assigned to the planets, and what probably originated as a scholium giving a great many ratios between the numbers. The list of notes, which we take from von Jan's edition from Naples III C 2, is called 'Notes of the system of the universe' (*Horoi systematos kosmikou*). It can be translated as a table, in which we have adopted Vincent's emendation of 'meta hyperbolaion' for the fixed stars, although we have no opinion on whether it is correct, and have added the missing fraction of 1/3 for Mars.

Fixed Notes	Numbers	Spheres of:
[meta] hyperbolaion	36	Fixed stars
nete hyperbolaion	32	Saturn
nete diezeugmenon	24	Jupiter
nete synemmenon	21[$\frac{1}{3}$]	Mars
paramese	18	Sun
mese	16	Venus
hypate meson	12	Mercury
hypate hypaton	9	Moon
proslambanomenos	8	fire, air, water, earth

This is followed by the means, concords, and ratios listed in the *Canobic Inscription* with their expansion in the scholium. The differences from the *Canobic Inscription* concern the lower bodies, in particular in separating Venus and Mercury. Vincent, from Paris Gr. 3027, places fire and air with the Moon and only water with earth. Although both of these seem to make more sense than the *Canobic Inscription*, at least in separating Venus and Mercury, *Harmonics* 3.16 confirms 12 for the Moon, since it must be at the interval of a fourth, 4/3, to Venus, not 16/9 as here, which in turn confirms the reading in the *Canobic Inscription*.

The other text published by von Jan and Vincent, under the name 'Music of Ptolemy' (*Ptolemaiou mousika*), the latter from Paris Gr. 449 suppl., consists of two lists of ratios and numbers; the numbers are those of the excerpt above, associated in the second list with bodies from the Moon upward to the fixed stars. It is headed, 'The origin of musical ratios is the number 8 and the notes of the system of the universe (*horoi tou kosmikou systematos*) are as follows.' This heading applies specifically to another form of the same two lists that appears written out continuously under the name of Anatolius in the *Theologoumena arithmetica* attributed to Iamblichus. It is certainly a later addition to a short excerpt from Anatolius (late 3rd cen.) on the significance of the number eight (because there are eight spheres), from his book *On the first ten numbers*, in which we learn, among other curiosities, under the number seven that Herophilus said the human intestine is 21 cubits long, which is three times seven. This gives a good idea of its level. Evidently, both forms of the two lists originated as a scholium to the excerpt from Anatolius contained in the *Theologoumena arithmetica*. Although they are without doubt

based indirectly upon Ptolemy's tones of the planets, they are so mindless and pedestrian that we shall not waste space translating them.

Something similar to the passage attributed to Anatolius turns up in Arabic in the *Epistle on Music* of the Ikhwān aṣ-Ṣafā (10th century), translated by A. Shiloah, concerning the significance of the number eight for the harmonic ratios of the celestial spheres. The numbers for the spheres: earth 8, air 9, Moon 12, Mercury 13, Venus 16, Sun 18, Mars $21\frac{1}{2}$, Jupiter 24, Saturn $24\frac{4}{7}$, fixed stars 32, are interpreted as diameters of the spheres, hence they are also relative distances, with various harmonic ratios between them, and it is remarked that Mercury, Mars, and Saturn are called malificent because they do not have a harmonic ratio, which is rather like *Harmonics* 3.16. Just what to make of this, I do not know, but it would be of interest if something directly from Ptolemy's *Harmonics* were found in Arabic, presumably in writers on music, perhaps on astrology or astronomy, as it is believed to have been translated under the title *Kitāb al-Mūsīqī*, although no copy is known to survive.

References

The Greek text of the *Harmonics* is edited by I. Düring in *Die Harmonielehre des Klaudios Ptolemaios, Göteborgs Högskolas Årsskrift* 36 (1930). The only earlier edition is by John Wallis (Oxford, 1682, 1699). Düring's German translation is published in *Ptolemaios und Porphyrios über die Musik, Göteborgs Högskolas Årsskrift* 40 (1934). A. Barker's English translation is in *Greek Musical Writings: II Harmonic and Acoustic Theory*, Cambridge, 1989. Both translations are of great value and are thoroughly, and helpfully, annotated. Barker's recent study, *Scientific Method in Ptolemy's Harmonics*, Cambridge, 2000, is likewise an important contribution to the understanding of this work. A new translation of the *Harmonics* by J. Solomon appeared (Leiden, 2000) after this paper was written. It correctly gives the tables of tones as sexagesimals.

The most valuable study of Book 3 of the *Harmonics* is Kepler's annotated translation, *Cl. Ptolemaei Harmonicorum librum III. a cap. III ad finem*, edited by Ch. Frisch in *Joanni Kepleri Opera omnia* V, Frankfurt, 1864, pp. 335-412. Kepler goes far

beyond Ptolemy in his interpretation and the annotation is far longer than the text. He completely rewrites the missing 3.14-16 and annotates them even more thoroughly than the surviving part. Nevertheless, his understanding of what Ptolemy himself wrote is usually correct and always of great interest. There is a chapter devoted to Kepler's translation, mostly concerning the rewritten 3.14 which Kepler believed to concern distances, by B. Stephenson in *The Music of the Heavens, Kepler's Harmonic Astronomy*, Princeton, 1994; Stephenson also considers ancient harmonic theories of the heavens, including the *Canobic Inscription*, with specimen lists of tones.

The *Canobic Inscription* is edited by J. L. Heiberg in *Claudii Ptolemaei Opera quae extant omnia II. Opera minora*, Leipzig, 1907, pp. 149-55. I have also used a copy of Heiberg's MS A: Venice, Marc. Gr. 313, fols 28v-29v. The early date of the *Canobic Inscription* is shown in N. Hamilton, N. M. Swerdlow and G. J. Toomer, 'The *Canobic Inscription*: Ptolemy's Earliest Work', in *From Ancient Omens to Statistical Mechanics. Essays on the Exact Sciences Presented to Asger Aaboe*, eds J. L. Berggren and B. R. Goldstein, Copenhagen, 1987, pp. 55-73. The excerpt of the tones from the *Canobic Inscription*, or from *Harmonics* 3.14, has been published by M. A. J. H. Vincent, *Notice sur divers manuscrits grecs relatifs a la music. Notices et extraits des manuscrits de la bibliothèque du roi et autres bibliothèques*, 16 (Paris, 1847), pp. 250-52; C. von Jan, 'Die Harmonie der Sphären,' *Philologus*, 52, 1893, pp. 29-30; *Musici scriptores graeci*, ed. C. von Jan, Leipzig, 1895, pp. 411-12 and 418-20. The form under the name of Anatolius in the *Theologoumena arithmetica* attributed to Iamblichus is edited by V. de Falco Leipzig, 1922, pp. 75-6. The *Epistle on Music* of the Ikhwān aṣ-Ṣafā is translated by A. Shiloah, *Documentation and Studies. Publications of the Department of Musicology and the Chaim Rosenberg School of Jewish Studies*, III, Tel-Aviv, 1978, with the passage of concern here at pp. 45-6. A part of the passage is quoted by H. G. Farmer, *Influence of Music from Arabic Sources*, London, 1926, 19, which says something of harmonics and the heavens in Arabic writings.

Ptolemy's *Tetrabiblos* is edited and translated by F. E. Robins, London, 1940. Concerning the short work on epistemology at-

tributed to Ptolemy, *On the Criterion*, I have nothing to say except to doubt its authenticity, or at least its pertinence to the subjects considered here. It is edited and translated along with twelve papers on related topics in *The Criterion of Truth*, eds. P. Huby and G. Neal, Liverpool, 1989. It contains not a single reference to the subjects of Ptolemy's other works, *all* in the mathematical sciences, and parallels that have been drawn with the *Harmonics* seem to me vague. If Ptolemy is its author, he could have better occupied his time with mathematics. Of course, one might say that of *Harmonics* 3.3-16 also.

The literature on Greek music and music theory is vast beyond comprehension. I know of no subject about which so much is written and yet so little is understood. My impression is that over four hundred years ago Vincenzio Galilei knew nearly as much about Greek music as we do today even though the translations of sources he had at his disposal were wretched. And to believe that we really know anything of the sound of Greek music from the few surviving notated examples is wishful thinking. I first discovered the subject when a student in Gustave Reese's *Music in the Middle Ages*, New York, 1940, and then in Kathleen Schlesinger's exhaustive *The Greek Aulos*, London, 1939, which is on a lot more than auloi, and R. P. Winnington-Ingram's *Mode in Ancient Greek Music*, Cambridge, 1936. Winnington-Ingram also wrote the articles on Greek music (ancient) in *Grove's Dictionary of Music and Musicians*, 5th ed., New York, 1954, and in the *New Grove*, New York, 1980, VII, pp. 659-72. It is not easy to say something new about this old subject, but in what may be called a Renaissance of interest, recent studies have actually done so, of which we may mention M. L. West, *Ancient Greek Music*, Oxford, 1992, W. D. Anderson, *Music and Musicians in Ancient Greece*, Ithaca, 1994, J. G. Landels, *Music in Ancient Greece and Rome*, London, 1999, and T. J. Mathiesen, *Apollo's Lyre: Greek Music and Music Theory in Antiquity and the Middle Ages*, Lincoln, Neb., 1999.

Neither Observation nor Astronomical Tables: An Alternative Way of Computing the Planetary Longitudes in the Early Western Middle Ages

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1 *Introduction: early medieval records of planetary longitudes*

Before the age of Arabic-Latin translations of scientific texts in the twelfth century, planetary astronomy (with the exception of lunar and solar cycles needed for the purpose of computus) was in the Western world mainly restricted to information found in ancient encyclopaedias, such as those of Pliny, Calcidius, Macrobius, Martianus Capella and Isidore of Seville.[1] None of these authors had taught how to locate the planets in the zodiacal signs at a given time. This knowledge is normally provided by astronomical tables which indicate the position of the planets at regular intervals over a certain period of time. Such material does not appear to have been known in the West before the Latin translations of the Arabic tables of al-Khwārizmī and the Toledan Tables in the first half of the twelfth century. A possible exception is the *Preceptum Canonis Ptolomei*, a set of astronomical tables, together

*I am most grateful to Vera Rodrigues, Charles Burnett and Bruce Eastwood for their comments on earlier versions of this paper, and to Jill Kraye and Kristine Haugen for generously giving their time to improving my English.

[1] Early medieval planetary astronomy has only recently begun to receive serious scholarly attention. See in particular W. M. Stevens, 'Astronomy in Carolingian Schools', in *Karl der Grosse und sein Nachwirken. 1200 Jahre Kultuur und Wissenschaft in Europa*, eds P. L. Butzer, W. Oberschelp and H. Jongen, I, Turnhout, 1997, pp. 417-87 (esp. pp. 439-56); S. McCluskey, *Astronomies and Cultures in Early Medieval Europe*, Cambridge, 1998, esp. pp. 114-27 and 157-64; and the collected studies of B. Eastwood, *The Revival of Planetary Astronomy in Carolingian and Post-Carolingian Europe*, Aldershot, 2002.

with rules for their use, which David Pingree has recently studied and edited.[2] The *Preceptum* was translated from Greek in about A.D. 534/535, was known to Cassiodorus (†580) and first occurs in a manuscript copied around the year 1000, probably at the monastery of Fleury (Saint-Benoît-sur-Loire): London, BL, Harley 2506. The relevance of this work with regard to early medieval astronomy is however to be questioned for three reasons. First, the translation is so literal, including transliterations of Greek words unintelligible to a Latin audience, that its use could only have been very limited.[3] Second, only the tables of the luminaries remain in the present state of preservation,[4] which means that the tables of the planets either were lost at an early date or were never available in the Latin version.[5] Third, there is no firm evidence that it was used, or even known, between Cassiodorus and the time of Harley 2506.

We do, however, have some early medieval records of the positions of the planets. In 820, Rabanus Maurus, the schoolmaster of Fulda who later became abbot of the same monastery (822-47) and bishop of Mainz (847-56), composed a treatise of computus based mainly on the works of Bede but also on other sources and personal observations. Among these is the following passage in chapter 48:

> Today, that is in the year of the Lord's Incarnation 820, on the 9th of July, the Sun is in Cancer 23°, the Moon in Taurus 9°,

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[2] D. Pingree, 'The Preceptum Canonis Ptolomei', in *Rencontres de cultures dans la philosophie médiévale. Traductions et traducteurs de l'Antiquité tardive au XIVe siècle (Actes du Colloque international de Cassino, 15-17 juin 1989)*, eds J. Hamesse and M. Fattori, Louvain-la-Neuve-Cassino, 1990, I, pp. 355-75. *Preceptum Canonis Ptolomei*, ed. (with translation and commentary) D. Pingree, Louvain-la-Neuve, 1997 (= *Corpus des Astronomes Byzantins*, VIII). This edition includes the canons only, not the tables themselves. See also D. Pingree, 'Avranches 235 dans la tradition manuscrite du *Preceptum Canonis Ptolomei*' (the article is in English), in *Science antique, science medievale. Actes du colloque international (Mont-Saint-Michel, 4-7 septembre 1998)*, eds L. Callebat and O. Desbordes, Hildesheim, 2000, pp. 162-9.

[3] D. Pingree, 'The *Preceptum...*', p. 366, and *Preceptum Canonis...*, p. 17 (as n. 2 above).

[4] D. Pingree, 'The *Preceptum...*' (as n. 2 above), pp. 367- 8.

[5] It is extremely curious that only one chapter of the canons (out of 103) refers to the five planets (cf. *Preceptum*, chap. 62, ed. D. Pingree (as n. 2 above), pp. 85-6).

> Saturn in the sign of Aries, Jupiter in Libra, Mars in Pisces, and the positions of Venus and Mercury are not visible because they are at the moment close to the Sun in daylight.[6]

According to Tuckerman's tables, the positions of Saturn, Jupiter and Mars are correct, and the last statement about Venus and Mercury makes it clear that Rabanus did observe (rather than compute) them.[7] On the other hand, there are some mistakes in the positions of the luminaries: on 9 July 820, the Sun was at Cancer 20°-21°, and the Moon travelled from about Taurus 21° early in the morning to about Gemini 4° in late evening. Three explanations have hitherto been proposed as to how Rabanus determined these positions. On the ground that 'he must have had a set of astronomical tables to arrive at *any* longitude expressed in such precise terms', David Pingree assumed that he used the *Preceptum*, but this leaves unexplained the large error with respect to the Moon.[8] More recently, Wesley Stevens suggested that Rabanus might have located the Sun with the help of an instrument, such as the *horologium nocturnum*, some drawings of which survive in early ninth-century manuscripts from Salzburg and Regensburg.[9] Finally, Bruce Eastwood alluded to

[6] 'Modo autem, id est anno dominicae incarnationis DCCCXX mense Iulio nona die mensis, est Sol in XXIII parte Cancri, Luna in nona parte Tauri, stella Saturni in signo Arietis, Iouis in Librae, Martis in Piscium, Veneris quoque stella et Mercurii, quia iuxta Solem in luce diurna modo sunt, non apparet in quo signo morentur' (Rabanus Maurus, *De computo*, 48, ed. W. M. Stevens, Turnhout, 1979 (= *Corpus Christianorum. Continuatio Medievalis*, 44), p. 259).

[7] B. Tuckerman, *Planetary, Lunar and Solar Positions A.D. 2 to A.D. 1649 at Five-Days and Ten-Days Intervals*, Philadelphia, 1964, p. 428. All true positions given hereafter are taken from this study. On 9 July 820 Saturn was at Aries 19°, Jupiter at Libra 29° and Mars at Pisces 16°. Presumably, then, Rabanus was able to recognize the planets and to locate them in the constellations, i.e. in the 'sidereal' zodiac. It should be noted that, whatever the value of the backward shift due to the precession of the equinoxes we admit, the three planets would remain in the same sign. It is probably for the purpose of observation that Rabanus introduced his chapter by listing the colour/appearance of each of the planets: 'Suus quidem unicuique color est Saturno candidus, Ioui clarus, Marti igneus, Lucifero gaudens, Vespero refulgens, Mercurio radians, Lunae blandus, Soli cum oritur ardens, postea splendens' (*De computo*, 48, ed. W. M. Stevens, p. 258).

[8] D. Pingree, 'The *Preceptum*...' (as n. 2 above), p. 372. See also 'Avranches 235...' (as n. 2 above), p. 163.

[9] W. M. Stevens, 'Astronomy in Carolingian Schools' (as n. 1 above), pp.

the possibility that he simply used computus texts, including his own.[10]

This latter solution appears to be the right one. The computation of the position of the Sun and the Moon formed an integral part of early medieval computus, and Rabanus himself, like others before and after him, had provided the relevant procedures in his treatise. For the Sun he states (1) that it enters a sign on the 15th kalends of the month, starting with Aries in March, and (2) that it transits a single sign in 30 days and $10\frac{1}{2}$ hours, or a single degree in one day.[11] A mere application of these rules leads to the result that the Sun (1) enters the first degree of Cancer every year on 17 June, and (2) reaches the 23rd degree of Cancer 22 days later, i.e. on 9 July.[12] The procedure for computing the position of the Moon, more complex because of its velocity (the Moon traverses about 13° along the zodiac every day and, therefore, a little more than 1° in two hours), relies on the (correct) assumption that the age of the Moon determines its position with respect to the Sun. Following Bede's model, Rabanus devoted two chapters to the

440-41. See also idem, 'Compotistica et astronomica in the Fulda School', in *Saints, Scholars and Heroes. Studies in Medieval Culture in Honour of Charles W. Jones*, eds W. M. Stevens and M. H. King, Collegeville, 1979, II, pp. 27-63, here pp. 47-8 (reprinted in W. M. Stevens, *Cycles of Time and Scientific Learning in Medieval Europe*, Aldershot, 1995, article IX).

[10] B. Eastwood, 'Astronomia, computo e astrologia', in *Storia della scienza*, vol. IV: *Medioevo, Rinascimento*, sezione II: *La scienza bizantina e latina*, ed. J. D. North, Rome, 2001, pp. 149-68 (p. 157).

[11] See especially *De computo*, 39: 'Nam Ariete Martio mense... Februario Pisces... Singulis autem signis triginta partes, ternae uero decades putantur, eo quod Sol triginta diebus et decem semis horis illa percurrat, a medio mensis, id est XV kalendarum die, semper incipiens' (ed. W. M. Stevens, pp. 250 and 251); chap. 40: '*Demonstratio signorum per Solis cursum...* Siquidem Sol moratur transiens in unoquoque signo XXX dies, decem horas ac semissen. Ad deinde in aliud signum ingreditur, quod similiter in XXX diebus et decem horis ac semisse pertransiens, tertium intrat' (pp. 251-2, see also chap. 28, p. 232, l. 13-18); chap. 42: 'Sol igitur cotidie partem unam zodiaci sui complet' (p. 254). These information are also found in Bede (*De natura rerum*, 17, and *De temporum ratione*, 16 and 18).

[12] On this computation of the position of the Sun in other contexts, see A. Borst, *Die karolingische Kalenderreform*, Hannover, 1998, pp. 435-6 and 478-9. The same result could also have been easily obtained from the data provided in Carolingian calendars, most of which indicate for each month the date of the entry of the Sun in the corresponding zodiacal sign; see for instance the 'Lorsch Prototype' of 789 edited *ibid.*, pp. 254-98, esp. p. 274: 'Sol in Cancrum' on 17 June.

question,[13] providing the following formula: the day of the Moon is to be multiplied by 4 and divided by 10; the resulting quotient corresponds to the number of signs separating the Sun and the Moon and the resulting remainder to the number of additional *puncti* (one *punctus* being equal to 3 degrees).[14] The computation using the correct day of the Moon, i.e. the 24th day on 9 July 820,[15] would be as follows:

$24 \times 4 = 96$
$96 : 10 = 9$ (remainder: 6)
= 9 signs and 6 *puncti* (or 18 degrees) counted from Cancer 23°
(position of the Sun)
= Taurus 11°

The small difference between Taurus 11° and Taurus 9° can be explained either as a mistake or as the result of some adjustments made to fit the time of the computation. We cannot be certain whether this reconstruction is valid or not, but there is no reason to believe, especially in the context of a handbook composed for his students, that Rabanus did not follow his own teaching.

In her exhaustive survey of astronomical phenomena mentioned in annals and chronicles written between 600 and 1200 in the area of

[13] *De computo*, 41 (*De Lunae cursu per signa*, ed. W. M. Stevens, pp. 252-4) and 42 (*Argumentum ad investigandum Lunae cursum*, pp. 254-5). The source is Bede, *De temporum ratione*, 17-18.

[14] Rabanus gives, among several examples, those for the 5th and 8th days of the Moon: 'Luna quotidie quattuor punctis siue crescens a Sole longius abit, seu decrescens Soli uicinior, quam pridie fuerat, redditur. Singula autem signa decem punctos habent, id est duas horas, sicut et superius admonuimus, quinque enim puncti in Luna horam faciunt. Et ideo si uis scire in quo signo Luna est, sume Lunam quam uolueris, utputa quintam; multiplica per quattuor, fiunt XX; partire per decem, bis denis uies; duobus ergo signis quinta Luna semper a Sole distat. Item sume octauam Lunam; multiplica per quattuor, fiunt XXXII; partire per decem, ter deni tries et remanent duo. Tribus ergo signis et duobus punctis octaua Luna semper a Sole dirimiter. Duos autem punctos sex partes intellege, id est quantum Sol in zodiaco sex diebus conficit itineris; punctus siquidem habet tres partes, quia signum quodque decem punctos, XXX habet partes' (*De computo*, 41, ed. W. M Stevens, p. 253). For a detailed commentary on this procedure, see F. Wallis, *Bede: The Reckoning of Time*, Liverpool, 1999, pp. 58-63 and 290-91.

[15] The preceding full Moon, i.e. the 14th or 15th day of the Moon, occurred on 29 June 820 in the evening (see H. H. Goldstine, *New and Full Moons, 1001 B.C. to A.D. 1651*, Philadelphia, 1973, p. 152).

present-day Belgium, Isabelle Draelants pointed out that eclipses were never reported with a high degree of technical precision, with the notable exception of four successive eclipses which occurred during the years 806 and 807.[16] The most detailed report of these is found in the *Annales regni Francorum*, which also mentions two conjunctions involving Jupiter and Mercury:[17]

Date	Event	Sun	Moon	Sun (true)
2 September 806	lunar eclipse	Virgo 16°	Pisces 16°	Virgo 14°
31 January 807	conjunction Moon-Jupiter	–	–	–
11 February 807	solar eclipse	Aquarius 25°	Aquarius 25°	Aquarius 27°
26 February 807	lunar eclipse	Pisces 11°	Virgo 11°	Pisces 12°
17 March 807	conjunction Sun-Mercury	–	–	–
22 August 807	lunar eclipse	Virgo 5°	Pisces 5°	Virgo 2°

For the eclipses, a comparison with Tuckerman's tables shows that the positions of the Sun are correct with, however, a difference of 1 to 3 degrees in each case. Again, this difference is fully

[16] I. Draelants, *Eclipses, comètes, autres phénomènes célestes et tremblements de terre au Moyen Age. Enquête sur six siècles d'historiographie médiévale dans les limites de la Belgique actuelle (600- 1200)*, Louvain-la-Neuve, 1995, pp. 75-6 and 115 (and n. 219).

[17] 'DCCCVII. Anno superiore IIII. Non. Septembr. [= 2 September 806] fuit eclypsis Lunae; tunc stabat Sol in XVIma parte Virginis, Luna autem stetit in XVIma parte Piscium; hoc autem anno pridie Kal. Febr. [= 31 January 807] fuit Luna XVIIma, quando stella Iovis quasi per eam transire visa est, et III. Id. Febr. [= 11 February 807] fuit eclypsis Solis media die, stante utroque sidere in XXV parte Aquarii. Iterum IIII. Kal. Mart. [= 26 February 807] fuit eclypsis Lunae, et apparuerunt acies eadem nocte mirae magnitudinis, et Sol stetit in undecima parte Piscium, et Luna in undecima parte Virginis. Nam et stella Mercurii XVI. Kal. Aprilis [= 17 March 807] visa est in Sole quasi parva macula, nigra tamen, paululum superius medio centro eiusdem sideris, quae a nobis octo dies conspicitur. Sed quando primum intravit vel exivit, nubibus impedientibus minime adnotare potuimus. Iterum mense Augusto, XI. Kal. Septembr. [= 22 August 807], eclypsis Lunae facta est hora noctis tertia, Sole posito in quinta parte Virginis et Luna in quinta parte Piscium. Sicque ab anni superioris Septembrio usque ad anni praesentis Septembrium ter Luna obscurata est et Sol semel' (*Annales regni Francorum inde ab a. 741 usque ad a. 829, qui dicuntur annales Laurissenses majores et Einhardi*, ed. F. Kurze, Hannover, 1895 (= *Monumenta Germaniae Historica, Scriptores Rerum Germanicarum in Usum Scholarum*), pp. 122-3). On this passage, see also the commentary by R. and M. Mostert, 'Using Astronomy...' (as n. 21 below), pp. 250-53.

explained if we assume that these positions were computed according to the same method as Rabanus's, that is, starting with the 15th kalends (or the 14th kalends in the month of February) as the date of the entry of the Sun into the corresponding sign and adding one degree per day:[18]

Position of the Sun on the (14th-)15th kalends	Computation for the relevant date
18 August = Virgo 1°	2 September (+ 15 days) = Virgo 16°
18 January = Aquarius 1°	11 February (+ 24 days) = Aquarius 25°
16 February= Pisces 1°	26 February (+ 10 days) = Pisces 11°
18 August= Virgo 1°	22 August (+ 4 days) = Virgo 5°

As for the positions of the Moon, they were clearly inferred from those of the Sun, by taking the degrees of the Sun and locating the Moon in the correct sign (the same sign in the case of a solar eclipse and the opposite sign in the case of a lunar eclipse), as anyone aware of the general mechanics of eclipses would have done.[19]

The two other celestial phenomena mentioned in the *Annales* are said to have been observed ('visa est' is used in both cases). There was indeed a conjunction of the Moon and Jupiter on 31 January 807. This conjunction was exact around the middle of the night and would therefore have been visible at almost any time between sunset on 30 January and sunrise on 31 January.[20] On the other hand, the Sun and Mercury were not in conjunction on 17 March 807 (the Sun was at Aries 1° and Mercury at Aries 19°); so the reported 'small black spot' ('parva macula, nigra tamen') on the Sun must have been another phenomenon.

Other early medieval reports of the planetary positions have also been preserved. There is the well-known example of the Leiden

[18] A. Borst, *Die karolingische Kalenderreform* (as n. 12 above), pp. 435-6. The use of the 14th of the kalends for the month of February is attested in Carolingian calendars, e.g. the 'Lorsch Prototype' (see n. 12 above), ed. *ibid.*, p. 259: 'Sol in Pisces' on 16 February.

[19] See e.g. Bede, De temporum ratione, 27. Rabanus's formula would have led to the same results, using as a basis the 15th day of the Moon for a lunar eclipse and the 30th day of the Moon for a solar eclipse.

[20] On 31 January 807, Jupiter was at Libra 3° and the Moon at Libra 11°. Tuckerman's positions are given at 16h00 Universal Time (= 4 p.m. Greenwich Civil Time); the Moon was then at Libra 3° around the preceding midnight.

Planetarium (ninth century), where the positions of the seven planets, depicted inside a circular zodiac, correspond to those of 18 March 816.[21] A series of astronomical manuscripts from the ninth century onwards contains records of the positions of Saturn, Jupiter, Mars and Venus in the signs for the years 770, 781 and 797.[22] In both cases, unfortunately, neither the context nor the

[21] As shown by R. and M. Mostert, 'Using Astronomy as an Aid to dating Manuscripts. The Example of the Leiden Aratea', *Quaerendo*, 20, 1990, pp. 248-61 (on pp. 253-5, with a reproduction of the *Planetarium* on p. 249) against an earlier dating (28 March 579) proposed by B. Eastwood, 'Origin and Contents of the Leiden Planetary Configuration (ms Voss. Q.79, fol. 93v), an Artistic Astronomical Schema of the Early Middle Ages', *Viator*, 14, 1983, pp. 1-40 (with the reproduction as Fig. 1, at the end). D. Pingree, 'The *Preceptum*' (as n. 2 above), p. 373, had independently suggested 816 as a possible date for the positions depicted in the *Planetarium*.

[22] (1) 'Anno DCCLXXXI, stella Iouis et stella Martis et stella Veneris in sidere Cancri, sed Venus fugiens statim reuersa est ad Phoebum.' (2) 'Anno DCCLXX, indictione VIII, prima die Ianuarii mensis stella Saturni necnon et stella Iouis in sidere Cancri uersabantur. Et cuicumque codicellus iste perueniat in manus, si tamen curiosus est errantium cognoscere siderum cursus, intellegat post annorum curricula, quod ad idem redeant sidus.' (3) 'Anno dominicae incarnationis DCCXCVII, indictione V, stellarum in mense Mar<tii> <stella Saturni> in Geminis et stella Iouis in Scorpione, stella Martis in Cancro, et stella Veneris in Tauro uersatur.' Extracts (1) and (2) appear in this order in Berlin, Staatsbibl., Phill. 1869 (*ca.* 840, Prüm), fol. 7v (as an insertion in a calendar, in the month of August); see edition in D. Lohrmann, 'Alcuins Korrespondenz mit Karl dem Grossen uber Kalendar und Astronomie', in *Science in Western and Eastern Civilization in Carolingian Times*, eds P. L. Butzer and D. Lohrmann, Basel-Boston-Berlin, 1993, pp. 79-114, on p. 113, and in A. Borst, *Die karolingische Kalenderreform* (as n. 12 above), p. 284, with commentary on pp. 478-9. Extracts (2) and (3) occur together in Paris, BNF, lat. 5543, fol. 130v; Paris, BNF, lat. 5239, fol. 123r; Strasbourg, Bibl. Nationale et Universitaire, 326, fol. 125r (on these three manuscripts, see below); and London, BL, Harley 3091 (s. ix, Nevers), fol. 21v. Lohrmann also notes extract (2) in a manuscript that I have not seen: Valenciennes, Bibl. Municipale, 343 (s. ix, St-Amand), fol. 29r. Extracts (1) and (2) are taken from Borst's edition. Extract (3) is my edition based on the above-mentioned manuscripts. Paris, BNF, lat. 5543; BNF, lat. 5239 and Strasbourg 326 (which derive from the same model, as will be shown below) give 'DCCXCIIII' as a date in extract (3), but this is evidently a misreading of 'DCCXCVII' (given in Harley 3091), as the year 797 (not 794) corresponds to the 5th indiction. In extract (3), the name of the planet staying in Gemini is omitted in all the manuscripts but it must be Saturn. (1) In 781, Mars and Venus are found together in Cancer from 5 August to 3 September, but Jupiter travelled then from Leo 7° to 13°. (2) On 1 January 770, Saturn and Jupiter were not in Cancer but they were together in the following sign, at Leo 9° and 19° respectively. (3) During March 797, Saturn

available data allow us to settle with certainty whether these positions were observed or somehow computed. Whatever the case, the examples of Rabanus Maurus and the *Annales regni Francorum* show that it was possible both to obtain precise positions of the luminaries by means of elementary rules of computus and to observe the position of the planets (at least Saturn, Jupiter and Mars) in the correct signs.

2 *In quo signo versetur Mars?*

But the determination of the planetary longitudes was not limited to these possibilities. There is another method which is found in a short text, without attribution or title, opening with the words 'In quo signo versetur Mars...'. This text was noted in 1936 by André Van de Vyver, who identified it as a source of the *Liber Alchandrei* (on which see below) and gave a list of manuscripts in which it occurs; but it seems not to have received any attention since.[23] The *In quo signo versetur Mars* (hereafter *IQSVM*) consists of five chapters describing a handy method for computing the position of each planet (Mars, Jupiter, Saturn, Venus and Mercury) in the zodiac for any time, past, present or future. This method has nothing to do with the usual procedures; instead, it is based on an empirical combination of the following three elements: (1) the position of the planets at the creation

travelled from Cancer 0° to 1°, Jupiter was stationnary at Sagittarius 0°, Mars was retrograde from about Cancer 21° to 17°, and Venus travelled from about Taurus 0° to 29°. As such, these positions are rather inaccurate. If, however, we take into account the precessional shift (see n. 7 above), all the positions reported in 797 would be correct (Saturn would be found in Gemini and Jupiter in Scorpio), and this could also apply, to some extent, to the positions given in 781 and 770. On 15 August 781, for instance, Jupiter was at Leo 9°, and Mars and Venus were in conjunction at Cancer 10°; the three planets could therefore have been seen around that date in the constellation of Cancer. It should be noted that Venus was clearly visible both in August 781 and in March 797, since it was at its maximal elongation from the Sun.

[23] A. Van de Vyver, 'Les plus anciennes traductions latines médiévales (Xe-XIe siècles) de traités d'astronomie et d'astrologie', *Osiris*, 1, 1936, pp. 658-91 (pp. 670-3). A brief description of the text is given in D. Juste, 'Les doctrines astrologiques du *Liber Alchandrei*', in *Occident et Proche-Orient: contacts scientifiques au temps des Croisades (Actes du colloque de Louvain-la-Neuve, 24 et 25 mars 1997)*, eds I. Draelants, A. Tihon and B. van den Abeele, Turnhout, 2000, pp. 277-311 (pp. 289-90, see also pp. 284-5).

of the world, (2) the zodiacal periods of the planets and (3) the time elapsed since the creation of the world. The procedure to determine the position of Saturn, for instance, is as follows (see edition in Appendix I):

1. One has to divide the number of years elapsed from the creation of the world by 30 (30 years = zodiacal period of Saturn).

2. The remainder is to be converted into months (i.e. multiplied by 12).

3. This result is to be distributed along the zodiac from Capricorn (= the sign of Saturn at the creation of the world) at the rate of 30 months per sign (since Saturn travels along the zodiac over a period of 30 years, it traverses a single sign in 30 months). The sign in which the last unit, i.e. a group of 30 months or fewer, falls is the sign of Saturn.

The procedure is the same for the other planets, taking into account, of course, the necessary adjustments due to their original sign and zodiacal periods. These two pieces of data require some brief comments. The distribution of the original signs (Mars in Scorpio, Jupiter in Sagittarius, Saturn in Capricorn, Venus in Libra and Mercury in Virgo) is identical to the diurnal domiciles of the planets as found both in astrological texts and in the ancient *thema mundi* or *genitura mundi* transmitted to the Latin world by Macrobius and Firmicus Maternus.[24] On the other hand, the zodiacal periods are problematic; for, with the exception of Saturn, they do not correspond to anything 'real' and appear to be arithmetically based on multiples of six following the arrangement of the planetary week: Mars 6 years, Mercury 12 years, Jupiter 18 years, Venus 24 years, Saturn 30 years. These values come into conflict with the 'true' mean periods of the planets, widely known in the early Middle Ages through the works of Pliny, Macrobius, Martianus Capella and Bede, as well as with the values given by Isidore of Seville, as indicated in the table below:[25]

[24] Macrobius, *Commentarii in Somnium Scipionis*, I.21.24; Firmicus Maternus, *Mathesis*, III.1.1. On the *thema mundi*, see A. Bouché-Leclercq, *L'astrologie grecque*, Paris, 1899, pp. 185-96.

[25] Pliny, *Historia naturalis*, II.32-9 (and Bede, *De natura rerum*, 13); Macrobius, *Commentarii in Somnium Scipionis*, 19.3-4; Martianus Capella, *De*

	Pliny–Bede	Macrobius	Martianus Capella	Isidore of Seville
Saturn	30 years	30 years	30 years	30 years
Jupiter	12 years	12 years	12 years	12 years
Mars	*ca.* 2 years	2 years	almost 2 years	15 years
Venus	348 days	*ca.* 1 year	*ca.* 1 year	9 years
Mercury	339 days	*ca.* 1 year	almost 1 year	20 years

It should be noted that the only basic data not provided in the text is the number of years elapsed from the creation of the world. But this is not a problem, for the chronology from the Creation to the Incarnation (to which the year of the computation is to be added) was a well-known element in early medieval computus, although the value differed depending on the translation of the Bible in use. The two main figures were 5199 years according to the Greek Septuagint and 3952 years according to the Hebrew tradition.[26]

The *IQSVM* occurs among astronomical material in nine manuscripts from the ninth to the twelfth century.[27] These are, in chronological order:

M Melk, Stiftsbibl., 412 (*olim* 370, *olim* G.32), pp. 29-30. This manuscript was copied around 840 at Auxerre and belonged to Heiric of Auxerre, who annotated it until *ca.* 875.[28]

nuptiis Philologiae et Mercurii, VIII.879-86; Isidore of Seville, *De natura rerum*, 23.4 and *Etymologiae*, III.66.2 (Isidore also gives 19 years for the Sun and 8 years for the Moon).

[26] See e.g. Bede, *De temporibus*, 22: 'conpletis ab Adam annis III.DCCCCLII, iuxta alios V.CXCVIIII' (ed. C. W. Jones, Turnhout, 1980 (= *Corpus Christianorum. Series Latina*, 123C), p. 607); Rabanus Maurus, *De computo*, 55: 'Secundum hebraicam ueritatem anni III milia DCCCCLVI (*sic*), secundum uero septuaginta interpretes anni V milia CXCVIIII' (ed. W. M. Stevens, pp. 281-2). On these chronological questions, see R. Landes, 'Lest the Millenium be Fulfilled: Apocalyptic Expectations and the Pattern of Western Chronography 100-800 CE', in The *Use and Abuse of Eschatology in the Middle Ages*, eds W. Verbeke, D. Verhelst and A. Welkenhuyzen, Leuven, 1988, pp. 137- 211.

[27] Van de Vyver (see n. 23 above) did not know Strasbourg 326 and Vatican City, BAV, Reg. lat. 123. I have also found a late copy in Vatican City, BAV, Urb. lat. 102 (s. xv), fols 192vb-193ra.

[28] T. Sickel, 'Un manuscrit de Melk venu de S. Germain d'Auxerre', *Bibliothèque de l'Ecole des Chartes*, 23, 1862, pp. 28-38; A. Van de Vyver, 'Les plus anciennes traductions...' (as n. 23 above), p. 671; B. de Gaiffier, 'Le calendrier

P Paris, BNF, lat. 5543, fols 131v-132r. This well-known manu script of computus and astronomy copied at Fleury is generally dated to the mid-ninth century (the date '847' appears on fol. 120v), but it could have been written in several stages until the first half of the tenth century.[29]

V Vatican City, BAV, Reg. lat. 309, fol. 3r. The manuscript, copied at the monastery of Saint-Denis, mainly belongs to the second half of the ninth century, but our text, together with two chapters on eclipses, has been added by a late ninth- or early tenth-century hand on fol. 3r, originally blank. The manuscript contains the most complete copy of the 'Seven-Book Computus' of 809.[30]

P_1 Paris, BNF, lat. 5239, fol. 124r. Another well-known manuscript of computus and astronomy, copied at Saint-Martial of Limoges in the middle or in the second half of the tenth century.[31]

V_1 Vatican City, BAV, Reg. lat. 596, fol. 51v. This is a composite manuscript, of which fols 10-23 were copied at Fleury at the beginning of the eleventh century. Fols 46-51, written in the tenth century in France—but not at Fleury according to Marco Mostert—, contain a treatise of computus and astronomy in 40

d'Héric d'Auxerre du manuscrit de Melk 412', *Analecta Bollandiana*, 77, 1959, pp. 392-425. C. Glassner, *Inventar der Handschriften des Benediktinerstiftes Melk*, I: *Von den Anfängen bis ca. 1400*, Vienna, 2000, pp. 191-3.

[29] See most recently B. Munk Olsen, *L'étude des auteurs classiques latins aux XI^e et XII^e siècles*, II, Paris, 1985, p. 264; and M. Mostert, *The Library of Fleury. A Provisional List of Manuscripts*, Hilversum, 1989, pp. 207-208. Both agree on the mid-ninth century as a date, but an unpublished and very detailed notice at the Institut de Recherche et d'Histoire des Textes (IRHT, Paris) indicates the tenth century for fols 129-45. See also A. Van de Vyver, 'Les plus anciennes traductions...' (as n. 23 above), p. 672, n. 66.

[30] F. Saxl, *Verzeichnis astrologischer und mythologischer illustrierte Handschriften des lateinischen Mittelalters*, I: *In römischen Bibliotheken*, Heidelberg, 1915, pp. 59-66; A. Wilmart, *Codices Reginenses Latini*, II, Roma, 1945, pp. 160-74; B. Munk Olsen, *L'étude des auteurs classiques...* (as n. 29 above), II, p. 270; D. Nebbiai-Dalla Guarda, *La bibliothèque de l'abbaye de Saint-Denis en France du IXe au XVIIIe siècle*, Paris, 1985, p. 231. For the chapters on eclipses, see n. 41 below.

[31] R. Derolez, *Runica manuscripta. The English Tradition*, Bruges, 1954, pp. 329-32; B. Munk Olsen, *L'étude des auteurs classiques...* (as n. 29 above), I, pp. 408-409; R. Landes, *Relics, Apocalypse, and the Deceits of History.* Ademar de Chabannes, 989-1034, Cambridge, Mass.-London, 1995, pp. 346-9.

chapters of which our text forms chapters 36-40.[32]

S Strasbourg, Bibl. Nationale et Universitaire, 326 (s. x-xi, Angoulême), fols 125v-126r.[33]

V_2 Vatican City, BAV, Reg. lat. 123, fols 170r-174r. This manuscript was copied in 1056 in Catalonia, most probably at the monastery of Ripoll. It contains a compilation in four books entitled *De Sole*, *De Luna*, *De natura rerum* and *De astronomia*, of which this manuscript seems to be the only copy.[34] Our text is divided among a group of chapters devoted to the planets on fols 170r (Mars), 171r (Mercury), 171v (Jupiter), 173r (Venus) and 174r (Saturnus).

P_2 Paris, BNF, lat. 12117 (*ca.* 1063, Saint-Germain-des- Prés), fol. 130r.[35]

V_3 Vatican City, BAV, Vat. lat. 643 (s. xii, Melk), fol. 71r.[36]

As we can see, the *IQSVM* existed in Auxerre in the first half of the ninth century and circulated principally in France (including

[32] A. Guerreau-Jalabert, *Abbon de Fleury: Questions grammaticales*, Paris, 1982, pp. 199-200; M. Mostert, *The Library of Fleury...* (as n. 29 above), pp. 272-3; *Codices Boethiani. A Conspectus of Manuscripts of the Works of Boethius*, III: *Italy and the Vatican City*, eds M. Passalacqua and L. Smith, London-Turin, 2001, pp. 486-8. There is also an unpublished and very detailed notice on this manuscript at the IRHT.

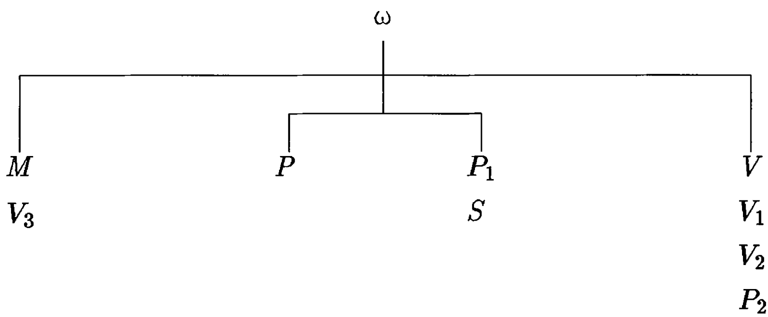
[33] R. Derolez, *Runica manuscripta...* (as n. 31 above), pp. 332-4; C. Leonardi, 'I codici di Marziano Capella', *Aevum*, 34, 1960, pp. 1-99 and 411-524 (p. 453); B. Munk Olsen, *L'étude des auteurs classiques...* (as n. 29 above), I, p. 409, and II, p. 269.

[34] F. Saxl, *Verzeichnis astrologischer...* (as n. 30 above), pp. 45-59; A. Wilmart, *Codices Reginenses Latini*, I, Roma, 1937, pp. 289-92; E. Pellegrin, *Les manuscrits classiques latins de la Bibliothèque Vaticane*, II.1, Paris, 1978, pp. 35-8; B. Munk Olsen, *L'étude des auteurs classiques...* (as n. 29 above), I, pp. 533-4; G. Puigvert, 'El manuscrito Vat. Reg. 123 y su posible adscripcion al scriptorium de Santa Maria de Ripoll', in *Roma, magistra mundi. Itineraria culturae Medievalis*, ed. J. Hamesse, III, Louvain-la-Neuve, 1998, pp. 285-316; A. García Avilés, *El tiempo y los astros. Arte, ciencia y religión en la alta edad media*, Murcia, 2001, pp. 107-115 and 129-43.

[35] C. Leonardi, 'I codici...' (as n. 33 above), p. 441; M. Vieillard-Troiekouroff, 'Art carolingien et art roman parisiens. Les illustrations astrologiques jointes aux Chroniques de Saint-Denis et de Saint-Germain-des-Prés (IXe-XIe siècles)', *Cahiers Archéologiques*, 16, 1966, pp. 77-105; B. Munk Olsen, *L'étude des auteurs classiques...* (as n. 29 above), II, p. 268.

[36] M. Vatasso, *Codices Vaticani Latini*, I, Roma, 1902, pp. 493-4; F. Saxl, *Verzeichnis astrologischer...* (as n. 30 above), pp. 70-71.

Catalonia) until the second half of the eleventh century. Variant readings in the text allow us to state more precisely the relationship between the manuscripts. M is clearly the model of V_3.[37] P_1 could be the model of S,[38] and both manuscripts have in common with P the omission of 'signo' in chapter 1, although it seems unlikely that P_1S derive from P.[39] $VV_1V_2P_2$ form a distinct family, characterized by several divergent readings and the addition of a title to each chapter.[40] Thus, a plausible *stemma* would be the following:



[37] Both agree on most readings, including mistakes such as 'parte faciet' for 'patefaciet' (chap. 4 in fine). The close relationship between the two manuscripts had already been established by A. Van de Vyver, 'Les plus anciennes traductions...' (as n. 23 above), p. 671, and C. W. Jones, *Bedae Pseudepigrapha: Scientific Writings Falsely Attributed to Bede*, Ithaca-London, 1939, p. 30.

[38] As suggested by several common spellings and mistakes, as well as by the final addition 'Virgo latitudinem signiferi scandes' (chap. 5).

[39] Two variants in P have not affected P_1S ('signatis' instead of 'assignatis' in chap. 1, 'annis' omitted in chap. 2). The relationship between the three manuscripts, which have also preserved the planetary positions of 770 and 797 (see n. 22 above), has long been recognized, see in particular C. W. Jones, *Bedae Pseudepigrapha...* (as n. 37 above), p. 128 (and references cited there), and his introductions to the edition of Bede's works, 3 vols., Turnhout, 1975-80 (= *Corpus Christianorum. Series Latina*, 123A-B-C), pp. 179, 182, 250, 253, 634 and 656; R. Derolez, *Runica manuscripta...* (as n. 31 above), pp. 329, 333 and 349; V. H. King, *An Investigation of Some Astronomical Excerpts from Pliny's Natural History Found in Manuscripts of the Earlier Middle Ages*, M. Phil. Thesis, Oxford, 1969, pp. 106, 108 and 110-1.

[40] These variants are: 'totidem' (instead of 'tot') in chap. 2 and 5, 'non congruentium' (instead of 'congruentium') in chap. 4 and 'assignandi' (instead of 'assignando') in chap. 5. The titles (omitted in V_2) are *In quo duodecim signorum Mars habeatur, In quo feratur Iuppiter, In quo Saturnus consistat, In quo moretur Venus* and *In quo meet Mercurius*. On the relationship between V and P_2, see also M. Vieillard- Troiekouroff, 'Art carolingien...' (as n. 35 above), *passim*.

Mistakes of copying in *M* make it clear that this manuscript cannot be the archetype of the text, and there is evidence that it derives from a model copied between 808 and 832, two dates corresponding to the *termini* of a short text on eclipses which immediately precedes the *IQSVM* in this manuscript as well as in VV_1V_3.[41] We have otherwise no clue as to the early history of the text, but two facts might suggest that it was not current before the first half of the ninth century. First, no known earlier records of planetary positions, all of which are dated from 770 to 820 as we have seen, were computed by means of this method. Second, the text is absent from the 'Seven-Book Computus' of 809, which includes virtually all the material on astronomy—including planetary astronomy— available at that time.[42]

The question arises as to whether such a curious method, based on zodiacal periods which conflict with those given in authoritative works, was actually practiced by early medieval scholars. The fact that it was copied in astronomical manuscripts is perhaps enough to prove that it was used; but better evidence is provided by two other examples.

3 *August 978: Abbo of Fleury computes the position of Mars*

At about the time when the *Preceptum Canonis Ptolomei* was being copied in Fleury, the abbot of the monastery was Abbo

[41] See T. Sickel, 'Un manuscrit de Melk...' (as n. 28 above), p. 32. The text in question is based on a 24-year cycle starting in 808: 'Solis eclipsis quemadmodum innotescat (title in VV_1 only). Deliquium Solis contigisse fertur anno ab incarnatione domini DCCCVIII (DCCCVII V), quicquid ergo inde aut abscessit annorum per XXIIII diuidatur et semper, ut nonnullis placet, XXIIII anno eclipsis inuenietur. Quo argumento sit uestigandum (title in VV_1 only). Eclipsis (EKΛYCIC *V*) etiam Lunae nosci posse existimatur si cognito solari deliquio inde CLXXVII dies computentur. Namque si sub ea summa XIIII Luna occurrerit, lunarem defestum (*sic*) pariter euenturum. Sin autem rursum CLXXVII alios supputandos donec eis aetas XIIII Lunae concordet, quod quotiens contingerit totiens eclipsin non defuturam (defecturam *M*)' (my edition, based on the four manuscripts; only significant variants are noted).

[42] On this still unpublished work, see A. Borst, 'Alkuin und die Enzyklopädie von 809', in *Science in Western and Eastern Civilization...* (as n. 22 above), pp. 53-75, and the detailed table of contents (from Vatican City, BAV, Reg. lat. 309) given in F. Saxl, *Verzeichnis astrologischer...* (as n. 30 above), pp. 60-65.

(988-1004), one of the most learned men of his time, who is generally considered the equal of Gerbert of Aurillac in his knowledge of the sciences of the *quadrivium*, including astronomy.[43] According to his biographer, Aimoin of Fleury (†after 1008), he is the author of some *disputationes* on the course of the Sun, the Moon and the planets.[44] These *disputationes* must be identified with the *De ratione spere* or *Sententia Abbonis de ratione spere* (inc.: 'Studiosis astrologiae primo sciendum est...'), a treatise in two parts which Abbo wrote in 978, when he was the schoolmaster in Fleury.[45] This treatise turns out to be the first Latin text of the Middle Ages entirely devoted to planetary astronomy. In the first part, Abbo gives a general description of the heavenly sphere and discusses a number of features pertaining to the planets drawn from Pliny and other sources (motions, *absides*, colours, elongation, distinction between planets and fixed stars, latitude, harmonic intervals, distance from the Earth to the Moon and the Sun), while the second part provides the basic data to compute their position. As far as the luminaries are concerned, Abbo's discussion offers no departure from the standard computus: the age of the Moon allows one to determine its position with respect to the Sun; and the position of the Sun is to be inferred from its

[43] On Abbo's scientific works and teaching, see A. Van de Vyver, 'Les œuvres inédites d'Abbon de Fleury', *Revue Bénédictine*, 47, 1935, pp. 125-69; G. R. Evans and A. M. Peden, 'Natural Science and the Liberal Arts in Abbo of Fleury's Commentary on the Calculus of Victorius of Aquitaine', *Viator*, 16, 1985, pp. 109-27; E.-M. Engelen, *Zeit, Zahl und Bild: Studien zur Verbindung von Philosophie und Wissenschaft bei Abbo von Fleury*, Berlin and New York, 1993; B. Eastwood, 'Calcidius's Commentary on Plato's *Timaeus* in Latin Astronomy of the Ninth to Eleventh centuries', in *Between Demonstration and Imagination. Essays in the History of Science and Philosophy Presented to John D. North*, eds L. Nauta and A. Vanderjagt, Leiden, 1999, pp. 171-209 (pp. 178-86).

[44] 'de Solis quoque ac Lunae seu planetarum cursu a se editas disputationes posterorum mandavit notitiae' (Aimoin of Fleury, *Vita s. Abbonis*, ed. J.-P. Migne, *Patrologia Latina*, 139, col. 390D).

[45] Ed. R. B. Thomson, 'Two Astronomical Tractates of Abbo of Fleury', in *The Light of Nature. Essays in the History and Philosophy of Science Presented to A.C. Crombie*, eds J. D. North and J. J. Roche, Dordrecht-Boston-Lancaster, 1985, pp. 113-33. On this treatise, see also A. Van de Vyver, 'Les œuvres inédites...' (as n. 43 above), pp. 140-50; B. Eastwood, 'Astronomy in Christian Latin Europe *c.* 500 - *c.* 1150', *Journal for the History of Astronomy*, 28, 1997, pp. 235-58 (pp. 251-3); S. McCluskey, *Astronomies and Cultures...* (as n. 1 above), pp. 152-3.

regular motion throughout the signs, starting with the entry into Aries on the 15th kalends of April (the exact hour of entry into the signs is even given according to the leap year cycle).[46] The last section is devoted to the five planets:[47]

A Scorpione Mars VI:	sexies m[ille] XXVIIII, sunt VI [milia] CLXXIIII.
A Sagittario Iuppiter XVIII:	decies octies CCCXLII sunt VI [milia] CLVI.
A Capricorno Saturnus XXX:	tricies CCV sunt VI [milia] CL.
A Libra Venus XXIIII:	vicies quater CCLVII sunt VI [milia] CLXVIII.
A Virgine Mercurius XII:	duodecies DXIIII sunt VI [milia] CLXVIII.

Signa in quibus singuli planetarum morentur scies si annos ab initio mundi secundum LXX per numeros hic eisdem planetis subiectos diviseris, hoc tantum retinens quod annorum quos idem numerus dividere non poterit sint colligendi menses et, ipso numero cuilibet planete subiecto, metiendi sumpto initio a signo ei prenotato de quo scire volueris.

Ut si de Marte vis, quia VI [milia] CLXXVII anni sunt, sexies mille sunt VI [milia] et sexies XXIX sunt CLXXIIII. Supersunt hoc tertio anno in mense Augusto menses XXX, quia hos annos in mense Martio cepimus et finivimus et sexies V [= quini] sunt XXX. In quinto igitur a Scorpione signo, hoc est in Piscibus, Martem modo esse credimus. Similiter de reliquis.

Mars 6 from Scorpio:	$6 \times 1029 = 6174$
Jupiter 18 from Sagittarius:	$18 \times 342 = 6156$
Saturn 30 from Capricorn:	$30 \times 205 = 6150$
Venus 24 from Libra:	$24 \times 257 = 6168$
Mercury 12 from Virgo:	$12 \times 514 = 6168$

You will know the signs in which each of the planets stays if you divide the <number of> years from the beginning of the world according to the Septuagint by the numbers here ascribed to the corresponding planets, keeping in mind only that the number of years which cannot be divided is the <number of> months to be taken into account and (this number having been ascribed to the <correct> planet) to be distributed <along the zodiac> starting

[46] *De ratione spere*, l. 98-122 for the Moon, and 123-69 for the Sun (ed. R. B. Thomson, 'Two Astronomical Tractates...' (as n. 45 above), pp. 127-31).

[47] *De ratione spere*, l. 170-85 (ed. pp. 132-3). I have slightly modified Thomson's punctuation.

> from the sign ascribed above to that <planet> whose sign you want to know.
>
> For example, if you want <to make the computation> for Mars, since there are 6177 years, 6 × 1000 = 6000 and 6 × 29 = 174. In this third year, in the month of August, there remain 30 months, for we have begun and finished these years in the month of March and 6 × 5 = 30. Then we believe that Mars is at the moment in the fifth sign from Scorpio, that is Pisces. The same <applies> to the other <planets>.

Abbo's source is immediately obvious. The basic data and all the details of the procedure are identical to those of the *IQSVM*. Abbo first sets up a table containing the original signs and zodiacal periods of the planets, as well as the planetary cycles valid for the time of the computation, i.e. from *annus mundi* 6174 (beginning of the cycle of Mars) to 6180 (end of the cycles of Mars, Saturn and Mercury).[48] In the second paragraph, he summarizes the procedure and makes it explicit that the chronology to be used is that of the Septuagint style (= 5199 years). Finally, he computes the position of Mars for the month of August 6177 (6177 - 5199 = A.D. 978):[49]

1. 6177 : 6 = 1029, remainder: 3 (Mars completed 1029 cycles of 6 years in 6174 years, 6177 - 6174 = 3).

2. Given that the year begins in March (Abbo uses the Annunciation style, i.e. 25 March) and that the current month

[48] As Thomson pointed out ('Two Astronomical Tractates...' (as n. 45 above), p. 132, commentary on lines 170-4), Abbo made a mistake in calculating the current cycle of Jupiter. It should read '18 × 343 = 6174' instead of '18 × 342 = 6156'. As set up by Abbo, the table is valid for *annus mundi* 6174 only. It should be noted that a similar table, containing the original signs and zodiacal periods of the planets, was included in Abbo's *Computus*, at least in two manuscripts: Berlin, Staatsbibl., Phill. 1833 (s. x-xi), fol. 38r, and Vatican, BAV, Reg. lat. 1573 (s. xi), fol. 52r, as well as in a very corrupt form in the printed version of the *Patrologia Latina*, 90, col. 228. I thank Bruce Eastwood for calling my intention to this; see his 'Calcidius's Commentary...' (as n. 43 above), p. 183 n. 24 (with a different interpretation of the purpose of this table) and p. 203 (for a reproduction of the relevant page of the Berlin manuscript).

[49] This allowed Van de Vyver to date the text ('Les œuvres inédites...' (as n. 43 above), p. 146); see also Thomson's comments on line 180 ('Two Astronomical Tractates...' (as n. 45 above), p. 132). The final sentence ('Martem modo esse credimus') makes it clear that August 978 is the current ('modo') date.

is August, the number of months is 24 (= 2 years) + 6 (6 months from March to August) = 30.

3. Since Mars takes 6 months to cross a single sign, in 30 months it traverses 5 signs, from Scorpio (original sign). Scorpio + 5 signs = Pisces.

This calculation is, in fact, not correct. Abbo misinterpreted the value of the remainder (3) which means three completed years in the cycle of Mars, rather than the third year, as he states ('hoc tertio anno'). From March 6174 to August 6177, there are 42 months, not 30, and the result should be Taurus instead of Pisces.

In his commentary on this section of the *De ratione spere*, Stephen McCluskey pointed out that such a calculation, which 'would often find the two inferior planets opposite to the Sun', was in contradiction to Abbo's previous statement about the maximal elongation of Mercury and Venus.[50] If Abbo noticed this contradiction, he said nothing as to how to accommodate the two theories. What is clear is that he took this system seriously and not just as something thrown in to entertain the reader. Doubtless the Fleury schoolmaster accepted the validity of the method, for he appears to have arranged certain data in order to give a better foundation for it. In the first part of his treatise, he carefully avoided mentioning the zodiacal periods of the planets current in his time—a piece of data that, as we have seen, he could hardly have failed to notice. But more significant is the fact that he altered the well-known concept of *absides*, by which Pliny, Martianus Capella and Bede had designated the signs where the planets reach their apogee (i.e., the remotest point from the Earth), in order to refer to the signs in which the planets stood at the Creation.[51] This shift might result partly from a reading of Mac-

[50] S. McCluskey, *Astronomies and Cultures...* (as n. 1 above), p. 153. Following Pliny, Abbo states that the elongation of Mercury is 32° (22° in Pliny, *Historia naturalis*, II.39) and that of Venus 2 signs at most (46° in Pliny, II.38): 'Mercurius..., qui numquam a Sole plus XXXII partibus poterit elongare' (*De ratione spere*, l. 32-3, ed. R. B. Thomson, p. 121); 'Mercurius numquam plus uno nisi duabus partibus; Venus vero plus uno signo sed numquam plus duobus recedit a Sole' (*ibid.*, l. 63-5, pp. 122-3). McCluskey (p. 152) believes that Abbo is the inventor of this method.

[51] 'Absides autem dicimus, loca ubi in principio conditi sunt planete, ut Mars in Scorpione, Iuppiter in Sagittario, Saturnus in Capricorno, Venus in

robius, as has been suggested,[52] but the omission of the luminaries as well as the unusual order of the planets (Mars, Jupiter, Saturn, Venus, Mercury) in Abbo's statement make it clear that his direct source was the *IQSVM*.

4 *The Alchandreana and the introduction of a variant version of computation*

The second context in which the *IQSVM* occurs is another late tenth-century text which I have already alluded to, the *Liber Alchandrei*. The text belongs to the oldest collection of astrological treatises of Arabic origin available in the West: the *Alchandreana*. This collection came to the Latin world from Spain and was fully elaborated in Catalonia, at least partly at the monastery of Ripoll, during the second half of the tenth century. Some of the texts appear to be direct translations or adaptations from Arabic, while others consist of new, more elaborate versions of the same material by different author.[53] The *Liber Alchandrei* belongs to this second stage and is distinctive for having been influenced by Latin science.[54] Its author knew of some computus sources, quoted passages from Martianus Capella and Pliny, and reproduced in full the *IQSVM* (= *Liber Alchandrei*, chap. 10), with

Libra, Mercurius in Virgine' (*De ratione spere*, l. 53-5, ed. R. B. Thomson, p. 122). Compare with Pliny, *Historia naturalis*, II.63-5; Martianus Capella, *De nuptiis Philologiae et Mercurii*, VIII.884-6; Bede, *De natura rerum*, 14. On the significance and importance of the concept of absides in the early Middle Ages, see B. Eastwood, 'Plinian Astronomical Diagrams in the Early Middle Ages', in *Mathematics and its Applications to Science and Natural Philosophy in the Middle Ages*, eds E. Grant and J. Murdoch, New York, 1987, pp. 141-72 (pp. 145-6 and 153-7).

[52] R. B. Thomson, 'Two Astronomical Tractates...' (as n. 45 above), p. 122, commentary on lines 53ff.; E.-M. Engelen, *Zeit, Zahl und Bild...* (as n. 43 above), p. 76; B. Eastwood, 'Astronomy in Christian Latin Europe...' (as n. 45 above), p. 251; S. McCluskey, *Astronomies and Cultures...* (as n. 1 above), p. 152.

[53] The corpus is studied and edited in D. Juste, *Les* Alchandreana *primitifs. Recherches sur les plus anciens traités astrologiques latins d'origine arabe (Xe siècle)* (forthcoming in the *Collection des Travaux de l'Académie Internationale d'Histoire des Sciences*, Turnhout). All passages from the *Alchandreana* cited hereafter are taken from this study. See also A. Van de Vyver, 'Les plus anciennes traductions...'(as n. 23 above), pp. 666-84, and D. Juste, 'Les doctrines astrologiques...' (as n. 23 above), *passim*.

[54] D. Juste, 'Les doctrines astrologiques...' (as n. 23 above), pp. 288-90.

an introductory sentence accommodating the material to the astrological context: 'Per has autem VII planetas quia, ut diximus et adhuc probabimus, humana fata disponuntur, regulam certam demus, qua in quo signo quaeque sit pernoscatur'. The text has otherwise been copied with only minor changes and, not surprisingly, it shares all the divergent readings of the group $VV_1V_2P_2$, which includes the copy made in Ripoll in 1056 (V_2).[55]

The insertion of the *IQSVM* into an astrological text does not require a long explanation. Due to the lack of astronomical tables, the method of the 'years of the world' represented for Western scholars of the tenth century the only way of computing the planetary longitudes, an element which appears indispensable in various places in the *Alchandreana*, notably in the prognostics based on the position of the planets in the signs and in the twelve mundane houses. But what is curious is that the corpus includes a variant version of the method of the 'years of the world'. This variant occurs in chapter 25 of the *Liber Alchandrei* and in two other texts: the *Proportiones competentes in astrorum industria* (chap. 42.14-23) and the *Quicumque nosse desiderat legem astrorum...* (chap. 14). The *Liber Alchandrei* and the *Proportiones* give almost verbatim the same version (though the latter is more complete) and all three texts basically agree with regard to the contents (see editions in Appendix II). For convenience, I shall distinguish between the method of the *IQSVM* and this variant by calling them 'System A' and 'System B' respectively.

[55] Variants between the *Liber Alchandrei* and the *IQSVM* as edited in Appendix I are: 1. Scorpio] Scorpione. 2. *Title:* In quo signo Iuppiter; Eisdem] Ex eisdem; partem *omitted*; tot] totidem; ut quorundam fert opinio *omitted*; non occurrerint] tribuendi defecerint; morari] morari dicunt. 3. *Title:* In quo signo Saturnus; transgressi] transgressi numerum; mensibus] menses; cognoscatur] cognoscantur; caetera deinde] deinde caetera; denique] deinde. 4. *Title:* In quo signo Venus; parte separare] separare parte; hinc] huic; congruentium] non congruentium. 5. *Title:* In quo signo Mercurii; Quin] Qui; ac *omitted*; tot] totidem; defuerint assignado XII] XII assignandi defuerint; inueniri] putant inueniri. Chapter 10 of the *Liber Alchandrei* is preserved in the following manuscripts: Paris, BNF, lat. 17868, s. x, fol. 3r-v; Munich, Bayerische Staatsbibl., Clm 560, s. xi, fols 63r-64r; London, BL, Addit. 17808, s. xi, fols 86v-87r; Orléans, Bibl. Municipale, 282, s. xi/xii, pp. 11-14 and 3^{bis}; Los Angeles (*olim* Malibu), J. Paul Getty Museum, Ludwig XII.5, s. xii, fols 78v-79r; London, BL, Cotton Appendix VI, s. xiii, fols 38va-39ra; Bernkastel-Kues, Cusanusstiftsbibl., 214, s. xiv, fol. 21r-v; Vatican City, BAV, Vat. lat. 4084, s. xiv, fol. 8v; Vatican City, BAV, Pal. lat. 1416, s. xv, fol. 119r-v.

'System B' essentially differs from 'System A' by giving alternative basic data. Here the original signs of the planets do not offer an altogether coherent astronomical or astrological picture: Saturn is in Libra, Jupiter in Aries, and the texts are in complete disagreement regarding the positions of Mars (Scorpio is given in the *Liber Alchandrei*, Capricorn in the *Proportiones*, Leo in the *Quicumque*) and Venus (Capricorn in the *Liber Alchandrei*, Libra in the *Proportiones*, Pisces in the *Quicumque*). On the other hand, the zodiacal periods correspond to the 'true' mean periods of the planets (Saturn 30 years, Jupiter 12 years, Mars $1\frac{1}{2}$ year, Venus 300 days). These values are not convenient to use in the process of calculation, and it is probably for this reason that the remainder of the first division, instead of being converted into months, is expressed in years or days corresponding to the period of transit of the planets in one sign (Saturn $2\frac{1}{2}$ years, Jupiter 1 year, Mars 45 days, Venus 25 days). Moreover, in the case of Venus, the computation becomes more complicated: one has (1) to divide the number of years from the creation of the world by 8; (2) to divide the remainder, converted into days, by 300; (3) to add 40 to the remainder; and (4) to count the result in the order of the signs starting from Libra at the rate of 25 days per sign. If the figures 300 and 25 days are easily recognizable, it is not clear what the initial division by 8 and the addition of 40 days refer to.[56] Finally, the procedure for Mercury is omitted in both the *Liber Alchandrei* and the *Quicumque*, but the author

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[56] The account of the procedure for Venus is that of the *Proportiones* (42.21), which preserves the most complete version. There are some variations in the two other texts. The *Liber Alchandrei* (25.6) omits steps 1 and 3. The author of the *Quicumque* (14.5) speaks of an initial addition of 60 (years) and omits step 3; he made a mistake here, for he had previously (in chapter 8) described the cycle of Venus as follows: 'Venus igitur moratur in unoquoque signo XXV diebus, et ideo constat percurrere eam signiferum CCC diebus, et expletis semper annis octonis ad unde cursus sui sumit exordium, reuertitur signum Librae [n.b. not Pisces], ita tamen ut in unaquaque annorum ogdoadis LX adiciat dies supra praedictum numerum' (8.7). '60' ('LX') is obviously a misreading of '40' ('XL'), or vice versa. On the cycle of 8 years for Venus, see n. 72 below. It should also be noted that the *Quicumque* gives a similar rule for Mars, which is said to return to the same sign every seven years (chap. 14.3), again in accordance with the description of the cycle of Mars provided in chapter 8: 'Mars denique moratur in unoquoque signo XLV diebus, permeat signiferum in DXL diebus, semper septenis annis peractis ad unde cursus sui sumit exordium, reuertitur signum Leonis' (8.4).

of the *Proportiones* states (chap. 42.23) that the planet should always be found close to the Sun, either in the preceding or in the following sign.[57]

Another feature of 'System B' is that it includes the computation of the longitude of the lunar nodes (i.e. the two intersections of the ecliptic with the lunar orbit, called the 'Head' and 'Tail of the Dragon'). The computation is given, together with a series of astronomical and astrological theories pertaining to the Dragon, in the same texts: *Liber Alchandrei*, 24; *Proportiones*, 42.1-5; and *Quicumque*, 9. Again, the procedure is identical and all three texts agree on the following parameters: the original sign of the Head of the Dragon is Leo, its zodiacal period is 18 years (and $1\frac{1}{2}$ year per sign), its motion is retrograde, and the Tail is always in the opposite sign (see Appendix II).

The author of the *Proportiones* was not satisfied with quoting only the text of 'System B'. Earlier in the treatise, he had devoted one chapter to the computation of the Head of the Dragon, Saturn, Jupiter, Mars and Venus for *annus mundi* 4795, explaining step by step—and successfully throughout—the mathematical procedure (chap. 32).[58] But this chapter provided him with an opportunity to raise doubts about the validity of the method.

[57] An attempt to compute the position of Mercury according to 'System B' appears in Vatican City, BAV, Vat. lat. 4084, a manuscript which has also preserved the *Liber Alchandrei* and the *Quicumque* (see n. 55, 92 and 94): 'Mercurius. Hisdem annis ab initio mundi sumptis, diuide per XXVIII et da unicuique singno (*sic*) XXVIII; cui XXVIII tribuendi defuerit, in eo morari Mercurium pronunciabis, sed a Virgine sumpto initio' (fol. 1r). This paragraph is part of a chapter giving a somewhat different version of 'System B', where the original signs correspond to the astrological exaltations of the planets (Saturn in Libra, Jupiter in Cancer, Mercury in Virgo, Mars in Capricorn, Venus in Pisces, the sign of the Head of the Dragon is omitted). The text is quite corrupt but '28 days' corresponds to the period of transit of Mercury in one sign, as indicated in the Alchandrean texts (*Liber Alchandrei*, 13; *Proportiones*, 1; *Quicumque*, 8). The same attempt to compute the position of Mercury is implied in a closely related manuscript: Vatican City, BAV, Reg. lat. 1324 (see n. 65 below).

[58] Here is the example for Saturn: 'Saturni autem cursum uel in quo signo uersetur ad praesens si forte ignoras, iterum ab initio mundi annos per XXX partire. Et primum de mille, centum sume, et ter sumptos XXX, supersunt X; de mille, tantum supersunt centum; de C, supersunt X; set, ad hunc modum, de IIII milibus, remanent XL; demptis XXX, supersunt X; de septingentis autem, remanent LXX; de septuaginta, X; de XCV, ter sumptis XXX, supersunt V. Residui quidem omnium numerorum simul iuncti, fiunt XXV. Hoc

For Mars, he says that the theory 'seems superstitious'; and for Venus, he points out the contradiction with its bounded elongation (which he estimates too generously at three or four signs) and invites the reader to observe the planet by night, as he himself claims to do.[59] In both cases, nonetheless, he describes the procedure in full.

This appears to be the only instance in which the validity of the method was called into question. A more serious problem was that the reader of the *Liber Alchandrei* is left with two incompatible systems of computation. The author himself said nothing as to which one was to be used, although he seems to have favoured 'System A' which occurs in chapter 10, while 'System B' is relegated in an incomplete form to the second part of the work. This preference was accepted by a scribe of northern France (possibly Fleury) who inserted an *exemplum* of the computation of Mars for the year 1040 or 1048 according to 'System A'.[60] But, apart from

uero per signa distinguens a Libra sumpto initio, cuique duos et dimidium tribue, et ubi defuerit numerus, ibi se receptat Saturnus' (*Proportiones*, 32.7-10). I do not know what the *annus mundi* 4795 corresponds to; moreover there is a problem here, for in the first sentence of the chapter, the author speaks of the year 4715: 'Annos ab initio mundi si uis computando colligere, quatuor milia septingentos quindecim poteris proferre, et tantum per omnia supra memorata sidera planetas scias cucurrisse' (*ibid.*, 32.1).

[59] 'De Marte uero, quoniam uidetur ratio superstitiosa esse, duxi congruum, ne omnino praetermitterem, describere compendiose' (*Proportiones*, 32.15); 'Veneris autem incertior caeteris habetur statio, incertior et difficilior exigitur sui cursus ratio. Verbi gratia. Cum Solem IIII praeoccupat signis, uidetur solito more exequi cursum caeterarum suae anticipationis et sic multos fallit imperitos reciprocando uestigia sui cursus, quia non est fas sibi Solem aut praeterire aut subsequi plus IIII signis. Et ut haec certius scias, disce experimento ipsius rei, ut ego iam didici, et inuenies in tempesta noctis diligenter oculos pascendo me nil mentiri' (*ibid.*, 32.20-2). The warning about the elongation of Venus is repeated in chap. 42.22 (see text in Appendix II).

[60] The *exemplum* is inserted into chapter 10, immediately after the rule for Mars: 'Verbi gratia. Accipe annos ab initio mundi qui modo sunt IIII.DCCCCLXX (?). Iunge his annos domini, qui modo sunt M.XL<VIII?>, et fiunt VI.XVIII anni. Hos, ut praefatum est, per senarium diuide ita: sexies M sex milia reddunt, remanent XVIII anni; bis sex faciunt XII, supersunt VI anni. Et quia annos amplius per senarium diuidere non uales, menses exuberantium annorum qui modo sunt XII applica supradictis signis ita. Da VI menses Scorpioni, VI Sagittario, item VI Capricorno, VI Aquario, VI etiam Piscibus, VI quoque Arieti, nil minus VI Tauro, VI item Geminis, Cancro VI, Leoni VI, Virgini etiam VI et Librae VI. Ecce deest quid Scorpioni iterato tribuas. Unde scias necesse esse in eo commorari Martem per supradictam

that, later versions based on the Alchandreana bear witness to hesitations and disputes about the two systems. A good example is provided by the treatise *De planetarum et signorum ratione*, which was published among the works of Bede, first by Johannes Hervagius (Basel, 1563) and most recently by Jacques-Paul Migne in Volume 90 of his *Patrologia Latina* (cols 941-4). No manuscript is known of this work, but the surrounding texts in Hervagius's and Migne's editions suggest an early date, since they are all prior to the end of the tenth century.[61] Here we find together 'System B' (from *Proportiones*, 42) and 'System A' (from *Liber Alchandrei*, 10, with a different order of planets, starting with Saturn) under the titles *Inquisitiones aliquot planetarum* and *Item de alio modo* respectively. The text is immediately followed (cols 943-6) by a series of rearranged chapters from the *Alchandreana* dealing, *inter alia*, with two different versions of the *thema mundi*, as well as with the domiciles, exaltations and falls of the planets (taken from *Liber Alchandrei*, 6, 9 and 27), in other words, the basic data of the computation.[62] In the copy of the *Liber Alchandrei* preserved in London, BL, Cotton Appendix VI (s. xiii), fols 38va-39rb, chapter 10 (entitled *Modus unus inueniendi planetas*) has been displaced just before chapter 25 (entitled *Alia acceptio*). In a twelfth-century manuscript of computus and astronomy from Hereford (Cambridge, Trinity College, O.7.41), a contemporary or somewhat later scribe added on fol. 59r-v, originally blank, several notes from the *Liber Alchandrei*, among which there is a

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rationem.' This passage is preserved in four manuscripts deriving from a lost copy (London, BL, Addit. 17808 and Cotton Appendix VI; Orléans 282; Vat. Pal. lat. 1416; see n. 55 above for the details). The date (1040 or 1048) cannot be definitely sorted out: the manuscripts agree on the date *annus domini* '1040' (except Cotton App. VI, which gives '1048') while the indicated chronology Creation-Incarnation varies between '4970' (Vat. Pal. lat. 1416), '4978' (Orléans 282) and '3970' (the two London manuscripts). Whatever the case, the calculation was made for *annus mundi* 6018 (although Vat. Pal. lat. 1416 gives '6010') and is correct throughout. This *exemplum* does not seem to be related to Abbo's computation of the position of Mars.

[61] All these texts are discussed in C. W. Jones, *Bedae Pseudepigrapha...* (as n. 37 above), pp. 85-90.

[62] The concordances of Pseudo-Bede's text with the *Alchandreana* were noted by A. Van de Vyver, 'Les plus anciennes traductions ...' (as n. 23 above), p. 669, n. 97, and are supplemented in D. Juste, 'Les doctrines astrologiques...' (as n. 23 above), p. 287, n. 33. On the two versions of the *thema mundi* and related Alchandrean chapters, see *ibid.*, pp. 288-9.

summarized version of the twofold *thema mundi* and of the computation mixing 'System B' (for Jupiter) and 'System A' (for the other planets).[63] In Vatican City, BAV, Reg. lat. 1324, written in the fifteenth century but reflecting a very early textual stage of the *Alchandreana*,[64] chapter 14 of the *Quicumque* is followed by eight tables (fol. 45r-v) for the division of numbers corresponding to the periods of the planets, mixing those of 'System A' and 'System B'.[65] Two astrological compilations, preserved in thirteenth- to fourteenth-century manuscripts and based on the *Alchandreana* as well as on more sophisticated material deriving from twelfth-century translations from Arabic, include a chapter devoted to the computation of longitudes and providing for each planet the rules of both systems.[66] Finally, the most spectacular

[63] '<A>nnos ab initio si computes ad presentem in quo es et, reiectis quotiens poteris XXX, quod superest annorum redigas in menses, et incipiens a Capricorno unicuique signorum XXX menses attribuas, ubi distributio defecerit, ibi Saturnum esse estimato. Eosdem si per XII reicias, et quod supererit per menses ab Ariete inchoans cuique signorum distribuas XII, ubi deerit numerus, ibi stellam Iouis attende. Eosdem si per senarium reicias et a Scorpione cuique signorum ex reliquo VI menses assignes, ubi deerit numerus, ibi est Mars. Eosdem per XXIIII annos eicere, itemque reliquum per XXIIII menses singulis dare signis iubemur, uidere Venerem ubi hic deest. Eosdem per XII separa, et de reliquo menses iterum per XII diuidens, a Virgine incipe, et ubi deerit, Mercurium attende' (fol. 59v).

[64] D. Juste, 'Les doctrines astrologiques...' (as n. 23 above), p. 282, n. 16.

[65] The tables are: (1) Venus = 25, (2) Mercury = 28, (3) Mars = 45, (4) Venus = 24, (5) Mars = 6, (6) Jupiter and Mercury = 12 (i.e. 'System B' for Jupiter and 'System A' for Mercury), (7) Dragon = 18, (8) Saturn = 30. For the second table (Mercury = 28), see n. 57 above.

[66] These compilations, totally different from each other, occur in Florence, Bibl. Medicea Laurenziana, Plut. 30.29 (s. xiii), fols 26ra-32ra (inc.: 'Omnis homo 12 participat signis...'), and Paris, BNF, lat. 7416B (s. xiii/xiv), fols 101ra-104va (fol. 100, which contained the beginning of the work, has been cut out). I give here below extracts about Saturn and Jupiter as they appear in the manuscripts: 'Item docendum est consequenter in quo signo queque planetarum reperiatur. Quod sic inuenies: Saturnum querens annos secundum Hebraicam ueritatem per 30a diuide, reliquos uero per 2 annos et medium diuidens signis distribue, sumptoque initio a Capricorno uel Libra singulis 30a dabis menses; cui autem desierint, in eo Saturnus erit. Iouem autem sic inuenies: perfectos mundi annos per 12 uel per 18 diuide et unicuique signo da 12 a Cancro incipe; et cui defuerit I, in illo erit. Aliter: prefatos mundi annos per 19 diuide, de reliquis 19 mensibus singulis signis tribue a Sagitario incipe, et Iouem inuenies...' (Florence, Bibl. Medicea Laurenziana, Plut. 30.29, fol. 29va-vb). 'In quo signo queque planeta fuerit certam regulam damus... Saturnus in quo signo moretur, annis ab initio mundi per 30 diuisis quo-

combination occurs in the *Opusculum de ratione spere ex summorum disciplinis philosophorum cum labore et diligentia excerptum*, a cosmological, geographical, astronomical and astrological compilation which probably dates from the eleventh century. In Book III, on planetary astronomy and astrology, we find chapter 14 of the *Quicumque* followed by the last two paragraphs of Abbo's *De ratione spere* ('Signa in quibus singuli planetarum... Martem esse modo credimus').[67]

5 *Origin of the method of the 'years of the world': related Greek, Syriac and Latin texts*

The final issue that I would like to address concerns the relationship between the two systems. Because 'System B' does not appear in Latin before the *Alchandreana*, I had suggested elsewhere that it was a revision of 'System A', reflecting an attempt to take into account more accurate astronomical data, i.e. the 'true' mean periods of the planets.[68] This is not, however, the case. The calculation of the planetary longitudes based on the periods of the planets from a given chronological starting point is not a medieval invention. The principle is already found in the Greek *Anthologiae* of Vettius Valens (second century A.D.),

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ciens potuerint metiantur, cumque trangressi numerum qui constent menses cognoscatur, atque ex hiis primo Capricornus deinde cetera signa sortiantur 30, ubi consistunt Saturnum nonnulli similiter colligendum existimant; deinde ubi 30 defuerint, ibi Saturnum uersari. Aliter: annos predictos per 30 diuide sumpto a Libra initio et quicquid infra 30 remanserit per duos annos et dimidium partire, da unicuique signo totidem; cui autem defuerint, in illo Saturnus moratur. Hic planeta 30 annis cursum suum explet, morans in uno signo 30 menses. Iupiter in quo signo uersetur supradictos annos per 12 diuide et quod infra remanserit initio ab Ariete sumpto unicuique signo annum tribue; ubi annus defuerit, ibi Iupiter morari. Aliter: annis ab exordio mundi per 18 separatis, menses exuberantium annorum si denuo per eandem partem diuisi 18, primum Sagitario deinde ceteris totidem tribuantur, ubinam Iupiter domicilium habeat, indicabunt; siquidem cui signo 18 tribuendi defuerint, Iouem in eo morari dicunt. Iupiter 12 annis suum conficit iter, morando in unoquoque signo 12 menses...' (Paris, BNF, lat. 7416B, fol. 101ra-vb).

[67] Oxford, Bodleian Library, Digby 83 (s. xii), fol. 37r-v. On the compilation, see L. Thorndike, *A History of Magic and Experimental Science*, I, Baltimore, 1923, pp. 705-709, and A. Van de Vyver, 'Les plus anciennes traductions...' (as n. 23 above), pp. 689-91.

[68] D. Juste, 'Les doctrines astrologiques...' (as n. 23 above), p. 289 and n. 40.

where the rules— also given for the Sun—are accompanied by examples which can be dated from A.D. 109 to 120.[69] Subsequently, the same method occurs in Byzantine manuscripts, in particular in a text where the positions have been computed for the year 906.[70] In this Greek tradition, the computation is based not on the years of the world but rather on the positions of the planets at the beginning of the era of Augustus (= 29 August 31 B.C.), with the exception of the position of the Sun which, in the Byzantine text, was calculated for *annus mundi* 6414.[71] The general procedure is otherwise the same and appears much closer to 'System B', especially in the use of the 'true' mean periods of the planets, in the more complex computation for Venus—which includes the initial division by 8[72] and the distribution of the final result along the zodiac at the rate of 25 days per sign—, as well as in the computation of Mercury, which aims only to determine its position with respect to that of the Sun, i.e. in the preceding or the following sign.[73]

It is clear, then, that 'System B' ultimately derives from the Greek tradition. This affiliation provides an explanation for the

[69] Vettius Valens, *Anthologiae*, I.18 (ed. D. Pingree, Leipzig, 1986, pp. 32-6) ; ed. with French translation and commentary J.-F. Bara, *Vettius Valens d'Antioche: Anthologies, Livre I*, Leiden etc., 1989, pp. 168-77. On this chapter, see O. Neugebauer, *A History of Ancient Mathematical Astronomy*, Berlin-New York, 1975, II, pp. 793-801. Following the first edition of the *Anthologiae* by W. Kroll (Leipzig, 1908), Neugebauer, Bara and Tihon (see n. 70 below) refer to this chapter as 'I.20'.

[70] This text has been edited, translated and studied in detail by A. Tihon, 'Le calcul de la longitude de Vénus d'après un texte anonyme du *Vat. gr.* 184', *Bulletin de l'Institut Historique Belge de Rome*, 39, 1968, pp. 603-24; and *ead.*, 'Le calcul de la longitude des planètes d'après un texte anonyme du *Vat. gr.* 184', *ibid.*, 52, 1982, pp. 51-82 (both are reprinted in *ead.*, *Etudes d'astronomie byzantine*, Aldershot, 1994, articles I and II).

[71] A. Tihon, 'Le calcul de la longitude des planètes...' (as n. 70 above), p. 11.

[72] This division is explained by the fact that Venus covers 5 synodic revolutions in 8 years. It should be noted that the procedure includes one substraction of 120 days which must be somehow related to the addition of 40 days found in 'System B'. All these refinements result from an attempt to account for the irregularities of the motion of Venus. See A. Tihon, 'Le calcul de la longitude de Vénus...' (as n. 70 above), pp. 62-9; O. Neugebauer, *A History* ... [as n. 69 above], pp. 796-800.

[73] O. Neugebauer, *A History...* (as n. 69 above), p. 800; A. Tihon, 'Le calcul de la longitude des planètes...' (as n. 70 above), pp. 22-3.

'odd' original signs: presumably, they corresponded to the 'true' positions of the planets at a given date.[74] The immediate source of 'System B' is not known with certainty, but it is likely that it came to the West through the same channel as the rest of the *Alchandreana*, i.e. through Arabic sources. Two independent elements might confirm this: the inclusion of the lunar nodes, and the fact that the relevant chapters of the *Proportiones* and the *Liber Alchandrei* are mingled with a set of meteorological prognostics based on the position of Saturn in the four triplicities (fire, earth, air, water).[75] The lunar nodes and the elemental triplicities—which both received considerable development in the *Alchandreana*—were rarely used in Greek astrology, whereas they are typical features of medieval Eastern, including Arabic, astrology.[76]

The presence of the method in the medieval East, beyond Greek sources, is moreover attested by an astrological collection in Syriac compiled in the twelfth century at the latest.[77] Here the computation is based on the 'years of Alexander' (= the Seleucid era of 311/312 B.C.)[78] and explicitly mixes elements from different sources.[79] It includes the lunar nodes and lists the planets in the order of the spheres (with an inversion of Mercury and Venus), i.e. two elements closer to 'System B'. On the other hand, the original signs correspond to the astrological domiciles of the planets,

[74] It is unlikely that the date might be inferred from the positions given in the *Alchandreana*, both because the three texts diverge on this point and because, as Neugebauer pointed out (*A History...* (as n. 69 above), p. 796), the positions given in Vettius Valens were already inaccurate.

[75] See *Liber Alchandrei*, 25.2, and *Proportiones*, 42.15, in Appendix II.

[76] See D. Juste, 'Les doctrines astrologiques...' (as n. 23 above), pp. 292-3 (on the elemental triplicities) and 300-301 (on the lunar nodes).

[77] The collection has been edited from a twelfth-century manuscript and translated by E. A. W. Budge, *The Syriac Book of Medicines. Syrian Anatomy, Pathology and Therapeutics in the Early Middle Ages*, 2 vols, London, 1913 (reprint St-Helier and Amsterdam, 1976), I, pp. 441-553 (edition) and II, pp. 520-655 (translation). Chapters 60 and 61 are devoted to the planetary longitudes (II, pp. 571-573).

[78] The Seleucidian era is usually called the 'Era of Alexander' in late Greek, Syriac and Arabic texts; see s. H. Taqizadeh, 'Various Eras and Calendars Used in the Countries of Islam', *Bulletin of the School of Oriental Studies (University of London)*, 10, 1939, pp. 107-32 (pp. 124-30).

[79] As is clear, e.g. from the sentence 'divide them by three, though some say divide them by twenty' (see the text n. 80 below).

as in 'System A', although, in this case, it is the nocturnal— not the diurnal—domiciles which are involved (Saturn in Aquarius, Jupiter in Pisces, Mars in Aries, Mercury in Gemini and Venus in Taurus—the Head of the Dragon is in Libra). Unfortunately, the text is in such a bad condition that it is not possible to infer the zodiacal periods with confidence.[80]

But the most detailed account of the method appears in another Latin text which I have not so far mentioned: the *Liber Nemrod.* This curious astronomical treatise, composed in the form of a dialogue between Nemrod and his disciple Ioanton, first occurs in a few manuscripts of the twelfth century, including Paris, BNF, lat. 14754, fols 202v-229v, on which what follows will be based. Five long chapters (fols 214v-217v) are devoted to the computation of the longitude of Mars, Mercury, Jupiter, Venus and Saturn, each of which contains four parts: a brief exposition of the procedure; a table showing the progression of the relevant planet throughout the zodiac according to its transit period in one sign and starting with its original sign; a lengthy discussion between Nemrod and Ioanton about various—not always relevant—subjects; and a second table giving the cycles of the planet for the whole history of the world, i.e. from Creation to *annus mundi* 7000. The original signs are not arranged in an entirely consistent way, but they seem to correspond to the diurnal domiciles of the planets: Mars in Scorpio, Mercury in Gemini (= nocturnal domicile!), Jupiter in Sagittarius, Venus in Sagittarius (instead of Libra), Saturn in Capricorn. The periods of the planets are given in two forms, the zodiacal periods and the periods of transit in one sign, as follows: Mars 72 years (6 years in one sign), Mercury 144 years (12 years), Jupiter 216 years (18 years), Venus 288 years (24 years), Saturn 360 years (30 years). The same periods appear in two

80 Here is the example of Saturn in Budge's translation (as n. 77 above, II, p. 572): 'Kronos, that is Zûkhâl. Take the years of Alexander the Greek at the time when thou makest thy enquiry, however many they may be, and divide them by three, though some say divide them by twenty, and the number which remaineth divide into thirties. And of the number which remaineth and which doth not amount to thirty, allot to each House two and a half. And [begin] to count with the Water-Carrier, and, where thy number stoppeth, there is Kronos.' The first division by 3 or 20 is hard to explain; the second division by 30 is not significant here, since the same divisor is given for each planet. Nevertheless, the last step involving $2\frac{1}{2}$ (years) per sign is formulated in the same way as in 'System B'.

other chapters of the treatise (fols 218v and 223v-224r).

As is immediately obvious, the basic data and the enumeration of the planets in the order of the planetary week are closely related to those of 'System A'. The additional features as well as the variations in contents and the contrasts in formulation,[81] however, make it unlikely that the *Liber Nemrod* derives from the *IQSVM*. Unfortunately, little is known about the origin and date of the *Liber Nemrod*. The text has been said either to descend from Syriac sources or to be an original Latin composition, while it has been variously dated from the late eighth to the late eleventh century.[82] A thorough study of its astronomical content is still needed, but there is one feature in the text which may be of interest to us: the recurrent use of the kalends of October as the beginning of the year (e.g. fols 204r, 225r-226v, 227v-228r and 229v). This dating system, unknown in the West, was that of the Julian Calendar of Antioch, used alongside the Seleucid era.[83]

[81] Compare the example for Mars with the *IQSVM*: 'Mars planeta quomodo currit in signis et quantis annis ibidem stat. Si cupis scire in quo signo currit Mars, numera annos ab origine mundi usque dum uenias ad annum quem cupis et diuide eos totos per LXXII; et dum perueneris ubi non complentur LXXII, diuide ipsos per sex; et quantis uicibus habueris, ipsa sunt signa in quibus currit Mars; et ubi non complentur VI, ipsum signum est initia, et numera VI de Scorpione, et cetera' (*Liber Nemrod*; Paris, BNF, lat. 14754, fol. 214v).

[82] The *Liber Nemrod* was first studied by C. H. Haskins (*Studies in the History of Mediaeval Science*, Cambridge, Mass., 1927, pp. 336-45) who adduced evidence in favour of a Syrian origin and dated the text between 791 and 826. He was followed by A. Van de Vyver, 'Les plus anciennes traductions...' (as n. 23 above), pp. 684-7 (who, however, preferred to date the text to around the First Crusade), and by R. Lemay, 'Le Nemrod de l'"Enfer" de Dante et le "Liber Nemroth"', *Studi Danteschi*, 1963, pp. 57-128. More recently, S. J. Livesey and R. H. Rouse, in a meticulous analysis based on all the known manuscripts ('Nimrod the Astronomer', *Traditio*, 37, 1981, pp. 203-265), rejected previous conclusions and argued that the text was based solely on Western sources. This study was in turn criticized by R. Lemay, 'De la scholastique à l'histoire par le truchement de la philologie: itinéraire d'un médiéviste entre Europe et Islam', in *La diffusione delle scienze islamiche nel Medio Evo europeo*, Roma, 1987, pp. 399-533 (pp. 488-526). No conclusive evidence about the origin of the *Liber Nemrod* is as yet available.

[83] See V. Grumel, *La chronologie*, Paris, 1958, pp. 174 and 210 (= *Traités d'Etudes Byzantines*, I); E. J. Bickerman, *Chronology of the Ancient World*, London, 1980 (revised ed.), pp. 71-2. It should be emphasized that this feature was unknown to Haskins, Van de Vyver and Lemay (see n. 82 above). The use of the kalends of October was noticed by Livesey and Rouse, but

The passages in question are not connected to the computation of the longitudes, but what is striking is that 1 October is also given as the beginning of the year in a text which occurs together with the *IQSVM* in six of the nine manuscripts listed above, including the oldest one (*M*).[84] This cannot be an accident, and provides us with strong evidence that 'System A' originated from a place where this dating system was in use, that is, broadly speaking, Syria, the region furthermore which appears to be the homeland not only of the Syriac collection, but also of Vettius Valens, who was a Syrian from Antioch. It is difficult to state more precisely the date, place and interrelationship of all these texts, but it is sufficient for the aims of this paper to note that both 'System A' and 'System B', and therefore the method of the 'years of the world' itself, originated from ancient or early medieval East and came to the Latin West through two distinct routes.

6 *Conclusion*

The method of the 'years of the world' appears to have been the standard way of computing the planetary longitudes in the early Western Middle Ages. It circulated as early as the first half of the ninth century and gained new popularity from the late tenth century onwards through Abbo of Fleury and the *Alchandreana*. By the end of the eleventh century, it was available in at least seven distinct Latin texts: the *IQSVM*, Abbo's *De ratione spere*, three treatises of the *Alchandreana*, the anonymous *Opusculum de ratione spere* and the *Liber Nemrod*. The existence of this method, as well as its unexpected success, has a bearing on two issues which may be of some importance for the history of science.

In recent years, scholars have been puzzled to find astrological texts and practices in times and places where the fundamental tool for astrologers, namely astronomical tables, was missing. In Muslim Spain, for instance, court astrologers were active as early

they seem to have played down its significance. They indicate that it appears (only?) on fol. 225v ('Nimrod the Astronomer' (as n. 82 above), p. 207, n. 13) and, building upon the 'Western ties to East' in the Early Middle Ages, they argue that 'it is possible that this Eastern practice was known in the West and was deliberately employed in a text like the Liber Nemrod which purports to relay ancient Eastern astronomy' (*ibid.*, pp. 208-209).

[84] See n. 89 below.

as the late eighth century, while the first set of astronomical tables was imported there from Baghdad only in the mid-ninth century. Again, during the second quarter of the twelfth century, numerous astrological treatises were translated from Arabic into Latin before astronomical tables began to circulate in Western Europe. These examples have been pointed out by Julio Samsó, Charles Burnett and David Pingree,[85] but the same might also apply to other times and places. The method of the 'years of the world' provides a possible and plausible answer to these questions. Julio Samsó, referring to the *De planetarum et signorum ratione* published in the *Patrologia Latina*, suggested that Andalusian astrologers of the eighth and ninth centuries might have used this method.[86] The fact that 'System B' came to the West through Spain adds credibility to this hypothesis.

Returning to the time and place covered in this paper, we may wonder what the intended purpose of the method of the 'years of the world' was. If the aim is plain in the case of the *Alchandreana*, both the *IQSVM* and Abbo of Fleury remain silent on this score. It is hardly necessary to recall that knowledge of the planetary longitudes is irrelevant for the ecclesiastical computus. Mere curiosity might be an explanation but, if so, it is not easy to explain why Abbo went so far as to manipulate well-established astronomical concepts in order to strengthen the foundations of the *IQSVM*. A more appropriate context might be astrology. Although no one has conducted a systematic investigation of the subject, it is a common assumption that horoscopic astrology was unknown in the early Western Middle Ages. Various reasons have been put forward to support this assumption: for instance,

[85] J. Samsó, *Las ciencias de los Antiguos en al-Andalus*, Madrid, 1992, pp. 32-3: 'Esto nos ayuda a resolver un problema evidente: si, como veremos, la introducción de las primeras tablas astronómicas en al-Andalus tuvo lugar durante el emirato de 'Abd al-Rahman II (821-52), ¿ cómo podían calcular los astrólogos del año 800 las longitudes planetarias que eran imprescindibles para levantar un horóscopo ?'; C. Burnett and D. Pingree, *The* Liber Aristotilis *of Hugo of Santalla*, London, 1997, p. 1: 'It is well known that among the earliest scientific texts translated from Arabic into Latin in the first half of the twelfth century were treatises on astrology; indeed, so eager were Westerners for such intriguing material that they read their Abū Maʿshar and their Māshā'allāh almost before they had any certain means of determining where the planets might actually be.'

[86] J. Samsó, *Las ciencias de los Antiguos...*, p. 35.

the condemnations of the Church, the scarcity of documentation or the general decay of learning from the sixth to the twelfth century. But the most recent view, which seems to prevail today, is that the text books potentially available, such as the *Astronomica* of Manilius and the *Mathesis* of Firmicus Maternus among others, were useless because of the lack of any means for determining the planetary longitudes and therefore for constructing a horoscope.[87] The method of the 'years of the world' runs counter to this view and consequently opens the way for a reconsideration of the subject. In this perspective, it is perhaps no coincidence that the oldest extant manuscript of the *Mathesis* of Firmicus Maternus was copied around the end of the tenth or the beginning of the eleventh century at Fleury,[88] which means that Abbo could have had access to it or to its model. Now it is fair to say that the knowledge of the planetary longitudes does not fully solve the problem, for the construction of a horoscope requires a second and equally important element: the knowledge of the rising sign, or the astrological ascendant, from which the twelve mundane houses are drawn. But if we look at the above-mentioned text using 1 October and occuring in six manuscripts of the *IQSVM*, we find that it provides indeed a handy method for computing

[87] E.g.: 'Some historians maintain that astrology was then eradicated by the Western Church continually renewing the condemnations by St Basil, St Augustine, and St Gregory. But if, after what might better be called a lengthy hibernation, astrology was reborn suddenly and with such profusion in the twelfth century, it was not because the attitude of the ecclesiastical authorities had changed. What had changed was that, thanks to the Arabs, Latins had learned to use astronomical tables and to calculate the longitude of the planets for any time whatever, past, present, or future—the precondition of an astrology which truly believed in itself' (G. Beaujouan, 'The Transformation of the Quadrivium', in *Renaissance and Renewal in the Twelfth Century*, eds R. L. Benson and G. Constable, Oxford, 1982, p. 480). See also idem, 'En quoi les recherches récentes sur l'astrologie et l'alchimie peuvent-elles améliorer notre compréhension de la philosophie médiévale ?', in *Sprache und Erkenntniss im Mittelalter. Akten des VI. Internationalen Kongressen fur Mittelalterliche Philosophie...* (Bonn, 29 August-3 Sept. 1977), ed. A. Zimmermann, Berlin and New York, 1981, p. 318; S. J. Tester, *A History of Western Astrology*, Woodbridge, 1987, p. 113; S. McCluskey, *Astronomies and Cultures...* (as n. 1 above), p. 148.

[88] Paris, BNF, lat. 7311, fols 4r-49r. See J. Vezin, 'Leofnoth: un scribe anglais à Saint-Benoît-sur-Loire', *Codices Manuscripti*, 3, 1977, pp. 109-120 (pp. 113 and 118).

the rising sign for any time, past, present or future.[89] Considered together, these two texts (the *IQSVM* and the '1 October' text) offer a short—yet complete—treatise devoted to the construction of a horoscope.

[89] The text opens 'Hortum signorum XII qualibet diei uel noctis hora quisque liquido deprehensurus est, si a kal. Octobris horas dierum...' It occurs in *M*, p. 29; *P*, fols 134v-135r; P_1, fol. 141r-v; V_1, fol. 51v; *S*, fols 117v-118r; V_3, fol. 70v. In MV_1V_3, it immediately precedes the text on eclipses (n. 41 above), while in PP_1S it is separated from the *IQSVM* by chapters dealing with planetary astronomy. The text is also preserved in a number of other manuscripts and appears in *Patrologia Latina*, 90, cols 213D-215A. I am preparing a new edition of it.

Appendix I: In quo signo versetur Mars (IQSVM) = 'System A'[90]

1. <Mars> In[91] quo signo uersetur Mars: annis ab initio mundi per senarium diuisis et annorum hunc numerum excedentium mensibus item per senarium separatis atque unicuique signo VI assignatis a Scorpio dumtaxat sumpto initio traditur manifestari; namque in eo signo Martem inueniendum cui VI tribuendi defuerint.

2. <Iuppiter>[92] ab exordio mundi annis per XVIII partem separatis, menses exuberantium annorum si denuo per eandem partem diuisi XVIII, primum Sagittario deinde signis caeteris tot tribuantur (ubinam Iuppiter domicilium habeat ut quorundam fert opinio) indicabunt; siquidem cui signo XVIII non occurrerint, Iouem in eo morari.

3. <Saturnus> Eorundem[93] annorum summam si XXX quotiens potuerint metiantur, cumque transgressi quot constent mensibus cognoscatur, atque ex his primo Capricornus caetera deinde signa sortiantur XXX, ubi consistat Saturnus nonnulli similiter colligendum existimant; denique ubi XXX defuerint, ibi Saturnum uersari.

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[90] The text is based on all nine manuscripts (see above for the sigla). I have imposed consistency in the use of *ae* (rather than *e* or *e* with a cedilla) and *ti-* (rather than *ci-*) before vowels. In these two cases, variant readings have been ignored. All other variants are noted in the apparatus. For the variants of *Liber Alchandrei*, 10, see n. 55 above.

[91] *Tit.*: De signo. Mars $P_1 S$; In quo duodecim signorum Mars habeatur $V V_1 P_2$ || diuisis] diuisus $P_1 S$ || excedentium] extendentium $P_1 S$ || item *om.* V_2 || (unicuique) signo *om.* $P P_1 S$ || VI] sex V_2 || assignatis] signatis P || Scorpio] Scorpione P_2 || initio *sus.* M || manifestari] est V_2 || VI] sex $V_1 V_2$

[92] *Tit.*: In quo feratur Iuppiter $V V_1 P_2$ || Eisdem... annis] Annis ab exordio mundi V_2 || Eisdem] Ex isdem $V P_2$ || mundi *om.* P_2 || annis *om.* P || (per) XVIII] Xam VIIIam V; XVIIIIam V_2; XVam IIIam V_3 || (diuisi) XVIII] XVIIII V_1 || Sagittario] Sagittarium $P_1 S$; Sag. P || tot] totidem $V V_1 V_2 P_2$ || Iuppiter] Iupiter V_3 || quorundam] quorum undam M || indicabunt] indicabit $P_1 S$ || occurrerint] occurerint P

[93] *Tit.*: In quo Saturnus consistat $V V_1 P_2$ || Annos ab exordio mundi sumam V_2 || quot] quod $M V_3$ || Capricornus] Capricornius $V P_2$ || consistat] consistit (?) P_2; constat S || Nonnulli] Nonnullis V || existimant] existiman S

4. <Venus> Eosdem[94] annos praeterea XXIIII parte separare iubemur et hinc summae congruentium annorum menses consideratos applicare signis, exordio sumpto a Libra, ita ut unumquodque XXIIII semper accipiat; etenim sic expensus numerus Veneris patefaciet stationem.

5. <Mercurius> Quin[95] etiam praefati saepe anni duodenario separati, annorumque superfluentium cogniti menses ac XII Virgini et ordine reliquis tot attributi, ubi se receptet Mercurius indicabunt; namque cuicumque signo defuerint assignando XII, in eo uersari Mercurium curiosi rerum talium opinantur. Sin autem prima partitio duodenario concludatur, in Virgine memoratum semper inueniri planetam.

Appendix II: Alchandrean versions = 'System B' [96]

Liber Alchandrei, 24.1-5 and 25[97]

24. 1 <Caput Draconis> Ut scias autem in quo signo quaeque planetarum, Caput Caudaue Draconis omni tempore uersetur, haec regula a tuo animo non labatur. 2 Annos ab initio mundi

[94] *Tit.*: In quo moretur Venus VV_1P_2 || Eosdem annos praeterea] Annos ab exordio mundi V_2 || XXIIII parte] XXam IIIIam partem V_2; XXa IIIIa parte V_1 || hinc] hic P_1S; huic VV_1V_2 || congruentium] non congruentium $VV_1V_2P_2$ || XXIIII] uiginti et IIIIor V_2 || sic] sit P_2 || patefaciet] parte faciet MV_3

[95] *Tit.*: In quo meet (et P_2) Mercurius VV_1P_2 || Quin etiam praefati saepe anni] Annis ab initio mundi V_2 || separati] separato M || ac] hac MV_3 || ordine] ordini P_1S || tot] totidem $VV_1V_2P_2$ || attributi] adtributi P_1S || indicabunt] indicabit P_1S || signo defuerint cuicumque V_2 || signo *om.* P_1S || assignando XII] asignando XII P_1S; assignandi XII VV_1P_2; *om. (and add. later hand)* V_2 || uersari] uersarii (?) P_1; uersarum (*or:* uersari in) V_1 || prima *om.* P_1S || partitio] partito P_1S || concludatur] concludantur P_1S || Virgine memoratum] Virginem memoratam P_2; Virginem memoratum V || inueniri planetam] inueri planeta V; Virgo *add.* MPV_3; Virgo latitudinem signiferi scandes *add.* P_1S

[96] All the texts are taken from my forthcoming edition (see n. 53 above).

[97] Manuscripts: Paris, BNF, lat. 17868, s. x, fol. 9r-v; Munich, Bayerische Staatsbibliothek, Clm 560, s. xi, fol. 72r-v (chap. 25 only); London, BL, Addit. 17808, s. xi, fols 94v-95v; Paris, BNF, lat. 14065, s. xi- xii, fol. 52r (chap. 25 only); Kremsmünster, Stiftsbibl., CC30, *ca.* 1200, fols 62r-64v; London, BL, Cott. App. VI, s. xiii, fol. 38rb-va and 39ra-39rb ; Vatican City, BAV, Vat. lat. 4084, s. xiv, fol. 11r (chap. 24 only); Florence, Bibl. Riccardiana, 917, s. xiv, fols 32r-33r; London, BL, Sloane 702, s. xv, fol. 26v (chap. 24 only). There is also a slightly different recension in London, Wellcome Institute, 21, s. xii, fol. 1ra.

per XVIII diuide quandiu possis; initio a Leone sumpto, quod autem infra XVIII remanserit, per annum et dimidium tantum concedens singulis distribuis; cui autem annus et dimidius non contigerit, in illo Caput Draconis tunc commorari indubitanter pronuntiabis. 3 Memento tamen post Leonem non Virginem set Cancrum, post Geminos et sic per uniuersum ordinem esse computandum, quia Draco, solus inter caeteras stellas quae per se feruntur, sequi cursum firmamenti perhibetur. 4 Perficit autem cursum suum XVIII annis, unumquodque signum anno et dimidio. 5 Caput uero in quo signo moretur reperto, Ventrem eius, qui ut Caput prospera protendit, semper scias esse in quarto, Caudam eius uenenosam in septimo. <...>.

25. 1 <Saturnus> Saturnus in quo signo moretur si nosse desideras, item annis ab initio mundi diuisis per XXX, a Libra sumpto initio quicquid infra XXX remanserit per duos annos et dimidium diuide; cui uero duo et dimidius tribuendi defuerint, in illo signo uersari indubitanter pronuntiabis. 2 Et hoc notandum est summopere quanta omnibus rebus Saturnus dominetur potestate: illo namque inter XII signa spatiante, quando astra ignea intrauerit, siccitate omnia constringit; si in terrea, grandine uniuersa percutit; si in aerea, uenti asperitate disperdit; cum autem in aquatica, aquarum inundatione cuncta laborant. 3 <Iuppiter> Iouem uero reperies si, eisdem annis per XII diuisis, hoc quod remanet initio sumpto ab Ariete unicuique signo annum unum tribues, quia ubi unus tribuendus deerit, ibi se receptat Iouis. 4 Notandum hic quod si quando contingit has duas, Saturnum scilicet et Iouem, Solem aut IIII signis praecurrere aut IIII retrogradare, quia calorem solitum Solis spatiorum longinquitate nequeunt mutuare, his qui tunc nascuntur, et Iouem antea semper mitem aliquantulum aduersari, et Saturnum semper aduersum magis tum minitari. 5 <Mars> Isdem item annis per annum et dimidium diuisis, quod remanserit initio a Scorpione sumpto per signa tribue; ubi uero XLV dies deerint tribuendi, ibi Martem habebis. 6 <Venus> Venerem uero, si eosdem annos in dies deductos per CCC diuisos, unicuique signo initio a Capricorno sumpto XXV tribuis de eo quod remanet.

Proportiones competentes in astrorum industria, 42[98]

In quo signo moretur quaeque planetarum. 1 <Caput Draconis> Si uis scire in quo signo omni tempore uersetur quaeque planetarum, siue in Capite siue in Cauda Draconis (sic), haec regula a tuo non labatur animo. 2 Annos ab initio mundi quandiu possis per XVIII diuide; sumpto initio a Leone, si quis uero numerus in eadem computatione superfuerit, non ualens peruenire usque ad supradictae diuisionis limitem, id est XVIII, da unicuique signo annum et dimidium; cui autem hoc non contigerit, in illo signo Caput Draconis morari indubitanter scias. 3 Memento tamen post Leonem, Virginem, set non Cancrum, post Virginem, Libram, non Geminos, et ita per uniuersum ordinem esse computandum, quia Draco, solus inter caeteras stellas quae per se feruntur, sequi firmamenti cursum perhibetur. 4 Perficit autem cursum suum XVIII annis, nec est dubium quoniam in unoquoque signo moretur annum et dimidium, id est XVIII mensibus. 5 Capite autem Draconis in quo moretur signo reperto, Ventrem eius, qui ut Caput prospera protendit, semper esse in quarto, Caudam uero in septimo quae semper uenenosa atque nociua dicitur. <...>.

14 <Saturnus> Saturnus in quo signo moretur si nosse desideras, item annos ab initio mundi per XXX diuide, sumpto a Libra initio, et quicquid infra XXX remanserit per duos annos et dimidium partire, dato unicuique signo totidem; cui autem signo defuerint anni duo et dimidius, in illo uere Saturnus moratur. 15 Et hoc est notandum quanta potestate praecellit omnes res: illo namque inter XII signa spatiante, quando ignea astra intrat, omnia comburit siccitate; quando autem terrea, saepe uastat grandine multa; cum uero aerea, uento et multa tempestate dispertit; cum in aquaticis manet, aquarum inundatione cuncta laborant. 16 Saturnus autem, usque expallet insidias Martis, et Mars cum omnibus expallet Saturnum. 17 <Iuppiter> Iuppiter indubitanter in quo signo moretur si uis scire, supradictos annos per XII diuide, et quod infra remanserit initio suo ab Ariete sumpto unicuique

[98] Manuscripts: London, BL, Sloane 2030, s. xii-xiii, fol. 93r; Kremsmünster, Stiftsbibl., CC30, *ca.* 1200, fols 33r-36v; Oxford, Bodleian Library, Ashmole 369, s. xiii, fol. 82r; Munich, Bayerische Staatsbibl., Clm 458, s. xv, fols 19v-21r; Paris, BNF, lat. 7349, s. xv, fols 85r-86r. A different recension appears in Oxford, Bodleian Library, Ashmole 345, s. xiv, fols 54v-55r.

signo unum annum tribue; cum autem defuerit annus, ibi se receptat Iuppiter. 18 Notandumque est quod cum euenit Saturnum et Iouem IIII signis Solem praecurrere non retrogradare, quia solitum calorem Solis longinquitate spatiorum nequeunt mutuare, et tunc Iuppiter qui antea erat mitis aliquantulum aduersabitur illis qui interim nascuntur, Saturnus uero multum minitabitur. 19 <Mars> Si autem in quo signo sit Mars diligenter inquiris, saepe fatos annos per annum et dimidium diuide, a Capricorno sumpto initio, et quod superfuerit per signa partire; et cui XLV deerunt, ibi Martem habeas. 20 <Venus> Ut uero satisfaciam lectori in difficilioribus quia et satisfeci in uilioribus. 21 Si quis curat scire quo in signo splendida rotetur Venus, supputationem longissimam et multo errore inuolutam, set nunc sollerti indagatione adbreuiatam, sic colligat: annos ab initio mundi per VIII computans semper eiciat donec ad VIII peruenire nequeat, et quicquid infra remanserit per trecentos diuidat, et ubi hic numerus defuerit XL addat, et cuilibet signo XXV distribuat et a Libra incipiat; et cui hic numerus defuerit, ibi Venerem receptare cognoscat. 22 Ad haec, sciat Venerem Soli semper esse uicinam, siquidem in longiorem nunquam recedat ab eo regionem, ut uel praecedat uel, retrograda, subsequatur plus IIII uel tribus signis. 23 <Mercurius> De Mercurio autem, quia nec difficilis habetur quaestio, non duxi necessarium ut longa subsequeretur descriptio quia sic uicinus semper uidetur et habetur Soli, ut numquam uel praecedat uel subsequatur tantum quin aut in praecedenti signo aut in subsequenti aut certe in eodem facillime possit inueniri.

Quicumque nosse desiderat legem astrorum..., 9.1-4 and 14[99]

9. <Caput Draconis> 1 Post planetarum autem praedictam rationem, Draconis inserere placet cursus, conuersos immo retrorsum aduersos per singula. 2 Nam cum planetae omnes per directum suos cursus peragant, prout supra tractatum est, ut ab Ariete in Taurum, a Tauro in Geminos, Draco semper abire dic-

[99] Manuscripts: Paris, BNF, lat. 17868, s. x, fols 14v (chap. 9) and 15r (chap. 14, in a summarized form); Burgo de Osma, Archivo de la Catedral, 7, s. xi/xii, fols 102v and 104r; Paris, BNF, lat. 7299A, s. xi/xii, fol. 95v (chap. 9.2-5 only); Vatican City, BAV, Vat. lat. 4084, s. xiv, fols 2v and 9v; Vatican City, BAV, Reg. lat. 1324, s. xv, fols 43v-44r and 45r. The first two paragraphs of chapter 14 (on the luminaries) are perhaps a later addition, for they appear only in the two Vatican manuscripts

itur retrorsum, incipiens a Leone, decurrit in Cancrum, indeque egrediens, ingreditur Geminos, a quibus transiens, intrat in Taurum, indeque Arietem, a quo egressus, ingreditur Piscium signum, sicque per caetera. 3 Moratur in unoquoque signo XVIII mensibus, uidelicet anno integro et dimidio, lustrat autem omnia signa annis XVIII, est autem longitudo illius spatio signorum VII. 4 Si autem uis scire ubi est Caput Draconis, computa annos ab exordio mundi iuxta Hebraicam supputationem et diuide eos omnes per XVIII, et quod superfuerit, quod non ad XVIII poterit peruenire, tolle ex praedictis quae superfuerant, quae non ad perfectionem XVIII annorum peruenire potuerunt et diuide per menses, dans unicuique signo XVIII menses, et ubi defecerit eadem supputatio XVIII mensium, ibi erit Caput Draconis. <...>.

14. [1 <Sol> Set et hoc sciri oportet quia quinto decimo kalendarum Aprilis intrat Sol in Ariete, in quinto decimo kalendarum Mai intrat Sol in Tauro, in quinto decimo kalendarum Iunii intrat Sol in Geminis, in quinto decimo kalendarum Iulii intrat Sol in Cancro, in quinto decimo kalendarum Augusti intrat Sol in Leone, in quinto decimo kalendarum Septembris intrat Sol in Virgine, in quinto decimo kalendarum Octobris intrat Sol in Libra, in quinto decimo kalendarum Nouembris intrat Sol in Scorpione, in quinto decimo kalendarum Decembris intrat Sol in Sagittario, in quinto decimo kalendarum Ianuarii intrat Sol in Capricorno, in quinto decimo kalendarum Februarii intrat Sol in Aquario, in quinto decimo kal. Martii intrat Sol in Piscibus. 2 <Luna> Si uis scire in quo sit Luna prima, procul dubio scias quia in secundo signo a Sole, ita si Sol fuerit in Ariete, Luna prima erit in Tauro.] 3 <Mars> Si uis scire in quo signo degat Mars, computa annos a mundi principio et ipsos diuide per VII; et quod non peruenerit ad septenarium, computa quot dies ibi sint, et diuide illos per XLV dies, da primitus Leoni XLV, Virgini similiter XLV, et ita per ordinem; et in quo signo defecerint XLV, in eodem signo erit Mars. 4 <Iuppiter> Si uis scire in quo signo sit Iuppiter, computa eosdem annos a mundi creatione et diuide eos per XII; et ubi defuerit duodenus numerus, in eodem signo erit Iuppiter, item da Arieti integrum annum, et Tauro similiter, et ita per ordinem. 5 <Venus> Si uis scire in quo signo sit Venus, iunge eisdem annis LX et diuide eos per octonarium numerum; et quod non peruenerit ad supplementum octonarii, partire illos per dies

CCC; quod minus CCC fuerit, diuide item eos per XXV, et da unicuique signo XXV a Piscibus sumpto initio, sicque per singulos dies; et in quo signo XXV defecerint, in eodem Venus degit. 6 <Saturnus> Si uis scire in quo signo sit Saturnus, diuide eosdem annos per XXX; et quod non peruenerit ad tricenum numerum annorum, partire illos per annos et dimidium; et in quo duobus annis et dimidio anno, in eodem signo erit Saturnus, et da duos annos et dimidium Librae, similiter caeteris.

Les *Tables faciles* de Ptolémée: une édition critique

Anne Tihon

Les *Tables faciles* de Ptolémée ont eu, sur l'astronomie mathématique postérieure, une influence au moins aussi grande que l'*Almageste.* Utilisées très largement à la fin de l'Antiquité, les *Tables faciles* seront l'outil privilégié des astrologues et des amateurs d'astronomie aussi bien dans les écoles tardives de l'antiquité que dans le monde byzantin. Leur influence n'est pas moins grande sur l'astronomie islamique, et on les retrouve comme modèles de nombreuses tables médiévales, aussi bien en latin qu'en langue vulgaire.

Dès les années 1974, le regretté O. Neugebauer m'avait suggéré d'entreprendre une édition critique des *Tables faciles.* Ce projet a été longtemps retardé, en raison de diverses circonstances, mais aussi du fait que le matériel nécessaire à une telle édition n'était pas encore disponible. Il fallait, en effet, d'abord éditer le *Grand Commentaire* de Théon d'Alexandrie, la seule œuvre ancienne qui tente d'expliquer comment les *Tables faciles* ont été calculées. L'édition du *Grand Commentaire* de Théon a été achevée en 1999,[1] et dès lors, il était possible de mettre en chantier une édition critique des *Tables faciles.*

Dans une étude sur les manuscrits en onciale des *Tables faciles* publiée en 1992, j'ai jeté quelques bases nécessaire à cette édition: description des plus anciens manuscrits et surtout, une classification des tables qui permet de les identifier rapidement.[2] En voici les grandes catégories:

A = tables astronomiques

B = tables spéciales pour le climat de Byzance

C = tables chronologiques

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[1] Les références bibliographiques complètes sont données en fin d'article. Voir Théon GC I, II-III et IV.

[2] Tihon, 'Manuscrits'.

G = tables géographiques
S = tables supplémentaires

L'édition des tables A est actuellement en cours de préparation, par les soins de Raymond Mercier et de moi-même. Il s'agit d'un travail long et difficile: les *Tables faciles* soulèvent beaucoup de questions. En effet, si l'on connaît relativement bien l'*Almageste*, notre ignorance est grande à propos des œuvres mineures de Ptolémée. L'*Almageste* n'a cessé d'être enseigné et commenté jusqu'à la fin de l'Antiquité (VI^e s.), comme le montrent les abondantes scolies qui accompagnent le texte dans les manuscrits (surtout les six premiers livres). Dès qu'un texte faisait l'objet d'un enseignement, il s'est bien conservé, éventuellement dans une édition revisée par un savant éditeur. Les *Hypothèses*[3] et les *Phaseis*,[4] n'étant pas matière d'enseignement, sont graduellement morts en grec et la tradition manuscrite n'en a conservé que des versions mutilées et détériorées, figées sans doute vers les IV^e–V^e siècles. Les *Tables faciles* ont échappé partiellement à ce destin parce qu'elles étaient utilisées constamment, mais le mode d'emploi de Ptolémée[5] a vite été supplanté par d'autres commentaires, en particulier ceux de Théon d'Alexandrie (ca 364 P. C).[6] A travers Théon et les autres commentateurs, à travers les scolies et la tradition manuscrite, nous n'avons des *Tables faciles* qu'une information déformée et incomplète. C'est pourquoi, il n'est pas inutile d'essayer de faire le point en posant une série de questions:

1. Pourquoi éditer les *Tables faciles* de Ptolémée?
2. Comment éditer les tables?
3. Que contient le manuel de Ptolémée?
4. Les *Tables faciles* des manuscrits sont-elles conformes à Ptolémée?
5. Théon a-t-il remanié les *Tables faciles*?
6. Quelles sont les relations des *Tables faciles* avec l'*Almageste* et les *Hypothèses des planètes*?
7. Quelles sont les relations des *Tables faciles* avec l'astronomie antérieure ou contemporaine de Ptolémée?

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[3] Edition du texte grec: voir Ptolémée, *Hypothèses*; édition du texte arabe: voir Goldstein, Morelon, 'Hypothèses'.

[4] Edition: voir Ptolémée, *Phaseis*; Morelon, 'Phaseis'.

[5] Edition: voir Ptolémée, *Tables faciles*.

[6] Edition: voir Théon, PC et GC.

8. Quel a été le rôle des *Tables faciles* dans l'astronomie byzantine?

9. Quelle a été l'influence des *Tables faciles* sur l'astronomie postérieure, en particulier dans le monde islamique?

1. Pourquoi éditer les *Tables faciles* de Ptolémée?

Malgré leur importance, les *Tables faciles* ne sont accessibles que dans l'édition de Nicolas Halma (1822 ss).[7] Malgré ses mérites et les services qu'elle a rendus, cette édition, faite dans l'esprit de l'époque, n'est pas du tout scientifique: Halma passe d'un manuscrit à l'autre, commet de nombreuses erreurs et ne respecte pas toujours la présentation extérieure des tables. Quant à la traduction française qu'il donne du mode d'emploi de Ptolémée, elle est souvent inexacte. On peut également recourir à la thèse de W. D. Stahlman, qui reprend et analyse les tables du *Vat. gr.* 1291, en y ajoutant les variantes de deux autres manuscrits (*Paris. gr.* 2399 et *Paris. gr.* 2493).[8] Mais cet excellent travail n'a jamais été publié, et il ne s'agit pas d'une vraie édition, puisque les tables n'y sont données qu'en transcription, sans mention des titres originaux. La nécessité d'une édition critique est depuis longtemps reconnue.

2. Comment éditer les tables?

L'édition critique d'un texte ancien obéit à certain nombre d'exigences bien connues des philologues (à défaut d'être toujours respectées). On rassemble les manuscrits et on les collationne en vue d'établir un *stemma codicum.* Celui-ci permet d'éliminer des copies et de ne retenir que les manuscrits qui sont les ancêtres de telle ou telle branche de la tradition manuscrite. Sur base des manuscrits retenus, on établit un texte qui est, idéalement, le plus proche de ce que l'auteur avait écrit.

Dans le cas des tables astronomiques, ce processus philologique ne peut être appliqué sans rencontrer d'énormes difficultés. Le nombre de manuscrits grecs contenant les *Tables faciles* se monte à une cinquantaine environ et le matériel à collationner—la masse des chiffres—est énorme. Plutôt que de collationner les manuscrits en aveugle, il était intéressant de sélectionner d'abord les

[7] La liste des tables avec leurs références dans Halma est donnée dans Tihon, 'Manuscrits', p. 58.

[8] Voir Stahlmann, *Astronomical Tables.*

manuscrits les plus anciens, à savoir quatre copies en écriture onciale des IXe-Xe siècles :[9]

F = Laurentianus 28/26
H = Leidensis BPG 78
M = Marcianus gr. 331
v = Vaticanus gr. 1291

Nous avons décidé de faire l'édition sur base de ces quatre manuscrits. Sur cette base, il sera plus facile d'examiner et de débrouiller la tradition manuscrite postérieure.

Ces quatre manuscrits remontent directement, semble-t-il, à la fin de l'antiquité. Le *Vaticanus gr.* 1291, qui a sans doute été copié au début du IXe siècle,[10] comporte des indices qui marquent une étape en 522/523, soit sous le règne de Justin Ier (518–527). Les historiens de l'art ont depuis longtemps montré que ses miniatures étaient un héritage direct de l'iconographie antique. Le *Vaticanus gr.* 1291 reproduit peut-être un exemplaire de luxe copié pour la cour de Constantinople au début du VIe siècle. Beaucoup moins luxueux, le *Leidensis* BPG 78 (H) contient des éléments chronologiques (tables de rois, listes de consuls) qui indiquent que son modèle a dû être une copie effectuée sous le règne d'Héraclius (610–641) ou une copie plus ancienne annotée et complétée à ce moment. Il est tentant de le mettre en rapport avec Stéphane d'Alexandrie, auteur d'un *Commentaire aux Tables faciles* inspiré du *Petit Commentaire* de Théon et rédigé vers 610-630.[11] Avec Stéphane d'Alexandrie se marque la transition entre Alexandrie et Constantinople. Les autres manuscrits n'offrent pas de repères chronologiques aussi nets, mais l'examen de leur contenu montre qu'ils contiennent, eux aussi, du matériel rassemblé dans les tout débuts de l'empire byzantin. Ces manuscrits offrent donc — sous réserve d'une découverte toujours possible dans les manuscrits plus récents — l'accès le plus proche de l'antiquité pour l'ensemble des *Tables faciles*.

[9] Description de ces manuscrits dans Tihon, 'Manuscrits', pp. 58–68.

[10] Selon notre examen personnel, le *Vat. gr.* 1291 a été copié sous le règne de Théophile (829–842) (Tihon, 'Manuscrits', pp. 62-63). Wright, 'Vatican' le date de la fin du VIIIe siècle. La question sera reprise par Timothy Jencz dans un article à paraître.

[11] Inédit, sauf quelques chapitres édités dans Usener, 'De Stephano'.

Les tables astronomiques permettent des vérifications mathématiques, de sorte que, en principe, on peut établir des tables sans faute. Ceci est très théorique. En réalité, il arrive souvent que l'on ne puisse pas décider à 1 minute près quelle valeur est meilleure que l'autre. Dans certains cas (par exemple la col. 4 de la table des parallaxes ou la colonne de l'équation du temps dans la table de la sphère droite[12]) , on ne comprend pas exactement comment Ptolémée s'y est pris pour la calculer. Enfin, des tables ≪parfaites≫ n'ont certainement jamais circulé. Pour ces raisons, nous avons décidé de ne pas essayer de reconstruire les tables, mais de reproduire fidèlement les tables d'un de ces manuscrits.

Le choix de ce manuscrit privilégié a fait l'objet de nombreuses hésitations. Finalement, F a été retenu. En effet, H est certainement intéressant: des recherches antérieures[13] ont montré que, dans les tables de parallaxes, il est le seul à avoir conservé des annotations mentionnées par Ptolémée, mais qui se sont perdues dans les autres manuscrits. Malheureusement, H est inutilisable, car de nombreuses tables y sont effacées et réécrites par une main postérieure, non fiable, probablement du XIVe siècle. De même, M est trop mutilé pour jouer ce rôle. Restent alors en compétition deux manuscrits complets et parfaitement lisibles: v et F.

Le premier, v, a pour lui d'être le plus ancien des deux, puisqu'il a dû être copié sous le règne de Théophile (829–842), tandis que F a été copié sous le règne de Léon VI le Sage (886–912). Mais les recherches préliminaires qui ont été faites sur la tradition manuscrite des *Tables faciles* semblent montrer que v est resté isolé dans la transmission postérieure.[14] Editer v reviendrait sans doute à éditer les tables sous une forme qui a peu circulé dans le monde byzantin. Au contraire, en choisissant F, on a une bonne chance d'éditer une version largement diffusée des *Tables faciles*. Le choix de F n'implique en aucune façon qu'il soit un ≪meilleur≫ témoin que v, H ou même M. Ce n'est qu'à la fin du travail d'édition qu'il sera possible de voir si, parmi ces quatre manuscrits, l'un d'eux se distingue comme ≪le meilleur≫.

Un apparat critique traditionnel est souvent pénible à lire.

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[12] Sur l'équation du temps dans les *Tables faciles*, voir Van Dalen, *Ancient and Mediaeval*, pp. 106 ss.

[13] Voir Chabàs–Tihon, 'Parallaxes'; Théon, GC II, pp. 111 ss.

[14] Schnabel, 'Die Entstehungsgeschichte', p. 223. L'avis de Schnabel est confirmé par mes premières analyses de la tradition manuscrite postérieure.

Il est franchement impossible dans le cas de tables. Pour cette raison, l'apparat critique sera donné après chaque table, d'abord, toutes les variantes de titres, ensuite, les variantes numériques, sous forme d'un tableau qui donne non seulement les variantes des quatre manuscrits, mais aussi la valeur calculée. Chaque table fait l'objet d'une vérification mathématique par les soins de R. Mercier. Mais il n'est pas toujours possible de décider que telle valeur est meilleure que l'autre. Aucune règle générale ne peut être donnée, car les conditions varient pour chaque table. La valeur conforme à la vérification mathématique et, en principe, la valeur ≪correcte≫, mais ce terme doit être utilisé avec de grandes précautions.

3. Que contient le manuel de Ptolémée?

Ptolémée a laissé un manuel très court pour accompagner les *Tables faciles*: aucune explication théorique, mais un simple mode d'emploi. Celui-ci s'est dissocié dans la tradition manuscrite des tables elles-mêmes. La traduction française de Halma étant très inexacte, une nouvelle traduction sera jointe à l'édition. Que nous apprend ce mode d'emploi? Tout d'abord le but de ces tables. Lisons le début du traité:

> L'organisation de ces *Tables faciles* pour les mouvements des planètes, mon cher Syrus, a été faite par nous conformément à leurs hypothèses moyennes et circulaires, en vue de pouvoir, au moyen des excentriques et des épicycles tracés dans un plan selon les explications démontrées par la *Syntaxe*, indiquer leurs mouvements en longitude considérés par rapport au zodiaque en accord avec les résultats obtenus par les calculs, tandis que les écarts en latitude, ne pouvant pas tomber dans de tels tracés, seront calculés méthodiquement au moyen de l'introduction de tables spécifiques.

Il s'agit de pouvoir trouver

(a) les longitudes par construction (dessin) en deux dimensions et par calcul;

(b) les latitudes, par calcul seulement. Il est clair que l'aspect ≪graphique≫ était important pour Ptolémée. Dans le traité, il explique sommairement comment trouver les longitudes du Soleil, de la Lune et des cinq planètes (avec des explications séparées pour Mercure) par méthode graphique, selon les modèles décrits dans l'*Almageste*. Il est donc vraisemblable que les simplifications

apportées aux *Tables faciles* par rapport à l'*Almageste* avaient pour but non seulement de faciliter le calcul, mais aussi le tracé des constructions.

Le texte du manuel des *Tables faciles* de Ptolémée ne permet pas à lui seul de tracer les figures, car les paramètres ne sont pas donnés: il faut les rechercher dans l'*Almageste*. On peut penser - mais ce n'est pas démontré- que des figures ou des maquettes (des équatoires avant la lettre) devaient accompagner ce traité. Des maquettes en dur (en bois ou en bronze) existaient et nous en avons la description dans l'*Hypotypose* de Proclus (mort en 484) en ce qui concerne le Soleil.[15] D'autre part, le texte de Ptolémée est suivi d'une collection de scolies dont une série décrit de manière plus précise et avec des paramètres les constructions en question.[16] Ces scolies comportent des figures. Ces méthodes graphiques ont été utilisées: la question serait plutôt de savoir pourquoi Théon les a complètement abandonnées au profit des calculs.

En proposant une représentation en deux dimensions des modèles de l'*Almageste*, les *Tables faciles* font pendant aux *Hypothèses des Planètes*, ou Ptolémée veut proposer, cette fois, une représentation tridimensionnelle des mêmes modèles sur base des principes expliqués dans l'*Almageste*. Depuis la redécouverte de la traduction arabe de la seconde partie du livre I des *Hypothèses*, on a souvent affirmé que le but de Ptolémée, dans les *Hypothèses*, était de construire un système du monde, qui traduise la réalité physique de l'Univers.[17] A en juger par la préface du traité, ceci semble fortement exagéré. Ptolémée déclare, en effet:

> Les *Hypothèses* des mouvements célestes, o Syrus, nous les avons exposées méthodiquement dans le traité de la Syntaxe mathématique, en démontrant par des raisonnements ce qui est raisonné, en accord partout avec les apparences, en vue de la démonstration du mouvement uniforme et circulaire, qui doit nécessairement se trouver sous-jacent aux choses qui participent au mouvement éternel et ordonné, et qui ne peuvent recevoir en aucune façon une augmentation ou une diminution. Ici, nous avons été porté à exposer ceci seulement, afin que cela soit imaginé le plus facilement possible par nous-mêmes et par ceux qui choisissent de les

[15] Proclus, *Hypotypose*, p. 72 ss.
[16] Tihon, 'Scolies' (scolies III-VI).
[17] Voir, par exemple Neugebauer, dans *HAMA*, p. 918.

> exposer en fabriquant un instrument, soit qu'ils fassent cela de manière plus simple en exécutant à la main les révolutions de chacun des mouvements vers leurs positions propres, soit que, par des procédés mécaniques, ils attachent ces mouvements les uns aux autres et (à la révolution) de l'univers. Ce n'est certes pas de cette manière qu'on a coutume de faire des sphères (célestes). *Car une telle manière, outre le fait qu'elle se trompe dans les hypothèses, fournit seulement l'apparence des choses et non ce qui est sous-jacent* (τὸ ὑποκείμενον), *de sorte qu'il y a démonstration de technique et non des hypothèses* (τῶν ὑποθέσεων). Mais (nous ferons cette sphère) selon ce que (prescrit) l'ordonnance et la différence des mouvements sous notre vue, avec l'irrégularité qui survient au moyen des mouvements uniformes et circulaires pour ceux qui regardent, même s'il n'est pas possible de les imbriquer tous de manière digne du propos annoncé, mais bien de démonter chacune séparément ...

Ce n'est pas le moment de discuter en détail ce texte difficile, mais remarquons que la traduction arabe (citée ici d'après la traduction francaise de R. Morelon[18]) semble avoir considérablement déformé le texte grec. En effet, le passage traduit par R. Morelon comme ceci:

> (cette sorte de sphère) ... montre seulement l'apparence des choses et *non leur situation vraie*, si bien qu'il y a là manifestation d'une habileté technique, mais *non de la situation dans sa vérité* (p. 16)

correspond au passage mis en italique dans notre traduction du texte grec. Les mots grecs utilisés par Ptolémée (ὑπόθεσις, hypothèse, ὑποκείμενον, sous-jacent) n'ont pas la signification de ≪ situation vraie ≫ ou ≪ situation dans sa vérité ≫ que leur a donnée le traducteur arabe. On reviendra plus loin sur les relations entre les *Tables faciles* et les *Hypothèses.*

4. Les *Tables faciles* des manuscrits sont-elles conformes à Ptolémée?

Cette question est très difficile, car on manque de points de comparaison, si ce n'est quelques fragments conservés notamment dans les papyrus d'Oxyrhynque.[19] Ces fragments, édités par A.

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[18] J'ai consulté sur ce passage plusieurs arabisants qui m'ont tous confirmé l'exactitude de la traduction francaise.

[19] Jones, *Papyri*, 4167 (IV s. p. C), 4168 (IVve s), 4169 (IIIe s.), 4170 (Ve s.?), 4171(date?), 4172(IVe-Ve s)), 4173 (IVe s.).

Jones, montrent des tables identiques à celles des manuscrits, mais aussi quelques tables assez différentes.

Il n'y a pas, à la lecture du manuel de Ptolémée, de contradictions flagrantes entre les tables des manuscrits et les indications de Ptolémée, mais celles-ci sont très sommaires. Dans les manuscrits, de nombreuses tables d'origine et de contenu divers ont été introduites parmi les tables originales. Il est possible d'éliminer les tables parasites, selon un principe simple (qui était déjà celui de Heiberg[20]): les tables qui ne sont pas mentionnées dans le manuel de Ptolémée ne sont pas dues à Ptolémée. Une fois établie la liste des tables authentiques, on peut commencer l'analyse.[21]

Certaines différences apparaissent d'emblée, à commencer par l'ordre des tables. Si l'on suit l'ordre du traité, on voit que Ptolémée donnait d'abord toutes les tables qui concernent le calcul des longitudes (Soleil, Lune, Cœur du Lion et planètes) et ensuite toutes les tables de latitudes: obliquité, latitude de la Lune, latitude des planètes. Cet ordre est respecté par Théon dans le *Petit Commentaire*, mais pas dans les manuscrits des *Tables faciles* qui, en rejetant toutes les tables des planètes après celles qui concernent le Soleil et la Lune, suivent plutôt l'ordre de l'*Almageste*. Dans les manuscrits, des regroupements de tables semblent avoir été opérés diversement. On trouvera ci-dessous (annexe I) un tableau qui donne l'ordre des tables A dans les quatre manuscrits. On remarque que l'ordre des tables semble figé pour A1-A5 et A13-A20, tandis qu'il varie pour la section A6-A12.

Dans les manuscrits, les *Tables faciles* sont conditionnées par la forme du codex qui limite le nombre de colonnes que l'on peut écrire sur une page. Ceci a sans doute influencé la présentation de tables que Ptolémée avait conçues, à ce qu'il semble, pour la forme du *volumen*. Ainsi, Ptolémée semble mettre dans une même table les mouvements moyens du Soleil, de la Lune, du Cœur du Lion et des cinq planètes. Ceci était possible sur un rouleau de parchemin, mais impossible sur une page de codex. Les manuscrits ont tous réuni en une table les longitudes du Soleil

[20] Heiberg, *Opera minora*, pp. CXC-CXCI; ce même principe est suivi par Van der Waerden, *Handlichen Tafeln*, p. 55 et Neugebauer, *HAMA*, p. 972. Voir Tihon, 'Manuscrits', § 2, pp. 50 ss.

[21] Voir à ce sujet Tihon, 'Manuscrits', § 2.

et de la Lune (A3), mais des indices montrent qu'il existait des exemplaires de tables séparées pour le Soleil et la Lune.[22]

La présentation particulière de chaque table est également sujette à variation. Par exemple, dans les tables de parallaxes (A13), la séquence des signes du zodiaque dans M diffère de FHv, mais on la retrouve dans le Pap. Oxyrhynchus 4168.[23] La table des mouvements horaires moyens du Soleil et de la Lune (A3, e) dans H est divisée en deux parties: heures à partir de midi et heures à partir de minuit. Quant aux titres des colonnes et des tables, ils varient eux aussi, d'autant plus que les abréviations se prêtaient souvent à plusieurs interpétations. Par contre, les nombres inscrits dans les tables semblent s'être remarquablement transmis, compte tenu de la masse des chiffres qu'il fallait recopier.

5. Théon a-t-il remanié les *Tables faciles*?

Pour comprendre les *Tables faciles*, nous disposons, outre le mode d'emploi de Ptolémée, de deux traités anciens, dus à Théon d'Alexandrie dans la seconde moitié du IVe siècle de notre ère:

1. le *Grand Commentaire*
2. le *Petit Commentaire*

Le *Petit Commentaire* de Théon est un mode d'emploi simple et clair, illustré d'exemples; le *Grand Commentaire* est beaucoup plus ambitieux: il explique comment les *Tables faciles* ont été calculées à partir de l'*Almageste*. Bien que plus de deux siècles séparent Théon (ca 364 p. C.) de Ptolémée, les explications de Théon sont absolument indispensables pour comprendre les tables, même si dans certains cas (col. 4 des parallaxes, ou table de correction des parallaxes) il ne recoupe certainement pas la manière dont Ptolémée a dû procéder. Toute vérification doit d'abord commencer par tester les procédés décrits par Théon - ce qui sera fait systématiquement dans notre édition.

Les *Commentaires à l' Almageste* de Pappus (ca 320 PC) et de Théon donnent eux aussi des indications sur les *Tables faciles*. Ainsi après avoir donné un exemple de calcul au moyen de l'*Almageste*, Théon donne systématiquement le même calcul effectué d'après les *Tables faciles*. Malheureusement, on ne dispose pas d'une édition moderne pour les livres V-XIII de Théon. En

[22] Voir Théon, PC, p. 208, l. 10-11; voir p. 312, note 4.
[23] Jones, *Papyri*, p. 132.

outre, nous disposons d'informations partielles trouvées au hasard des recherches dans les manuscrits: fragments de commentaires non identifiés, scolies anonymes ou notes diverses.

Al-Battānī, lorsqu'il se réfère aux *Tables faciles*, parle des ≪tables de Théon≫, et beaucoup de modernes l'ont suivi sans autre vérification. Or, dans les manuscrits grecs et les scolies anciennes, les tables ne sont jamais attribuées à Théon. Dans nos quatre manuscrits HFMv, en fait, elles n'ont ni titre général, ni attribution d'auteur. Al-Battānī disposait-il d'une source grecque ou les tables portaient le nom de Théon? Ou s'agit-il d'une assimilation -fréquente chez les auteurs arabes—entre l'œuvre commentée et le commentaire? Il est impossible de le dire.

L'analyse du *Grand Commentaire* de Théon ne donne aucun indice de remaniement des tables par Théon, au contraire. Théon, en effet, se borne à donner un exemple pour montrer que la méthode expliquée par ses soins correspond bien à ce qu'on trouve dans la table. Or, dans une certain nombre de cas, les méthodes de Théon ne permettent pas de retrouver les résultats inscrits dans la table. C'est le cas pour la table de correction des parallaxes (A7), la col. 4 de la table des parallaxes (A13) ou la col. de l'équation du temps dans la table de la sphère droite (A1). Ceci montre qu'il n'a pas calculé lui-même, ni modifié les tables en question.

L'influence de Théon n'est cependant pas à exclure sur la transmission des *Tables faciles*: en fournissant dans le *Grand Commentaire* un certain nombre de méthodes rapides et sûres de vérification, il a sans doute contribué à une transmission correcte des chiffres, et peut-être à une certaine systématisation de ceux-ci. Par exemple, le système d'interpolation qu'il explique pour les tables d'anomalie permettaient aux copistes un contrôle rapide. On peut donc se poser la question: Théon est-il conforme aux tables, ou les copistes ont-ils conformé les tables aux instructions de Théon? Bien que je penche pour la première hypothèse, on ne pourrait en être certain qu'en découvrant une version des tables antérieure à Théon.

6. Quelles sont les relations des *Tables faciles* avec l'*Almageste* et les *Hypothèses des planètes*?

Les liens entre l'*Almageste* et les *Tables faciles* sont à examiner cas par cas. Les *Tables faciles* sont supposées dériver de

l'*Almageste* et reposer sur les mêmes constructions, mais il y a de notables exceptions comme le calcul des prosneuses dans les éclipses, la latitude des planètes ou l'équation du temps. D'autres petites variations existent aussi comme un écart de 1' ou 2' dans les mouvements moyens des planètes,[24] ou des différences dans les limites écliptiques.[25] Ces variations (qu'il eût été facile d'éliminer) restent inexpliquées.

Les relations entre les *Tables faciles* et les *Hypothèses* méritent qu'on s'y attarde.

Outre le parallélisme déjà souligné dans les obectifs des deux traités, il y a de nombreuses similitudes: même origine, mêmes positions de départ. L'origine des *Tables faciles* est l'ère de Philippe (12 novembre 324 a. C.) appelée dans les *Hypothèses* l'ère de la mort d'Alexandre. Il semblerait que l'appellation ≪ère de Philippe≫ (i e Philippe Arrhidée) ne soit pas attestée avant les *Tables faciles.* Pourquoi avoir choisi cette ère et non un point de départ plus proche, comme par exemple, le commencement du règne d'Auguste? Selon le *Grand Commentaire*, Ptolémée aurait choisi cette date parce que le 1er Thoth de la 1ère année du règne de Philippe, le Soleil se trouvait ≪à peu près≫ à 0° du Scorpion, là où l'équation du temps a sa valeur minimale. Cette explication n'est pas très plausible. Ptolémée, dans l'*Almageste*, utilise plusieurs fois des observations datées en années de la mort d'Alexandre. On peut penser qu'il existait déjà avant lui des tables utilisant cette ère, mais cette question reste encore sans réponse.

Revenons aux *Hypothèses des Planètes.* Dans l'étude du livre IV du *Grand Commentaire*, nous avons déjà souligné que les divergences qui apparaissent entre les *Tables faciles* et les *Hypothèses* à propos des *epochai* des planètes proviennent sans doute d'erreurs paléographiques.[26] On trouvera en annexe II une comparaison plus développée. Comme on l'a déjà souligné, les valeurs données par le *Grand Commentaire* (qui sont pratiquement celles des *Tables faciles*) sont plus fiables, car elles ont fait l'objet d'une explication détaillée et les erreurs y étaient plus faciles à repérer. Dans les *Hypothèses*, les données sont énumérées sans justifica-

[24] Voir par exemple Théon, GC IV, pp. 72, 79, 82.
[25] Voir Théon, GC III, p. 296.
[26] Voir Théon, GC IV, p. 86.

tions, et les corruptions y sont nombreuses.

7. Quelles sont les relations des *Tables faciles* avec l'astronomie antérieure ou contemporaine de Ptolémée?

Question particulièrement complexe! L'astronomie de Ptolémée n'est pas surgie du néant: il y avait avant lui et de son temps beaucoup de tables astronomiques de différents types. La publication par A. Jones des papyrus d'Oxyrhinque en montre un échantillonnage. D'autres papyrus, encore inédits, montrent que des tables de même type que celles de Ptolémée existaient, mais avec une autre ère de départ des différences dans les données. Parmi les tables S, il y a une table de vitesse lunaire (S13) qui repose manifestement sur un modèle à épicycle: elle utilise un paramètre non ptoléméen (11; 42° pour la vitesse minimale de la Lune), mais certaines données de la table sont calculées à l'aide du *prokanonion* des *Tables faciles* (A6).[27] Un tel mélange montre qu'il n'y avait pas de cloison étanche séparant l'astronomie de Ptolémée du reste des activités astronomiques de son temps.

Mais malgré tout nos sources sont extrêmement limitées. Les abondantes scolies de l'*Almageste*, très précieuses lorsqu'il s'agit de reconstituer des méthodes de calcul ou des points de détail, reflètent surtout l'enseignement postérieur à Ptolémée: elles ne s'écartent que très rarement des données présentes dans l'*Almageste* et ne semblent pas avoir accès aux sources de Ptolémée. Elles donnent souvent des informations concernant les *Tables faciles*. Malheureusement, il n'existe presque pas de scolies anciennes aux *Tables faciles*.

8. Quel fut le rôle des *Tables faciles* dans l'astronomie byzantine?

Aux Ve et VIe siècles, les *Tables faciles* étaient abondamment utilisées dans les écoles de l'antiquité tardive: on en trouve de nombreuses traces à Alexandrie, mais aussi à Athènes, à Apamée, et même dans l'Italie byzantine avec le *Preceptum canonis Ptolomei* composé vers 524–534[28]. La transition avec le monde byzantin se fit donc pratiquement sans rupture. Mais la prise d'Alexandrie par les Arabes en 640 marque un grand changement. L'enseignement des sciences, dont nous avons tant de témoignages vivants sous forme de notes de cours ou de scolies, se fait beaucoup plus rare, les manuels sont plus limités, les notes plus réduites– -sans

[27] Cette table fera l'objet d'une étude à paraître.
[28] Pingree, *Preceptum*.

doute aussi parce que manquait désormais un support d'écriture bon marché comme le papyrus.

Dès le début du VIIe siècle, vers 610-617, Stéphane d'Alexandrie écrit un *Commentaire aux Tables faciles* inspiré du *Petit Commentaire* de Théon. On peut penser que c'est à ce savant que l'on doit les tables spéciales pour le climat de Byzance qui figurent dans v, F et de nombreux autres manuscrits.[29] Au IXe siècle, des scolies ajoutées au *Vat. gr.* 1291 expliquent l'utilisation des *Tables faciles.*[30] Vers les années 1003–1007, des chapitres anonymes[31] recourent eux aussi aux *Tables faciles* de même qu'un *quadrivium* anonyme.[32] Vers 1032, un scoliaste anonyme compare les tables de Ptolémée, *Almageste* et *Tables faciles*, aux tables arabes d'Alim (i.e. ibn al-Aʿlam).[33] A la fin du XIIIe siècle démarre la grande restauration de l'astronomie de Ptolémée avec la *Stoicheiôsis* (≪ Eléments ≫) de Théodore Métochite (ca 1300), qui bien sûr connaît les *Tables faciles.* Son élève, Nicéphore Grégoras, calcule l'éclipse de soleil du 16 juillet 1330 aussi bien par l'*Almageste* que par les *Tables faciles.*[34] Un calcul anonyme de l'éclipse de Lune du 19 avril 1334, qui pourrait être l'œuvre de Nicolas Rhabdas, recourt à la fois à l'*Almageste* et aux *Tables faciles.*[35] Georges Lapithe, auteur présumé de notes sur les *Tables Tolédanes*, connaît aussi les *Tables faciles.*[36]

Vers 1352, Théodore Méliténiote dans le livre II de sa *Tribiblos astronomique* explique en détail les méthodes de Ptolémée, l'*Almageste* et les *Tables faciles.*[37] En 1368, Isaac Argyre adapte les *Tables faciles* dans un des traités sur les *Tables nouvelles* en vue de calculer les syzygies.[38] Au début du XVe siècle, Jean Chortasmenos donne de nombreux exercices astronomiques basés sur l'*Almageste* et les *Tables faciles*: ses élèves, parmi lesquels figuraient Bessarion et Marc Eugenikos, étaient longuement entraînés aux calculs avec les tables de Ptolémée, et avec les tables

[29] Tihon, *Manuscrits*, p. 54
[30] Mogenet, 'Vat. gr. 1291'.
[31] Inédits (voir Botte).
[32] Heiberg, *Quadrivium.*
[33] Mogenet, 'Scolie inédite'; Tihon, 'Alim'; Mercier, 'Parameters'.
[34] Nicéphore Grégoras, *Eclipse.*
[35] Voire le mémoire inédit d'A. Stoffel.
[36] Pingree, 'Toledan Tables'.
[37] Méliténiote, *Tribiblos*, livre II.
[38] Inédit (voir Laurent et Wampach).

perses.[39] A cette époque, les résultats obtenus par les tables de Ptolémée sont évidemment erronés. Qu'à cela ne tienne: on les ajustera en ajoutant d'office 6° à la longitude obtenue pour le Soleil (ou pour une syzygie), voire, en essayant de ressusciter la théorie de la trépidation des équinoxes expliquée par Théon dans le *Petit Commentaire*!

Le succès des *Tables faciles* est confirmé par les nombreux manuscrits des tables, dont plus de trente s'échelonnent entre la fin du XIIIe siècle et le début du XVe siècle, et par les nombreuses copies du *Petit Commentaire* (souvent rattaché aux tables à cette époque). Ces manuscrits étaient lus, annotés, et circulaient dans les milieux intellectuels byzantins.

Les Byzantins ont-ils modifié les tables? Ils les ont à coup sûr prolongées, mais non modifiées–à l'exception d'Isaac Argyre dans son traité des *Tables Nouvelles.* Ils en ont ajouté d'autres, comme par exemple, une table donnant le mouvement du Soleil et de la Lune par fractions d'heures, ou les tables déjà signalées pour le climat de Byzance, et beaucoup de matériel concernant les calendriers. Mais l'inventaire complet des additions byzantines ne pourra être fait qu'après examen de chaque manuscrit en particulier.

9. Quelle a été l'influence des *Tables faciles* sur l'astronomie dans le monde islamique?

Question trop vaste pour être développée ici! Il faudrait bien sûr étudier de plus près la transmission des *Tables faciles* dans le monde oriental non islamique, notamment en syriaque: Sévère Sebokt utilise les *Tables faciles* et nous savons que celles-ci étaient bien diffusées en Syrie et en Mésopotamie.[40] Il suffit de lire al-Battānī pour voir qu'il utilise souvent les commentaires de Théon et que ses tables de parallaxes et d'équation du temps sont baties sur le modèle des *Tables faciles.* Les *Tables faciles* de parallaxes seront le modèle de presque toutes les tables de parallaxes qui circuleront au Moyen Age. En donnant une nouvelle édition des *Tables faciles*, nous espérons donner aux spécialistes de l'astronomie arabe un outil de comparaison plus efficace et plus fiable que l'ancienne édition de l'abbé Halma.

[39] Tihon, 'L'astronomie byzantine'.
[40] Tihon, 'Manuscrits', p. 68 (palimpseste Vat. Syr. 623); p. 78.

Annexe I. Ordre des tables A dans les manuscrits H v F M

Le tableau qui suit ne tient pas compte des tables B,C,G,S qui s'intercalent diversement dans les manuscrits. Le signe + signifie que plusieurs tables sont réunies en une seule. L'ordre des folios de M a été perturbé par des erreurs de reliure.

H	v	F	M
A1	A1	A1	
A2	A2	A2	
A3	A3	A3	
A4	A4	A4	A4
A5	A5	A5	A5
	A12		
A6 + A7	A6	A6 + A7	A6
	A10	A10	A10
	A7		A7
			A11
A8	A9	A9	A8
A9	A8	A8	A9
A10			
A11	A11	A11	
A12		A12	
A13	A13	A13	A13
A14	A14	A14	
A15	A15	A15	A15
A16	A16	A16	A16
A17	A17	A17	A17
A18	A18	A18	A18
A19	A19	A19	A19
A20	A20	A20	A20
			A15 (partie)
			A14 (partie)
			A15 (partie)
			A13 (partie)
			A14 (partie)
			A17
			A3
			A20

Annexe II. Positions des planètes au départ de l'ère de Philippe: comparaison entre les Tables faciles *et les* Hypothèses des Planètes.

La comparaison entre les données des *Tables faciles* et des *Hypothèses* n'est pas évidente, car elles sont présentées selon des modalités différentes. Une comparaison a été établie par O. Neugebauer dans *HAMA* p. 912, table 20, mais celle-ci doit être revue, à la lumière notamment du *Grand Commentaire* de Théon. La comparaison qui suit suppose la connaissance du modèle de Ptolémée pour les planètes. Soit

u = apogée de l'excentrique compté depuis 0° du Bélier en sens direct

k = position du centre de l'épicycle comptée en sens direct depuis l'apogée de l'excentrique

n = position de la limite nord de l'excentrique comptée en sens rétrograde depuis l'apogée

$n = u - 2^e$ addition TF

la 2^e addition = angle entre la direction de l'apogée de l'excentrique et le point nord de l'excentrique compté en sens rétrograde, soit:

Saturne: 40°; Jupiter: 340°; Mars: 0°; Vénus: 0°; Mercure: 180°.

m = position de la limite nord de l'épicycle comptée en sens rétrograde depuis l'apogée de l'épicycle.

$k + 1^{ère}$ addition $= m$

la $1^{ère}$ addition = angle entre la direction de l'apogée de l'épicycle et le point nord de l'épicycle, compté en sens rétrograde, soit:

Saturne: 220°; Jupiter: 160°; Mars: 180°; Vénus: 270°; Mercure: 90°.

i = position de la planète sur l'épicycle comptée depuis l'apogée de l'épicycle en sens direct

y = position de l'astre comptée en sens direct depuis la limite nord de l'épicycle

$k + i + 1^{ère}$ addition $= y$

Pour permettre la comparaison avec *HAMA*, p. 912 (table 20), voici les équivalences des symboles utilisés:

	HAMA
u	$\lambda A'$
n	λN
k	κ
m	θ
y $k+u$	ω λ
i	α

Conventions utilisées:
H gr = *Hypothèses*, texte grec
H ar = *Hypothèses*, texte arabe
GC = *Grand Commentaire*
TF = *Tables faciles*
< ... > = nombres recalculés, non attestés explicitement.
N (en italique) = nombres donnés par Neugebauer (HAMA, p. 912, table 20)

Dans le tableau, GC = valeurs explicitement données dans le *Grand Commentaire*. En général, elles sont identiques à celles des *Tables faciles* et dans ce cas les valeurs TF ne sont pas données. En cas de divergence, les deux valeurs sont notées. Lorsque le *Grand Commentaire* fait défaut, seules les valeurs TF sont notées.

	u	k	$k+u$
Mercure	185; 24 H gr 185; 24 H ar 185; 24 GC *185; 24 N*	52; 16 H gr 42; 16 H ar 42; 16 GC *42; 16 N*	<237; 40> H gr <227; 40> H ar 227; 40 GC *227; 40 N*
Vénus	50; 24 H gr 50; 24 H ar 50; 24 GC *50; 24 N*	177; 12 H gr 177; 16 H ar 177; 17 GC[41] *177; 16 N*	<227; 36> H gr <227; 40> H ar 227; 40 GC *227; 40 N*
Mars	110; 44 H gr 110; 54 H ar 110; 54 GC *110; 54 N*	356; 20 H gr[42] 356; 7 H ar 356; 7 GC *356; 20 N*	<107; 04> H gr <107; 01> H ar 107; 01 GC *107; 14 N*
Jupiter	156; 24 H gr 156; 24 H ar 156; 23 GC *156; 24 N*	292; 43 H gr 292; 23 H ar 292; 20 GC[43] *292; 43 N*	<89; 07> H gr <88; 47> H ar 88; 43 GC *89; 07 N*
Saturne	228; 24 H gr 228; 24 H ar 228; 24 GC *228; 24 N*	210; 38 H gr[44] 210; 38 H ar 211; 02 GC *210; 38 N*	<79; 02> H ar <79; 02> H ar 79; 26 GC *79; 02 N*

[41] Sur ce chiffre, voir Théon, GC IV, p. 82.

[42] Sur ce chiffre, voir Théon, GC IV, p. 87.

[43] Dans Théon, CG IV, p. 86, les chiffres du *Grand Commentaire* et des Hypothèses (arabe) ont été intervertis par erreur.

[44] Sur ce chiffre, voir Théon, GC IV, p. 87.

	n	i	m	y
Mercure	5; 24 H gr 5; 24 H ar <5; 24> TF *5; 24 N*	<204; 25> H gr <214; 25> H ar 214; 39 GC 214; 40 TF *214; 25 N*	142; 16 H gr 132; 16 H ar <132; 16> TF *132; 16 N*	346; 41 H gr 346; 41 H ar <346; 55> GC <346; 56> TF *346; 41 N*
Vénus	50; 24 H gr 50; 24 H ar <50; 24> TF *50; 24 N*	<81; 23> H gr <81; 19> H ar 82; 01 TF *81; 19 N*	87; 12 H gr 87; 16 H ar <87; 17> TF *87; 16 N*	168; 35 H gr 168; 35 H ar <169; 18> TF *168; 35 N*
Mars	110; 44 H gr 110; 54 H ar <110; 54> TF *110; 54 N*	<120; 26> H gr <120; 39> H ar 120; 39 TF *120; 26 N*	176; 20 H gr 176; 20 H ar <176; 07> TF *176; 20 N*	296; 46 H gr 296; 46 H ar <296; 46> TF *296; 46 N*
Jupiter	<176; 24> H gr 176; 24 H ar <176; 23> TF *176; 24 N*	<138; 48> H gr <138; 53> H ar 138; 57 TF *138; 33 N*	92; 43 H gr 92; 43 H ar <92; 20> TF *92; 43 N*	231; 31 H gr 231; 16 H ar <231; 17> TF *231; 16 N*
Saturne	<188; 24> H gr 188; 24 H ar <188; 24> TF *188; 24 N*	<148; 38> H gr <148; 38>H ar 148; 16 TF *148; 38 N*	70; 38 H gr 70; 38 H ar <71; 01> TF *70; 38 N*	219; 16 H gr 219; 16 H ar <219; 18> TF *219; 16 N*

Les valeurs de u n'ont pratiquement pas de variantes, sauf une erreur paléographique pour Mars et une variante de 1' pour Jupiter. Les valeurs de k dans les *Hypothèses* sont fort perturbées par des fautes d'origine paléographique (voir à ce sujet GC IV, p. 86). Il en résulte évidemment des erreurs dans la colonne $k + u$. Les valeurs de n dérivent de u et ont donc les mêmes petites erreurs. Les valeurs de m dérivent de k et sont perturbées en fonction des erreurs de k. Les valeurs de i et de y posent plus de problèmes: i n'est pas donné comme tel dans les *Hypothèses* et a été calculé d'après

$$i = y - k - 1^{ère} \text{ addition}$$

sur base des valeurs données explicitement pour y et k. Mais comme les valeurs de k sont perturbées, celles de i le sont également. Dans les *Tables faciles*, i est explicitement donné, mais y a été recalculé selon

$$k + i + 1^{ère} \text{ addition.}$$

Si on compare les résultats dans la col. y, les différences sont difficiles à apprécier. Dans le cas de Saturne, on remarquera que les *Tables faciles* ont une augmentation inexpliquée de + 2' par rapport à l'*Almageste* (voir GC IV, pp. 72-73). Si on

supprime cette augmentation, on obtient la valeur des *Hypothèses*: 219; 16°. Pour Jupiter, la différence n'est que de 1', et pour Mars les valeurs concordent. Restent Vénus et Mercure, dont les différences sont plus difficiles à expliquer sur base paléographique. Faut-il supposer une révision des paramètres ou les considérer comme des erreurs de graphie? La question reste posée.

Liste des ouvrages cités

Botte: Botte, G., *Un traité byzantin d'astronomie (XIe siècle)*, Louvain, 1968, pp. 108–110 (mémoire inédit).

CAB : Corpus des astronomes byzantins.

Chabàs–Tihon, 'Parallax': Chabàs, J. et Tihon, A., 'Verification of parallax in the Handy Tables', *Journal for the History of Astronomy*, 24, 1993, pp. 123–41.

Goldstein: Goldstein, B. R., *The Arabic Version of Ptolemy's Planetary Hypotheses*, Transactions of the American Philosophical Society, 57, 4, Philadelphia, 1967.

Halma: Halma, Nicolas B., Θέωνος ἀλεξανδρέως (υπόμνημα. *Commentaire de Théon d'Alexandrie sur le livre III de l'Almageste. Tables manuelles des mouvemens des astres. Traduites pour la première fois du grec en francais sur les manuscrits de la bibliothèque du Roi*, 3 parties, Paris, 1822–5.

Heiberg, *Opera minora* : *Claudii Ptolemaei opera quae exstant omnia, Vol. II: Opera astronomica minora*, ed. J.-L. Heiberg, Leipzig, 1907.

——. *Quadrivium*: Heiberg, J. L., *Anonymi logica et quadrivium, cum scholiis antiquis*, Kgl. Danske Videnskabernes Selbkab., Hist. -Filol. Meddelelser, 15, 1, Copenhague, 1929.

Jones, *Papyri*: Jones, A., *Astronomical Papyri from Oxyrhynchus*, 2 vols, Philadelphia, 1999.

Laurent: Laurent, F., *Isaac Argyre. Traité relatif aux calculs de syzygies*, Louvain, 1969 (mémoire inédit).

Méliténiote, *Tribiblos*, livre II: Leurquin, R., *Théodore Méliténiote. Tribiblos Astronomique*, livre II, 2 vols, CAB V-VI, Amsterdam, 1993.

Mercier, 'Parameters': Mercier, R., 'The parameters of the Zij of ibn al-A'lam', *Archives Internationales d'Histoire des Sciences*, 122, 39, 1989, pp. 22- 50.

Mogenet, 'Scolie inédite': Mogenet, J., 'Une scolie inédite sur les rapports entre l'astronomie arabe et Byzance', *Osiris*, 14, 1962, pp. 198–221.

——. 'Vat. gr. 1291': Mogenet, J., 'Les scolies astronomiques du Vat. gr. 1291', *Bulletin de l'Institut Historique Belge de Rome*, 40, 1969, pp. 69- 91.

Morelon, *Hypothèses*: Morelon, R., 'La version arabe du livre des Hypothèses de Ptolémée', Mélanges de l'Institut dominicain du Caire, 21, 1993, pp. 7–85.

——. 'Phaseis': Morelon, R., 'Fragment arabe du premier livre du Phaseis de Ptolémée', *Journal for the History of Arabic Science*, 5, 1981, pp. 3- 22.

Neugebauer, *HAMA*: Neugebauer, O., *A History of Ancient Mathematical Astronomy*, 3 vols, Berlin-Heidelberg-New York, 1975.

Nicéphore Grégoras, Eclipse: Mogenet, J., Tihon, A., Royez, R., Berg, A., *Nicéphore Grégoras. Calcul de l'éclipse de Soleil du 16 juillet 1330*, CAB, I, Amsterdam, 1983.

Pingree, *Preceptum*: *Preceptum Canonis Ptolomei*, ed. D. Pingree, CAB, VIII, Louvain-la-Neuve, 1997.

——. 'Toledan Tables': Pingree, D., 'The Byzantine Version of the Toledan Tables: the work of George Lapithes?', *Dumbarton Oaks Papers*, 30, 1976, pp. 87–132.

Proclus, *Hypotypose*: Proclus Diadochus. *Hypotyposis astronomicarum positionum*, ed. K. Manitius, Leipzig, 1909.

Ptolémée, *Almageste*: *Claudii Ptolemaei opera quae exstant omnia, Vol. I: Syntaxis Mathematica*, ed. J.-L. Heiberg, 2 vols, Leipzig, 1898–1903.

——. *Hypothèses des Planètes*: *Claudii Ptolemaei opera quae exstant omnia, Vol. II: Opera astronomica minora*, ed. J.-L. Heiberg, Leipzig, 1907, pp. 70–145.

——. *Phaseis*: *Claudii Ptolemaei opera quae exstant omnia, Vol. II: Opera astronomica minora*, ed. J.-L. Heiberg, Leipzig, 1907, pp. 1–67

——. *Tables faciles*: *Claudii Ptolemaei opera quae exstant omnia, Vol. II: Opera astronomica minora*, ed. J.-L. Heiberg, Leipzig, 1907, pp. 159–185.

Schnabel, 'Entstehungsgeschichte': Schnabel, P., 'Die Entstehungsgeschichte des kartographischen Erdbildes des Klaudios Ptolemaios', *Sitzungsberichte der preussichen Akademie der Wissenschaften, Ph. Hist. Kl.*, Berlin, 1930, pp. 214–50.

Stahlmann, *Astronomical Tables*: Stahlmann, W. D., *The Astronomical Tables of Codex Vaticanus graecus 1291*, dissertation doctorale inédite, Brown University, 1960.

Stoffel, A., *Calcul de l'éclipse lunaire du 19 avril 1334 d'après l'*Almageste *et les Tables faciles*, Louvain-la-Neuve, 1982-1983 (mémoire inédit).

Théon GC IV: Tihon, A., *Le "Grand Commentaire" ..., livre IV*, Studi e Testi, 390, Vatican, 1999.

——. CA VI-XIII: *Claudii Ptolemaei Magnae Constructionis idest Perfectae caelestium motuum pertractationis lib. XIII. Theonis Alexandrini in eosdem commentariorum lib. XI*, Bale, 1538 (édition de Joachim Camerarius).

——. CA, I-IV: Rome, A., *Commentaires de Pappus et de Théon d'Alexandrie sur l'Almageste*, II et III, Studi e Testi, 72 et 106, Vatican, 1936-1943.

——. GC I: *Le "Grand Commentaire" de Théon d'Alexandrie aux* Tables faciles *de Ptolémée, livre I*, ed. et trad. A. Mogenet avec un commentaire par A. Tihon, Studi e Testi, 315, Vatican, 1985.

——. GC II-III: Tihon, A., *Le "Grand Commentaire"..., livres II et III*, Studi e Testi, 340, Vatican, 1991.

——. GC IV: Tihon, A., *Le "Grand Commentaire" ..., livre IV*, Studi e Testi, 390, Vatican, 1999.

——. PC: Tihon, A., *Le "Petit commentaire" de Théon d'Alexandrie aux Tables faciles de Ptolémée (Histoire du texte, édition critique, traduction)*, Studi e Testi, 282, Vatican, 1978.

Tihon, 'Alim': Tihon, A., 'Sur l'identité de l'astronome Alim', *Archives Internationales d'Histoire des Sciences*, 122, 39, 1989, pp. 3–21.

——. 'L'astronomie byzantine': Tihon, A., 'L'astronomie byzantine à l'aube de la Renaissance', *Byzantion*, 66, 1996, pp. 244–80.

——. 'Manuscrits': Tihon, A., 'Les Tables faciles de Ptolémée dans les manuscrits en onciale', *Revue d'Histoire des Textes*, 22, 1992, pp. 47–87.

——. 'Scolies': Tihon, A., 'Les scolies des Tables faciles de Ptolémée', *Bulletin de l'Institut Historique Belge de Rome*, 43, 1973, pp. 49–110. Usener, 'De Stephano': Usener,H., 'De Stephano Alexandrino', dans id., *Kleine Schriften*, III, Leipzig–Berlin, 1914, pp. 247-322.

Van Dalen, *Ancient and Mediaeval*: Van Dalen, *Ancient and Mediaeval Astronomical Tables: Mathematical Structure and Parameters Values*, Den Haag, 1993.

Van der Waerden, B. -L., 'Die Handlichen Tafeln des Ptolemaios', *Osiris* 13, 1958, pp. 54–78.

Wampach: Wampach, B., 'Les traités sur les Tables Nouvelles du moine Isaac Argyre. Texte, traduction et commentaire', Louvain, 1978–9 (mémoire inédit).

Wright, 'Vatican': Wright, D. H., 'The date of the Vatican illuminated handy tables of Ptolemy and of its early additions', *Byzantinische Zeitschrift*, 78, 1985, pp. 355–62.

Arabic and Latin Astrology Compared in the Twelfth Century: Firmicus, Adelard of Bath and 'Doctor Elmirethi' ('Aristoteles Milesius')*

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A common view in the history of astrology is that, in the Middle Ages, Latin translations of Arabic texts on the subject completely replaced and consigned to oblivion what survived of the native Latin astrological tradition. This cannot be true, for at least two reasons. First of all, in translating Arabic texts Latin writers had to use Latin terminology, and preexistent Latin accounts of astrology were an obvious source for this terminology. Secondly, the principal Latin textbook on astrology, the fourth-century *Mathesis* of Firmicus Maternus, far from disappearing from sight, actually had a resurgence of popularity in the eleventh and twelfth centuries precisely when new texts were being introduced from the Arabic.[1] In one eleventh-century manuscript it was combined with the first of these Arabic translations, the *Liber Alchandrei philosophi*,[2] and William of Malmesbury in the early twelfth century brackets Firmicus and 'Alhandraeus' as paradigms of excellence in astrology.[3] An interesting example of the perceived

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[1] Thirteen eleventh to thirteenth-century manuscripts of the *Mathesis* are mentioned in the preface to the second volume of Iulius Firmicus Maternus, *Matheseos libri VIII*, eds W. Kroll, F. Skutsch and K. Ziegler, 2 vols, Leipzig, 1897-1913.

[2] A single hand has copied the first two books of the *Mathesis* followed without a break by the *Liber Alchandrei* on fols 20-88r of MS Munich, Bayerische Staatsbibliothek, clm. 560. Since this same scribe writes at the junction of the two works 'Sequentia ad opus Iulii Firmici ut arbitror non pertinent' it seems that he found the same combination in his exemplar.

[3] William of Malmesbury, *Gesta regum Anglorum* (written 1114-23 with additions 1140), II, c. 167. Contemporary with William was the anonymous author of the English legal text, the *Quadripartitus*, whose prose is peppered

relevance of native Latin astrology to the new Arabic science is provided by one of the twelfth-century manuscripts of Firmicus, namely MS Soest, Wissenschaftliche Stadtarchiv 24, fols 1-32, which was copied before 1185 A.D.[4] Here a contemporary annotator[5] has added in the margin the Arabic equivalent of many of the astrological terms in Firmicus's text. The terms are as follows:

	Firmicus[6]	annotation[7]	Arabic	Adelard[8]
1. house	II, 2 domicilia	1v ElBeit	al-bayt	domicilia
2. exaltation	II, 3 altitudines	1v ElsceRaf	al-sharaf	regna
3. decan	II, 4 decani	2r ElWeg, i.e. facies	al-wajh	facies
4. ninth-parts	II, 4, 4 munifices	2r EnnoWaRat	al-nawbah-rāt	novene
5. degree	II, 5 partes	2r Edareia, gradus	al-daraja	gradus
6. term	II, 6 fines	2r Elhad	al-ḥadd	fines
7. 12th-part	II, 13 duode-catemoria	3r Idnasceria arabice	ithnāʿasha-rīya	duodeca-temoria
8. ascension	II,14,3 horoscopus	3v, 32r Matale arabice	maṭāliʿ	ortus/horoscopus
9. ascendant	II, 15, 2 ortus	3v tale	ṭāliʿ	ortus/horoscopus
10. fridarie	II, 25 anni vitae	5v Affraadot arab. 6r Afraadet	al-firdārāt	af(f)raadet
11. projecting rays	II, 29 antiscia	6r Muteaton	muṭīʿa	amicus

(table to be continued)

with phrases from *Mathesis*, I-III: see *Quadripartitus, englisches Rechtsbuch von 1114*, ed. F. Liebermann, Halle, 1892, and R. Sharpe, 'The Prefaces of *Quadripartitus*' in *Law and Government in Medieval England and Normandy: Essays in Honour of Sir James Holt*, eds G. Garnett and J. Hudson, Cambridge, 1994, pp. 148-72.

[4] This manuscript includes only *Mathesis*, II-IV, 22, and is not mentioned by Kroll, Skutsch and Ziegler, or in the more recent edition of P. Monat (3 vols, Paris, 1992-7). For a description of this manuscript see B. Michael, *Die mittelalterlichen Handschriften der Wissenschaftlichen Stadtbibliothek Soest*, Wiesbaden, 1990, pp. 153-8. I am very grateful both to Dr Michael and to the librarian of the Stadtbibliothek at Soest for providing me with photographs of the glosses.

[5] To all appearances the hand of the annotator is the same as that of the text, but the annotations were not written simultaneously with the text.

12. lot of fortune	IV, 17 locus fortunae	29r Cehem 29v Cahada	sahm al-saʿāda	cehem fortune
13. lot of the demon	IV, 18 locus demonis	29v Elgaib arab<ice> non apparentis	(sahm) al-ghayb	cehem abscentie/pars non apparentie
14. the significant planet	IV, 19 oikodespotēs/dominus geniturae	30r Eldelil vel Elmubtez	al-dalīl, al-mubtazz	—

In the case of nos 7, 8, 10 and 13 the annotator specifies that he is giving the Arabic equivalent by adding 'Arab(ice)'. That he personally was comparing Arabic astrological doctrine with Firmicus's Latin doctrine is obvious from his comments that

1) 2v (*Mathesis*, II, 11, concerning the year-number attributed to each sign) 'hanc lectionem apud Arabes non inveni' ('I have not found this reading among the Arabs')

2) 3r (*Mathesis*, II, 12, concerning the winds attributed to each sign) 'secundum Arabes tria prima orientalia, alia tria meridiana, que sequntur occidentalia, postrema septentrionalia' ('according to the Arabs, the first three <i.e. the triplicity of Aries> are eastern, another three are southern, those that follow are western, the last are northern')

3) 5v (*Mathesis*, II, 25, concerning the two lengths of life given by each of the planets, depending on their good or bad state) 'hunc locum dividunt Arabes in tres donationes, magnam, mediocrem et minimam' ('the Arabs divide this feature into three gifts: a large, a middling and a smallest')

4) 5v (*Mathesis*, ibidem) 'hec adiectio de mensibus et horis apud Arabes non est' ('this addition of months and days <to the years of life given by the planets> is not found among the Arabs')

5) 6r (*Mathesis*, II, 26 concerning the *chronocrator* or *dominus temporum*) 'hec lectio vel apud Arabes non est vel loco eius

[6] The book and chapter (and, were relevant, the section number of the chapter) of the *Mathesis* are given. Note that the glossed term in Firmicus is usually in the plural.

[7] The small capitals are as in the manuscript.

[8] The equivalent terms in Adelard of Bath's translation of Abū Maʿshar's *Small Introduction*, whose significance is explained below.

alia est que dicitur afraadet' ('this term is not found among the Arabes, or instead of it there is another <term> which is called "afraadet"')

6) 9r (*Mathesis*, III, 1, 18, concerning the horoscope of the world, in which Firmicus places Aries in the MC) 'magistri Caldeorum Arieti locum dant horoscopi' ('the masters of the Chaldaeans give the place of the ascendant to Aries') = *Iudicia Aristotelis.*

7) 9r (*Mathesis*, III, 2 ff., in which predictions are made from each of the planets separately) 'decreta Arabum secundum collationem planetarum et signorum et omnium supradictorum collatorum fiunt, nihil vero singulariter' ('the judgements of the Arabs are made according to a comparison of the <indications of> the planets, the signs and all the data which have been brought into consideration above; no <judgement is made> according to one <indication> alone')

8) 30r (*Mathesis*, IV, 19, concerning the *oikodespotēs*) 'Eldelil vel Elmubtez apud Arabes dicitur qui in genitura secundum omnia supradicta amplius potest' ('The <planet> which has the greatest power in a birth-horoscope according to all the above-mentioned criteria is called *eldelil* or *elmubtez* among the Arabs')

9) 32r (*Mathesis*, IV, 22, concerning the full and empty degrees) 'apud Arabes aliter' ('The Arabs give a different account').[9]

Moreover, he indicates his Arabic sources, both after his first annotation, and at the end of Firmicus's text:

1) 1v (*Mathesis*, II, 1, 3, listing the male and female signs) 'ex calore ma<sculina>, ex frig<ore> fe<minina> secundum Elmireth' ('the < *signs are* > male as a result of heat, female as a result of cold, according to Elmireth')

[9] The frequent addition of 'deest' ('it is lacking') in the margin (see Appendix below) may also suggest that the annotator is looking for equivalents for Firmicus's doctrine in his Arabic sources, but failing to find them.

2) 32r 'Abumascer Elmuscal Elkaber Elmuscal Escager'.

The second reference is to two works of Abū Maʿshar: the *Great Introduction to Astrology* (*al-mudkhal al-kabīr*) and the *Small Introduction to Astrology* (*al-mudkhal al-ṣaghīr*).[10] As is well known, there were two translations of the *Great Introduction*, one made by John of Seville in 1133, the other by Hermann of Carinthia in 1140.[11] In neither of these translations, however, is the Arabic name of the text retained; nor are the Arabic terms kept in transliteration; the implication is that the Soest annotator knew of Abū Maʿshar's work, either directly, or by report, in its Arabic form. The second work by Abū Maʿshar tells us more. For the *Small Introduction* was translated, with that title ('Ysagoga minor') by Adelard of Bath, several years before the translations of the *Great Introduction*.[12] Another work of Adelard's—his original *De opere astrolapsus*—gives us a clue to the remaining reference, 'Elmireth'. For in it Adelard refers to 'doctor Almirethi' as the man responsible for the astrolabe on which he bases his discussion.[13] In the latter context it has been suggested that 'Almirethi' is Maslama al-Majrīṭī, the late tenth-century mathematician of Madrid who was responsible for adapting several Oriental astro-

[10] These are the Arabic titles given to the first two works of Abū Maʿshar listed in the tenth-century bibliography (*Fihrist*) of Ibn al-Nadīm: see F. Sezgin, *Geschichte des arabischen Schrifttums*, VII, Leiden, 1979, p. 142. For the other titles found in manuscripts of the works see the following two notes.

[11] See R. Lemay, ed., Abū Maʿšar al-Balḫī [Albumasar], *Liber introductorii maioris ad scientiam judiciorum astrorum* 9 vols. Naples, 1995-6, in which both versions, along with the Arabic original, are edited.

[12] See Abū Maʿšar, *The Abbreviation of the Introduction to Astrology together with the Medieval Latin Translation of Adelard of Bath*, eds C. Burnett, K. Yamamoto and M. Yano, Leiden, 1994. For convenience, and because of Adelard's designation of the work as 'Ysagoga minor', the *Abbreviation of the Introduction* (*Mukhtaṣar al-mudkhal*) will be called 'the Small Introduction' throughout this article.

[13] Adelard, *De opere astrolapsus*, ed. B. Dickey in 'Adelard of Bath: An Examination Based on Heretofore Unexamined Manuscripts', Ph.D dissertation, University of Toronto, 1982, p. 177 (referring to the zodiacal calendar on the back of the astrolabe): 'Unde et in tali superficie ambitum zodiaci ecentralem magis describi convenit, ex Geminorum scilicet parte sublimatum, ex opposita submissum, quod in uno solo, videlicet doctoris Almirethi (*variants* Almirecti, Almurethi), astrolabio observatum esse repperi'. It is not clear whether the final '-i' in 'Almirethi' is part of the transliteration, or is rather a Latin genitive ending.

nomical texts to the needs of the scholars of al-Andalus,[14] but al-Majrīṭī is not up to now known to have written anything on astrology (except for some of his astronomical tables, which had astrological uses), and 'Elmireth' both here and in the *De opere astrolapsus* may refer to an individual (and possibly an Arabic teacher of Adelard) as much as to a written work.

The Arabic text of the *Small Introduction* includes all the terms mentioned by the annotator, except three:

1) 'al-nawbahrāt'. The 'ninth-parts' are not mentioned in either of the two extant copies of the Arabic *Small Introduction*, but must have been in the copy used by Adelard, since they are discussed, as 'novene', in his translation, in the last chapter (7 [22-4]), beyond the point where the Arabic manuscripts break off.[15]

2) 'al-dalīl' and 'al-mubtazz' are not mentioned in the *Small Introduction*, which does not give instructions on how to make predictions, but only definitions and descriptions of the astrological elements themselves. The two terms are, however, very common in Arabic astrology, and may have been mentioned by 'Elmireth'.

Of these terms, only 'afraadet' and 'cehem' can be found in the text of Adelard's translation of the *Small Introduction*. 'Idnasheria', 'elgaib' and 'elnowarat' additionally appear in the margins of the earliest manuscript of the translation, London, British Library, Sloane 2030. However, the other terms cannot be found in any manuscripts of the Latin *Small Introduction*. This suggests that they were added by a scholar who knew the *Arabic* text of the *Small Introduction*.

[14] This is the suggestion of Dickey, 'Adelard of Bath', p. 13, followed by E. Poulle in 'Le traité de l'astrolabe d'Adélard de Bath', in *Adelard of Bath: An English Scientist and Arabist of the Early Twelfth Century*, ed. C. Burnett, London, 1987, pp. 119-32 (see p. 123). See also P. Kunitzsch, 'On Six Kinds of Astrolabe: A Hitherto Unknown Latin Treatise', *Centaurus*, 36, 1993, pp. 200-208, who mentions this identification, and suggests, on the grounds of terminology and the spelling of the Arabic transcriptions, that the text that is the subject of his article also may be by Adelard.

[15] The *nawbahrāt*, additionally, are discussed in the *Great Introduction*, V, 17.

Further annotations indicate that the *Small Introduction* was being compared with Firmicus's text, either in its Arabic version, or in the Latin translation:[16]

Soest annotation	*Small Introduction*, Arabic text	*Small Introduction*, Latin text
32r Elmuscal Elkaber	1 [4] kitābunā al-musammā bi-l-mudkhal ilā ʿilm aḥkām al-nujūm ('in our book called 'The Introduction to the Science of the Judgement of the Stars'); 2 [13] kitāb al-mudkhal	1 [4] in ysagoga maiore ('in the Larger Introduction'); 2 [13] in maiore ysagoga[17]
1v Elsceraf, dominans, eius *oppositum serviens*	cf. 1 [84] hubūṭuhu fī muqābila burj sharafihi mithla darajat al-sharaf ('its fall is in the same degree of the sign opposite its exaltation')	1 [84] *servitus* autem regni, eodem gradu, *oppositum*
3r secundum Arabes tria prima *orientalia*, alia tria *meridiana*, que sequntur *occidentalia*, postrema *septentrionalia*	1 [86-9] sharqīya ... janūbīya ... gharbīya ... shamālīya ('east ... south... west ... north <triplicities>')	*Orientalia* ... *meridiana* ... *occidentalia* ... *septentrionalia*[18]
5v hunc locum dividunt Arabes in tres *donationes*, magnam, mediocrem et minimam	7 [4] wa-ammā sinūhā fa-hiyā thalāthat anwāʿ, al-kubrā wa-l-wusṭā wa-l-ṣughrā ('Their years are of three kinds, greatest, middle and smallest')	Dividuntur autem anni isti in tres *donationes*, maximam et mediam et minimam
6r Muteaton, secundum *equal*itatem suorum *dierum*	1 [93] wa-hādhihi al-muttafiqa fī ṭūl al-nahār yuqāl lahā al-muqtadira al-muttafiqa fi'l-quwwa ('Those agreeing in the length of daylight are said to be powerful, agreeing in power')	1 [93] Hec itaque quorum *dies equales* sunt et sibi amica sunt[19]et in bonis effectibus consentiunt et in potentia conveniunt

[16] Terms and phrases common to the Soest annotator and Adelard are highlighted in italics; the notes provide comments on the annotator's apparent knowledge of an Arabic text.

Other comments also can be confirmed from the *Small Introduction*: the 'years' given by the planets are not increased by months or days; instead of Firmicus's 'full and empty degrees' with special significations, there is a division into 'dark', 'shadowy', 'bright', 'indifferent' and 'empty' degrees etc.[20] However, as we have seen, the *Small Introduction* was not the sole source of Arabic astrology for the annotator. For, aside from his definition of the *dalīl* and *mubtazz*, he has added further comments on the practical application of the rules of astrology. The most extensive of these have been written in the lower margins of Firmicus's text, and are not tied to specific passages in the *Mathesis*:[21]

> 1) 1v 'Ab <Ariete> incipiens, unum semper dic mascul<inum>, aliud semper femin<inum>. Homo natus in masculino, causam habet diutinæ vite ex signo, mulier, brevis. Mulier in fem<inino>, longæ, homo, brevis. Aliqua (?) pregnans consulat mathematicum quem partum sit editura; si masculinum signum in hora questionis ascendat, est quedam causa quod conceperit masculum, si femina, quod feminam. Sol et Luna distribuunt planetis sicut clientibus suis mansiones' ('Beginning from <Aries>, always say "one male, the next female". A man born in a male sign has a reason for a long life as a result of this sign, a woman, a short life. A woman born in a female sign <has a reason for> a short life, a man, a long one. A pregnant woman should consult the astrologer on the baby she is about to give birth to. If a male sign ascends at the time of the question, this is a reason <to suggest> that she has conceived a male child, if a female, a female child. The Sun and the Moon divide the mansions amongst the planets as if among their clients').
>
> 2) 2r 'Proprietas signi substantialis est ut sit masc<ulini> vel fem<inini> generis; accidentalis, ut nunc sit primum, nunc secundum. Item, signorum aliud instabile, aliud stabile, aliud

[17] This suggests that the Arabic text that Adelard was using gave the title 'al-mudkhal al-akbar' or 'al-mudkhal al-kabīr'.

[18] The adjective 'meridianus' for 'south' is particularly distinctive, since the alternatives 'meridionalis' and 'australis' are commonly found.

[19] 'et sibi amica sunt' is not in the Arabic MSS, but 'amicus' must be a translation of 'muṭīʿa' as it is in the two sentences that follow this one.

[20] *Small Introduction*, 7 [25-8], beyond the point where the extant Arabic manuscripts break off.

[21] The lower margins are badly rubbed and it is not possible to read all the words. 'æ' represents the e-*caudata*.

mediocre. Instabilia, Aries et eius triplicitas, quia, Sole existente in aliquo istorum, nondum stabilis est proprietas sui quarti temporis, scilicet veris, estatis, autumpni vel hiemis. Aliter: si consularis de inceptione alicuius negocii, iuxta qualitatem signi quod erit tunc (?) in ortu, procedit status negocii in bono vel in malo: si bene se habeat dominus orientis, in bono, si male, in malo. Instabile notat quod negotium non stat per diem. Stabilia dicuntur Taurus et eius triplicitas, quia cum Sol est in illis, iam stat natura sui quarti totius. Aliter: cum stabile sit in ortu, quando quid de negotio queritur, si dominus eius bene se habeat vel male, informat stabilitatem negocii per annum. Mediocria sunt Gemini et eorum triplicitas, quia Sol ens in illis quartarum temporis proprietates mediocriter conservat. Aliter: hec si sint tempore questionis in ortu, mediocriter durat pro qualitate esse dicti inceptio negotii' ('The substantial property of a sign is that it is of a male or female kind; the accidental, that it is now first, now second. Similarly, some of the signs are unstable, others stable, others middling.[22] Unstable: Aries and its triplicity,[23] because, when the Sun is in one of them, the property of the period of its season—i.e. spring, summer, autumn or winter—is not yet stable. Put another way: if you are consulted about beginning any activity, the state of that activity will proceed for good or bad according to the quality of the sign which is then in the ascendant: if the lord of the ascendant is in a good condition, for good; if in a bad, for bad. An unstable sign indicates that the activity does not continue for <as much as> a day. Taurus and its triplicity are called 'stable', because, when the Sun is in them, the nature of its whole quarter is now stable. Put another way: when a stable sign is in the ascendant, when the question concerns any activity, if its lord is in a good or bad condition it gives information concerning the stability of the activity for <as much as> a year. The middling signs are Gemini and its triplicity, because, when the Sun is in them, it preserves the properties of the periods of the seasons to a middling extent. Put another way: if these <signs> are in the ascendant at the time of the question, the inception of the activity endures according to the quality of the aforementioned state').[24]

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[22] This is an unusual terminology for 'tropical', 'fixed' and 'of two bodies'. The terminology in the Latin *Small Introduction* is 'convertibile/conversivum', 'firmum', and 'biforme/bicorpor'.

[23] Note that the annotator uses 'triplicity' in the wrong sense here, since a 'triplicity' should refer to a group of three signs forming a triangle when joined within the zodiacal circle, and not (as here) to a group of four signs forming a square within the circle.

[24] A similar discussion, but with different terminology, is used in a gloss

These quotations both come from the so-called 'Iudicia Aristotelis', the earliest manuscript of which is probably Paris, BNF, lat. 16208, fols 76r-83v. See fol. 76ra-b:

> Et sic curre per ordinem, unum prius dicendo masculini, alterum semper feminini generis esse. Quare dicantur masculini generis esse vel feminini multe sunt cause, set duas de pluribus in medio ponamus. Masculi dicuntur quia si contingat aliquem nasci in ipso, est quedam causa quod ille homo diu vivat; si autem mulierem in ipso nasci contigerit, quedam causa est quod non diu vivat, quare dico quedam causa est qua multe sunt cause in quibus vita longa vel brevis consideratur. Similiter dicuntur feminini quia si mulier in eo nascitur, est quedam causa quod diu vivat; si homo, non. Est alia quidem causa quare dicantur masculini vel femini. Dicimus, si aliqua pregnans veniat querere de se an pariat masculum, signum quod est in ortu tunc fuerit masculini generis est quedam causa quod proles illa sit futura masculina; si autem fuerit in ortu tunc signum feminini generis, quedam causa est quod sit feminina soboles. De hoc plena regula dabitur in sequentibus.
>
> Et hec proprietas quam modo diximus, scilicet 'signorum alia sunt masculini generis, alia feminini', est substantialis, quia proprietatum alie sunt substantiales, alie accidentales. Substantiales sunt sine quibus numquam sunt signa ut hec proprietas sic est substantilis, ut signum quod est masculinum numquam sit femininum neque femininum masculinum; accidentales sunt sine quibus esse contingit, verbi gratia, signum quod in ortu est primum, et aliud quod sequitur secundum et, erecto quod primum erat, incipit esse primum quod erat secundum, ut sit Aries primum, Taurus secundum, erecto Ariete, incipit esse Taurus in ortu, et est primum Taurus quod prius erat secundum.
>
> Item signorum alia sunt instabilia, alia stabilia, alia mediocria. Instabilia Aries, Cancer, Libra, Capricornus; stabilia, Taurus,

(presumably by the translator of the text, John of Seville) to Alcabitius, *Introductorius*, 1 [17]: 'Dicuntur <signa> autem mobilia quia, quando Sol ingreditur aliquod istorum <signorum>, movetur, id est mutatur, tempus; fixa vero dicuntur quia quando Sol est in eis, tempus figitur et eodem statu perseverat; alia sunt communia, id est medietas esse eius unius temporis est, alia vero medietas est alterius temporis. Verbi gratia: quando Sol signum ingreditur Arietis, mutatur tempus, id est convertitur hyems in ver; et quando intrat Taurum, figitur, id est vernale tempus permanet; quando vero Sol ingreditur Geminos, fit tempus commune, id est dimidium erit veris et dimidium estatis, et sic de ceteris.' Note, however, that John's terms—'mobilia', 'fixa' and 'communia'—are different.

Leo, Scorpius, Aquarius; mediocria, Gemini, Virgo, Sagittarius, Pisces. Ut autem levius invenias, scias Arietem esse instabile, Taurum stabile, Geminos mediocre, Cancrum instabile; deinde sic ordinem dispone, ut post instabile ponas stabile et inde mediocre. Instabilia secundum nos ideo dicuntur quia, si negotium aliquod incipiatur, in ipso existente in oriente cito finitur negotium illud, sive in bono finiatur sive in malo. In bono si bene fuerit dominus orientis, in malo si male. Stabilia ideo dicuntur, quia diu stat negotium quod in ipso incipitur, sive stet ad bonum sive ad malum. Mediocria dicuntur quia non diu stat negotium neque cito finitur. Stabilia dicuntur annorum, quia diu stat negotium, instabilia dierum, quia cito transit, mediocria mensium, quia est in medio negotium eorum inter instabile et stabile.

Ab astrologis autem qui de temporibus tractant dicuntur instabilia quia tempora in eis incipiunt et tamen non sunt firma in eis, /76rb/ quia retinent adhuc de precedenti tempore. Stabilia dicuntur quia tempora incepta iam firma sunt in eis, suam tantum habentibus proprietatem, mediocria quia participant sua proprietate et transeunt ad aliam proprietatem. Verbi gratia, Aries est instabile quia ibi incipit ver et non est firmum ver; Taurus est stabile quia ibi est firmum ver, Gemini est mediocre quia iam desinit ver et ad estatem transit.[25]

Note that in the Paris manuscript a cursive hand has added the title in the margin: 'Liber Arystotelis Milesii medici perypathetici in principiis iudiciorum astrorum in interrogationibus'. 'Milesii' is strikingly close to 'Elmireth/Almirethi', while the substitution of a Classical name for an Arabic one is a characteristic of the works of Adelard and his colleagues.[26] However, the *Iudicia Aristotelis* do not seem to include information concerning the relation of the sex of the signs to their heat and coldness, attributed by the Soest annotator to 'Elmireth'.[27]

[25] The same terminology 'stabilia, instabilia' and 'mediocria' occurs in the *Iudicia Ptolomei* and in Raymond of Marseilles, which are both related to the *Iudicia Aristotelis*, but the verbal correspondence is with the *Iudicia Aristotelis*.

[26] For a discussion of Adelard's use of 'Theb<a>idis' for 'Thābit' in his *Liber Prestigiorum Thebidis* and 'Medi' for 'Miṣrī' ('Egyptian') in the *Small Introduction*, see my 'Thābit ibn Qurra on Talismans and the Spirits of the Planets' (in the press).

[27] It is possible to deduce from the descriptions of the signs in *Small Introduction*, 1 [9-81] that male signs are always hot, female cold, but Abū Maʿshar does not state this in the form of a causal relationship; explaining the reasons is a characteristic of the *Iudicia Aristotelis*.

Further indications that Adelard or one of his pupils was responsible for the annotations are:

1) In the early manuscripts of Adelard's translations from Arabic the original Arabic terms regularly appear in the margins. For example, in his translation of Euclid's *Elements* ('Adelard I') in MS Bruges, 529, a wide range of Arabic terms is included,[28] while in the translation of the *Small Introduction* in its earliest copy (MS London, British Library, Sloane 2030) 'idnasheria',[29] 'dileleti habeci' (possibly 'dalālāti l-kawākib'?) and 'ale elmeizrin' ("alā miṣrīyīn') are given in the margin on the first page (fol. 83r), and a list of the Arabic names for all the 16 conditions of the planets is given in the lower margin of 84v.

2) Where the same terms occur in Soest 24 and the manuscripts of Adelard's translations, the transliteration is almost exactly the same. To 'Almirethi' in the *De opere astrolapsus* and the examples from the *Small Introduction* can be added 'matale' in Adelard's translation of the astronomical tables of al-Khwārizmī, MS Chartres, 214 ('matale elburug'). Common characteristics can be observed in this transliteration: *alif* ('a') is usually represented by 'e', especially when long (hence the definite article is 'el'[30] rather than 'al'); *jīm* ('j') is 'g'; *sīn* ('s') is 'c', which becomes 'z' when adjacent to voiced consonants; *shīn* ('sh') is 'sc'. The transcriptions reflect the spoken Arabic of the Maghreb, rather than a strict letter-by-letter equivalence and show, for the most part, the spoken assimilation of the 'l' of the definite article to the 'sun-letters' (sibilants and dentals).[31]

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[28] These terms are listed in H. L. L. Busard, *The First Latin Translation of Euclid's Elements Commonly Ascribed to Adelard of Bath*, Toronto 1983, Addendum Ib, pp. 394-5.

[29] In both Sloane 2030 and the Soest manuscript it is unclear whether the scribe has written 'iduasheria' or 'idnasheria'.

[30] Whether the fact that the 'e' is always capitalized in the Soest annotation is significant is unclear.

[31] For further discussion of transliterations found in Adelard of Bath's translations see P. Kunitzsch, 'Letters in Geometrical Diagrams: Greek-Arabic-Latin', in *Zeitschrift für Geschichte der arabisch-islamischen Wissenschaften* 7, 1991/2, pp. 1-20, especially tables 3 and 4.

3) Soest 24 includes a quotation from Adelard's translation of the *Small Introduction*, on fol. 76v,[32] along with two texts by Roger of Hereford. The texts by Adelard and Roger occur in a separate codex in the manuscript, copied in the fourteenth century in what has been identified as an English hand.[33] It is possible, however, that this copy was made from a manuscript which was in the same place as the twelfth-century codex containing the annotations on Firmicus.

The association with Adelard of Bath is, therefore, strong. Adelard had at his disposal both an astrolabe which he calls that 'of doctor Almirethi' and Arabic manuscripts which included at least works of Abū Maʿshar, al-Khwārizmī, Thābit ibn Qurra, and Euclid. The Arabic annotations appearing in manuscripts of his works suggest that he was working with Arabic speakers.[34] In the Soest manuscript there are no noticeable mistakes in the transliterated Arabic words that can be ascribed to copying errors. In other words, the annotations are likely to have been made by someone who knew the Arabic words themselves, either in their spoken or in their written form.[35] Also, they were apparently copied into the manuscript *after* the text itself, which suggests that they were not simply copied over from a previous exemplar. We are bound, therefore, to ask whether we are dealing with the annotations of Adelard himself.

Adelard's activity lasts from the early years of the twelfth century to at least 1149.[36] Some astronomical notes on the last folio of the twelfth-century codex of Soest 24 refer to a present date of

[32] The excerpt immediately follows a copy of Alcabitius' *Liber introductorius*, and begins with the words 'Sumptum de isagoga Adelardi: Primum cehem fortune et prosperitatis...' It includes *Small Introduction*, 6 [3] and [6], ending with an explanation of Adelard's terms: 'cehem dicitur pars, horoscopus dicitur prima domus et proprie gradus ascendens qui est ipsius domus principium'.

[33] Michael, *Die mittelalterlichen Handschriften*, p. 153.

[34] See C. Burnett, *The Introduction of Arabic Learning into England*, pp. 40-44.

[35] The only problem is 'sc' for 'dkh' in 'Elmuscal' (twice); it is not likely that a scribe would have mistaken a 'd' for an 's'.

[36] See Adelard of Bath, *Conversations with his Nephew*, eds C. Burnett et al., pp. xi-xix.

15 October 1185.[37] However, these notes are not written in the same hand as the text of Firmicus and its annotations. They use Hindu-Arabic numerals (which Adelard is otherwise not known to have used) and could have been added later in the space provided by the empty last page (or cover) of the codex. The text of Firmicus and the annotations could well date from within the first half of the twelfth century, showing as they do a considerable use of the e-*caudata*, the digraph for 'de', small capitals within the text (especially N and R) and open gallows-shaped *pieds-de-mouches*. However, the script does not have the distinctive traits found in the most authoritative manuscript of Adelard's *Questiones naturales*—MS Paris, BNF, 2389—and Avranches, Bibliothèque municipale 253 (which contains the oldest fragment of the *Small introduction*).[38] On the other hand, it bears some striking resemblances to the script of the scholar who put together MS Cambridge, Trinity College, R.15.16.[39] Both scripts show the same trailing descenders to the long-s, open-bowed 'g', and dropped-9 form of the 'cum' abbreviation and the occasional 'R' within words and high 'a', as well as having the same general appearance. It is quite plausible that the scribe of Trinity College is the same man as that of Soest 24, but writing at a later date, when e-*caudata* tended to be dropped, the *pieds-de-mouches* where filled in, and Arabic numerals were starting to be used. It has long been recognised that Trinity, R.15.16 is closely related to Adelard's circle, since it refers to 'Alardus' as a 'present-day' ('modernus') geometer and includes a text that draws from, or from the same sources as, the *Helcep Sarracenicum* dedicated to Adelard by his pupil 'Ocreatus'.[40] The twelfth-century codex of

[37] See W. Becker, *Frühformen indisch-arabischer Ziffern in einer Handschrift des Soester Stadtarchivs*, Soester Beiträge zur Geschichte von Naturwissenschaft und Technik: Uni-GH Paderborn/Abt. Soest, 1995.

[38] See C. Burnett, 'Avranches, B.M. 235 et Oxford, Corpus Christi College, 283', in *Science antique—Science médiévale*, ed. L. Callebat and O. Desbordes, Hildesheim, 2000, pp. 63-70. Most distinctive of the script in these two manuscripts is the abbreviations for –m and –n which curve upwards sharply towards the right.

[39] For specimens of this script see C. Burnett, 'The Instruments which are the Proper Delights of the Quadrivium: Rhythmomachy and Chess in the Teaching of Arithmetic in Twelfth-Century England', *Viator*, 28, 1997, pp. 175-201 (see pp. 190-1).

[40] See C. Burnett, '*Algorismi vel helcep decentior est diligentia*: the Arith-

the Soest manuscript must also be brought into this circle.

The question remains as to the use that Adelard and/or his colleagues made of Firmicus's text. A full answer to such a question goes beyond the scope of this article. Nevertheless, it can be observed from the first table above that Adelard at least used Firmicus's terms for the planetary houses ('domicilia'), the 12th-parts ('duodecatemoria'), and the ascendant ('horoscopus').[41] In other cases he preferred a literal translation of the Arabic. It is significant that in three cases where Adelard *does* use such a literal translation rather than Firmicus's term, the annotator of the Soest manuscript specifically mentions those terms as being the exact translations of the Arabic: 'Elweg, id est facies', 'Edareia, gradus' and 'Elgaib, Arabice non apparentis'.[42] Whether the *Mathesis* may prove to be important for determining the terminology of the translations from Arabic or not, the annotations in the Soest manuscript remain a significant testimony to the fact that at least one Latin scholar compared the new Arabic and the native Latin tradition of astrology, without judging one to be superior to the other.[43]

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metic of Adelard of Bath and his Circle', in *Mathematische Probleme im Mittelalter*, ed. M. Folkerts, Wiesbaden, 1996, pp. 221-331, where further examples of the script of Trinity, R.15.16 are given.

[41] This is not to imply that there were no other sources for this Latin terminology.

[42] The primary (and common) meaning of 'wajh' is 'face', of 'daraja', 'step', and of 'ghayb', 'invisible'.

[43] It is significant that the annotator never uses phrases such as 'falsum est', 'erravit auctor' etc., but simply notes where Arabic doctrine differs from Firmicus's.

Appendix

Included here are all the annotations to Firmicus's text in Soest 24 that have not been quoted in the course of the article, with the exception of 'nota' (and its abbreviations), 'deest' (2v, 3v twice, 5v twice, 6v, 9r),[44] and corrections to Firmicus's text.

1. 1v: (*Mathesis*, II, 2) A domicilio dominus eius oppositum alienus (cf. *Small Introduction*, p. 94 (marginal note): 'domicilium alienationi oppositionis')

2. Ibid. (*Mathesis*, II, 2, 5: Saturnus habet domicilium) i.e. ibi habet maiorem vim et sic deest (*continued in margin*) non dico maiorem gloriam; gloria pertinet ad honorem et famam, sed mansio (?) ad vigorem rei de qua queritur. 'Gloria' is the term used for 'exaltation' in the *Iudicia Aristotelis*, cf. 76rb (c. 2): 'et gloriam eorum, que etiam appellatur erectio et exaltatio, et depositionem que casus appellatur et deiectio sive descensus seu mestitia.' 'Mansio' in the *Iudicia Aristotelis* is one of the twelve places; cf. ibid., c. 17 (f. 78rb-78va): 'Nunc sciendum quemque hominem habere .xii. mansiones in celo sive nascatur sive ad questionem veniat, prima turris que in ortu est in nativitate alicuius vel in questione vocatur turris vite...'

3. 2r (*Mathesis*, II, 4, 3: primus decanus Martis et secundus Solis) ordinem nota

4. 5r (*Mathesis*, II, 22, 7: Exagona hoc idem sunt quod trigona) complexio enim contraria

5. 6r (*Mathesis*, II, 27 'De distributione (divisione Soest MS) temporum') sed nec hec

6. 6r (*Mathesis*, II, 28, 'De anni divisione') sed nec hec

7. 6v (*Mathesis*, II, 29, 7) T<abula> Quomodo singule partes in singulas mutant antiscia

[44] The significance of these mentions of 'it is lacking' is not clear, since there is nothing lacking in Firmicus's text at these points, nor are the 'deest's keyed to particular words or phrases in the text. It rather seems that the annotator is not finding equivalent passages in the text he is comparing—i.e., perhaps, in an Arabic text. The repeated phrase on fol. 6r 'sed nec hec' ('but not these <doctrines>'?) may have a similar significance, and the 'quere' ('look for it!') on 30r might suggest that the annotator wants to search for an equivalent in the Arabic or another comparable text. 'Deest' and 'queritur' both appear in no. 2 below, whose meaning is not clear.

8. 30r (*Mathesis*, IV, 19 'De domino geniturae (Soest MS adds 'inveniendo')) Quere

9. 31v (*Mathesis*, IV, 20, 2 'De annis climacteris') Quemadmodum anni clematerici (*sic*)[45] inveniantur

10. 31v (*Mathesis*, IV, 20, 3) Quando anni clematerici transeant

11. 32r (after the end of *Mathesis*) A short glossary arranged in approximately alphabetical order:

Abumascer Elmuscal Elkaber (or 'Elkiber') Elmuscal Escager.

Antiscia: partis in partem radiatio missa

Apotelesmata: decreta

Apocatastasis per cirosim et cataclismum: redintegratio per exustionem et diluvium (cf. *Mathesis*, III, 1, 9)

Horoscopus: prima nuper orti signi inspectio vel Arietis vel medii celi.

Matale: Ortus

[45] 'clematerici' is also written in the text.

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India and Iran

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On the Dimension of the Astral Bodies in Zoroastrian Literature: between tradition and scientific astronomy

Antonio Panaino

Although Pahlavi Zoroastrian literature directly preserves only a few remnants of a larger astrological and astronomical production, we can still find therein some scattered information and a lot of traditional astral beliefs, frequently mixed (not without contradictions) with up-to-date astronomical data. This peculiar aspect of the Sasanian astral culture reflects the ambiguous and contradictory attempts of a priestly and intellectual class trying to make use of Greek and Indian astronomical and astrological sciences without any open and radical refusal of the standard theological doctrines concerning the sky and celestial phenomena. Thus, as has already been noted by Henning,[1] we can find in Pahlavi texts a number of 'modern' astronomical data intermingled, for instance, with the traditional idea that the heaven of the stars was closer to the earth than those of the moon and the sun [Panaino 1995]. I do not doubt that in this as in some other cases the writers (or at least some of them) knew the truth, but we can imagine that they tried to avoid a direct confutation of the Avestan doctrines. We have also to recall that the still-extant Pahlavi texts do not belong to professional astronomers/astrologers (who, as we know also from later Arabic, Byzantine and Latin sources, not only existed but were very active and productive). Rather, they represented a sort of religious commentary where astral problems were treated by means of a mélange of theology and current scientific (or pseudo-scientific) doctrines, some of them foreign in origin, as David Pingree (starting with a very seminal contribution in 1963) has pointed out in a number of works which have substantially defined the complex role of Sasanian Iran in the

[1] [Henning 1942], in particular p. 230.

transmission of astral sciences in antiquity.[2]

The survival of an ancient astral lore is in any case very important because it contributes to our understanding of the remotest picture of the heavenly bodies and phenomena developed in ancient Iran, apparently before the impact of Mesopotamian, Greek and Indian sciences. We are still able, at least in some cases, to trace for some Pahlavi texts their direct Avestan literary prototypes (or in any case the textual models derived from the same *Vorlage*), as for instance in the current description of the *Axis mundi*,[3] which appears in the *Bundahišn*. In fact it is very difficult to deny a direct parallel between *Yašt* 12, 25 (the Avestan hymn to the god Rašnu)[4] and a very similar passage from *Great Bundahišn* (or *Iranian Bundahišn*) VB, 1, which was rightly proposed by Bailey and MacKenzie;[5] nonetheless, we can say a little more about it.

The Av. text runs as follows [Geldner 1889, 166]:

yaṯciṯ ahi rašnuuō aṣ̌āum	'When you are, O pious Rašnu,
upa taērəm haraiθiiå barəzō	over the Peak (Taēra) of Mount Haraitī (Haraitī Bərəz)[6]

[2] See in particular the chapter 'The Recovery of Sasanian Astrology' in [Pingree 1997, 39–50], summarizing the main points of the research on this field.

[3] [Panaino 1994–5], [Panaino 1996].

[4] The Avestan hymn to Rašnu (*Rašn Yašt*, the XIIth of the Av. Corpus) probably is, in its attested version, a later composition, where the god is invoked (starting from stanza 12) with a quite repetitive litany in every part of the earthly, atmospheric and celestial worlds in which he is assumed to be present. The sequence of these invocations, however, clearly represents a sort of ascension made by the Iranian divinity from the earth—starting from the seven Karšvar-s (Pahl. Kišwar-s), i.e. the seven parts of the earth—up to the paradise of the supreme creator, Ahura Mazdā, by following a particular path which deserves attention, because it conforms to (and at the same time confirms) a number of astral patterns attested in Old Iranian culture. Such a strictly religious document presents us in fact with a number of cosmographic and uranographic conceptions that, on one hand, reflect an Indo-Iranian background and, on the other hand, had an enormous impact on the later Zoroastrian conception of the world, in particular in Pahlavi astrological and astronomical texts. See also [Windfuhr 1983], [Panaino 1994–5].

[5] [Bailey 1971, 138–9], [MacKenzie 1964, 517, n. 38].

[6] This is not the place where we can re-discuss all problems connected with the cosmic geography of the Avesta people; we can simply summarise the data [Gnoli 1980, 146-149] by noting that Mount Harā (or Haraitī) corresponds to

yaṯ mē aiβitō ˣ*uruuisiṇti*[7]	round which my
starasca måsca huuarəca	stars, Moon and Sun turn,
zbaiiamahi ...	(then) we invoke you ...'

The fact that the corresponding Pahlavi text in *Gr.Bd.* VB 2 contains a passage which can be considered as a direct quotation from an Avestan source is openly supported not only by the content itself, but also by the fact that the Pahlavi translation is introduced with the standard formula used for the quotations (translated) from the sacred scriptures. In fact the passage starts with 'as He says' (*čiyōn gōwēd*);[8] then we give the text (with transliteration, transcription and translation) of *Gr.Bd.* VB 1-2 (**DH**,[9] 173v, 13-16; **TD1**,[10] 45, 9-13 [= fol. 22r, 9-13]; **TD2**,[11] 55, 3-8; see also the parallel tradition according to the *Ind.Bd.*,

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the Harburz system (probably to be identified with the Hindūkuš), which, according to *Gr.Bd.* VB, 1 [MacKenzie 1964, 517], is around the world, while the mountain Tērag (corresponding to the Av. Taēra, and perhaps to be located in the Alburz range) is in the centre of the world, and corresponds to the *Axis Mundi.*

[7] MSS: °*səṇti* F1, E1; *uruuasəṇti* J10, O3; the emendation in *°*siṇti* was already suggested by Geldner in the *apparatus criticus.* We may also note that the Av. verb *uruuis-* 'to turn' (here translated with Pahl. *waštan, ward-*) is particularly fitting for the movement of the astral bodies (cf. [Bartholomae 1904, cols. 1533–4], [Kellens 1995, 60]). In addition we may quote the Av. adj. compound *dūraēuruuaēsa-* [Bartholomae 1904, col. 751], actually attested with Av. *paṇtā-/paθ-*, m. 'way' (cf. O.P. *paθi-*, f.; Ved. *pánthā-*, *path-*, *pathi-*, m.), e.g. in the expression *dūraēuruuaēsəm paiti pantąm* 'along the far-winding way' (*Yt.* 8, 35; see the commentary in [Panaino 1990, 123-124]) in a context referred to Tištrya (the star Sirius), but also in an elliptical construction with Av. *aδβan-*, m. 'path' (cf. [Bartholomae 1904, col. 62]) in the sentence *āaṯ tē nūrąm frauuazəṇti dūraēuruuaēsəm aδβanō uruuaēsəm nāšəmna* 'thus, these now move forth along the far-winding (way), reaching the turning point of the path' (*Yt.* 13, 57–8) in a context referred to Sun, Moon and stars. It has to be noted that in its turn, O.Av. *aduuan-*, m. 'idem', which occurs with perfect synonymy with *paθ-* (cf. [Kellens & Pirart 1990, 201]) is attested in *Y.* 44, 3, with reference to the course of the Sun and of the stars: *kasnā x́ᵛəṇg strə̄mcā dāṯ aduuānəm* 'who determined the path of the Sun and of the stars?' (cf. [Kellens & Pirart 1988, 149]; [Insler 1975, 66–7]; [Humbach 1991, I, 157]). In *Yt.* 12, 3 we find also *paitiša hū adβanəm* 'in front of the course of the sun'.

[8] On this expression see [Henning 1942, 231, n. 8], who wrote 'When the book cited happens to be the Avesta (...), the subject of *gōvēd* is the author of the Avesta, namely Ohrmazd according to Zoroastrian teachings (cf. Dinkard, pp. 9–10).'

[9] **DH** = [*Codex DH* 1971].

[10] **TD1** = [*Bondahesh* 1970].

[11] **TD2** = [Anklesaria 1908].

V 3-4: **M51**,[12] 230r, 12-16; **K20**,[13] 13, 5-10):

(1) kwp Y hlbwlc pyt'k AYK[14] pyl'mwn[15] Y gyh'n' kwp Y tylk' Y[16] mdy'n' Y gyh'n'.[17] hwlšyt' gltšn'[18] cygwn 'p̄ysl[19] pyl'mwn Y gyh'n' BYN[20] 'pyckyh[21] Y hcpl[22] kwp Y hlbwlc[23] pyl'mwn[24] Y tylk' LAWHL wltyt'. (2) cygwn YMALLWNyt[25] AYK tylk' <Y>[26] hlbwlc[27] MNW ZK Y[28] L hwlšyt' W m'h[29] <W> st'lk'n[30] MN AHL LAWHL wltyt'.

(1) *kōf ī harburz paydāg kū pērāmōn ī gēhān. kōf ī tērag ī mayān ī gēhān. xwaršēd gardišn čiyōn abesar pērāmōn ī gēhān. andar abēzagīh ī azabar ī kōf ī harburz pērāmōn ī tērag abāz wardēd.* (2) *čiyōn gōwēd kū tērag <ī> harburz kē ān ī man xwaršēd ud māh ud stāragān az pas abāz wardēd.*

'It is revealed that the mountain Harburz is around the world (and) the mountain Tērag is the middle of the world. The revolution of the sun is like a crown around the world. In (a state of) purity above the mountain Harburz it turns back around Tērag.' (2) 'As He says: "Tērag of the Harburz, behind which my sun, moon and stars turn back".'[31]

[12] **M51** = Unpublished ms. of the *Indian Bundahišn* from the Munich Library. I have to thank my friend and colleague Dr. Carlo Cereti (University of Rome) who kindly placed at my disposal the readings of M51. Dr. Cereti is now preparing an edition of this very important codex of the Munich Library (see [Bartholomae 1915, 56–72]).

[13] **K20** = [Westergaard 1851].

[14] pyt'k AYK only in K20; desunt in DH, TD1, TD2.

[15] K20; TD1, DH, TD2: pyl'mw.

[16] Only M51.

[17] DH, TD2, K20; TD1: gytydy.

[18] DH, TD1, TD2; K20: gltyn'.

[19] DH, TD1, TD2; M51; M51; K20: MYAsl.

[20] TD1, K20; DH, TD2 omittunt.

[21] DH, TD1, TD2; M51, K20: 'pyck Y MN wl.

[22] DH, TD1, TD2; K20: Y MN wl.

[23] DH, TD2, K20; TD1: hlblc.

[24] DH, TD2, K20; TD1: pyl'mw.

[25] DH, TD2; TD1, K20: YMRRWNyt.

[26] DH, K20.

[27] DH, TD2, K20; TD1: hlblc.

[28] DH, K20.

[29] DH, TD1, TD2; M51, K20: BYRH.

[30] K20; DH, TD1, TD2: st'lk'.

[31] See [MacKenzie 1964, 517]; [Anklesaria 1956, 64–5]; for the version of the *Indian Bundahišn*, see also [Justi 1868, 7 (translation), XIII, 5–9 (text)]; [West 1880, 22].

The Pahlavi sentence is clearly a *quasi*-word-for-word literary quotation—with only an inversion of the order of stars, moon and sun, which actually is the opposite in the Pahlavi text; very striking appears the presence of Pahl. *man*, which perfectly translates Av. *mē*, and which permits us to identify the subject of the sentence 'He says...' as Ahura Mazdā himself, being the sole and highest god, in such a source, in order to affirm that the astral bodies are his own belongings. We have to note, on the contrary, that the personal pronoun is omitted in another parallel passage as attested in *Gr.Bd.* IX, 6 (**DH** 179v, 16–17 [= fol. 40, 16–17]; **TD1**, 63, 16–17 [= fol. 31r, 16–17]; **TD2**, 77, 8–10); but here, on the other hand, the astral order is the same as in the Av. passage:

> tylk' Y hlbwlc ZK MNW-š st'l W m'h W hwlšyt ptš B̲YN wltynd W ptš LAWHL YATWNyt'.
>
> *tērag ī harburz ān kē-š stār ud māh ud xwaršēd padiš andar wardēnd ud padiš abāz āyēnd.*
>
> 'Tērag of the Harburz is that (mountain) through which stars, moon and sun turn and through which they come back.'[32]

Thus, if it is impossible to affirm that any of these texts is the direct translation of the Avestan stanza of *Yt.* 12, 25 (we have to recall that more than two thirds of the original Avesta has disappeared) we can surely state that this very Avestan passage confirms the existence of an Av. *Vorlage* which was accessible to translations, adaptations and eventually changes, already in Avestan and of course also in Pahlavi writings. The identification of Avestan models becomes very significant because we can distinguish and precisely determine the impact and preservation of traditional doctrines in later astronomical documents, notwithstanding that these old Iranian texts were subjected to modifications and reinterpretations.[33]

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[32] Cf. [Anklesaria 1956, 94–5].

[33] As in the case of the idea, attested just in *Gr.Bd.* VB, 3 (see [Panaino 1998, 76–7], with all the additional bibliographical references), that the astral bodies (moon, stars and planets) are bound to the windows which lie (180 in the East and 180 in the West) around the chain of Mount Harburz. This doctrine—as Pingree has first noted ([Pingree 1963], in particular p. 242; [Pingree 1964, 123]; [Pingree 1975, 6])—finds its model in Indian astronomical

Another example which for instance can be placed among the patent contradictions between past and present 'doctrines' attested in Pahlavi texts is the one concerning the dimension of the astral bodies. In this case again we will have the opportunity to trace and compare different and concurrent traditions.

Among the Avestan fragments quoted in the Avestan-Pahlavi Dictionary, better known as *Frahang ī Ōīm,* whith reference to the contents of the *Nīkādūm* Nask,[34] we find the following sentence:[35]

Av. text:

> *nitəmacit̰* [36] *auuaēšąm stārąm yaθa narš *maδəmiiehe*[37] *vaγδanəm.*

Pahl. translation:

> ZKc Y nytwm MN OLEš'n stl'n' cnd GBRA 1 Y mdy'nk wγd'n.
>
> *ān-iz ī nidom az awēšān starān čand mard ēk ī mayānag waγdān.*

literature, where the pole of the world (*dhruva*) is bound to the stars and the planets by wind-bonds [Panaino 1998, 52–57]. In a recent work, where the Pahlavi texts reflecting these Indian concepts have been discussed, I have underlined the importance of a few sources concerning the bonds which link the Moon to some seas; in the discussion of the important passages attested in the *Wizīdagīhā ī Zātspram*, III, 18–19 [Panaino 1998, 80–1] I have omitted to take into consideration ch. III, 20, where, after the statement that the tides are produced by the bonds tied with the waters, it is stated: *pāymār az pēš māh dō wād frāz tazēnd andar sadwēs māništ dārēnd ēk ul-āhang ud ēk frōd-āhang pad ān ī ul-āhang bawēd purr ud pad ān ī frōd-āhang bawēd ōgar.* 'Il est désigné: au-devant de la lune courent deux vents, et ils ont leur demeure dans (le lac) Sadwēs, l'un expirant et l'autre inspirant, par celui qui expire il y a le flux et par celui qui inspire il y a le reflux' (see [Gignoux & Tafazzoli 1993, 44–5]). The presence of two winds (*wād*), one which breathes out (*uk-āhang*) and the other breathes in (*frōd-āhang*), closely evokes that of the Pravaha wind (i.e. the Provector) that (e.g. in the *Sūryasiddhānta* and some Purāṇas) impels the astral bodies along the diurnal motion; in the Siddhāntas are mentioned also other winds producing the anomalies of the Sun, the Moon and the planets (see [Pingree 1990]; cf. [Panaino 1998, 57]). On the cosmological and cosmographical function of Mount Meru in Indian astronomical literature, see, e.g., the *Āryabhaṭīya*, 11–12 [Shukla 1976, 121–3].

[34] See also [West 1892, 471–5].

[35] [Darmesteter 1893, 16] (as number 9); [Reichelt 1900, 191]; [Reichelt 1901, 176]; [Klingenschmitt 1968, xix, 81] (discussed under number 224).

[36] [Darmesteter 1893, 16, n. 3] reads *nitəmcit̰* (thus as acc. sg.), but suggesting a correction in *nitəməcit̰*.

[37] MSS *maδmiiehe*; correction suggested by [Reichelt 1901, 176] and [Klingenschmitt 1968, 81].

> 'The smallest ones of those stars (are) like the head[38] of a middle-sized man.'

The idea that the stars can be small or big, and that their (supposed) dimension can be compared with that of human beings or in other cases with terrestrial things, parts of the body or places, is confirmed by the Pahlavi texts, and it is actually attested in the famous astronomical chapter of the *Great Bundahišn*, II, 16 (**DH**, 166v, 19–167r, 2; **TD1**, 25, 17–26, 1–3 [= fol. 12r, 17–12v, 1–3]; **TD2**, 29, 7–12):

> MN OLEš'n' st'lk'n'[39] ZK Y ms cnd *KYPA-1[40] Y htk-ms'd, ZK Y mdy'n[41] cnd c'hlkw'n'[42] *wp[t]šn', ZK Y ks cnd LOYŠE Y TWRA <Y> ktkyk. W m'h cnd 'splys-1 <Y> 2 h'sl,[43] cygwn KRA h'sl-1 Y PWN z̲myk plsng-1 <Y> ptm'nyk hwm'n'k.[44] hwlšyt' 'ndcnd 'yl'nwyc.

> *az āwēšān stāragān, ān ī mas čand *gōhr-ēw ī kadag-masāy. ān ī mayān čand *čahragwān wafišn. ān ī keh čand sar ī gāw <ī> kadagīg. ud māh čand asprēs-ē <ī> 2 hāsar; čiyōn harw hāsar-ē ī pad zamīg frasang ī paymānīg homānāg. xwaršēd and-čand ērān-wēz.*

> 'Among these stars, the large ones are like a piece of rock the size of a room, the medium-sized ones are like a rolling wheel, the smallest ones like the head of the domesticated ox. The moon is the size of a racecourse of two *hāsars* (*hāθras*), each geographical *hāsar* being about as much as a parasang of average length. The sun is the size of Ērān-vēz.'[45]

The present passage clearly reflects a traditional doctrine, as we can deduce from the persistence of the idea that the smallest stars

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[38] It is peculiar that here the Daēvic stem *vaδa*-, n., is used, and not *sara*-, n. (Ahuric), which should be the expected one with reference to the stars, which are positive divine beings in the Zoroastrian tradition.

[39] DH, TD1, TD2.

[40] DH, TD1, TD2: cc'-1. [Anklesaria 1956, 34] reads *dālman ē* 'an eagle'; for the emendation see [Henning 1942, 233–4, n. 7].

[41] DH, TD2; TD1: mdy'nk.

[42] DH, TD2; TD1: c'hlk' 'n'; [Anklesaria 1956, 34] reads *chehār-kunān paṭīkh*^{v} 'a four-sided granary'.

[43] TD1; DH: hy'sl.

[44] DH; TD1: hm'n'k.

[45] Translation from [Henning 1942, 233–4]; cf. [Bailey 1971, 136–7]; [Anklesaria 1956, 34–5].

can be compared to a head. In the Av. fragment the head was that of a middle-sized man, here that of a domesticated ox, as already noted by [Henning 1942, 234, n. 3]. The concept of 'size' of the stars (expressed by the second element of the compound -*masāy*)[46] has to be interpreted as referring to their apparent brightness.

In addition we have to note that these popular traditions were current in Iran. For instance, in the *Pahlavi Rewāyat to the Dādestān ī Dēnīg* we find the following proportions:[47]

Ch. 65, 3:

> W stl 'ywk' 'ywk' 'nd cnd ktk'-ms'd PWN 22 YATWNd W OZL-WNd AP̱-š'n' 22 'sp̄' 'hncyt ...
>
> *ud star ēk ēk and čand kadag-masāy pad wīst ud dō āyēnd ud šawēnd u-šān wīst ud dō asp āhanǰēd;...*
>
> 'And each of the stars individually (is) house-sized, and 22 come and go, and 22 horses draw them;... '

Ch. 65, 8:

> W gltk' Y m'h KHDE 2 plsng dlhn'd W 2 plsng p'hn'd...
>
> *ud girdag ī māh hammis dō frasang drahnāy ud dō frasang pahnāy...*
>
> 'And the disk of the moon (is) altogether two parasangs in length and two parasangs in width... '

Ch. 65, 13:

> gltk' Y hwlšyt 'nd cnd 'yl'nwyc 'yl'nw<y> c 'nd cnd 7 'ywk' Y hwnyls...

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[46] It has to be noted that Pahlavi *kadag-masāy* corresponds to an Av. *katō.masah-* (see [Henning 1942, 233, n. 7]; cf. [Bartholomae 1904, col. 434], where *katō*°, the first member of the compound, has to be derived from the stem *kata-*, m. 'Kammer, Keller' [Bartholomae 1904, col. 432]; [Duchesne-Guillemin 1936, 153, par. 193]. Pahlavi *kadag-masāy* occurs also in *Dēnkard*, VII, 4, 41 (see [Molé 1967, 48–9]) where it refers to a stone that Zoroaster received from Ohrmazd. It is possible that the celestial origin of such a stone is evoked through the mention of this rare compound.

[47] [Williams 1990, vol. I, 234–5; vol. II, 113–14].

*girdag ī xwaršēd and čand ērān-wēz ērān-wēz and čand haft ēk ī *xwanirah...*

'The disk of the sun (is) as great as Ērān-wēz;[48] Ērān-wēz (is) as great as one seventh part of Xwanirah.'[49]

The stars referred to in the present *Pahlavi Rewāyat* are the 'biggest' ones and in fact their dimension (house-sized [*kadag-masāy*]) corresponds to that of the *Bundahišn*'s already quoted passage (like a piece of rock having the size of a room [*kadag-masāy*]). This parallel is very important because thanks to it we can conclude that the 22 stars carried by a corresponding number of horses, mentioned in ch. 65, 3 of the *Pahlavi Rewāyat*, were the brightest ones. Unfortunately this very number of stars, to my knowledge, is not otherwise attested in Pahlavi literature; thus their identification remains quite hypothetical.[50]

The dimension of the moon is also confirmed: the *Bundahišn* in fact established that the moon is the size of two *hāθras* (corresponding to two parasangs)[51] and the same figure (two parasangs) is given in the *Rewāyat*. In the case of the Sun we find again that its disc corresponds to the size[52] of the Ērān-wēz, and more

[48] On the Ērān-wēz, the Av. Airyana Vaējah 'the Iranian space', i.e., the mythical region of the Aryan peoples, see [Gnoli 1966, 67–75]; [Gnoli 1967, 81–101]; [Gnoli 1980, 88–90]; [Gnoli 1985, 15–30].

[49] It corresponds to Av. *x*ᵛ*aniraθa-*, the central karšvar; see [Bartholomae 1904, col. 1864]. On the Indo-Iranian (and perhaps Mesopotamian) background of the subdivision of the world in seven parts, see [Kirfel 1920, 30*, 34*]; [Honigmann 1929, 8]; [Panaino 1995, 220–21].

[50] We could imagine that this very figure comprehended some of the 15 stars of first magnitude in the Ptolemaic catalogue, plus some of second magnitude; as we can see below, it has to be noted that the *Bundahišn* apparently attests to only 3 of the 6 expected magnitudes. In the first chapter of the Zarādušt's *Kitāb al-Mawālīd*, ([Kunitzsch 1993]; see also [Panaino 1996] for some Sasanian and Iranian aspects of this text) we find only 16 bright stars, only 10 of which are of 1st magnitude.

[51] On these measures see [Henning 1942, 235–8]. Henning suggested (op. cit., p. 243, n. 4) that the original Av. passage should have said: 'The moon is *cərətu.masah-*', i.e. 'having the measure of a complete full round of course' (= 1400 metres; in fact an Av. *hāθra-* corresponds to 700 metres (and is the length of a racecourse), while a *cərətu-* is double that).

[52] A primitive and peculiar measure for the Sun has been attributed also to Heraklitos (apud Aëtius, 2, 21 [περὶ μεγέθους ἡλίου]); 14 [A 54] εὖρος ποδὸς ἀνθρωπηΐου. 'For width (the sun) is like the man's foot'; 22B3 DK (= [Diels 1956]); Dox. 351 a 11f., b 21 (= [Diels 1969]); see [Colli 1993, 62–3].

precisely to one-seventh of Xwanirah, the central region of the traditional Iranian cosmography. On the other hand, we may recall that the comparison between the Sun's size and that of an earthly region is attested for instance in Greek culture. According to a tradition attributed to Anaxagoras, the sun should have been larger than the Peloponnesus (see Diogenes Laertius, II, 8: Οὗτος ἔλεγε τὸν ἥλιον μύδρον εἶναι διάπυρον καὶ μείζω τῆς Πελοποννήσου· οἱ δέ φασι Τάνταλον. 'He declared the sun to be a mass of red-hot metal and to be larger than the Peloponnesus, though others ascribe this view to Tantalus' [Hicks 1980, I, 137]).

These old Iranian doctrines did not stop the diffusion of absolutely contradictory concepts which, paradoxically, we can find in the same Pahlavi text; in fact, just a few lines before chapter II, 16, in paragraph 5, the *Bundahišn* openly referred to the 'magnitudes' (*wuzurgīh*, wcwlgyh) of the stars by following an astronomical pattern, which ultimately goes back (but see below) to Hipparchus (**DH**, 166r, 17–18; **TD1**, 25, 6–8[= fol. 11v, 6–8]; **TD2**, 27, 6–9):

> W 'htl'm'l'n' Z̲NE st'lk'n KON st'lk <Y> 'wyd'p̄'nyk KRYTWNd W ms W ks W mdy'nk' l'd YMLLWNd[53] AYK wcwlgyh Y nhwstyn',[54] wcwlgyh <Y dtykl, wcwlgyh Y> [55] stykl.
>
> *ud axtarāmārān ēn stāragān nūn stārag ī awiyābānīg xwanēnd ud meh ud keh ud mayānag rāy gōwēnd kū wuzurgīh ī naxustīn ud wuzurgīh <ī dudīgar, wuzurgīh ī> sidīgar.*
>
> 'The astronomers nowadays call these stars *inerrantes*,[56] and instead of 'large', 'small', 'medium', they use the expressions 'first magnitude', '<second> magnitude', 'third <magnitude>'.[57]

There is no doubt that Pahl. *wuzurgīh* is a perfect translation of Gr. μέγεθος (in Arabic it will be translated as *al-ʿuẓm* or *al-*

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[53] TD2: YMRRWNd.

[54] DH; TD1: nzdstyn'; TD2: nwstyn'

[55] A blank space in DH and TD2. In TD1 three points between nzdstyn and stygl.

[56] On the stars *awiyābānīg* see [Henning 1942, 232, n. 3]; [Kunitzsch 1968]; [Kunitzsch 1970]; [Kunitzsch 1972]; [Kunitzsch 1981]; [Kunitzsch 1993, 244–5]; [Pingree 1989, 227–39]. See also [Burnett & Pingree 1997, 44] (*Liber Aristotilis* III, ii, 1, 10); [Panaino 1996].

[57] [Henning 1942, 232]; [Anklesaria 1956, 32–3].

qadr).[58] On the other hand, it is clear, as [Henning 1942, 232, n. 4] noted, by focusing on the philological condition of this sentence, that the scribes (and perhaps the writer himself) did not understand the passage very well. Probably they were not conversant with the astronomical concept of 'magnitude' (*wuzurgīh*), while for them that of standard physical and visible greatness was more usual. However, notwithstanding the poor condition of the present passage, I would like to draw the attention of the reader on the fact that here we find only three magnitudes; first, second and third. This very simple distinction finds counterparts in other astronomical traditions, so it is not necessarily the fruit of a textual mistake or of a bold omission and excision suffered by the original Pahlavi text. Actually we know, as [Toomer 1984, 16] has clearly pointed out, that the magnitudes range[59] from 1 to 6 according to a system which certainly preceded Ptolemy,[60] but which is 'only conjecturally attributed to Hipparchus'. In the Commentary to Aratus by Hipparchus, the earliest and 'almost only source from which we can hope to obtain information about the terminology at his time and before'—as Neugebauer states—'there is no evidence... for numerical classes of stellar magnitudes; only qualitative distinctions are made like "bright" and "small" stars, etc.' In addition some astrologers used simpler patterns, as in the summary of the Θησαυροῖ attributed to Antiochus (but more precisely of Rhetorius),[61] who apparently seems to distinguish only three categories of brightness, as we deduce from the ms. Parisinus Graecus 2425 [*CCAG*, VIII, III, 105: 36–7]: Περὶ τῶν παρανατελλόντων ἀπλανῶν λαμπρῶν ἀστέρων καὶ ἀμυδρῶν καὶ μέσων καὶ τῆς τούτων δυνάμεως καὶ τί ἀποσημαίνουσιν ('On the stars rising simultaneously which are bright, faint and middle, and on their power and what they signify').[62] I do not know if

[58] See [Kunitzsch 1974, 166, n. 111]. Cf. Cf. al-Bīrūnī, *Tafhīm* [Wright 1934, 68].

[59] [Neugebauer 1975, I, 291] however remarks that 'the magnitudes themselves are not always integers but occasionally modified "greater" or "less", indicating a brightness between two magnitudes'. See also *ibid.*, n. 3 with reference to Manitius' Commentary to Aratus.

[60] See [Grasshoff 1990, 69–71]; cf. [Boll 1916, 87–8].

[61] See [Gundel 1936, 133, n. 2]; cf. also [Gundel & Gundel 1966, 250, 6]; on this subject see in particular [Pingree 1977].

[62] See also [Gundel 1936, 133, n. 2], with reference also to [CCAG, I, 151], chapter 11 and to the *Isagoge* of Porphyrius (*ibid.*, p. 200): λαμπροὺς ἀστέρας

the present division in three groups of magnitudes, as attested in the Pahlavi texts, reflects a simplified vulgarisation of a complete Ptolemaic scheme with six magnitudes, or if it is based on a simpler division in three categories according to a primitive and local tradition. On this regard we could recall that al-Bīrūnī in the *Tafhīm* wrote[63] that 'astrologers give the name of glory' (*šaraf*) to the two first degrees of magnitudes;[64] in any case, all these sources confirm that different arrangements of the magnitudes were possible, in particular among astrologers.

However, it is interesting to note also that the 'modern' conception of 'magnitude' was formulated in the *Bundahišn* in a way which did not clash with the traditional astral lore; actually to say 'first', 'second' or 'third' magnitude, or to say 'large' (*an ī mas*), 'medium-sized' (*ān ī mayān*) or 'small' (*ān ī keh*) for a star was a way in which a 'technical' language was distinguished from popular and traditional speech, but on the practical side the two definitions closely corresponded. This observation could thus alert us to the fact that—from a strictly linguistic point of view—the Greek representation too of the concept of (increasing or diminishing) brightness of the stars by a word meaning 'greatness' [μέγεθος],[65] which involves a very concrete and physical idea, belongs to an old pattern.

In addition it is very bizarre but also very significant that in later times, in the context of the Persian *Riwāyats*, we find an evident mixture of traditional lore and astronomical sciences with reference to the size of the astral bodies [Unvâlâ 1922, 59, 4–5]:

> *wa ʿarz <ō> ṭul-e xoršīd čahār ṣad farsang wa ʿarz <ō> ṭul-e māh do sad farsang wa ʿarz ṭul sitāre-ye bozorg si-o-se gaz wa ʿarz ṭul sitāre-ye xord panj gaz wa asmāni farsang-e yek hazār o do ṣad gaz ast wa gaz yek dast.*

'The length (and) breadth of the sun is 400 *farsangs*. The length

καὶ ἀμυδροὺς καὶ στυγνοτέρους 'the bright stars and the faint ones and those more gloomy'. Cf. again [Neugebauer 1975, I, 291, n. 4].

[63] Al-Bīrūnī, *Tafhīm*, according to [Wright 1934, 68].

[64] Corresponding in Lat. texts to *honor*, *exaltatio*. See again [Kunitzsch 1974, 166, n. 111].

[65] In Homer it means only stature (of men and women); but it can mean also size in general, loudness (of sounds), might, power, extension, length (in metrics). See [Liddell & Scott 1968, 1089].

> and breadth of the moon is 200 *farsangs*; the length and breadth of a big star is 33 *gaz*; the length and breadth of a small star is 5 *gaz.* A heavenly *farsang* is equal to 1,200 *gaz*; and one *gaz* is (equal to the length of) a hand.'[66]

These figures have to be compared with those attested in the *Rivāyat* of Shapuhr Baruchi [Unvâlâ 1922, II, 59, 9–12]:

> *wa ʿarz ō ṭul-e xoršīd čahār sad farsang ʿarz ō ṭul-e māh do sad farsang ʿarz wa ṭul-e sitāre-ye bozorg si-o-se *<gaz>*[67] *wa ʿarz ō ṭul-e sitāre-ye miyāne panj gaz; wa ʿarz ō ṭul-e sitāre-ye xord ham-kadār sar-e gāw ast wa šomar-e farsang-e asmāni duwāzda hazār gaz ast.*
>
> 'The length and breadth of the sun is 400 *farsangs.* The length and breadth of the moon is 200 *farsangs*; the length and breadth of a big star is 33 (*gaz*); the length and breadth of a middle star is 5 *gaz*; the length and breadth of a small star is like the head of a bull. The measure of a heavenly *farsang* is equal to 12,000 *gaz*; and a *gaz* is of the measure of one hand.'[68]

Clearly, in both texts the concepts of magnitude and size are mixed, and it is noteworthy that the big stars here are expressly called *bozorg*, i.e. with an adjective etymologically linked with Pahl. *wuzurgīh* 'magnitude', the abstract noun derived from Pahl. *wuzurg* (cf. O.P. *vazṛka-* 'big, great'); in the second Persian *Rivāyat*[69] we still find a direct echo of the Pahlavi description concerning the three different sizes of the stars (big (*bozorg*), medium

66 Cf. [Dhabhar 1932, 427].

67 The ms. has *farsang.*

68 This explanation differs from the meanings listed by [Steingass 1892, 1087], who mentions a cubit, a length of 24 finger-breadths, or six hands, etc. Cf. [Dhabhar 1932, 427].

69 The fact that the *Rivāyat* of Shapuhr Bharuchi is closer to a Pahlavi text is confirmed not only by the complete sequence of three different types of stars, as in *Gr.Bd.* VB and in the *Pahlavi Rewāyat to the Dādēstān ī Dēnīg*, but also because here the distances between the world and the star-station, the star-station and the moon-station, the moon-station and the sun-station are given as 34,000 *frasangs*, as in the *Pahlavi Rewāyat*, ch. 46, 7 (cf. [Williams 1990, I, 162–163; II, 73]; [Zaehner 1972, 361, 365]), while in the preceding *Rewāyat* they are given as 33,000 *frasangs.* On the other hand it has to be noted that the Pahlavi and the Persian Letters present some general problems in the order of the stations, which probably reflect a flaw in the transmission of this doctrine.

(*miyāne*) and small (*xord*), and the comparison of the smallest ones with the head of a bull), with the only difference that in the Persian document we find references to the *gaz* unit of measure.

Notwithstanding the evolution of astronomical sciences, later Parsi doctors and savants still followed (and mixed) old and new patterns, astral lore and sciences; thus they have maintained a properly Iranian tradition. In fact such a distinction (in a sort of scale of greatness) of the stars' brightness is apparently not yet attested, as far as I know, in Mesopotamian sources, nor do I know of any clear attempt at comparison for their brightness.[70] Also there, for instance, we find a strict link between the idea of brightness and that of the (supposed) dimension of the astral body. Bright meteors are called in fact 'big stars' (*kakkabu rabû*), e.g., in the 'Diaries' [Sachs & Hunger 1988, 27], while in the MUL.APIN compendium [Hunger & Pingree 1989] and in the 'Astrological Reports' [Hunger 1992] the stars (but also the planets) can be defined as *rabû* 'big', *ṣehru/ṣeheru* 'small', *nebû* 'bright', and *unnunu/unnutu* 'faint'. Moreover, as Hunger kindly informs me, 'Venus is sometimes called *nabat kakkabi* "the bright one of the stars" which means the brightest of the stars', while 'the expression "big star" is used sometimes for a meteor, sometimes for Jupiter'. In addition we may note that again in MUL.APIN, e.g. in tabl. I iv 21–22 [Hunger & Pingree 1989, 63], the 'bright star of the Old Man' (*kakkabu nebû ša Šībi*), i.e. α Persei, the brightest star of Perseus (Šību), is distinguished by the 'dusky stars of the Old Man' (*kakkabū ummulūtu ša Šībi*), while tabl. II i 26–27, 36–42, 54–59 [Hunger & Pingree 1989, 77–84] concerns the systematic observation of fixed stars and planets with particular attention to their change in 'glow' (*zīmu*) and in colour (*risnu*). On the other hand it is clear that we do not have anything which can be compared to a primitive magnitude scheme and which can be assumed as a prototype for the Greek magnitude order.[71] On this aspect, the Iranian astral culture seems to

[70] I have to deeply thank for their help and kind information on this subject my dear friends and colleagues Prof. Hermann Hunger (University of Wien) and Prof. Simo Parpola (University of Helsinki).

[71] We have in fact to recall, as [Neugebauer 1975, I, 291] noted, that 'we do not know where and when the concept of six stellar magnitudes originated'. On the other hand I think it is worthy of note that in Ptolemy we find 9 stars which are 'faint', i.e. the same adjective used in Babylonian astronomical

have—independently and in a very primitive form (and without any probable impact on the western system)—foreseen a real astronomical problem by distinguishing the brightness of the stars in three groups of different size (the greatest ones as a piece of rock the size of a room, the medium-sized ones like a rolling wheel, the smallest ones like the head of the domesticated ox or of a middle-sized man), by connecting their visible light with a proportional physical dimension.

Why they did not abandon this pattern is another problem we have tried to explain.

Bibliography

Ervad T. D. Anklesaria (ed.), *The Bûndahishn Being a Facsimile of the TD Manuscript No. 2...*, Bombay, 1908.

B. T. Anklesaria, *Zand-Ākāsīh. Iranian or Greater Bundahishn*, Bombay, 1956.

H. W. Bailey, *Zoroastrian Problems in the Ninth-Century Books*, 2nd ed., Oxford, 1971.

C. Bartholomae, *Altiranisches Wörterbuch*, Strassburg, 1904.

——. *Catalogus Codicum manuscriptorum Bibliothecae Regiae Monacensis*, vol. I, part VII, *Codices Zendicos Complectens*, Munich, 1915.

F. Boll, *Antike Beobachtungen farbiger Sterne*, Munich, 1916.

The Bondahesh. Being a Facsimile Edition of the Manuscript TD1 (Iranian Culture Foundation, 88), Teheran, 1970.

C. Burnett & D. Pingree, *The Liber Aristotilis of Hugo of Santalla*, London, 1997.

Catalogus Codicum Astrologorum Graecorum (*CCAG*), vols I and VIII, Brussels, 1896–1912.

texts. [Le Boeuffle 1987, 172] suggests a distant link with a 'Chaldaean' pattern of associating the fixed stars with the planets according to their own colour.

The Codex DH. Being a Facsimile Edition of Bondahesh, Zand-e Vohuman Yasht and Parts of Denkard (Iranian Culture Foundation, 89), Teheran, 1971.

G. Colli, *La sapienza greca. III. Eraclito*, Milan, 1993.

B. N. Dhabhar, *The Persian Rivayats of Hormazyar Framarz and others*, Bombay, 1932.

J. Darmesteter, *Le Zend-Avesta*, vol. III, Paris, 1893.

H. Diels, *Die Fragmente der Vorsokratiker. Griechisch und Deutsch*, 3 vols, Berlin, 1956.

——. *Doxographi Graeci*, Berlin, 1969.

J. Duchesne-Guillemin, *Les composés de l'Avesta*, Liège-Paris, 1936.

F. K. Geldner, *Avesta, the Sacred Books of the Parsis. II. Vispered and Khorda Avesta*, Stuttgart, 1889.

P. Gignoux & A. Tafazzoli, *Anthologie de Zādspram, Édition critique du texte pehlevi traduit et commenté*, Paris, 1993.

G. Gnoli, 'Airyō.šayana', *RSO*, 41, 1966, pp. 67–75.

——. *Ricerche storiche sul Sīstān antico*, Rome, 1967.

——. *Zoroaster's Time and Homeland*, Naples, 1980.

——. *De Zoroastre à Mani. Quatre leçons au Collège de France*, Paris, 1985.

G. Grasshoff, *The History of Ptolemy's Star Catalogue*, New York, 1990.

W. Gundel, *Neue astrologische Texte des Hermes Trismegistos. Funde und Forschungen auf dem Gebiet der antiken Astronomie und Astrologie*, Munich, 1936.

W. Gundel & H. G. Gundel, *Astrologumena. Die astrologische Literatur in der Antike und ihre Geschichte*, Wiesbaden, 1966.

W. B. Henning, 'An Astronomical Chapter of the Bundahisn', *Journal of the Royal Asiatic Society*, parts 3/4, 1942, pp. 229–248 (repr. in idem, *Selected Papers*, vol. II, Acta Iranica 15, Teheran-Liège, 1977, pp. 95–114).

R. D. Hicks (tr.), *Diogenes Laertius, Lives of the Eminent Philosophers*, 2 vols, Cambridge MA-London, 1980.

E. Honigmann, *Die sieben Klimata und die* ΠΟΛΕΙΣ ΕΠΙΣΗΜΟΙ. *Eine Untersuchung zur Geschichte der Geographie und Astrologie im Altertum und Mittelalter*, Heidelberg, 1929.

H. Humbach, *The Gāthās of Zarathushtra and the Other Old Avestan Texts*, 2 vols, Heidelberg, 1991.

H. Hunger, *Astrological Reports to Assyrian Kings*, Helsinki, 1992.

H. Hunger & D. Pingree, *MUL.APIN. An Astronomical Compendium in Cuneiform* (*Archiv für Orientforschung* Beiheft 24), Horn, 1989.

S. Insler, *The Gāthās of Zarathustra* (Acta Iranica 8), Teheran-Liège, 1975.

F. Justi, *Der Bundehesh*, Leipzig, 1868 (repr. Hildesheim-New York, 1976).

J. Kellens, *Liste du verbe avestique*, Wiesbaden, 1995.

J. Kellens & E. Pirart, *Les textes viel-avestiques*, vol. I, Wiesbaden, 1988.

——. *Les textes viel-avestiques*, vol. II, Wiesbaden, 1990.

W. Kirfel, *Die Kosmographie der Inder nach den Quellen dargestellt*, Bonn-Leipzig, 1920.

G. Klingenschmitt, *Farhang-i ōīm. Edition und Kommentar [Maschinenschrift]* (Inaugural-Dissertation der Philosophischen Fakultät der Friedrich-Alexander-Universität zu Erlangen-Nürnberg; unpublished text), Erlangen, 1968.

P. Kunitzsch, 'Zum "liber hermetis de stellis beibeniis"', *ZDMG*, 118, 1, 1968, pp. 62–74.

——. 'Neues zum "liber hermetis de stellis beibeniis"', *ZDMG*, 120, 1970, pp. 126–130.

——. 'Zur Tradition der "Unwettersterne"', *ZDMG*, 112, 1972, pp. 108–117.

——. *Der Almagest. Die Syntaxis Mathematica des Claudius Ptolemäus in arabisch-lateinischer Überlieferung*, Wiesbaden, 1974.

——. 'Stelle bebenie—al-kawākib al-biyābānīya. Ein Nachtrag', *ZDMG*, 131, 2, 1981, pp. 263–267.

——. 'The Chapter on the Fixed Stars in Zarādusht's Kitāb al-mawālīd', *Zeitschrift für Geschichte der arabisch-islamischen Wissenschaften*, 8, 1993, pp. 241–9.

A. Le Boeuffle, *Astronomie Astrologie. Lexique latin*, Paris, 1987.

H. G. Liddell & R. Scott, *A Greek-English Lexicon*, Oxford, 1968.

D. N. MacKenzie, 'Zoroastrian Astrology in the *Bundahišn*', *Bulletin of the School of Oriental and African Studies*, 27, 1964, pp. 511–29 (repr. in D. N. MacKenzie, *Iranica Diversa*, ed. C. G. Cereti and L. Paul, vol. I, Rome, 1999, pp. 7–25).

M. Molé, *La légénde de Zoroastre selon les textes pehlevis*, Paris, 1967.

O. Neugebauer, *A History of Ancient Mathematical Astronomy*, 3 vols, New York, 1975.

A. Panaino, *Tištrya. Part I. The Avestan Hymn to Sirius* (Serie Orientale Roma, LXVIII, 1), Rome, 1990.

——. 'Uranographia Iranica I. The Three Heavens in the Zoroastrian Tradition and the Mesopotamian Background', in *Au Carrefour de Religions. Mélanges offerts à Philippe Gignoux*, ed. R. Gyselen, Bures-sur-Yvette, 1995, pp. 205–25.

——. 'Uranographica Iranica II. Avestan *hapta.srū-* and *mərəzu-*: Ursa Minor and the North Pole?', *Archiv für Orientforschung*, 42–3, (Horn) 1994–5 [1996], pp. 190–207.

——. 'Sopravvivenze del culto iranico della stella Sirio nel *Kitāb al-mawālīd* di Zarādušt ed altre questioni di uranografia sasanide', in *Alle soglie della Classicità. Il Mediterraneo tra tradizione e innovazione. Studi in onore di Sabatino Moscati*, vol. I, ed. E. Acquaro, Pisa–Rome, 1996, pp. 343–54.

——. *Tessere il cielo. Considerazioni sulle Tavole astronomiche, gli Oroscopi e la Dottrina dei Legamenti tra Induismo, Zoroastrismo, Manicheismo e Mandeismo* (Serie Orientale Roma, LXXIX), Rome, 1998.

D. Pingree, 'Astronomy and Astrology in India and Iran', *Isis*, 54, 1963, pp. 229–46.

——. 'Indian Influence on Sasanian and Early Islamic Astronomy and Astrology', *Journal of Oriental Research, Madras*, 34–5, 1964–6 (1973), pp. 118–26.

——. 'Māsha'allāh: Some Sasanian and Syriac Sources', in *Essays in Islamic Philosophy and Science*, ed. George F. Hourani, Albany NY, 1975, pp. 5–14.

——. 'Antiochus and Rhetorius', *Classical Philology*, 72, 1977, pp. 203–23.

——. 'Classical and Byzantine Astrology in Sassanian Persia', *Dumbarton Oaks Papers*, 43, 1989, pp. 227–88.

——. 'The Purāṇas and Jyotiḥśāstra: Astronomy', *Journal of the American Oriental Society*, 110, 1990, pp. 274–80.

——. *From Astral Omens to Astrology, From Babylon to Bīkāner*, Rome, 1997.

H. Reichelt, 'Der Frahang i oīm (Zand-Pahlavi Glossary)', *WZKM*, 14, 1900, pp. 177–213.

——. 'Der Frahang i oīm (Zand-Pahlavi Glossary)', *WZKM*, 15, 1901, pp. 117–186.

A. J. Sachs & H. Hunger, *Astronomical Diaries and Related Texts from Babylonia*, vol. I, Vienna, 1988.

K. S. Shukla, *Āryabhaṭīya of Āryabhaṭa*, New Delhi, 1976.

F. Steingass, *A Comprehensive Persian-English Dictionary*, London, 1892.

G. Toomer, *Ptolemy's Almagest*, London, 1984.

M. R. Unvâlâ, *Dârâb Hormazyâr's Rivâyat*, 2 vols, Bombay, 1922.

E. W. West, *Pahlavi Texts*, Part 1, Oxford, 1880.

——. *Pahlavi Texts*, Part IV, Oxford 1892.

N. L. Westergaard, *Bundehesh. Liber Pehlevicus...*, Copenhagen 1851.

A. W. Williams, *The Pahlavi Rivāyat Accompanying the Dādēstān ī Dēnīg*, 2 vols, Copenhagen, 1990.

G. L. Windfuhr, 'Where Guardian Spirits Watch by Night and Evil Spirits Fail: the Zoroastrian Prototypical Heaven', in *The Word of the Lord Shall Go Forth. Essays in Honor of D. N. Freedman in Celebration of His Sixtieth Birthday*, ed. C. L. Meyers and M. O'Connor, Philadelphia, 1983, pp. 625–45.

R. R. Wright (tr.), *Kitāb al-tafhīm li-awā'il ṣināʿt al tanjīm. The Book of Instruction in the Elements of the Art of Astrology, by Al-Bīrūnī*, London, 1934.

R. C. Zaehner, *Zurvan, a Zoroastrian Dilemma*, New York, 1972.

Jambudvīpa: Apples or Plums?

DOMINIK WUJASTYK

One of the common Sanskrit names for India is *Jambudvīpa*. This is commonly translated into English as 'The Continent (or Island) of the Rose Apple Tree', a poetically charming expression which evokes idyllic images of a land of trees heavy-laden with sweet, ripe, rosy fruits.[1] This translation is supported by the standard Sanskrit–English dictionary of Monier-Williams, which has:

> *jambu*, °*bū*, f. the rose apple tree (Eugenia Jambolana or another species), ... (°*bu*) n. the rose apple fruit....[2]

The St. Petersburg lexicon too identifies *jambu* with *Eugenia jambolana* Lam., adding,

> von den Engländern *rose apple* genannt....[3]

The *Comparative Dictionary of the Indo-Aryan Languages* [Turner 1966, #5131] propagates the same identities: '*jambú*... the rose-apple tree Eugenia jambolana'. The *Addenda and Corrigenda* to *CDIAL* note under #5136 that *jāmṇu*, the Gaṛhwālī cognate of *jambula*, is a rose-apple. Mayrhofer's etymological dictionaries make the same identifications between Sanskrit *jambu*, Latin *Eugenia jambolana*, and English 'the Rose Apple tree' [Mayrhofer 1953–1972, i.418, Mayrhofer 1986– , i.571]. The same is true of Apte [1992, 728a]: 'The rose apple tree and its fruit'. In general, then, the dictionary sources most used by Sanskrit translators agree that Sanskrit *jambu* is the *Eugenia jambolana*, Lam., and that this is the 'Rose Apple' of English usage.

[1] By no means especially culpable examples include [Basham 1967, 488], [Gombrich 1975, 125] [Caillat 1987, 510], [Dundas 1992, 78], [Klostermaier 1994, 119], [Hardy 1995, 28], and [Fynes 1998, i.7 *et passim*].

[2] [Monier-Williams et al. 1899, 412].

[3] [Böhtlingk & Roth 1855–1875, iii.39a].

In what follows, I shall argue that *jambū* is not the 'Rose Apple', and therefore, India is not the 'Rose Apple Continent'.

I offer this chapter with respect and affection to my distinguished colleague and friend, David Pingree, who has done so much to reveal what Indians thought about Jambudvīpa.

Indigenous Sanskrit sources on the Jambu

The word *jambū* does not not occur in the vedic *saṃhitās*, appearing occasionally in the later vedic epic literature, and then with increasing frequency in the epic literature. The name *Jambūdvīpa* is used in the *Mahābhārata*, for example, to mean 'this world'.[4]

The *jambū* is frequently mentioned in the earliest Sanskrit medical texts. Caraka and Suśruta mention it twenty or thirty times each, and it appears in nearly thirty medical recipes in Vāgbhaṭa's *Aṣṭāṅgahṛdayasaṃhitā* [Singh & Chunekar 1972, 164–5]. Its properties are described and its dark colour is alluded to.

> The Jambu fruit is heavy, constipating, cold, and strongly excites the wind humour.[5]
>
> [Of a patient bitten by a lethal spider:]
> The person's teeth, lips, and mouth are dark, their back and neck bend, and blood the colour of a ripe jambu flows from the bite.[6]

Recipes with jambu are used against diarrhoea, urinary disorders, infant vomiting, earache, against vaginal disorders, wounds, poison, and insect and spider bites. It is often used in combination with mango (Skt. *āmra*, *Mangifera indica, L.*) or catechu (Skt. *khadira*, *Acacia catechu, L.*).

Narahari (fl. ca. 1500), in his famous pharmacopoeia *Rājanighaṇṭu*, describes the *jambu* as a large, dark tree with sweet, fragrant leaves. Its dark blue fruits are worthy of a king, and are much loved by parakeets. Narahari also gives a

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[4] Anuśāsana. 13.110.128a *et passim*, especially in *Bhīṣmaparvan.*

[5] *jāmbavaṃ guru viṣṭambhi śītalaṃ bhṛśavātalam*
Aṣṭāṅgahṛdayasaṃhitā Sū.6.127.

[6] *śyāvauṣṭhavaktradantatvaṃ pṛṣṭhagrīvāvabhañjanam*
pakvajambūsavarṇaṃ ca daṃśāt sravati śonitam
Aṣṭāṅgahṛdayasaṃhitā Utt.37.53.

Sanskritized form of the Tamil name of the plant, *nāval*.[7]

Later Sanskrit authors on materia medica continued to describe the Jambu in traditional terms, and also began to distinguish up to five kinds of the tree.[8]

Early foreign notices on the Jambu tree

Chinese diarists such as Faxian (fl. AD 399–414) and Xuanzang (fl. AD 629–645) mention the Jambu tree, and the name *Jambūdvīpa*, as a matter of course.[9] Clearly the name was in common use from the early centuries of the Common Era.

The Jambu tree was also known to the Muslim traveller and diarist Ibn Baṭṭūṭa, who arrived in north India in AD 1333. Ibn Baṭṭūṭa was an extraordinary traveller, covering over 77,640 miles in the course of his long journeys from his home in Morocco through the Middle East, Iraq, Persia, various parts of Africa, India, the Maldives, Burma, Sumatra, and China.[10] His learned, insightful, and sometimes witty observations are a treasure of information about the territories he visited during the fourteenth-century. At the beginning of his eight years in India, Ibn Baṭṭūṭa travelled to Delhi, and during his narrative of this journey to the Tughluq capital, he took the opportunity to describe the trees he witnessed in North India which differed from those he had seen elsewhere on his travels. He mentions the Jambu tree as follows:[11]

> Next the jambol (*jumūn*). Its trees are large and their fruit is like the olive. It is black in colour and like the olive has one stone.

Ibn Baṭṭūṭa clearly knew the tree at first hand, and had no doubts about its name or identity.

Amongst the earliest European commentators on Indian botanical matters was the sixteenth-century physician Garcia

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[7] *jaṃbūs tu surabhipatrā nīlaphalā śyāmalā mahāskandhā* |
rājārhā rājaphalā śukapriyā meghamodinī navāhvā ||
[Purandare 1986, 186], also cited in [Warrier et al. 1994–1996, v.225–6]. On Narahari's date see [Meulenbeld 1999–2002, IIa.265–70]; on the Tamil name, see [Burrow 19612375, #2378].

[8] [Warrier et al. 1994–1996, 5.228].

[9] [Watters 1904–1905*passim*], [Legge 1886, 34, 48, 80].

[10] [Husain 1976, liii] gives the estimate of the miles covered by Ibn Baṭṭūṭa. An accessible account of Ibn Baṭṭūṭa's travels is given by [Dunn 1986].

[11] [Husain 1976, 17].

da Orta (ca. 1501–68).[12] In the course of his wide-ranging discourses on the medicinal flora of India he speaks of a sharp-tasting olive-like fruit which he calls '*jamboloés*':[13]

> They are called Jambolôes and grow in the country on a bush like a myrtle, and [have] leaves resembling those of the arbutus.

Da Orta goes on to claim that the fruit is not considered very wholesome by the people of India. Markham, the learned translator of da Orta's work, notes that this fruit is:[14]

> Eugenia jambolana, Lam., a Myrtle-bloom. Ibn Batuta also compares the fruit to the olive. It is the 'Black Plum' of Anglo-Indians, and *jambu*, *jamam*, and *jambul*, all meaning 'eatable', 'toothsome', to the people of India.

Da Orta's remarks on this tree are not quite as clear as Markham's precise and accurate note would suggest. As we shall see below, da Orta's account of the tree was open to misinterpretation.

Hendrik van Reede tot Drakenstein (1636–1691) was a distinguished servant of the Dutch colonial service in western India [Heniger 1986]. After building a military career at sea in the 1650s, he participated in the conquest of Malabar between 1658

[12] See [Boxer 1963].

[13] Translation from [Markham 1987, 236]. Da Orta's Portuguese of this and the following passages reads as follows [da Orta 1563, 121v–122r]:

> Chamanse jamboloés he naçem no campo em hũa mata que pareçe como murta he nas folhas pareçe madornho mas asi esta fruita como a iaqua náo se tem por fruita muito sadiam dagente desta terra mas esta que vos mostro he muyto estimda nesta terra veo de malaqua a esta terra a pouquo tempo porq̃ ha muytos na quesas partes mas dizei a que vos pareçe este pomo pois he detamanho de hũ houo de pata, e algũ tanto maior jà vedes como acor delle he feito de branco he vermelho, e cheira a aguoa rosada de maneira que aos dous sentidos he apreziuel agora he necesario porq̃ parece bem auista, e ao cheiro que seia ao gosto he porisso prouaio.... Em malaqua he chama jambos he asilhe cahmão nesta terra.

The 'arbutus' is a family of fourteen evergreen shrubs and small trees, ranging in height from about 10m to 30m, with smooth red or silver-grey flaking bark and pendant berries, known from Europe, America, and Asia Minor [Griffiths 1994, 86b–87a].

[14] [Markham 1987, 236, n.1].

and 1663, gaining preferment for his actions. During his period as Commander of Malabar, between 1670 and 1677, van Reede began work on a survey of the medicinal and economic plants of the region. This work was eventually published in twelve magnificently illustrated folio volumes, as the *Hortus Malabaricus* [van Rheede 1678–1703]. This work was of such importance that it came to rank equal with da Orta's work of a century earlier as a source of botanical knowledge from the East. Van Reede wrote in Latin, and alongside lavish engravings gave local names of plants in Malayalam, Konkani, and Arabic, as well as Latin. Perhaps most important of all, van Reede cooperated extensively with local experts [Heniger 1986, ch. 10]; [Grove 1995, ch. 2]. He convened an expert council with a broad membership of Dutch and Malayali specialists. In particular he relied upon the brahmans Ranga Botto, Vinaique Pandito and Apu Botto from Cochin, and more especially the Ezhava physician Itti Achudem from the coastal region of Mouton, all of whom signed statements which were reproduced in the *Hortus*. Perhaps even more valuable for the accuracy of his work was his collaboration with the traditional Keralan collectors of medical plants, the members of the Chogan ('tree climber') community [Heniger 1986, 146].

In volume 5, the *Hortus Malabaricus* describes the *E. Jambolana*:[15]

> Perin-Njara. Bedṛśī Brah. Arbor haec Malabarensibus Perinjara seu Inalel, Brahmanis Madanaka, Lusitanis *Graõ do gato*, Belgis *Paters-ballen* nuncupatur. ... Fructus prunorum emuli, oblongor-rotundi, exterius gibbi, interius leviter sinuosi.

Van Reede says the tree is called 'Perin-njara' (in Malayalam) and its fruits, likened to prunes or plums, seem definitely to be those of the Jambu. Inexplicably, however, van Reede asserts that Brahmins call the tree 'Madanaka', not 'Jambu'.[16]

Thus, all early commentators—Sanskrit, Islamic, Portuguese, Dutch—describe a large tree with a black olive-like fruit, but da Orta calls it 'Jamboloés' and van Reede calls it the 'Perin-njara'.

[15] [van Rheede 1678–1703, v.29]; [Dalgado 1896, 45–6].

[16] 'Madana' is the Sanskrit name of a medicinal plant, but 'Madanaka' is not. 'Bedṛśī' is mysterious, but presumably intended to be Konkani.

The Rose Apple

Van Reede describes in the following terms a tree which he calls *E[ugenia]. Malaccensis*:[17]

> Náti-schambu. Bram. jāmbu. Nati-Śchambu, quae altera species Iambu est, ling. Bram. *Shambu*, arbor quoque excelsa. ...Fructus...cum maturi, ad unam partem candidi, ad aliam rubescentes, ac inter candidum colorem striis rubris perfusi.

This is clearly a description of a 'Rose Apple', but it claims that Brahmins call the tree a Jambu, while its name implies that it is a native of Malacca.

Returning to Da Orta, we find that he contrasted 'Jamboloés' with another plant:[18]

> But this that I show you is much esteemed here. It is not long since it came here from Malacca, where it is abundant. I ask what you think of the fruit. It is the size of a chicken's egg or rather larger. You see that the colour is white and yellow, and the scent like that of rose water, so that both senses are gratified.... In Malacca the name is Jambos, and it is so called also in this country.

Da Orta liked this tree so much that he planted several in his own garden. He made jam from the fruit and flowers, and he thought the fruit fit for a 'prince of Spain'. Markham notes that this Malaccan tree is,

> *E. malaccensis*, Linn., the 'Malay–Apple' of Anglo–Indians, and *Malaka-jamrul* of Southern India, and known also by the name of *kavika.* The Anglo-Indian 'Rose-Apple' is the E. Jambos of Linnaeus.[19]

This succinct statement by Markham puts the matter well. It is *Eugenia jambos*, L., a tree with fragrant pinkish fruit, probably imported to India, like *Eugenia malaccensis* L., from SE Asia not long before da Orta's time, which is called the Rose-Apple. And it is the Malaccan import which da Orta is describing as being 'much esteemed'. It is even possible that da Orta does not

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17 [van Rheede 1678–1703, i.18]; [Dalgado 1896, 45].

18 [da Orta 1563, 121v], beginning 'mas esta que vos mostro...', tr. [Markham 1987, 236–7]

19 [Markham 1987, 236–7].

distinguish the *E. malaccensis* from the *E. Jambos*, both having been imported from SE Asia, and both having similar fruit with pink, perfumed flesh.[20] However, da Orta's statement that the Malaccan tree is called 'Jambos', both in Malacca and in India, has led to confusion ever since.

Just to be clear: the well-known, traditional tree native to India, which figures in Sanskrit *purāṇic* literature and whose Sanskrit name is '*jambu*', is the *Eugenia jambolana*, Lam. All the Sanskrit–English dictionaries cited above are correct on this point. However, the English name of this tree is 'Jambul' or 'Black Plum', not 'Rose Apple'.

Linnaeus' confusion

Why did Linnaeus give the name *E. Jambos* to a tree that was not the India Jambu? Throughout his writings, he makes an effort to echo local traditions of nomenclature in his scientific binomial taxonomy. It seems clear in this case that Linnaeus thought that the Rose Apple of Malacca was indeed the Jambu, and assigned his name accordingly. This suggests that he was using some source which suggested this erroneous view.

It seems beyond reasonable doubt that the source of this view was da Orta. He actually says of the Rose Apple, 'In Malacca the name is Jambos and it is so called in this country'.[21]

But this error is compounded by van Reede, who is one of the principal authorities cited by Linnaeus. Once Linnaeus had given the Latin name *Eugenia Jambos* to the Rose Apple, it was inevitable that non-specialists would draw the conclusion that this tree was the Sanskrit Jambu.

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[20] There is pervasive confusion in the botanical literature concerning the difference between *E. jambos* (today, *Syzygium jambos* (L.) Alston) and *E. mallacensis* (today, *S. malaccense* (L.) Merrill & Perry [Griffiths 1994, 1136]). Hooker, for example, calls *E. Malaccensis* a 'shrub, 6–8ft high' (1872–97, 2.472) and *E. jambos* 'a moderate-sized tree' (2.474). However, Griffiths [1994] describes the former as 'a tree to 25m', and the latter as 'Bush or small tree to 8m'. Hooker says the former has red flowers, while the latter has white, but many sources do not name the colours of the flowers. Both species are at different times called 'Malay Apple'. It is entirely possible that da Orta's Malaccan tree is in fact the *Syzygium jambos*.

[21] 'Em malaqua he chama jambos he asilhe cahmão nesta terra.' [da Orta 1563, 122r], tr. [Markham 1987, 237].

	Black plum	Rose apple
Fruit	olive-like, black, staining, astringent	like a small apple, rosy-coloured flesh, perfumed
Latin (former)	*Eugenia jambolana*, Lam.	*Eugenia jambos*, L.
Latin (current)	*Syzygium cumini*, (L.) Skeels	*Syzygium jambos*, (L.) Alston
Other English	Jambul, Jambolan, Java plum, Jamoon	Jambu mawar
Sanskrit	*jambu*	(unstable)

Table 1: Nomenclature

Looking back even further, it seems a possible hypothesis that Indian communities in South East Asia named the Rose Apple 'Jambu'.[22] That tree, and its emigrant name, were later imported to India during or shortly before the sixteenth century. Both da Orta and van Reede understood their informants to be calling the Malaccan Rose Apple 'Jambu', and here the confusion began.

Due to the modernization of botanical terminology, the revised scientific name for *Eugenia jambolana* Lam. is today *Syzygium cumini*, (L.) Skeels; another English name is 'Jambolan'. Likewise, the current scientific name for the Rose Apple, formerly *E. jambos* L., is *Syzygium jambos*, (L.) Alston; another English name is 'Jambu Mawar'.[23]

These variants of nomenclature are are summarized in Table 1.

Thus, *E. jambos*, the Rose-Apple, is essentially a foreigner in most of India, and may have been little known before the

[22] On the pre-modern Indian diaspora in SE Asia, [Kulke & Rothermund 1998, 143–51, 345] provide an up to date overview and bibliography, and [Frodin 1984] gives an invaluable survey of the botanical writings available for this and all parts of the world. SE Asia is relatively poorly served by botanical literature. The Jambu is unfortunately not mentioned in [Hui-Lin Li 1979].

[23] According to the usually authoritative [Griffiths 1994, 1136]. But the *Index Kewensis* [Royal Botanic Gardens, Kew 1993] appears to use the name *E. jambos* L. as the current name. Such difficulties are common: 'Frequent changes in plant names have brought plant taxonomy into disrepute among plant scientists working in areas other than taxonomy...' [Sivarajan 1991, 215].

sixteenth century, when it reached India from SE Asia. This relatively late arrival in mainland India explains the lack of an established Sanskrit terminology for the tree, and the fact that all common names for the tree, such as the Gujarati *gulābajāṃba* [Bāpālāla 1968, 580] or the Hindī *gulābajamūn* [Warrier et al. 1994–1996, v.229], are in New Indo-Aryan languages. The fact that these terms borrow from the traditional name used for the Black Plum is an illustration of the recognized fact that traditional herbalists and vaidyas often succeeded in recognizing family resemblances between plants through their knowledge of plant morphology and medical function.[24]

Later development

By and large, the confusion that appears in Sanskrit-English lexicons and translations is absent in the literature on Indian economic botany and pharmacology. Thus at the end of the nineteenth century, Dymock et al. identify *E. jambolana* as the tree which symbolizes India: '[the E. jambolana, Hindi *Jámun*, Marathi *Jámbú*] is one of the four colossal mythic trees which mark the four cardinal points, standing to the south of Mount Méru... Jambudvipa takes its name from this tree', but nowhere do they call this tree the 'Rose Apple' [Dymock et al. 1890, ii.25–6].

Likewise, Watt clearly distinguishes the two plants [Watt 1908, 526]:

- E. (Zyzygium) Jambolana, Lam. ...the Black plum, Jaman-plum, jámbu, jáman, jám, jámbul, kuda, nâval, nerale, zebri, thabyebyu, etc., and perin-njara, according to van Reede.
- E. (Jambosa) Jambos, Linn. ...The Rose-apple, gulab-jáman (i.e., rose-jáman) and nati-schambu, according to van Reede.

However, this clear and correct statement was Watt's second attempt at the problem. In his earlier encyclopedic work, even

[24] Sivarajan & Balachandran [1994] offer many examples of the botanical or medical appropriateness of substitute plants chosen by traditional vaidyas when an original is not available.

Watt seems to have been slightly confused. He correctly calls *E. Jambolana*, Lam. the Black Plum, and *E. Jambos*, Linn. the Rose-Apple. However, he wrongly ascribes the Sanskrit name *jambu* to the latter tree, as well as attaching to it the mythological associations concerning the Jambu as the 'world tree', which properly belong to *E. Jambolana*. In spite of this, he is aware that several authorities confuse *E. Jambolana* with the Rose-Apple, since he says, 'the Rose-apple is wrongly referred by Yule, Mason, and other to E. Jambolana' [Watt 1972, iii.284–6]. This much is perfectly correct, but Watt is wrong in simultaneously describing *E. Jambos*, the Rose-Apple, as the Sanskrit *jambu*.

It is particularly unfortunate that the widely-used and normally trustworthy *Hobson-Jobson* reproduces all the worst confusions of its sources [Yule & Burnell 2000, 448, 449]. The authors recognise that 'there is great confusion in application, both colloquially and in books', but their own account is unfortunately no less confused.

Other later sources in the genre of economic botany are, generally speaking, clear on this distinction.[25]

Botanical distribution and description

The botanical descriptions in [Hooker 1872–97, ii. 471, 474, 499], [Griffiths 1994, 1136], [Kirtikar et al. 1987, ii.1049–51] and similar works that are based on direct observation and an involvement with the Indian medico-botanical milieu, make it clear that the tree *E. jambos* L. is uncommon in peninsular India, being indigenous in Burma and SE Asia, (with some specimens in the Sikkim Terai). The *E. jambolana*, on the other hand, is common and generally distributed throughout India, Burma, and Sri Lanka. *E. jambos* is a moderate-sized tree, while *E. jambolana* is of considerable size, and has a thick, crooked trunk. The latter is evidently the more visually striking tree, known throughout India, whose image is used as a traditional symbol of the continent.

However, when we reach botanical texts written merely from secondary sources, without direct exposure to the Indian botan-

[25] See, e.g., [Dutt 1980, 165], [Warrier et al. 1994–1996, v. 229] (where the Sanskrit name for the Rose apple is given as *cāmpeya*), [Sivarajan & Balachandran 1994, 188], and [Bāpālāla 1968, 581-6]. [Meulenbeld 1974, 555–6] collects many references.

ical landscape, we find abundant confusion. For example, Mrs Grieve's *A Modern Herbal*, first published in 1931 but still in print, presents a hopelessly conflated account of the two trees [Grieve 1980, 446]. This conflation is also evident in the respected and popular handbook *Potter's New Cyclopaedia of Botanical Drugs & Preparations* by [Wren et al. 1994, 158], which has been in print throughout the twentieth century.

By contrast, even relatively modest guides written by authors with field experience in India, like [Santapau 1993, 100] and [Bole & Vaghani 1986, 59], are clear and correct. In fact, these latter texts do not even mention *E. jambos*, since it is not commonly found in the Indian peninsula even today. Even casual enquiries on the ground in India today produce clear and unambiguous information about the Jambu tree.

We have seen that most botanists are clear about the correct identity of the Jambu. So are the Sanskrit sources on medical botany. So are people living in India. In fact, the only people at all confused about the Jambu today appear to be Western herbalists and Sanskritists trained outside India.

India, therefore, the *Jambudvīpa*, is not the 'Land of the Rose Apple Tree'. It is more correctly the 'Isle of the Jambul', or 'Black Plum Island'.

Bibliography

V. S. Apte, *The Practical Sanskrit-English Dictionary*, Kyoto, 1992, reprinted from the revised & enlarged edition, Poona 1957.

G. V. Bāpālāla, *Nighaṇṭu Ādarśa*, Vārāṇasī, 1968.

A. L. Basham, *The Wonder that was India*, London, 3rd edn., 1967.

O. Böhtlingk & R. Roth, *Sanskrit-Wörterbuch*, St. Petersburg, 1855–1875.

P. V. Bole & Y. Vaghani, *Field Guide to the Common Trees of India*, Bombay, Delhi, Oxford, etc., 1986, 4th reprint.

C. R. Boxer, *Two Pioneers of Tropical Medicine: Garcia d'Orta and Nicolás Monardes*, London, 1963.

T. Burrow, *A Dravidian Etymological Dictionary*, Oxford, 1st edn., 1961.

C. Caillat, 'Jainism', in M. Eliade (ed.), *The Encyclopedia of Religion*, New York, vol. 7, 1987.

G. da Orta, *Coloquios dos Simples, e Drogas he Cousas Mediçinais da India...*, Goa, 1563.

D. G. Dalgado, *Vires Plantarum Malabaricarum ou Virtudes das Plantas do Malabar Extrahidas do "Hortus Indicus Malabaricus" de Henrique van Rheede. Com Classificaçã das Plantas Segundo o Moderno Systema da Nomenclatura Botanica; Classificação Therapeutica das Plantas Mais Uteis de Goa e Savantvadil e Apontamentos Posologicos*, Rangel, Bastora, Goa, 1896, publicado como Supplemento ao "Archivo Medico da India".

P. Dundas, *The Jains*, London & New York, 1992.

R. E. Dunn, *The Adventures of Ibn Battuta: a Muslim Traveler of the 14th Century*, London and Sydney, 1986.

U. C. Dutt, *Materia Medica of the Hindus... with a Glossary of Indian Plants by George King. Revised Edition... by Binod Lall Sen and Ashutosh Sen and Pulin Krishna Sen*, Krishnadas Sanskrit Studies, vol. II, Varanasi, 3rd edn., 1980.

W. Dymock, C. J. H. Warden et al., *Pharmacographia Indica: a History of the Principal Drugs of Vegetable Origin Met With in British India*, London, Bombay, Calcutta, 1890.

D. G. Frodin, *Guide to Standard Floras of the World : an Annotated, Geographically Arranged Systematic Bibliography of the Principal Floras, Enumerations, Checklists, and Chronological Atlases of Different Areas*, Cambridge, New York, etc., 1984.

R. C. C. Fynes, *Hemacandra: the Lives of the Jain Elders, Translated with an Introduction and Notes*, Oxford, 1998.

R. Gombrich, 'Ancient Indian Cosmology', in C. Blacker & M. Loewe (eds.), *Ancient Cosmologies*, London, 3rd edn., 1975, pp. 110–142.

M. Grieve, *A Modern Herbal*, Harmondsworth, 1980, first published 1931.

M. Griffiths, *The New Horticultural Society Index of Garden Plants*, London, 1994.

R. Grove, *Green Imperialism: Colonial Expansion, Tropical Island Edens and the Origins of Environmentalism, 1600–1860*, Cambridge, 1995.

F. Hardy, *The Religious Culture of India: Power, Love and Wisdom*, New Delhi, first south asian edition edn., 1995, first published Cambridge: Cambridge University Press.

J. Heniger, *Hendrik Adriaan van Reede toto Drakenstein (1636–1691) and Hortus Malabaricus: a contribution to the history of Dutch colonial botany*, Rotterdam, Boston, 1986.

J. D. Hooker, *Flora of British India*, L. Reeve, London, 1872–97, reprinted, Dehra Dun, 1984. 7 v.

Hui-Lin Li, *Nan-fang Ts'ao-mu Chuang. A Fourth Century Flora of Southeast Asia: Introduction, Translation, Commentaries*, Hong Kong, 1979.

M. Husain, *The Reḥla of Ibn Baṭṭūṭa (India, Maldive Islands and Ceylon). Translation and commentary*, vol. 122 of *Gaekwad's Oriental Series*, Baroda, 2nd edn., 1976, first edition: 1953. Covers only the S. Asian section of Ibn Baṭṭūṭa's voyages.

K. R. Kirtikar, B. D. Basu et al., *Indian Medicinal Plants*, Dehradun, 2nd edn., 1987, first published in Allahabad, 1933.

K. K. Klostermaier, *A Survey of Hinduism*, Albany, 1994.

H. Kulke & D. Rothermund, *A History of India*, London, 3rd edn., 1998.

J. Legge, *A Record of Buddhistic Kingdoms: Being an Account by the Chinese Monk Fâ-Hien of his Travels in India and Ceylon*

(A.D. 399–414) in Search of the Buddhist Books of Discipline, Oxford, 1886, reprinted New York: Paragon and Dover, 1965.

C. Markham, *Colloquies on the Simple[s] and Drugs of India, by G. da Orta, Translated [into English]*, Delhi, 1987 edn., 1987, first published London, 1913.

M. Mayrhofer, *Kurzgefaßtes etymologisches Wörterbuch des Altindoarischen; A Concise Etymological Sanskrit Dictionary*, Heidelberg, 1953–1972.

——. *Etymologisches Wörterbuch des Altindoarischen*, Heidelberg, 1986– .

G. J. Meulenbeld, *The Mādhavanidāna and its Chief Commentary: Chapters 1–10. Introduction, Translation, and Notes*, Leiden, 1974.

——. *A History of Indian Medical Literature*, Groningen, 1999–2002, 5v.

M. Monier-Williams, E. Leumann et al., *A Sanskrit–English Dictionary Etymologically and Philologically Arranged, New Edition*, Oxford, 1899, 1970 reprint.

V. N. Purandare (ed.), *Rājanighaṇṭusahito Dhanvantarīyanighaṇṭuḥ*, vol. 33 of *Ānandāśramasaṃskṛtagranthāvalih*, Poona, 1986.

Royal Botanic Gardens, Kew, *Index Kewensis on Compact Disc*, Oxford, 1993.

H. Santapau, *Common Trees*, New Delhi, 1993, 5th reprint.

T. B. Singh & K. C. Chunekar, *Glossary of Vegetable Drugs in Bṛhattrayī*, Varanasi, 1972.

V. V. Sivarajan, *Introduction to the Principles of Plant Taxonomy*, New Delhi, 2nd edn., 1991.

V. V. Sivarajan & I. Balachandran, *Ayurvedic Drugs and Their Plant Sources*, New Delhi, Bombay, Calcutta, 1994.

R. L. Turner, *A Comparative Dictionary of the Indo-Aryan Languages*, London, New York, Toronto, 1966, with *Indexes* compiled by Dorothy Rivers Turner (OUP, London, 1969), *Phonetic Analysis* by R. L. & D. R. Turner (OUP, London, 1971), and *Addenda and Corrigenda* edited by J. C. Wright (School of Oriental and African Studies, London, 1985).

H. van Rheede, *Hortus Indicus Malabaricus...*, Amsterdam, 1678–1703, 12v.

P. K. Warrier, V. P. K. Nambiar et al. (eds.), *Indian Medicinal Plants: a Compendium of 500 Species. Vaidyaratnam P. S. Varier's Arya Vaidya Sala, Kottakal*, Madras, 1994–1996, 5 v.

G. Watt, *The Commercial Products of India, Being an Abridgement of "The Dicionary of the Economic Products of India"*, London, 1908.

——. *A Dictionary of the Economic Products of India*, Delhi, 1972, 6 v. in 10. First published, 1889–96.

T. Watters, *On Yuan Chwang's Travels in India (A.D. 629–645)*, London, 1904–1905, 2.v. Edited after the author's death by T. W. Rhys Davids and S. W. Bushell. Reprinted in a single volume, Delhi: Munshiram Manoharlal, 1973.

R. C. Wren, E. M. Williamson et al., *Potter's New Cyclopaedia of Botanical Drugs and Preparations*, Saffron Walden, 1994, reprint of revised 1988 edition.

H. Yule & A. C. Burnell, *Hobson-Jobson: a Glossary of Colloquial Anglo-Indian Words and Phrases...*, New Delhi, 2000, originally published in 1903.

Setting up the Water Clock for Telling the Time of Marriage

Sreeramula Rajeswara Sarma

0.1 From about the fourth century AD up to recent times the water clock of the sinking bowl type (*Ghaṭikā-* or *Ghaṭī-yantra*) has been the chief device in India for measuring time.[1] The instrument consists of a hemispherical bowl (*ghaṭikā* or *ghaṭī*) with a minute perforation at the bottom. When this bowl is placed on the surface of water in a larger vessel or basin (*kuṇḍa, kuṇḍikā, kuṇḍī*), water slowly percolates into the bowl through the perforation. When the bowl is full, it sinks to the bottom of the vessel with a clearly audible thud. The weight of the vessel and the size of the perforation are so regulated that the bowl sinks sixty times in a nychthemeron (*ahorātra*). Thus the time taken for filling the bowl once is one-sixtieth part of a nychthemeron, or twenty-four minutes. This was the standard unit of time measurement in India and is called *ghaṭikā* or *ghaṭī* after the name of the bowl. The *ghaṭikā* is subdivided into sixty *vighaṭikās*, which are also called *palas*.[2]

When the bowl sinks to the bottom of the vessel, indicating the completion of one *ghaṭikā*, this fact is broadcast with blasts on a conch-shell or strokes on a drum. In the early medieval period, the conch and drum were replaced by the gong, which was designated in the Middle Indic as *ghaḍiyāla* (from *ghaṭikālaya*, 'water clock house').

[1] On the different types of water clocks, see Needham, p. 115; on the sinking bowl type in India, see S. R. Sarma, 'The Bowl that sinks and tells Time'.

[2] This sinking bowl type replaced the outflow type which too measured the standard unit of twenty-four minutes. Here the instrument and also the time unit were styled *nālikā, nālī* or *nāḍikā, nāḍī*. Even after the outflow clock was discarded, the old names continued to be used along with *ghaṭikā* and *ghaṭī*.

0.2 Āryabhaṭa I was the first to describe the bowl of this water clock in his *Āryabhaṭasiddhānta.*[3] He describes it in the following words:

vṛttaṃ tāmramayaṃ pātraṃ kārayed daśabhiḥ palaiḥ/
ṣaḍaṅgulaṃ tadutsedho vistāro dvādaśānane//
tasyādhaḥ kārayec chidraṃ palenāṣṭāṅgulena tu/
ity etad ghaṭikāyantraṃ palaṣaṣṭyāmbupūraṇāt//
sveṣṭaṃ vānyad ahorātre ṣaṣṭyāmbhasi nimajjati/
tāmrapātram adhaśchidram ambuyantraṃ kapālakam//

Kripa Shankar Shukla's translation of this passage reads thus:

> 'One should get a hemispherical bowl made of copper, 10 *palas* in weight, six *aṅgulas* in height, and twelve *aṅgulas* in diameter at the top. At the bottom thereof, let a hole be made by a needle eight *aṅgulas* in length and 1 *pala* in weight.
>
> 'This is the *ghaṭikā-yantra*, (so named) because it is filled by water in a period of 60 *palas* (i.e. one *ghaṭī*).
>
> 'Any copper vessel made according to one's liking with a hole in the bottom, which sinks into water 60 times in a day and night, is the water instrument called *Kapāla.*'

Shukla obviously thought that this passage describes two different instruments, called respectively *Ghaṭikā-yantra* and *Kapāla-yantra.*[4] But the fact is that these are two different methods of producing one and the same *Ghaṭikā-yantra.* The first method is to take ten *palas'* weight of copper and with that to produce a hemispherical bowl, having a height of six *aṅgulas* and an upper diameter of twelve *aṅgulas.* There should be a fine perforation at the bottom of the bowl. The small size of the perforation is defined in a peculiar manner. It should be made, as Āryabhaṭa says, 'with [an object, say a needle, having a weight of] one *pala*

[3] This work is now lost, but luckily Āryabhaṭa's descriptions of various instruments survive in Rāmakṛṣṇa Ārādhya's commentary on the *Sūryasiddhānta.* Kripa Shankar Shukla published these passages in his paper 'Āryabhaṭa I's Astronomy with Midnight Day-Reckoning'.

[4] Accordingly, *Indian Astronomy: A Source-Book* reproduces these verses and their translation under two different heads, viz. 10.11.1: *Ghaṭikā-yantra* (p. 91) and 10.5.5: *Kapāla-yantra* (p. 89).

and a [length of] eight *aṅgulas.*' He does not specify what kind of metal is to be used for the needle. But since the weight and the length are prescribed, the implication is that a wire of uniform thickness and a specific length has to be produced from a given amount of the metal. Such metal could only be gold; it is malleable enough to be drawn into wires of uniform thickness. Indeed, in his *Arthaśāstra*, Kauṭilya prescribes that the outflow water clock (*nālikā*) should have a 'perforation by [a needle made of] four *māṣakas* of gold and four *aṅgulas* in length.'[5] It is highly probable that the unspecified metal in Āryabhaṭa's verse is also gold. But a thin gold needle cannot pierce through a copper bowl. Hence Āryabhaṭa's specification (and also Kauṭilya's) should be understood to mean that 'the perforation should be such that a gold wire, one *pala* in weight and eight *aṅgulas* in length, can pass through it.'[6]

Goldsmiths of that time may have been able to draw fine grades of gold wire, but whether they could draw a wire measuring exactly eight *aṅgulas* from a lump of gold weighing exactly one *pala* is open to question. However, the main purpose of the instrument is to measure one-sixtieth part of the nychthemeron. For this purpose, the bowl should be such that it fills with water and sinks into the basin sixty times in a day and night; in other words, the bowl should fill in a period of one *ghaṭikā*, or of sixty *palas*. Therefore, Āryabhaṭa goes on to suggest that the measurements of the bowl and of the needle are not so important; what is important is that the bowl should be able to fill with

[5] This system is employed for the first time in the *Kauṭilīya Arthaśāstra* for defining the perforation of the outflow water clock (*Nālikā-* or *Nāḍikā-yantra*); cf. 2.20.34: *suvarṇamāṣakāś catvāraś caturaṅgulāyāmāḥ kumbhacchidram āḍhakam ambhaso vā nālikā.*

[6] Similar specifications were given in other texts as well. The *Jyotiṣkaraṇḍaka* (verses 11-14) lays down that the hole of the outflow water clock (*nāligā*) should be such that ninety-six hairs from the tail of a three years' old female elephant calf, or twice that number from the tail of a two years' old female elephant calf, or a gold needle of four *māṣas*' weight and four *aṅgulas*' length should pass through it. Al-Bīrūnī (Vol. I, p. 334) cites from 'the book *Srûdhava* by Utpala the Kashmîrian': 'If you bore in a piece of wood a cylindrical hole of twelve fingers' diameter and six fingers' height, it contains three *manā* of water. If you bore in the bottom of this hole another hole as large as six plaited hairs of a young woman, not of an old one nor of a child, the three *manā* of water will flow out through this hole in one *ghaṭī*.' It may be noted that this is also an outflow water clock.

water sixty times in a day and night. This is the import of verse no. 31, where particle '*vā*' indicates that this is an alternative method. The word '*kapāla*' here is not the designation of another instrument, but denotes the hemispherical shape of the bowl. In this light, I retranslate the verse:

> 'Or, alternatively, any hemispherical (*kapāla*) copper vessel made according to one's liking with a perforation in the bottom, which sinks into water sixty times in a day and night, is the water clock (*ambu-yantra*).'[7]

0.3 Lalla, while retaining the weight and size of the bowl, changed the dimension of the perforation:

> *daśabhiḥ śulbasya palaiḥ pātraṃ kalaśārdhasannibhaṃ ghaṭitam/*
> *hastārdhamukhavyāsaṃ samaghaṭavṛttaṃ dalocchrāyam//*
> *satryaṃśamāṣakatrayakṛtanalayā samasavṛttayā hemnaḥ/*
> *caturaṅgulayā viddhaṃ majjati vimale jale nāḍyā//*[8]

> 'The bowl, which resembles half a pot (i.e. hemispherical), which is made of ten *palas* of copper, which is half a cubit (i.e. twelve *aṅgulas*) in diameter at the mouth and half (i.e. six *aṅgulas*) as high, which is evenly circular, and which is bored by a uniformly circular needle, made of three and one-third *māṣas* of gold and of four *aṅgulas* in length, sinks into clear water in one *ghaṭikā* (*nāḍī*).'

It is difficult to say what caused this shift in the size of the perforation—from the gold needle of one *pala* weight and eight

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[7] Brahmagupta too does not prescribe any measurements for the bowl and the perforation. All that is required is that the bowl should sink sixty times in an *ahorātra*; cf. *Brāhmasphuṭa-siddhānta*, 22.41:

> *ghaṭikā kalaśārdhākṛti tāmraṃ pātraṃ tale 'pṛthu cchidram/*
> *madhye tajjalamajjanaṣaṣṭyā dyuniśaṃ yathā bhavati//*

'The *Ghaṭikā-yantra* is a copper vessel of the shape of a hemisphere. At the centre of the bottom is a small perforation so made that the bowl sinks sixty times in a day and night.' *Sūryasiddhānta*, 13.23 also gives a similar definition, without specifying the measurements:

> *tāmrapātram adhaśchidraṃ nyastaṃ kuṇḍe 'malāmbhasi/*
> *ṣaṣṭir majjaty ahorātre sphuṭaṃ yantraṃ kapālakam//*

[8] *Śiṣyadhīvṛddhidatantra*, Part I, 21 (*Yantrādhikāra*), 34–35.

aṅgulas' length to a gold needle three and one-third *māṣas* in weight and four *aṅgulas* in length.[9] According to Śrīdhara, who is a near contemporary of Lalla, sixty-four *māṣas* make one *pala*.[10] Then the perforation prescribed by Lalla would be 5/48th of that prescribed by Āryabhaṭa.[11] While the size and weight of the bowl remained the same, such stark reduction in the size of the perforation would have greatly increased the duration of the time needed for the bowl to become full and then to sink, and consequently the duration of the *ghaṭikā* as well. One is therefore led to suspect that these specifications for the size of the perforation in terms of a gold wire of certain weight and length are fictitious and have no connection with actual practice. Yet this latter specification for the size of the perforation is repeated in several subsequent works.

0.4 Bhāskara II dismisses these specifications of the size as impractical. According to him a bowl of any size with any arbitrary hole can serve as the water clock. It need not even have a duration of one *ghaṭikā*; a bowl of any duration will do for measuring time. Thus he declares in the *Siddhāntaśiromaṇi*:

> *ghaṭadalarūpā ghaṭitā ghaṭikā tāmrī tale 'pṛthucchidrā/*
> *dyuniśanimajjanamityā bhaktaṃ dyuniśaṃ ghaṭīmānam//* [12]
>
> 'A copper bowl, formed like a hemisphere, having a small (*apṛthu*) hole at the bottom. The duration of a day and night divided by

[9] Of course, Lalla also adds that the vessel and the perforation can be of any dimension, provided the bowl sinks in one *ghaṭī*: cf. *Śiṣyadhīvṛddhidatantra*, Part I, 21 (*Yantrādhikāra*), 36:

> *athavā svecchāghaṭitaṃ ghaṭīpramābhiḥ prasadhitaṃ bhūyaḥ /*
> 'Or, it is a vessel manufactured according to one's liking [with a perforation at the bottom] which has been adjusted by the measure of a *ghaṭī* by repeated [trials].' (Bina Chatterjee's translation, slightly modified.)

[10] *Pāṭīgaṇita*, Rule 10.

[11] Let A be the area of the cross section of the wire (and therefore the area of the hole produced by the wire) of 1 *pala* weight and 8 *aṅgulas*' length. Then its volume will be 8A. If a gold wire of 1 *pala* (64 *māṣas*) has a volume of 8A, then a gold wire of $3\frac{1}{3}$ *māṣas* will have the volume of $\frac{8A}{64} \cdot \frac{10}{3} = A \cdot \frac{5}{12}$. The gold wire of $\frac{10}{3}$ *māṣas* is 4 *aṅgulas* long. Then the area of its cross section $B = A \cdot \frac{5}{12} \cdot \frac{1}{4} = A \cdot \frac{5}{48}$.

[12] *Siddhāntaśiromaṇi, Golādhyāya, Yantrādhyāya*, 8, pp. 366–367.

> the number of immersions [of this bowl] gives the measure of the water clock.'

This statement is elaborated in his auto-commentary *Vāsanā-bhāṣya*: 'Here, we ignore the definition of the water clock given by certain scholars as '*daśabhiḥ śulbasya palair...*' because it is illogical (*yukti-śūnya*) and difficult to implement (*durghaṭa*).'[13]

Here Bhāskara is attacking Lalla directly by quoting the latter's definition of the water clock. In his view, such specifications for the size of the bowl and that of the perforation are impractical. On the other hand, by stating that a bowl with any duration of immersion will do, Bhāskara goes to the other extreme. If the bowl sinks, say, in thirty-four minutes and thus measures a period of thirty-four minutes each time it sinks, these periods have to be converted into *ghaṭikās* of twenty-four minutes. By not insisting that the bowl should sink in one *ghaṭikā*, as Āryabhaṭa and Brahmagupta did, Bhāskara underestimates the Indian artisan's ability to fabricate bowls of exactly one *ghaṭikā* duration.

0.5 There is yet another problem with Bhāskara II's statement. His diction (*ghaṭadalarūpā ghaṭitā ghaṭikā tāmrī tale 'pṛthucchidrā*) follows very closely that of Brahmagupta (*ghaṭikā kalaśārdhākṛti tāmraṃ pātraṃ tale 'pṛthu cchidram*).[14] Since the manuscripts rarely mark the *avaghraha* symbols, these have to be inserted according to the context by the modern readers. There is no way of knowing whether the author himself desires an *avagraha* at a certain point or not, unless this is spelt out clearly in a subsequent statement by the author himself, or in an auto-commentary or in a reliable commentary written by one who is close to the author in time or spirit. In the present case, it is uncertain whether Brahmagupta and Bhāskara II consider the perforation to be '*pṛthu*' or '*apṛthu*'. Bhāskara's auto-commentary is silent on this issue.

However, Bhāskara II's commentator Munīśvara (b. 1603) expressly prohibits the insertion of an *avagraha* here. Thus he argues in his *Marīci* commentary: 'The bowl should be so made

[13] Ibid, p. 367: *atra daśabhiḥ śulbasya palair ity ādi yad ghaṭīlakṣaṇaṃ kaiścit kṛtaṃ tad yuktiśūnyaṃ durghaṭaṃ cety etad upekṣitam /*

[14] See n. 6 above.

that it has a large hole (*pṛthucchidra* = *mahārandhra*) at the bottom. Through this statement it is indicated that the hole should be made in such a manner that, when the bowl is placed on the water of the basin and when water enters [the bowl], the hole is not blocked by any dirt that may be in the water of the basin. Because of the possibility of a small hole getting blocked by dirt and the like, assuming here a coalescence of the vowel *a* (*akāra-praśleṣa*) [by reading *apṛthu*] is not proper.'[15]

We do not know if this is the intention of Bhaskara II or if this is just Munīśvara's view.[16] But a large hole will certainly make the bowl sink very rapidly. If it sinks, for example, in ten minutes, then it has to be lifted, emptied and placed again on the surface of the water each ten minutes! There is no practical advantage in using a water clock of such short duration.[17] It is more likely, therefore, that Bhāskara intended to say '*apṛthu*'. All the extant specimens that I have seen have very small holes.

0.6 Besides the question of the size of the perforation, another misunderstanding seems to have arisen about the volume of the bowl. The basic requirement, as we saw, is that the bowl should sink sixty times in a day and night, or that it should become full of water in one *ghaṭī* of sixty *palas*. Āryabhaṭa expressed this as '*palaṣaṣṭyā ambupūraṇa*'. This seems to have been misunderstood to mean that the bowl should have the capacity to hold sixty *palas* of water. In his commentary on the *Āryabhaṭīya*, Bhāskara I mentions such a view, albeit as a *pūrvapakṣa*.[18] Since his discussion throws up many problems, it is necessary to quote his statement in full and then to analyse it:

> *kathaṃ punar divasasya ṣaṣṭibhāgaḥ sādhyate ity atrābhidhīyate/ atra kecid bruvate suvarṇarajatatāmrāṇām anyatamaṃ pātram*

[15] *Siddhāntaśiromaṇi*, p. 367: *tale 'dhobhage pṛthucchidrā mahārandhrā ghaṭikā kāryā / etena jalapūrṇapātre nikṣiptādhaśchidre jalāgamane jalāntargatamalādikaṃ vastu pratibandhakaṃ na bhavati tathā chidraṃ kāryam iti sūcitam/ sūkṣmacchidre malādikena tatpratibandhasambhavād akārapraśleṣo na yuktaḥ/*

[16] The only other published commentary by Nṛsiṃha is silent on this issue; see p. 442 of Nṛsiṃha's commentary. See Bibliography.

[17] For preventing the blockage of the hole, it is generally prescribed that the water should be very clean.

[18] *Āryabhaṭīya*, p. 174.

ardhavṛttākāraṃ ṣaṣṭipalapānīyadhārakaṃ pūrakaṃ nisrāvakaṃ vā ghaṭiketi/ naiṣa niyamo [19] *yāvat palāni ṣaṣṭiḥ pānīyaṃ prasravaty āpūryate vā tāvatā nāḍikākāla iti/ prājñās tu naivam iti mayante/ kathaṃ tarhi/ ahorātraprasrutasya pānīyasya ṣaṣṭibhāgo ghaṭikāpramāṇa iti sthūlaḥ kalpaḥ/ sūkṣmas tu samāyām avanau nirdiṣṭākārasya śaṅkor ghaṭikāchāyām aṅkayitvā ghaṭikā sādhyate/ ghaṭikāchidraṃ ca chāyākālavaśād yuktyā yojayitavyam/*

'How then is the one-sixtieth part of a nychthemeron to be determined?' To this question, [the following] has to be said. In this connection some say: 'The *Ghaṭikā*[*-yantra*] is a vessel [made out] of one of the metals like gold, silver or copper, hemispherical in shape (lit. semicircular), which holds (*dhāraka*) sixty *palas* of water and which is filled with or discharges [the same amount of water].' Actually there is no such rule that the duration of the *ghaṭikā* (*nāḍikā-kāla*) is so long as it takes the vessel to discharge or to fill with sixty *palas* of water. Wise persons think that this is not so. How then?

'It is only a rough method (*sthūlaḥ kalpaḥ*) to say that the one-sixtieth part of the water that has been discharged in the course of a nychthemeron is the measure of one *ghaṭikā*. The more accurate method is to measure the *ghaṭikā* by marking the shadow of one *ghaṭikā*, cast by a gnomon of specified shape that has been set up on a level ground. The perforation in [the bowl of] the *Ghaṭikā-yantra* should be made skilfully according to the period measured by the shadow.'

Though Bhāskara I does not himself approve of it, there are some who hold that the *Ghaṭikā-yantra* should be a hemispherical vessel, which has the capacity to hold sixty *palas* of water and which either discharges or is filled with the same amount of water. Thus according to these persons both the outflow type of water clock which discharges water and also the sinking bowl type which is filled with water should have the same hemispherical shape and a volume of sixty *palas*. It is not known who these persons were who thought (already before 629 AD when Bhāskara wrote the commentary) that the bowl should have the capacity of sixty *palas*, but this notion continued to be held in later times as well.

Thus Śrīpati in his *Siddhāntaśekhara* prescribes that the bowl should hold sixty *palas* of water:

[19] The Edition reads *naiṣa niyamaḥ* and closes the sentence here. I combine this sentence with the next one in order to draw a coherent meaning.

śulbasya digbhir vihitaṃ palair yat
ṣaḍaṅguloccaṃ dviguṇāyatāsyam/
tad ambhasā ṣaṣṭipalaiḥ prapūryaṃ
pātraṃ ghaṭārdhapramitaṃ ghaṭī syāt//

satryaṃśamāṣatrayanirmitā yā
hemnaḥ śalākā caturaṅgulā syāt/

viddhaṃ tayā prāktanam atra pātraṃ
prapūryate nāḍikayāmbunā yat// [20]

'A vessel, resembling half a pot in shape (i.e. hemispherical), made of ten *palas* of copper, six *aṅgulas* in height and twice the same in the diameter of the mouth, which can be filled with sixty *palas* of water, is the *Ghaṭī-yantra.* It should be pierced beforehand by a four *aṅgulas* long gold needle that has been made of three and one-third *māṣas* [of gold]. Then it fills with water [and sinks] in one *ghaṭikā* (= *nāḍikā*).'

The expression '*tad ambhasā ṣaṣṣṭipalaiḥ prapūryaṃ*' can also be interpreted to mean that 'it should fill with water in a period of sixty *palas*' but then it would be redundant to say '*prapūryate nāḍikayāmbunā yat*'. Therefore, the first expression should pertain to the volume of the bowl. If the bowl really contains sixty *palas* of water, one *pala* would be either 8 or 7.6 cubic *aṅgulas. Pala* occurs as a unit of weight and also as a unit of time; but this is a rare use of *pala* as a unit of liquid measure. But the chances are that this too is a fictitious specification for the capacity of the bowl.

On the other hand, the time unit *pala* is occasionally referred to as '*pānīyapala*' in contexts where it is quite certain that it is a time unit. Thus Āryabhaṭa II speaks of six *asus* in one *pānīyapala*; and Bhaskara II of 3600 *pānīyapalas* in a day and night.[21] Probably here it means 'a *pala* that is measured by means of water clock' as distinct from '*pala*', the unit of weight.

0.7 A third problem is the occasional confusion between two types of water clocks, namely the sinking bowl type and the outflow type that preceded it chronologically. While the former fills

[20] *Siddhāntaśekhara* 19.19-20.

[21] See *Mahāsiddhānta*, 1.6: *prāṇāḥ pānīyapale tā; Siddhāntaśiromaṇi, Golādhyāya, Yantrādhyāya,* 8, *Vāsanābhāṣya: dyuniśanimajjanasaṃkhyayā yadi ṣaṭtriṃśacchatāni pānīyapalāni labhyante tadaikena kim iti trairāśikam.*

itself with water through a perforation and sinks in a fixed duration of time, the latter empties itself of water through a perforation in a fixed duration of time. In the previous paragraph we have seen that the *pūrvapakṣins* cited by Bhāskara I treat the vessels in both the types as hemispherical with the same volume. Or is it Bhāskara I himself who sees both the vessels as identical in shape and size? While the bowl in the sinking bowl type is certainly hemispherical or nearly so, this cannot be true of the outflow type, where the vessel must have had the shape of a regular cylinder or that of a truncated cone. Such confusion between the two types occurs elsewhere also. As we shall see below in 3.4.1-2, the *Dharmasindhu*, while discussing the sinking bowl type of water clock, cites in support a passage from the *Bhāgavata*, which describes the outflow type.

Sanskrit astronomical texts describe a large number of instruments. But no other instrument received such detailed specifications as the *Ghaṭikā-yantra* did. Again, of all the instruments described in these texts, the *Ghaṭikā-yantra* was the only one which was manufactured in great numbers and was used by astronomers and laymen alike. It is indeed strange that there should be such confusion in describing this simple instrument in all the highly 'scientific' texts on astronomy.

0.8 In spite of this theoretical confusion in the texts, countless specimens of this water clock were produced throughout the centuries and these kept reasonably correct time of one *ghaṭikā* of twenty-four minutes. But it is doubtful if any artisan has ever produced a bowl according to the textual prescriptions. In the few specimens that survive in modern collections,[22] rarely any bowl has the exact shape of a hemisphere; some are more conical, some are shallower, than a precise hemisphere. The sizes and weights too do not conform to the textual prescriptions, and vary considerably.

The holes were obviously made by a trial and error method, by comparing the new bowl with another bowl that shows correct time or with a sundial, as Bhāskara I recommends,[23] and by

[22] While cataloguing the extant specimens of Indian astronomical and time measuring instruments, I had occasion to study several water clocks.

[23] See 0.6 above.

suitably enlarging the hole or by reducing its size rather than by means of a gold wire of given dimensions. Sometimes the hole can become larger by constant use. Then the size of the hole can be reduced by hammering the area around the hole. In one case, a gold buff was added to the hole in order to reduce its size.[24]

0.9 There are evidences of timekeeping establishments in royal palaces, Buddhist monasteries, temple courtyards or town squares, maintained by royal or private endowments. Here time was constantly measured by this water clock and broadcast by conch and drum, or by the gong.[25] The Chinese traveller I-Tsing who spent some ten years from ca. 675 to 685 AD at the famous Buddhist monastery of Nalanda, gives a detailed description of the time keeping establishment there.[26] At the beginning of the eleventh century, al-Bīrūnī describes the time-keeping establishment at Purshor (modern Peshawar) and adds that 'Pious people have bequeathed for these clepsydrae (i.e. water clocks), and for their administration, legacies and fixed incomes.'[27] Epigraphic and literary records show that the institution of time keeping with the water clock and announcing it by means of the gong was adopted through all the centuries by royal courts (of the Tughluqs, Mughals, Rajputs, and even petty jamindars up to the beginning of the twentieth century), and also at places of worship belonging to the Hindu, Jain and Muslim faiths.[28]

1.0 Common householders could not afford the permanent installation of a water clock in their houses, for it needed the constant attendance of at least two people, one to announce the time when the bowl sinks and another to lift the bowl and place it again upon the water.[29] But householders too required the

[24] The bowl is now in the Museum of the Clockmakers' Company in London.

[25] For a photograph of the full ensemble of the bowl, vessel and gong, see Virendra Nath Sharma, 'Astronomical Instruments at Kota'.

[26] I-Tsing, *A Record of the Buddhist Religion*, pp. 144-145.

[27] Al-Bīrūnī, *Alberuni's India*, Vol. 1, pp. 337-338.

[28] Cf. S. R. Sarma, 'Astronomical Instruments in Mughal Miniatures'; idem, 'Indian Astronomical and Time-Measuring Instruments'; S. R. Sarma and Ishrat Alam, 'Announcing Time: The Unique Method at Hayatnagar 1676.'

[29] Actually, one needs at least six persons, namely two for each watch of eight hours.

water clock on special occasions like marriages, in order to know precisely the astrologically auspicious moment (*śubha-muhūrta* or *śubha-lagna*, or simply *muhūrta* or *lagna*). Usually the Purohita who performed the marriage brought the water clock with him and set it up ceremoniously in the client's house.[30]

1.1 The ritual connected with the setting up of the water clock and its invocation is described, albeit briefly, in an unpublished manuscript entitled *Ghaṭikāyantraghaṭanāvidhi*. This manuscript cites Nārada as the authority for this ritual. The extant version of the *Nāradasaṃhitā* (before 1365) does describe the ritual but the wording is somewhat different. Likewise Govinda Daivajña's *Pīyūṣadhārā* commentary (AD 1603) on his paternal uncle Rāma Daivajña's *Muhūrtacintāmaṇi* (AD 1600) and Kāśīnātha Upādhye's *Dharmasindhu* (AD 1790-91) describe the ritual, but with different wording. Unfortunately, the relevant passages in all these four sources are corrupt. But with the help of these sources, a hitherto unknown ritual connected with the water clock can be reconstructed. In the following pages, I shall first describe the ritual as gleaned from these passages. After that, I shall reproduce the text of the manuscript, and also three parallel passages from the other texts, suitably emended as far as possible, and provide a translation in English.

1.2 The *Ghaṭikāyantraghaṭanāvidhi* is a small paper manuscript of just three folios, now deposited in the Sarasvatī Bhavana of Sampūrṇānanda Saṃskṛta Viśvavidyālaya at Varanasi. The title *Ghaṭikāyantraghaṭanāvidhi* is mentioned at the beginning of the work. There are in all thirteen verses. These are followed by the expression '*atha maṅgalāṣṭakam*'. But before providing these eight auspicious verses, the manuscript breaks off abruptly. There is no colophon, nor any other indications to identify the author, scribe or the date. However, because of its close relation to some

[30] Cf. Edgar Thurston, *Ethnographic Notes in Southern India*, part II, p. 565: 'This form of time-measurer, made of half a cocoanut or copper, is still in use among native physicians, astrologers and others in Malabar. ... At the present day it is used on the occasion of marriage among higher Hindu castes. The brahmin priest brings the cup, and places the bridegroom in charge of it. It is the duty of the latter to count the gadis (= *ghaṭīs*) until the time fixed for his entrance into the wedding-booth.'

texts which belong to the early years of the seventeenth century, it is possible that the text of the present manuscript (if not the manuscript itself) also belongs to the early seventeenth century.

1.3 The ritual consists of (i) setting up the basin (*kuṇḍa*) on a sacred ground; (ii) placing the bowl therein at sunrise or sunset; (iii) the *mantra* in praise of the water clock; (iv) prognostication (*phala*) according to the cardinal direction to which the bowl moves when placed on the surface of the water in the basin; (v) and prognostication according to the direction in which the bowl finally sinks; and (vi) the recitation of the so-called *pala-vṛttas.*

1.4 All the four texts begin with the measurements of the bowl and the definition of the perforation. Here all the texts are highly corrupt. The confusion we have already noticed in astronomical texts is multiplied here many times. Though the *Yajamāna* is not expected to fabricate the bowl of the water clock as part of the ritual, the dimensions of the bowl and its perforation became part of the ritual text. And this too was apparently recited in course of the ritual. In a story contained in the *Kathāratnākara* which Hemavijaya Gaṇin composed in AD 1600 in Ahmedabad, there is an account of a Brāhmaṇa setting up the water clock for telling the time of his daughter's marriage, which runs thus:

> 'The Brāhmaṇa, who is especially well-versed in the whole range of astral science, wore a forehead mark made of saffron and rice-grains—
>
> 'The round vessel is made of ten *palas* of copper. In the *ghaṭikā* [bowl] the height should be made of six *aṅgulas.* The diameter there should be made to the measure of twelve *aṅgulas.* The good cherish a water clock that holds sixty *palas* of water.
>
> '—dropped the bowl, made fully according to the aforementioned prescriptions, in a basin filled with clean water at the time of the setting of the divine Sun.'[31]

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[31] *Kathāratnākara*, pp. 539-40:
viśeṣato niḥśeṣajyotiḥśāstrakuśalo vinirmitakuṅkumataṇḍulatilakaḥ sa vipraḥ
daśatāmrapalāvartapātre vṛttīkṛte sati/
ghaṭikāyāṃ samutsedho vidhātavyaḥ ṣaḍaṅgulaḥ//
viṣkambhaṃ tatra kurvīta pramāṇād dvādaśāṅgulam/
ṣaṣṭ[y]ambhaḥpalapūreṇa ghaṭikā sadbhir iṣyate//
ityādiparipūrṇapramāṇopetaṃ ghaṭikāpātraṃ svacchanīrabhṛte kuṇḍe bhaga-

The weight and the size of the bowl are not relevant to the story. Yet these are mentioned but not the hole, which plays a role in the story: it gets blocked—not by some dirt in the water of the basin, but by a rice grain that got itself detached from the Brāhmaṇa's forehead mark and fell into the bowl—and the auspicious moment for the marriage lapses.

1.5 While the ritual sometimes preserves archaic technical processes, frequent recitation in the ritual of passages that are not immediately relevant and therefore are not clearly understood can lead to their distortion as well.[32] Both these processes occur in the case of the *Ghaṭikā-yantra.* Even today, in temples at Mathura, the midnight hour of Kṛṣṇa's birth is measured by this water clock. Thus ritual preserves an archaic practice of time measuring. But the passages containing the measurements of the bowl get distorted by constant repetition, because the Brāhmaṇa priests do not produce the bowl themselves, and the measurements are of no interest to them. These distortions can be seen in ample measure in the four texts which will be presented below. The *Ghaṭikāyantraghaṭanāvidhi* and the *Nāradasaṃhitā* provide the dimensions of the bowl and its perforation, not once but twice, ostensibly as two alternative methods (*prakārāntara*) but actually the same method from two different sources, and both times the prescriptions are garbled. The *Dharmasindhu*, on the other hand, mixes up the prescriptions for the sinking bowl type and the outflow type. Nevertheless, these passages are of interest from the viewpoint of cultural history and the process of text transmission. In my translation of these passages, I shall try to explain how certain distortions may have taken place.

2.1 It is prescribed that the ground where the basin is to be set up should be sloping to the east and to the north and be smeared with cow dung. On this ground, some grains of rice are sprinkled and a jewel or a piece of gold is placed. The basin, of copper or clay, is placed upon the grains of rice. According to the

vato bhānor astagamanasamaye mumoca/

[32] For an engaging account of similar distortions in modern times, see Madhav M. Deshpande, 'Contextualizing the Eternal Language: Features of Priestly Sanskrit'.

Nāradasaṃhitā, the basin is wrapped in a pair of clothes. The *Pīyūṣadhārā* adds that it should be decorated with sandal paste and flowers. Then it is filled with clean water.

2.2 The measuring of time, i.e. the counting of the *ghaṭīs*, starts either at sunrise or sunset as the case may be. This moment is defined in our passages as 'when half of the Sun's orb has risen or set.' At this moment the bowl should be placed on the water in the basin. Before placing the bowl, Gaṇeśa and the Sun are worshipped, so also the personal deity and the teacher of the householder who is the bride's father.

2.3 While placing the bowl upon the water in the basin, the bowl is addressed with a sacred formula which is said to have been composed by Nārada. However, the text of the formula varies in all the four sources; the one cited in the *Pīyūṣadhārā* shows the greatest divergence. The original text may be that which occurs in the *Nāradasaṃhitā*; a variation of this can be seen in the *Ghaṭikāyantraghaṭanāvidhi* and an elaboration of the same in the *Pīyūṣadhārā*.

Nāradasaṃhitā:

mukhyaṃ tvam asi yantrāṇāṃ brahmaṇā nirmitā purā//
bhavābhayāya dampatyoḥ kālasādhanakāraṇam/

Dharmasindhu:

mukhyaṃ tvam asi yantrāṇāṃ brahmaṇā nirmite purā/
bhava bhāvāya dampatyoḥ kālasādhanakāraṇam//

Ghaṭikāyantraghaṭanāvidhi:

yantrāṇāṃ mukhyarūpāsi brahmaṇā nirmite ghaṭi/
dampatyoḥ śubhakālāptihetave bhava śobhane//

Pīyūṣadhārā:

yantrāṇāṃ mukhyayantraṃ tvam iti dhātrā purā kṛtam/
dampatyor āyuvṛddhyardhaṃ putrādidhanahetave/
jalayantraka me tasmād iṣṭasiddhiprado bhava//

2.4 When the bowl is placed on the water, it does not remain stationary where it is placed. It keeps turning until it settles at some place, generally towards the edge of the basin. The cardinal direction where it settles is said to be indicative of future portents. Likewise, the direction where it sinks is used for prognostication.

Besides the *Ghaṭikāyantraghaṭanāvidhi*, only the *Dharmasindhu* contains this section. According to both the sources, the result is not beneficial if the bowl settles or sinks in any one of the following four directions, viz. south-east, south, south-west and north-west. Furthermore, if the bowl sinks in the west, it is also not beneficial; on the other hand, sinking in this direction is said to be beneficial; it is even said that the girl becomes the favourite — obviously of the in-laws.

2.5 In the *Ghaṭikāyantraghaṭanāvidhi*, this prognostication is followed by three verses the contents of which do not have any apparent relation with one another or with the subject of the water clock. Fortunately, in the story from the *Kathāratnākara* which has been mentioned above, there is a reference to the recitation of '*pala-vṛttas*': 'The Brāhmaṇa placed the bowl of the water clock in a vessel containing clear water at the time of the setting of the divine Sun. Because he was busy reciting the *pala-vṛttas* such as '*mā kānte pakṣasyānte parvākāśe svāpsīḥ...*'[33] This is the first line of the first of the three verses in our manuscript. The story narrates that, after placing the bowl upon the water of the basin, the Brāhmaṇa recited this and similar *pala-vṛttas*. Therefore all the three verses in our manuscript must be *pala-vṛttas* that are recited after placing the perforated bowl upon the water in the basin. What then does the expression '*pala-vṛtta*' mean and why must these be recited?

The first of these three verses occurs also in Bhāskara I's commentary on the *Āryabhaṭīya*.[34] There Bhāskara explains that the time taken to utter sixty long syllables (*guru-akṣaras*) is one *vināḍikā*, and then cites the first of our three verses which consists exactly of sixty long syllables. '*Pala*' being a synonym of *vināḍikā*, *pala-vṛtta* designates a verse consisting of sixty long syllables, the reciting of which takes one *pala* of time, i.e. twenty-four seconds.

Indeed all the three verses at the end of our manuscript are such '*pala-vṛttas*'. Why should they be recited? The *Ghaṭikāyantra* can measure the period of one *ghaṭikā* of twenty-four minutes. For measuring smaller periods of time, it is likely that one

[33] *Kathāratnākara*, p. 540: *ghaṭikāpātraṃ svacchanīrabhṛte kuṇḍe bhagavato bhānor astagamanasamaye mumoca/ mā kānte pakṣasyānte parvākaśe svāpsīḥ ityādi-pala-vṛtta-paṭhanato ...*

[34] *Āryabhaṭīya*, p. 175.

recited the appropriate number of these *pala-vṛttas*.[35] It is possible that in the early seventeenth century when the *Kathāratnākara* was composed and also when the *Ghaṭikāyantraghaṭanāvidhi* may have been put together, the main function of these verses was forgotten and that they were just recited, without being employed for measuring the fractions of the *ghaṭikā*, just as one recited the verses about the measurements of the bowl and of its perforation. Nevertheless, the manuscript preserves three verses which were meant to measure time, no matter whether they were so used or not. The first verse is also the oldest; for it was quoted by Bhāskara I in his commentary which he completed in AD 629.[36] The second verse seeks the blessings of all the heavenly bodies for the couple and the third verse celebrates the ten incarnations of Viṣṇu. In Sanskrit prosody, such verses containing fifteen long syllables in each quarter are named variously as *Kāmakrīḍā, Līlākhela* and so on. The last two verses are benedictory in nature. It is quite appropriate to recite them in the context of a marriage ritual. But the first verse is of a different nature. It is not clear how it came to be connected with this ritual.

I now reproduce the four passages. Orthography and *sandhi* are silently corrected. Occasionally numerical expressions are followed by numeral symbols, e.g. *daśa 10*; such numbers are omitted. Wherever the wording has been emended, the original reading of the manuscript (MS) or of the edition (Edn) is shown in the footnotes.

3.1.1 *Ghaṭikāyantraghaṭanāvidhiḥ* [37]

śrīgaṇeśāya namaḥ/ atha ghaṭikāyantraghaṭanāvidhiḥ/

śuddhaṃ tāmravinirmitaṃ daśapalaiḥ [38] *pātraṃ ghaṭārdhākṛti* [39]

[35] S. R. Sarma, 'Measuring Time with Long Syllables'.

[36] Principal Vaman Shivaram Apte's *The Practical Sanskrit-English Dictionary*, revised and enlarged edition, Poona 1957, Appendix A: Sanskrit Prosody, p. 1, cites this verse to illustrate the metre *Līlākhela*, attributing it to the *Sarasvatīkaṇṭhābharaṇa*. But the verse does not occur in this text.

[37] For kindly providing me with a xerox copy of this manuscript, I am grateful to Professor Vidya Niwas Misra, the then Kulapati of the Sampūrṇānanda Saṃskṛta Viśvavidyālaya, and to Dr B. N. Misra, the then Librarian of the Sarasvatī Bhavana.

[38] MS: *rasa6palaiḥ*

[39] MS: *°kṛtiṃ*

mūlād ūrdhvaṣaḍaṅgulaṃ samaghanaṃ vistārato dvādaśa/[40]
viddhaṃ svarṇaśalākayā tripacayā (?) māṣaikayā tadghaṭīṃ
mittvā[41] *vai palaṣaṣṭivārapatanāc*[42] *cet pūrita sā ghaṭī//1//*

atha prakārāntaram/

ardhodayaṃ vāstamayaṃ suvīkṣya
yantraṃ pradadyāj jalapūrṇapātre/

ṣaḍaṅgulotsedhasamaṃ suvṛttaṃ
kṛtaṃ mukhaṃ yad [d]virasāṅgulaṃ tat//2//

palais tu tāmrair daśabhir jalasya
pūrṇaṃ palaiḥ ṣaṣṭibhir ambuyantram/

satryaṃśamāṣatritayaiva vṛt[t]a-
śalākayā madhyamabhāgaviddham//3//

samudrasaṃkhyāṅgulidīrghayopa-
deśān mayūraṃ naravānarādyam (!)/

gurvakṣaraiḥ khendumitair asus taiḥ
ṣaḍbhiḥ palaṃ tair ghaṭikā khaṣaḍbhiḥ[43] *//4//*

atha ghaṭīsthāpanabhūlakṣaṇam/

prāgudagpravaṇe deśe gomayenopalepite/

mṛṇāyaṃ (?) kuṇḍikāpātraṃ[44] *sthāpayed avraṇaṃ śubham//5//*

kuṅkumāktena sūtreṇa pariveṣṭya parasparam/

svacchena variṇā pūrṇaṃ taṇḍulānāṃ tathopari //6//

niścale salile sthapyaṃ kuṇḍikāyāṃ jalopari/

sthāpayed ghaṭikāyantraṃ sūryabimbārdhadarśanāt/

gaṇeśārkau ca saṃpūjya guruṃ natveṣṭadevatām//7//

[atha] ghaṭīsthāpanamantraḥ/

yantrāṇāṃ mukhyarūpāsi brahmaṇā nirmite ghaṭi/

dampatyoḥ śubhakālāptihetave bhava śobhane//8//

atha prāgādimadhyāntaghaṭībhramaṇaphalam āha/

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[40] MS: *vistārataḥ saptakaṃ* 7

[41] MS: *bhittvā*

[42] MS: *°paṭhanāc*

[43] MS: *khaṣaṣṭhiḥ*

[44] MS: *kuṇḍikāyugmaṃ*, which is obviously wrong. Only one basin is needed. Probably the corruption stems from '*vastrayugma*' in *Nāradasaṃhitā* 92.

yadrūpaṃ bhramaṇaṃ karoti ghaṭikā prāgādimadhyaṃ kramāt
saubhāgyaṃ[45] *nidhanaṃ vadhūmṛtisamaṃ yuktā ca rogais tanuḥ*[46]*/*
kanyā vallabhatām upaiti[47] *gaṇikātulyā yadā(?) dimaṃ*
syāt sādhvī sutavittabandhusahitā madhyasthitāryapradā//9//
uttareśānapūrvāsu ghaṭī pūrṇā śubhapradā/
dikṣu śeṣāsu kanyāyāḥ magnā vaidhavyadā smṛtā//10//
mā kānte pakṣasyānte paryākāśe deśe svāpsīḥ
kāntaṃ vaktraṃ vṛttaṃ pūrṇaṃ candraṃ matvā rātrau cet/
kṣutkṣāmaḥ prātaṃś cetaś ceto rāhuḥ krūraḥ prādyāt[48]
tasmād dhvānte harmyasyānte śayyaikānte kartavyā//11//
mārtāṇḍas tārānāthaḥ kṣoṇīsūnuḥ sūnuś cendor
vāgīśo daityācāryaś chāyāputro rāhuḥ ketuḥ/
nakṣatrair aśvinyādyais tārāyuktaiś cābhiḥ sarve
kuryāsuḥ[49] *kalyāṇaṃ vo nityārogyaṃ lakṣmīm āyuḥ//12//*
lokakṣemayāsīn matsyaḥ kūrmaḥ kroḍaḥ puṃsiṃho[50]
yo hrasvākāro rāmo[51] *rāmaḥ kṛṣṇo buddhaḥ*[52] *kalkī/*
evaṃ nānārūpaṃ nānākāraṃ nānānāmānaṃ
yogidhyeyaṃ devaṃ devānāṃ[53] *vande 'haṃ govindam//13//*

atha maṅgalāṣṭakam/

3.1.2 Translation

Salutation to śrī Gaṇeśa.

Now the method of setting up the water clock.

A pure vessel, made of copper of ten *palas* in weight, of the shape of a hemisphere, measuring six *aṅgulas* from the bottom to the top, evenly dense, in width twelve *aṅgulas*; pierced by a golden needle, made of one *māṣa* increased by three (*tripacayā*?). After

[45] MS: *saubhāgyaṃ na*
[46] MS: *rogaiś canuḥ*
[47] MS: *upaiti ka*
[48] MS: *prādyāt krūraḥ*
[49] MS: *kuyāsuḥ*
[50] MS: *puṃsīghe*
[51] MS: *romo*
[52] MS: *bauddhaḥ*
[53] MS: *devānāṃ devaṃ*

measuring with that vessel, if it sinks sixty times (or, if it is filled in sixty *palas* of time), then it is a [proper] water clock.//1// [54]

Now another method. [55]

After carefully observing the rise of the Sun's orb up to the half, or the setting of the same, the instrument (i.e. the bowl) should be placed in a basin filled with water. The bowl is so made that its height is equal to six *aṅgulas*, and the circular opening is twelve *aṅgulas* in diameter.//2//

The water clock (*ambu-yantra*) is made of ten *palas* of copper; it is filled by water in sixty *palas*.[56] It is pierced at the central portion (i.e. centre of the bottom) by a round needle made of three and one-third *māṣas* of gold,//3//

and 4 *aṅgulas* length; ... [57] Ten long syllables (*gurvakṣaras*) make one breath (*asu*); six of these make one *pala*; sixty of these make one *ghaṭikā*. //4//

Now the characteristics of the ground on which the water clock is to be set up.

On a ground, sloped to the east and north,[58] which has been smeared with cow-dung, a vessel called *kuṇḍa*, faultless (*avraṇa*) and auspicious, should be placed ... //5//

upon grains of rice and should be encircled with thread dyed in saffron; then it should be filled with clear water.//6//

The water clock (i.e. the bowl) should be placed on the placid water in the basin, when the Sun's orb is half visible, after worshipping Gaṇeśa and the Sun, and after bowing to the teacher and to the personal deity.//7//

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[54] The verse is highly corrupt; besides messing up the measurements, the last line telescopes two separate prescriptions, viz. *ṣaṣṭivārapatana* 'sinking sixty times in a nychthemeron' and *palaṣaṣṭyā pūraṇa* 'filling in a period of sixty *palas*'.

[55] What follows it not exactly another method, but rather the same information from another source.

[56] This can also mean 'it is filled by sixty *palas* of water'.

[57] Here the verse is contaminated by some irrelevant material '*upadeśān mayūraṃ naravānarādyam*', apparently from *Sūryasiddhānta* 13.21b: *toyayantrakapālādyair mayūranaravānaraiḥ//*

[58] Sloping to the east and north is considered to be auspicious; Cf. *Rāmāyaṇa* (vulgate edition), 2.99.24: *prāgudagpravaṇāṃ vediṃ viśālāṃ dīptapāvakām.* In its stead the Critical Edition chose a reading which makes no sense. 2.93.23:

prāgudakśravaṇāṃ vedīṃ viśālāṃ dīptapāvakām/
dadarśa bharatas tatra puṇyāṃ rāmaniveśane//

The sacred formula (*mantra*) for placing the water clock:

'O Water Clock, you have been created by Brahmā as the foremost among the [time measuring] instruments. O auspicious one, be the means for measuring the auspicious time [for the wedding] of the couple.' //8//

Now he tells the fruit of the rotation of the bowl, starting from the east etc., and ending in the middle.

According as the bowl rotates in cardinal directions from the east up to the middle of the basin, it causes respectively the good fortune of having the husband alive and devoted (*saubhāgya*), death, near death of the bride (*vadhūmṛtisama*), the body full of diseases, the girl becomes the favourite [of all], resembles a courtesan, (?) virtuous, endowed with sons, wealth and relatives. Staying in the middle, [the bowl] grants noble [sons].//9//

If the bowl becomes full (*pūrṇā*) [and sinks] in the north, north-east, or in the east, it bestows auspiciousness; if it sinks (*magnā*) in the remaining directions, it is said to inflict widowhood on the girl.//10//

[Now the *pala*-verses]:

Do not, O pretty one, at the end of the bright fortnight, sleep at a place open to the sky. Should it turn night, the cruel Rāhu, starving with hunger and roaming hither and thither, may eat you up, taking your pretty round face for the full moon. Therefore, after darkness, make your bed at a secluded place inside the house.//11//

May the Sun, the Moon, Mars, Mercury, Jupiter, Venus, Saturn, Rāhu and Ketu, all these, together with the lunar mansions beginning with Aśvinī, and all these stars, produce auspiciousness, constant good health, prosperity, and longevity [for the couple].//12//

For the welfare of the world, there [manifested the incarnations of] the Fish, the Tortoise, the Boar, the Man-Lion, One who had a Short Stature, [Paraśu] Rāma, Rāma, Kṛṣṇa, Buddha and Kalkin. I bow to Govinda, the god of gods, who in this manner assumed diverse forms, diverse shapes and diverse names, and who is meditated upon by sages.//13//

Now the eight auspicious verses.

3.2.1 *Nāradasaṃhitā* [59]

tallagnaṃ jalayantreṇa dadyāj jyotiṣikottamaḥ/
ṣaḍaṅgulamitotsedhaṃ dvādaśāṅgulam āyatam//86//
kuryāt kapālavat tāmrapātraṃ tad daśabhiḥ palaiḥ/
pūrṇaṃ ṣaṣṭir jalapalaiḥ ṣaṣṭir majjati vāsare//87//
māṣatrayatryaṃśayutasvarṇavṛttaśalākayā [60]*/*
caturaṅgulāyatayā [61] *tathā viddhaṃ parisphuṭam//88//*
kāryeṇābhyadhikaḥ (?) ṣaḍbhiḥ palais tāmrasya bhājanam/
dvādaśa mukhaviṣkambha utsedhaḥ ṣaḍbhir aṅgulaiḥ//89//
svarṇamāṣena vai kṛtvā caturaṅgulakātmakaḥ/
madhyabhāge tathā viddhā nāḍikā ghaṭikā smṛtā//90//
tāmrapātre jalaiḥ [62] *pūrṇe mṛtpātre vāthavā śubhe/*
gandhapuṣpākṣataiḥ sārdham [63] *alaṃkṛtya prayatnataḥ//91//*
taṇḍulasthe svarṇayute vastrayugmena veṣṭite/
maṇḍalārdhodayaṃ vīkṣya raves tatra viniḥkṣipet//92//
mantreṇānena pūrvoktalakṣaṇaṃ yantram uttamam/
mukhyaṃ tvam asi yantrāṇāṃ brahmaṇā nirmitā purā//93//
bhavābhayāya dampatyoḥ kālasādhanakāraṇam/

3.2.2 Translation

The best of the astrologers should measure (*dadyāt*) [64] that auspicious moment by means of the water clock. With a height of six *aṅgulas*, with a width of twelve *aṅgulas*,//86//

let a copper bowl be made, like a hemisphere, with ten *palas* of weight. It is filled in the duration of sixty *palas* (or, with sixty *palas* of water), and sinks sixty times in a day and night. [65] //87//

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[59] *Nāradasaṃhitā*, 29.86-95, pp. 181-184.
[60] Edn: *māṣamātratryaṃśayutaṃ svarṇa°*
[61] Edn: *caturbhir aṅgulair āpas*
[62] Edn: *jalaḥ*
[63] Edn: *sārdhair*
[64] '*lagnaṃ dadyāt*' is an interesting expression.
[65] See section 0.7 above.

It should be pierced with a circular gold needle of three and one-third *māṣas* in weight and four *aṅgulas* in length. Then it is accurate.//88//

A copper bowl should be made with more than six *palas* (sic!). The diameter of the opening is twelve and the height six *aṅgulas*. //89//

Having made with one (sic!) *māṣa* of gold [a needle that is] four *aṅgulas* [in length], [with that] when the bowl (*ghaṭikā*) is pierced thus in the middle, it is then known as the water clock (*nāḍikā*).//90//

In an auspicious copper basin, or in a clay basin that has been filled with water, having decorated it with effort by means of sandal paste, flowers and coloured rice,//91//

the basin which is placed upon grains of rice, to which a gold piece is added and which is covered by a pair of clothes, one should place the bowl after having seen the rise of half of the Sun's orb.//92//

With this formula, one should deposit the best of the instruments, endowed with the aforementioned characteristics: 'You have been created a long time ago by Brahmā as the foremost among the [time measuring] instruments. For the safety (*abhaya*) of the couple, you become the means of measuring the time [of their wedding].' //93//

3.3.1 *Pīyūṣadhārā* [66]

sa ceṣṭakālaḥ kathaṃ sādhanīya ity ata āha kaśyapaḥ/
evaṃ guṇagaṇān vīkṣya lagnaṃ niścitya yatnataḥ/
siddhāntoktena mārgeṇa lagnakālaṃ prasādhayet//1//
jalayantreṇa tallagnaṃ dadyāt tenārcito dvijaḥ/
mukhaṃ dvādaśabhir vṛttam [67] *aṅgulaiś ca ṣaḍ unnatam//2//*
ghaṭārdhavat tāmrapātraṃ kuryāt tad daśabhiḥ palaiḥ/
ṣaṣṭir majjaty [68] *ahorātre ghaṭikāpātram uttamam//3//*
māṣatrayatryaṃśayutasvarṇavṛttaśalākayā/
caturbhir aṅgulair āyatayā viddhaṃ sphuṭaṃ nyaset//4//
raver ardhodayaṃ dṛṣṭvā vāpy ardhāstamayaṃ tathā/
pūrvoktalakṣaṇaṃ yantraṃ mantreṇānena niḥkṣipet//5//

[66] *Pīyūṣadhārā*, p. 424. The verse numbers have been added.
[67] Edn: *vṛttaṃ dvādaśabhir*
[68] Edn: *bhajed*

niḥkṣipej jalapūrṇapātra ity arthaḥ// yad āha Nāradaḥ//

> *tāmrapātre jalaiḥ pūrṇe gandhapupṣair alaṃkṛte/*
> *taṇḍulasthe ratnayute śucibhūmāv ahaspateḥ* [69] */*
> *maṇḍalārdhodayaṃ vīkṣya jalapātre viniḥkṣipet//6// iti//*

mantram apy āha sa eva/

> *yantrāṇāṃ mukhyayantraṃ tvam iti dhātrā purā kṛtaṃ/*
> *dampatyor āyuvṛddhyarthaṃ putrādidhanahetave/*
> *jalayantraka me tasmād iṣṭasiddhiprado* [70] *bhava//7// iti//*

3.3.2 Translation

How is that desired auspicious moment of time to be determined: in reply to this question, Kaśyapa spoke thus:

In this manner, after considering all the good points, and having chosen, with effort, the auspicious moment (*lagna*) according to the method taught by the Siddhāntas, the time of that auspicious moment should be calculated (*prasādhayet*).//1//

Let the Brāhmaṇa, who had been honoured by him (i.e. the householder) measure that moment (*lagnaṃ dadyāt*) by means of a water clock.[71] Let a copper bowl be made with ten *palas* weight, like a hemisphere, with the circular mouth measuring twelve *aṅgulas* in diameter and six *aṅgulas* in height. If it sinks sixty times in a day and night, it is the best water clock. //2-3//

The bowl that has been clearly pierced by a circular needle of gold, of three and one-third *māṣas*' weight and four *aṅgulas*' length, should be placed [on the water].//4//

After having seen the rise of half of the Sun's orb, or the setting of the half likewise, the instrument having the aforementioned characteristics should be deposited, with this sacred formula. //5//

'Should be deposited' means 'in a basin filled with water'. Thus spoke Nārada:

In a copper basin, which is filled with water, which is decorated with sandal paste and flowers, which is situated upon grains of rice on a pure ground, and which is endowed with jewels (*ratnayuta*), after noticing the rise of half of the Sun's orb, [the bowl] should be deposited. //6// He also taught the sacred formula:

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[69] Edn: *aharpateḥ*

[70] Edn: *°pradaṃ*

[71] Note the distinction between the two expressions '*lagnaṃ prasādhayet*' and '*lagnaṃ dadyāt.*'

'You have been created a long time ago by Brahmā as the foremost among the [time measuring] instruments. Therefore, for increasing the longevity of the couple and for conferring on them sons, wealth and the like, O water clock of mine, grant them the fulfilment of their desires.' //7//

3.4.1 *Dharmasindhuḥ* [72]

atha lagnaghaṭīsthāpanam/ daśapalamitatāmraghaṭitaṃ ṣaḍaṅgulonnataṃ dvādaśāṅgulavistṛtaṃ ghaṭī-yantraṃ kuryād iti Sindhuḥ/

dvādaśārdhapalonmānaṃ caturbhiś caturaṅgulaiḥ/

svarṇamāṣaiḥ kṛtacchidraṃ yāvat prasthajalaplutam//

iti tu śrībhāgavate tṛtīyaskandha uktam/ asyārthaḥ asītiguñjātmakaḥ karṣaḥ/ asyaiva suvarṇasaṃjñā/ karṣacatuṣṭayaṃ palam/ tathā ca ṣaṭpalatāmraviracitaṃ pātraṃ viṃśatiguñjonmitasuvarṇanirmitacaturaṅguladīrghaśalākayā mūle kṛtacchidraṃ kuryāt/ tena chidreṇa yāvat prasthaparimitaṃ jalaṃ praviśati tena ca prasthajalapūraṇena tat pātraṃ jale magnaṃ bhavati tat pātraṃ ghaṭīkālapramāṇam/ tatra prasthamānaṃ tu ṣoḍaśapalātmakam/

palaṃ suvarṇāś catvāraḥ kuḍavaḥ prastham āḍhakam/

droṇaṃ ca khārikā ceti pūrvapūrvacaturguṇam//

ity ukteḥ/ granthāntare caturmuṣṭiḥ kuḍavaś catvāraḥ kuḍavāḥ prastha iti/ kecit ṣaṣṭisaṃkhyākaguruvarṇoccār[aṇ]e palasaṃjñakālaḥ ṣaṣṭipalakālo nāḍikety āhuḥ/ evaṃ pramāṇīkṛtaṃ ghaṭīyantraṃ sūryamaṇḍalasyārdhodaye 'ste vā jalapūrṇe tāmrapātre mṛtpātre vā kṣipet/ tatra mantraḥ/

mukhyaṃ tvam asi yantrāṇāṃ brahmaṇā nirmite purā/

bhava bhāvāya dampatyoḥ kālasādhanakāraṇam//

anena mantreṇa gaṇeśavaruṇapūjanapūrvakaṃ ghaṭīyantraṃ sthāpayet/ evaṃ sthāpitā ghaṭī āgneyayāmyanairṛtavāyavyādidiggatā na śubhā/ madhyasthitānyadiggatā ca śubhā/ evam āgneyādi pañcadikṣu pūrṇā na śubhā// iti ghaṭīvicāraḥ

3.4.2 Translation

Now the setting up of the water clock [for measuring] the auspicious moment.

[72] *Dharmasindhu*, pp. 510-511.

The *Sindhu* declares that the water clock should be made of ten *palas* of copper, six *aṅgulas* high and twelve *aṅgulas* wide.

'[A vessel made of] half of twelve *palas*' weight, in which a hole has been made [with a needle of] four *māṣas* of gold and four *aṅgulas* [in length], till it is filled by (?) one *prastha* of water.'

Thus it has has been said in the third *Skandha* of the sacred *Bhāgavata.* [73] Its meaning is [as follows]. Eighty *guñjas* make one *karṣa.* The same has the designation of *suvarṇa.* Four *karṣas* are one *pala.* Thus, a vessel should be made of six *palas* of copper; it should be pierced at the base by means of a needle made of twenty *guñjas*' weight of gold and four *aṅgulas* in length. Through this perforation, by the time a *prastha* measure of water enters, that bowl sinks in the water, because of the *prastha* measure of water that filled it. Then that vessel becomes the standard measure for the period of one *ghaṭī.* There the unit of one *prastha* contains sixteen *palas.* [74]

For it has been said: one *pala* is four *suvarṇas*; then *kuḍava, prastha, āḍhaka, droṇa* and *khārikā,* are respectively each four times the previous unit.

In another text, it has been said that four fistfuls are one *kuḍava,* four *kuḍavas* are one *prastha.*

Some others say that the time taken for uttering sixty long syllables is one *pala*, and that the duration of sixty *palas* is one *nāḍikā.* The water clock, thus calibrated, should be placed in a copper basin or clay basin, full of water, when half of the Sun's orb has risen or set. There this sacred formula is recited.

'You have been created long time ago by Brahmā as the foremost among the [time measuring] instruments. For the sake of the

[73] *Bhāgavata* 3.11.9.

[74] What is described in the *Bhāgavata* is the outflow type of water clock. Here the perforation is made by a gold needle of four *māṣas* (= twenty *guñjas*) in weight and four *aṅgulas* in length, cf. *Kauṭilīya Arthaśāstra* and *Jyotiṣkaraṇḍaka*, cited in n. 4 and n. 5 above. The volume of the water discharged by this clock in a fixed period of time is one *prastha.* In this type of clock, water does not enter (*pra-viśati*) but flows out (*niḥsarati*). Similar confusion occurs elsewhere also. For example, while the *Jyotiṣkaraṇḍaka* describes the water clock of the outflow type, the commentator Malayagiri interprets the passage in the sense of the sinking bowl type. I shall discuss these two types elsewhere in greater detail. Finally, after a long and tire some excursion into metrology, the *Dharmasindhu* comes to the conclusion that the water that enters into the vessel has the volume of one *prastha* which is equal to sixteen (*ṣoḍaśa*) *palas.* Recall Śrīpati's view that the bowl should hold sixty (*ṣaṣṭi*) *palas* of water!

state of [their] becoming a married couple (*dampatyoḥ bhāvāya*), you be the means of measuring time.'

With this sacred formula, preceded by the worship of Gaṇeśa and Varuṇa, the bowl should be placed [on the water in the basin]. If the bowl thus placed moves to the south-east, south, south-west, or north-west of the basin, it is not auspicious. If it stays in the middle, or moves to other directions, it is auspicious. Likewise, if it fills [and sinks] in the five directions starting from the south-east, it is not auspicious. Thus the discussion of the water clock.

Bibliography

Al-Bīrūnī. *Alberuni's India*, tr. Edward C. Sachau. reprint: New Delhi, 1983.

Āryabhaṭīya. *Āryabhaṭīya of Āryabhaṭa* with the Commentary of Bhāskara I and Someśvara, critically edited with Introduction and Appendices, by Kripa Shankar Shukla. Indian National Science Academy, New Delhi, 1976.

Bhāgavata. *The Bhāgavata* (*Śrīmad Bhāgavata Mahāpurāṇa*), critically edited by H. G. Shastri. B. J. Institute of Learning and Research, Ahmedabad, Volume 1 (Skandhas 1-3), 1996.

Brāhmasphuṭasiddhānta. Brahmagupta, *Brāhmasphuṭasiddhānta*, ed. Sudhakara Dvivedin with his own *ṭīkā*. Benares, 1902.

Deshpande, Madhav M. 'Contextualizing the Eternal Language: Features of Priestly Sanskrit' in: Jan E. M. Houben (ed), *Ideology and Status of Sanskrit: Contributions to the History of the Sanskrit Language*. Brill's Indological Library, Volume 13. E. J. Brill, Leiden, New York, Cologne, 1996, pp. 401-36.

Dharmasindhu. Kāśīnātha, *Dharmasindhu* with the commentaries by Vaśiṣṭha Datta Miśra and Sudāma Miśra. Kashi Sanskrit Series No. 183. Vārāṇasī, 1968.

Ghaṭikāyantraghaṭanāvidhiḥ. MS no. 37074, Sarasvatī Bhavana Library, Sampūrṇānanda Saṃskṛta Viśvavidyālaya, Varanasi.

Indian Astronomy: A Source-Book (based primarily on Sanskrit Texts), compiled by B. V. Subbarayappa and K. V. Sarma. Nehru Centre, Bombay, 1985.

I-Tsing. *A Record of the Buddhist Religion as practised in India and the Malay Archipelago (A.D. 671-695)*, tr. J. Takakusu. Oxford 1896. Reprint: Delhi, 1966.

Jyotiṣkaraṇḍaka. Vallabhīyācāryīyaṃ Śrījyotiṣkaraṇḍakaṃ Prakīrṇakaṃ, with the *Vṛtti* by Malayagiri. Ratlam, 1928.

Kathāratnākara. Hemavijaya Gaṇī, *Kathāratnakara*, published by Hiralal Hansraj. Jamnagar, 1911.

Kauṭilīya Arthaśāstra, ed. and tr. R. P. Kangle, Vol. 1 (Bombay 1960). Second edition: Bombay, 1969.

Mahāsiddhānta. The Pūrvagaṇita of Āryabhaṭa's (II) Mahāsiddhānta, ed. and tr. Sreeramula Rajeswara Sarma, Marburg, 1966.

Nāradasaṃhitā. Nāradamunipraṇītā Nāradasaṃhitā, sānvaya vimalā ṭīkā sahitā, Vyākhyākār: Rāmajanma Mśira. Vārāṇasī, 1984.

Needham, Joseph. *Science and Civilisation in China*, Vol. 3: Mathematics and the Sciences of the Heavens and the Earth. Cambridge, 1959.

Nṛsiṃha. *Siddhāntaśiromaṇi* of Bhāskara with his auto-commentary *Vāsanābhāṣya* and the *Vārttika* of Nṛsiṃha Daivajña, ed. Murali Dhara Chaturvedi, Sampūrṇānanda Saṃskṛta Viśvavidyālaya, Varanasi, 1981.

Pāṭīgaṇita. The Patiganita of Sridharacharya, with an ancient Sanskrit commentary, ed. and tr. Kripa Shankar Shukla. Lucknow University, Lucknow, 1959.

Pīyūṣadhārā. Rāma Daivajña, *Muhūrtacintāmaṇi*, with the *Pīyūṣadhārā* commentary by Govindajyotirvid and Hindi translation by Kedaradatta Jod. Second enlarged edition, Varanasi, 1979.

Sarma, Sreeramula Rajeswara. 'Astronomical Instruments in Mughal Miniatures.' *Studien zur Indologie und Iranistik*, 16-17, 1992, pp. 235-76.

——. 'Indian Astronomical and Time Measuring Instruments: A Catalogue in Preparation.' *Indian Journal of History of Science*, 29, 1994, pp. 507-28.

——. 'The Bowl that sinks and Tells Time.' *India Magazine, of her People and Culture*, 14.9, September 1994, pp. 31-6.

——. 'Measuring Time with Long Syllables: Bhāskara I's Commentary on Āryabhaṭīya, Kālakriyāpāda 2.' *Indian Journal of History of Science*, 36, 2001, pp. 51-54.

Sarma, S. R. and Ishrat Alam. 'Announcing Time: The Unique Method at Hayatnagar 1676.' *Proceedings of the Indian History Congress*, 52nd Session, New Delhi, 1992, pp. 426-31.

Sharma, Virendra Nath. 'Astronomical Instruments at Kota.' *Indian Journal of History of Science*, 35, 2000, pp. 233-44.

Siddhāntaśekhara. Śrīpati, *Siddhāntaśekhara*, ed. by Babuaji Misra. Calcutta, 1932-1947.

Siddhāntaśiromaṇi. Bhāskara, *Siddhāntaśiromaṇi*, with his auto-commentary *Vāsanābhāṣya* and the *Marīci* commentary of Munīśvara, ed. Dattātreya Āpaṭe, *Golādhyāya*, Anandasrama Sanskrit Series, No. 122. Poona, 1943, 1952.

Śiṣyadhīvṛddhidatantra. Śiṣyadhīvṛddhida Tantra of Lalla with the commentary of Mallikārjuna Sūri, ed. and tr. Bina Chatterjee. Indian National Science Academy, New Delhi, 1981.

Shukla, Kripa Shankar. 'Āryabhaṭa I's Astronomy with Midnight Day-Reckoning,' *Gaṇita*, 18.1, June 1967, pp. 83-105.

Thurston, Edgar. *Ethnographic Notes in South India.* Reprint: Delhi, 1975.

Planet Worship in Ancient India

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The aim of the present paper is first to explain the development of the concept of *graha* as 'planet'[1] in ancient India, then to look at the rite of planet worship (*grahayajña*) in the group of ritual texts (*gṛhyasūtras*), and finally, to make clear the historical position of the section called *Grahaśānti* (appeasement for *grahas*) in the *Yājñavalkyasmṛti*, one of the most influential texts on *dharmaśāstra*.

1 *Grahas as heavenly bodies*

Putting aside references to *graha* in medical contexts,[2] the concept of *graha* as heavenly body experienced at least the following stages of development.

1. A demon which eclipses the Sun and the Moon was called *svarbhānu* and, probably, *graha* (from the Sanskrit root $\sqrt{grah}$, "to seize").

2. The demon got the name Rāhu and, somewhat later, the tail of the truncated Rāhu was called Ketu.

3. Five planets were regarded as *grahas* because they possess man and do him harm.

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*The original version of this article was read at Prof. Yasuke Ikari's seminar in Kyoto University in February 10, 1995. Since then several changes have been made and some parts were incorporated in some other articles.

[1] When the word 'planets' without modification is used in this paper it is in its pre-Copernican sense, namely, it includes the sun and the moon with the five planets known to naked-eye astronomy. In Sanskrit texts, when 'five planets' excluding the Sun and the Moon are specifically intended, they are called *tārāgraha* ('star-planets') or *aṅgārakādi* ('those which begin with Mars').

[2] I have discussed this problem in Yano [2003].

4. The Sun and the Moon joined the *grahas*, and a group of seven *grahas*—or nine *grahas* including Rāhu and Ketu—was established, although the order of enumeration was not yet fixed.

5. The week-day order of the seven *grahas* (Sun, Moon, Mars, Mercury, Jupiter, Venus, Saturn) and the group of the nine *grahas* were established.

1.1 *Svarbhānu as an eclipse demon*

Since a solar or lunar eclipse is one of the most conspicuous heavenly phenomena, it is difficult to imagine that it escaped the attention of the Vedic poets. In fact a demon called Svarbhānu in *Ṛgveda* 5.40 can be interpreted as the cause of eclipses:

> When the demon Svarbhānu wounded you by darkness, oh Sun (*sūrya*), the creatures watched as if they were bewildered, not knowing (their) place.[3]
>
> When the demon Svarbhānu wounded the Sun by darkness, Atris found it out, because the others were unable to.[4]

This is the only occurrence of Svarbhānu in the *Ṛgveda* and there is no evidence that this demon was identified as *graha*. In the epics, however, Svarbhānu is explicitly called '*graha*'.[5]

1.2 *Rāhu and Ketu as grahas*

The *Atharvaveda* may be the oldest text where *graha* appears together with *rāhu*:

> Weal for us be the planets (*grahāḥ*) belonging to the moon, and weal the Sun (*ādityá*) with Rāhu; weal for us smoke-bannered death, weal the Rudras of keen brightness. (AVŚ 19.9.10) [6]

[3] yát tvā sūrya súarbhānus tamasā́vidhyad āsuráḥ/
ákṣetravid yáthā mugdhó bhuvanāny adīdhayuḥ//(RV5.40.5)

[4] yaṃ vai sūryaṃ súarbhānus tamasā́vidhyad āsuráḥ/
átrayas tám ánv avindan nahí anye áśaknuvan//(RV5.40.9)

[5] *Mahābhārata* 6.13.40ab: parimaṇḍalo mahārāja svarbhānuḥ śrūyate grahaḥ.

Rāmāyaṇa 3.22.11cd: jagrāha sūryaṃ svarbhānur aparvaṇi mahāgrahaḥ.

[6] śáṃ no gráhāś cāndramasā́ḥ śám ādityáś ca rāhuṇā́/
śáṃ no mṛtyúr dhūmáketuḥ śáṃ rudrā́s tigmátejasaḥ//

This is borrowed from Whitney's translation.[7] He added a question mark when he translated *grahāḥ* (plural) as 'planets' in the seventh verse of the same hymn where the word is modified by 'moving in the sky' (*divícarā*). In this quoted verse, however, he omitted the question mark. On the other hand, he thinks it 'extremely unlikely' that the 'smoke-bannered death' (*mṛtyur dhūmaketuḥ*) means a comet.

The sixteenth century commentator Sāyaṇa had no qualms about interpreting *grahāḥ* as 'the planets beginning with Mars', but I see no strong reason to support him. What is clear here is only that the Sun and the Moon in this context are 'one who is seized' either by *graha* or by Rāhu and they themselves are not the 'seizers'.

Since the expression *nákṣatram ulkā́bhihatam* ('the meteor-smitten asterism') is found in the ninth verse of the same hymn, it seems that the author of the hymn intended to give mantras for appeasing the inauspicious phenomena in the sky.

Besides the passage of the *Atharvaveda* quoted above, the oldest text which mentions Rāhu as an eclipse demon is the *Chandogya-Upaniṣad*, where is said "Just like the Moon who was released from Rāhu's mouth...".[8] The *Maitrāyaṇī-Upaniṣad* enumerates Rāhu and Ketu along with Saturn (*śani*) as one of the semi-deities.[9] The date of these portions is problematical.

What has been shown above is that there is no strong evidence in Sanskrit literature of the Vedic period which shows the identification of *graha*s as planets. People in the Vedic period might have watched those stars whose behavior was different from those of the fixed stars, but they failed to classify them as a group of *graha* or planets. It is only after the period of Greek settlement in Bactria (third century B.C.) that explicit references to planets are attested in Sanskrit texts.

1.3 *The oldest reference to planets*

The *Arthaśāstra*, a book on politics ascribed to Kauṭilya who was a minister of Mauryan empire and sometimes compared with

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[7] W.D. Whitney [1962, Vol.2, 914].

[8] ChUp 8.13.b: candra iva rāhor mukhāt pramucya ...

[9] MaitUp 7.6a: śanirāhuketūragarakṣoyakṣanaravihagaśarabhebhā dayo 'dhastād udyanti.

Machiavelli, is one of the oldest texts which explicitly mentions Jupiter and Venus by the names Bṛhaspati and Śukra, respectively:

> Its ascertainment (is made) from the position, motion and impregnation of Jupiter, from the rising, setting and movements of Venus and from the modification in the nature of the sun. From the Sun (is known) the successful sprouting of seeds, from Jupiter the formation of stalks in the crops, from Venus rain.[10]

The date of the text has not been well established. Probably its oldest part was composed about two hundred years before the Christian Era. It is evident that this passage concerns weather prognostics. Pingree [1981,10] regards such prognostics as of Babylonian origin.

1.4 *Nine grahas without fixed order of reference*

The *Gārgyajyotiṣa* (somewhere near the start of the current era)[11] arranges the nine *grahas* in the following strange order: the Moon, Rāhu, Jupiter, Venus, Ketu, Saturn, Mars, Mercury, and the Sun. Here Ketu (almost always in plural form) is not yet a name for the tail of Rāhu but for comets. The great epic *Mahābhārata* abounds in the enumeration of *grahas*,[12] but the order of the enumeration is not fixed, nor is the week-day order attested. Ketu is not always included in the group of *grahas*. Often it is called *dhūmaketu* ('smoke-bannered', see above) and no reference to the single 'Ketu' without similar modification is found in the *Mahābhārata*. These evidences show that there was a period in India when all the nine *grahas* were known but the order was not yet fixed.

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[10] Arthaśāstra 2.24.7–8: tasyopalabdhir bṛhaspateḥ sthānagamanagarbhādānebhyaḥ śukrodayāstamayacārebhyaḥ sūryasya prakṛtivaikṛtāc ca. sūryād bījasiddhiḥ, bṛhaspateḥ sasyānāṃ stambakaritā, śukrād vṛṣṭiḥ. iti.

[11] The *Yugapurāṇa*, a part of the *Gārgya-jyotiṣa*, was edited and translated by John E. Mitchiner, Calcutta 1986. According to Mitchiner the sage Gārgya who narrates the two verses Mbh.13.18.25–6 may be Garga himself, who is the author of the *Gārgya-jyotiṣa* consisting of 64 divisions.

Cf. Pingree [1981, 71]: 'In the early fourth century Garga was named as an authority on omens along with Parāśara by Mīnarāja in his *Vṛddhayavanajātaka* (67,5).'

[12] MBh.2.11.20; 3.3.19; 6.3.11–17; 13.151.12 etc. Thanks are due to my friend Prof. M. Tokunaga, by whom the whole text of the *Mahābhārata* was digitized.

To this period belongs the *Śārdūlakarṇāvadāna*, one of the Buddhist texts contained in the *Divyāvadāna*. The passage which is our present concern runs as follows:

> Now, oh Puṣkarasārin, I will talk about *grahas*. Hear about them. They are Venus, Jupiter, Saturn, Mercury, Mars, Sun, and the Lord of stars (Moon).[13]

1.5 *The week-day order of grahas*

It was only after the transmission of Hellenistic astrology that the order of the seven planets in India was fixed in that of the week-days. This order is the outcome of a combination of the Greek cosmological idea of concentric spheres and the Egyptian belief in the deities presiding over each of the 24 hours (ὥρα in Greek, which was transliterated as *horā* in Sanskrit). In order to get the present order of the week-days, the geocentric spheres must be arranged in the (descending) order of Saturn, Jupiter, Mars, the Sun, Venus, Mercury, and the Moon. Then when the "lord" of the first hour of the first day is taken to be Saturn, the lord of the second is Jupiter, that of the third Mars, of the fourth the Sun, of the fifth Venus, etc., until the lord of the twenty-fifth hour—i.e., the first hour of the second day—is found to be the Sun. In the same way, the lord of the forty-ninth hour is the Moon, and so on. This order was known sometime in the second century B.C., but the evidence of the earliest use of the week-day belongs to a considerably later period. Ptolemy (fl. A.D. 150) used the order of the concentric spheres, but nowhere in his works does he mention the week-days or the week-day order. The first usage is found in Dio Cassius (born A.D. 155), and it was at the end of the second century that the week-day became a civil custom.[14] According to Pingree,[15] Vettius Valens (in A.D. 120) refers to the days of the week beginning with Sunday.

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[13] *Śārdūlakarṇāvadāna*, p. 53: atha khalu bhoḥ puṣkarasārin grahān pravakṣyāmi. tac chrūyatām. tadyathā śukro bṛhaspatiḥ śanaiścaro budho 'ṅgārakaḥ sūryas tārādhipatiś ceti.

It is interesting to note that the week-day order of the seven planets is found in a passage in one of the Chinese translations of this text. See my forthcoming article 'Scientific Relation between China and India' in the *Enciclopedia Italiana Storia della Scienza.*

[14] Jacobi [1876, 306].

[15] Pingree, *Yavanajātaka*, II, p. 405.

The first Sanskrit text which describes Greek astrology in a systematic way is the *Yavanajātaka* or "Greek Genethlialogy." The original text (now lost in Greek) was first translated into Sanskrit prose in A.D. 149/150 and it was versified in A.D. 269/270 by one Sphujidhvaja.[16] The prose version is no longer extant. Sphujidhvaja enumerates seven planets on many occasions, but it is only toward the end of the work (chapter 77) that the week-day order is attested. This order does not seem to have been widespread in that period in India. Neither Rāhu nor Ketu appears in the *Yavanajātaka.* About a half century later, however, Mīnarāja in his *Vṛddhayavanajātaka* (or "Greater Greek Genethlialogy," about A.D. 300–325) describes the planets in the order of the week-days, together with Rāhu, without mentioning Ketu.[17] Varāhamihira (mid-sixth century) does not identify Ketu as the tail of Rāhu but as comets. According to him some people counts 101 Ketus while others say it is 1000.

The oldest Indian inscription which gives a date with the week-day is that referring to the month of Āṣāḍha, the 12th day of the bright half-month (śukla-pakṣa), the day of Suraguru, which corresponds to Thursday, 21 July, A.D. 484.[18] The first astronomical text which defines the week-day is the *Āryabhaṭīya* of Āryabhaṭa (born A.D. 476). His definition is:

> These seven Lords of the *horā* beginning with Saturn are (more and more) speedy in this (descending) order (of the concentric spheres). Every fourth one in order of swiftness is the Lord of the (succeeding) day (which begins) with the sunrise.[19]

What we can safely say is, therefore, that the week-day and the order of the days of the week gradually became known to Indian people at the end of the third century, and it was about a century later that it became widespread.[20]

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[16] This text was first edited by D. Pingree.

[17] *Vṛddhayavanajātaka*, ed. by D. Pingree, Gaekwad Oriental Series, Nos. 162 and 163, 1976.

[18] Cf. J.F. Fleet; *Corpus Inscriptionum Indicarum*, Vol. 3, pp 80–4.

[19] sapta ete horeśāḥ śanaiścarādyā yathākramaṃ śīghrāḥ/
śīghrakramāc caturthā bhavanti sūryodayād dinapāḥ// (ABh 3.16)

[20] After finishing this paper I was informed by Prof. Toshifumi Goto of Stephen Markel's paper: 'The Genesis of the Indian Planetary Deities', *EAST and WEST*, ISMEO, Vol. 41, Nos. 1–4, December 1991, pp.173–88, which touches upon some of the problems which I have discussed.

Thus all the passages in Sanskrit texts which describe planets in the week-day order should be dated later than the end of the third century; several examples of such passages will be given below.[21] Old elements are of course preserved and repeatedly appear in the later texts, so we cannot use such elements as the means of fixing the lower limit of the date of a document, while new elements can surely serve as the criterion to judge the upper limit of the text, or at least the part of the text, which contains them.

2 *Texts on Planetary Worship*

The Hellenistic concept of planetary deities exerting influence on human beings and their deeds must have been very appealing to the Indian people. It did not take long before the planets assumed the name *graha*, because they 'possess' man and have a benefic and malefic influence. Now Indian ritualists had to devise some ritual rules to prevent the evil influence of *graha*s. This was how the rite of appeasing *graha*s (*grahaśānti* or *grahayajña*) suddenly emerged. The rules, which were codified in Sanskrit, found a due place in a group of the supplementary texts (*pariśiṣṭa* or *śeṣasūtra*) of the *gṛhyasūtra*s (rules for domestic rites).[22] The following *gṛhya* texts describe in some way or other the ceremony of planetary worship:[23]

Jaiminigṛhyasūtra 2.9,
Bodhāyanagṛhyaśeṣasūtra 1.17,
Āgniveśyagṛhyasūtra 2.5,
Vaikhānasagṛhyasūtra 4.13–14,
Āśvalāyanagṛhyapariśiṣṭa Chapter 2,
Āśvalāyanagṛhyapariśiṣṭa pp.313–324,
Hiraṇiyakeśigṛhyaśeṣasūtra 1.3.10,
Kāthakasaṃkaraṇa 4, 5, 6 (Grahabrāhmaṇa).

The same rite is found in the *Śāntikalpa* belonging to the *Atharvaveda* and in the *Atharvavedapariśiṣṭa*. Later it became one of

[21] Other examples are: *Rāmāyana* 1.17 (Rāma's horoscope), *Mudrārākṣasa* 4.19 (horoscopic prediction), and a part of the *Atharvaveda-pariśiṣṭa*.
[22] For this group of texts, see Einoo [1993], p. 61.
[23] I owe much to Shingo Einoo and Yasuke Ikari for bibliographical information on the *gṛhya* texts.

the topics of the manuals of professional astrologers, for example, the *Bṛhadyātrā* of Varāhamihira, Chapter 18. Some Purāṇa texts also contain chapters describing similar rites: *Matsyapurāṇa*, Chapter 93 (Grahaśāntivarṇanam), and Chapter 239 (Grahayajñādīnāṃ vidhānavarṇanam), *Viṣṇudharmottarapurāṇa* 2.105, *Agnipurāṇa* 266, *Garuḍapurāṇa* 100, *Bhaviṣyapurāṇa-Uttara* 141, *Padmapurāṇa* 5.82.

In the Purāṇic texts the planets are sometimes arranged in a different geocentric order from the lowest to the highest: to wit, the Sun, the Moon, Mercury, Venus, Jupiter, Mars, Saturn. This strange order is based on the Purāṇic cosmology. In some chapters including those on the *grahayajña*, however, the week-day order is found.

2.1 *The Grahayajña section of the Gṛhyasūtras*

Many varieties of the order of the nine *graha*s are found in the section of the rite of worshipping *graha*s (*grahayajña* or *grahapūjā*) in the *gṛhyasūtra*s.[24] Thus the time in which this rite originated belongs to the fourth stage mentioned above. Even in the same text we can find different arrangements in different contexts. Table 1 shows this variety in the three texts *Jaiminigṛhyasūtra* (JGS), *Vaikhānasagṛhyasūtra* (VGS), and *Āgniveśyagṛhyasūtra* (AGS).

Let us look at JGS 2.9. In five cases (i.e., contexts 3, 4, 8, 10, and 11) the seven planets are arranged in the order of the Sun, Mars, the Moon, Mercury, Jupiter, Venus, and Saturn. This seems to have been the original order. When the colours attributed to *graha*s are mentioned, they are classified as red (the Sun, Mars), white (Venus, the Moon), yellow (Mercury, Jupiter), and black (Saturn, Rāhu, Ketu). This order was followed by the *Baudhāyanagṛhyaśeṣasūtra* in all contexts and by AGS in this particular context. But the same text of JGS shows the week-day order in three contexts (i.e., 5, 6, and 7). Such parts definitely belong to the later period, as was shown in the previous section. Interpolations of this kind are frequently found in other texts, for instance, the *Pariśiṣṭa* and the *Śāntikalpa* of the *Atharvaveda*.

[24] Among the *gṛhya* texts, the week-day order is attested in: Baudhāyanadharmasūtra, JGS, VGS, AGS, ĀśvGP, and KāṭhS.

context	JGS 2.9	VGS 4.13.14	AGS 2.5
1. naming	—	*SuMoMaMeJuVeSa*RK	—
2. colour	SuMaVeMoMeJuSaRK	[*SuMoMaMeJuVeSa*RK]	SuMaVeMoMeJuSaRK
3. place	SuMaMoMeJuVeSaRK	*sve sve sthāne*	—
4. banner	SuMaMoMeJuVeSaRK	*krameṇa*	*SuMoMaMeJuVeSa*RK
5. devatā	*SuMoMaMeJuVeSa*RK	—	—
6. adhide-vatā	*SuMoMaMeJuVeSa*RK	[=JGS]	—
7. region	*SuMoMaMeJuVeSa*RK	—	—
8. faggot	SuMaMoMeJuVeSaRK	*krameṇa*	*SuMoMaMeJuVeSa*RK
9. odana	SuMaMoJuVeMeSaRK	*krameṇa*	*SuMoMaMeJuVeSa*RK
10. mantra	SuMaMoMeJuVeSaRK	[=JGS]	*SuMoMaMeJuVeSa*RK
11. gifts	SuMaMoMeJuVeSaRK	*SuMoMaMeJuVeSa*RK	*SuMoMaMeJuVeSa*RK
12. altar	—	MoVeRMaSaKJuMeSu[25]	—

Table 1: Order of Navagrahas in Gṛhya Texts[26]

It is also evident from this table that the *grahayajña* section of VGS and AGS belong to the comparatively later period. Especially to be noted is the use of the word *krameṇa* ('in the order <of the weekday>') in VGS. Such a term was necessary here since the author explicitly stated that he would follow the order of *grahas* at the beginning of the paragraph. It was because the order was thus fixed that VGS could even omit the word *krameṇa* when enumerating the *graha*s concerning the color, *adhidevatā*, and *mantra.* The expression '*sve sve sthāne* (in their own place)' in the third context also presupposes the established order.

The seventh context (region) gives us interesting information about the ancient Indian concept of the geographical division of their land. The first Sanskrit text which mentions the classification of the regions in India according to the ruling planets is the *Vṛddhayavanajātaka* of Mīnarāja [27] belonging to the mid-fourth century.

3 *The Grahaśānti section of the Yājñavalkyasmṛti*

3.1 *The rites for Vināyaka and planets*

The *Grahaśānti* section of the *Yājñavalkyasmṛti* (YS 1.295–309) is immediately preceded by the section called *Vināyakakalpa* (YS

[25] Grouped by 5 altars. cf. ŚāntiK 11.1f.

[26] Abbreviations: Su=Sun, Mo=Moon, Ma=Mars, Me=Mercury, Ju=Jupiter, Ve=Venus, Sa=Saturn, R=Rāhu, K=Ketu. The week-day order is italicized.

[27] VYJ 2.9–10. Pingree ed. I, page 11.

1.271–294) which prescribes a rite to be offered to Vināyaka. The two rites are closely related to each other. We observe three stages of development of the concept of Vināyaka(s).[28]

1. Four Vināyakas in the *Mānavagṛhyasūtra* who are all evil and inflict harm upon men.
2. Those with double functions, causing obstacles and removing them.
3. A single Vināyaka, son of Ambikā, who is identified with Gaṇeśa, the remover of obstacles. This is the very Gaṇeśa who is usually invoked at the outset of Sanskrit texts.

That YS belongs to the second stage is evident from YS 1.271:

> Vināyaka was committed to the rulership over the *gaṇas* by Rudra and Brahmā <respectively> for the purpose of *destruction and success* of enterprises.[29]

In the third stage, when Vināyaka was identified with Gaṇeśa, the rite of *vināyakaśānti* faded away and it was absorbed in the new rite of *grahaśānti*.[30] That the two functions of Vināyaka were taken over by *grahas* and Gaṇeśa can be conjectured from the following statement in YS 1.293–294:

> Thus worshipping Vināyaka and *grahas* according to rules, one gets the <good> results of *karma* and gets the highest bliss. One who <pays> worship offering <golden> *tilaka*[31] to the Sun as well as to the great Lord Gaṇapati (Gaṇeśa) gets success.[32]

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[28] Kane (HDhS II-1, pp. 213–216) admits the first two stages.

[29] vināyakaḥ karmavighnasiddhyarthaṃ viniyojitaḥ/
gaṇānām ādhipatye ca rudreṇa brahmaṇā tathā//

[30] One of the later texts which describe Vināyaka rite is the *Nīlamatapurāṇa*, a Kashmirian purāṇa of probably the ninth or the tenth century. Cf. NM 604, 698–700, 842, 847, and 990. It is interesting that 'gaṇeśa', occurring only in NM 994 and 1033, plays a small role in NM. For a detailed study of Gaṇeśa, see Robert L. Brown, ed. *Ganesh*, State University of New York Press, 1991. See also Yano [1994].

[31] The meaning of *tilaka* is not clear to me. I follow the *Mitākṣara* commentary.

[32] evaṃ vināyakaṃ pūjya grahāṃś caiva vidhānataḥ/
karmaṇāṃ phalam āpnoti śriyaṃ cāpnoty anuttamām//
ādityasya sadā pūjāṃ tilakaṃ svāminas(*) tathā/
mahāgaṇapateś caiva kurvan siddhim avāpnuyāt//
(*) In the *Bālakrīḍā* commentary Viśvarūpa reads *tilakasvāminas*.

The close relationship of the two rites is also attested in the *Mānavagṛhyasūtra* (2.14). The *Śāntikalpa*, just like YS, refers to *graha* at the end of *Vināyakakalpa* (SK 10.7). It is to be noted that both the *Mānavagṛhyasūtra* and the *Śāntikalpa* enumerate *graha*s in the weekday order.

3.2 *The date of the Yājñavalkyasmṛti*

At the time when the *Yājñavalkyasmṛti* was composed the weekday order of the planets was well established. Thus it is for brevity's sake that YS refers to the order of *graha*s by expressions such as 'in order' (*kramāt*: YS 1.297d, 302d, 306b), 'in the order of *grahas*' (*grahakramāt*: YS 1.305a), and 'according to the number' (*yathāsaṃkhyam*: YS 1.300d). Thus we can say that the upper limit of this section of the *Yājñavalkyasmṛti* is the beginning of the fourth century. This fact was first noticed by Jacobi [1876].

3.3 *Contents of YS 1.295–308*

In what follows a summary of the *grahaśānti* section of the *Yājñavalkyasmṛti* is given. Almost all the verses in this section are quoted by the *Agnipurāṇa* (AP) and the *Garuḍapurāṇa* (GP) and some verses by the *Matsyapurāṇa* (MP), the *Bhaviṣyapurāṇa* (BPU), and the *Viṣṇudharmottarapurāṇa* (VD). Table 2 at the end of this paper shows the collation.

295 Purpose of the rite:[33]
The object of *grahayajña* is not only to get wealth and happiness but also for the purpose of 'cursing' (*abhicāra*) enemies.

296 Names of *graha*s:
The nine *graha*s are enumerated in the week-day order plus Rāhu and Ketu. This verse is indispensable since in the rest of this section this order is presupposed and the nine *graha*s are referred to only by this order instead of by their names. The names are standard ones: *sūrya* (Sun), *soma* (Moon), *mahīputra* (the son of the earth, i.e., Mars), *somaputra* (the son of the Moon, i.e., Mercury), *bṛhaspati* (Jupiter), *śukra* (Venus), *śanaiścara* (Saturn),

[33] This verse is quoted by the *Bhaviṣyapurāṇa* (141.2), and the *Matsyapurāṇa* (93.1).

Rāhu, and Ketu.

	East			
	Mercury arrow	**Venus** pentagon	**Moon** square	
North	**Jupiter** oblong	**Sun** circle	**Mars** triangle	South
	Ketu triangular flag	**Saturn** bow	**Rāhu** winnowing basket	
		West		

Figure 1: *Navagrahamaṇḍala*

297–299 Images of and offerings to *graha*s:
The materials which are used to compose the images (*pratimā*) of the *graha*s are prescribed: red copper (Sun), crystal (Moon), red sandalwood (Mars), gold (Mercury and Jupiter), silver (Venus), iron (Saturn), lead (Rāhu), and white copper (Ketu). Such prescriptions for the planetary images are not found in *gṛhya* texts except in the *Āśvalāyanagṛhyapariśiṣṭa* (2.3)[34] where the materials are almost the same as those in YS, the only difference being the use of saffron for Mercury instead of gold. According to the *Śāntikalpa* (13.3), red copper (Sun and Mars), gold (Mercury and Jupiter), silver (Moon and Venus), and black iron (Saturn, Rāhu, and Ketu) are used.

Instead of making the statues one can draw the figures representing *graha*s in a *maṇḍala* (YS 1.298). The figures to be drawn are not prescribed in YS, but they seem to have been a common knowledge since almost all the *gṛhya* texts agree on the following figures: circle (Sun), square (Moon), triangle (Mars), arrow (Mercury), oblong (Jupiter), pentagon (Venus), bow (Saturn),

[34] athārcanāṅgāni tāmraṃ sphaṭikaṃ raktacandanaṃ kuṅkumaṃ suvarṇaṃ tad eva rajataṃ lohaṃ siīsakaṃ kāṃsyam iti navapratimādravyāṇi ...

winnowing basket (Rāhu), and triangular flag (Ketu).[35] These drawings are to be located as shown in Figure 1.

300–301 Prayers (*mantra*) to *grahas*:
While the offerings prescribed in the preceding verse are presented, one should recite the *mantras* which are prescribed in these two verses. When we compare the *mantras* used in different texts, we get the impression that the *mantras* utilized in *purāṇa* texts are mostly common with those in the *Yājñavalkyasmṛti*, while they are different from those used in *gṛhya* texts.

302–303 Faggots to be burned:
These two verses prescribe different faggots to be burned for *grahas* with offerings of honey, ghee, *dadhi*, and milk. It is interesting to note that some of the faggots (i.e. *parāśa, khadira, pippala*, and *śamī*) mentioned here are also used in the *Suśrutasaṃhitā* in the context (Uttaratantra chapters 27–37) of curing the diseases caused by *grahas*, which, in this case, are not planetary.

304–305 Cooked rice (*odana*) to be offered to *grahas*.

306 Gifts to priests:
It seems that when a specific *graha* was chosen as the object of appeasement, the gift (*dakṣiṇā*) prescribed for each *graha* was to be given to the priest(s) who performed the *śānti* ritual.[36]

307 Specific *graha* to be appeased:
It is not necessary to appease all the *grahas* at one time. Instead, one should perform this rite intending it for the specific *graha* who is regarded as exerting malefic influence on him. This is exactly the idea of astrological propitiation which is still alive in modern India. A *graha* should be worshipped when it is 'in a bad state' (*duḥstha*). This word seems to presuppose knowledge of horoscopic astrology. According to the *Mitākṣara* commentary, a planet has bad influence when it is in 'the eighth house', i.e. the 'house of death', as is taught in the standard manuals of astrology.

308 Closing verse:
That this rite was especially intended for kings is evident from this concluding verse. The same can be said about *Vināyakakalpa*.

[35] These figures are seen in the modern booklets of planet worship (*grahapūjā*) which one can purchase at Hindu temples.

[36] *Dakṣiṇā* related to Rāhu and Ketu show some variation.

Cf. YS 1.275ab: 'The prince who was possessed by him (i.e. Vināyaka) does not attain the kingdom'. It is also to be noted that what is taught in the following part of YS concerns king's government (*rājadharma*).[37]

That the *Yājñavalkyasmṛti* set a model of planet worship for some later texts is clear from Table 2. The collation of parallel passages was made pāda (quarter-verse) by pāda; those passages marked with an asterisks are slightly different from YS.

[37] The *Mitākṣara* text adds one verse (which we number as 308′) after 308. The *Bālakrīḍā* and Aparārka do not contain this verse. ŚāntiK 18.5 concludes with a verse which is almost identical to 308′.

topic	YS 1	AP 164	GP 1.101	MP 93, VD 1.101, BPU 141
purpose	295ab	1ab*	1ab*	YS=MP 2ab≈BPU 2ab
of rite	295cd	1cd*	[1cd]	YS=MP 2c*d*≈BPU 2cd
names of	296ab	2a*b*	2a*b[=AP]	
grahas	296cd	2cd	2cd	
images	297ab	3ab	3ab	
	297cd	3c*d*	3cd*	
navagraha-	298ab	4a*b*	[4]	
maṇḍala	298cd	4cd	[5] 5ef	
offerings	299ab	5ab*	6ab	
	299cd	5cd	6c*d	
mantra	300ab	6ab	7ab	
	300cd	6cd	7c*d*	
	301ab	7ab*	8a*b*	
	301cd	7cd	8c*d*	
faggots	302ab	8ab*	9ab*	YS=MP 27ab=VD 2ab=BPU 30cd
	302cd	8cd	9c*d	YS=MP 28cd=VD 2cd=BPU 31ab
	303ab	9a*b	—	YS≈MP 28ab
	303cd	9cd*	10ab*	AP = MP 28cd
cooked	304ab	10ab*	10cd	
rice	304cd	10c*d	11a*b	
	305ab	11a*b	11c*d	
	305cd	11cd	—	
gifts to	306ab	12ab*	12ab*	
priests	306cd	12c*d*	12cd*	
results	307ab	13a*b	[12ef concl.]	YS=VD 10ab
	307cd	13c*d*		YS≈VD 10cd; YS 307d≈MP 80c
	308ab	14ab		YS=VD 9ab
	308cd	14cd		YS≈VD 9cd
	308′ab			YS 308′=ŚāntiK 18.5
	308′cd			

Table 2: Purāṇic Parallels of YS 1.295–308 (*grahaśānti*)

Sanskrit Texts Used

Āgniveśyagṛhyasūtra, ed. by Ravi Varma, Trivandrum, 1940.

Agnipurāṇa, Kashi Sanskrit Series 174, 1966.

Atharvaveda, Śaunaka recension, ed. Vishva Bandhu, 5 parts, Hoshiarpur, 1960–64.

Atharvaveda-pariśiṣṭta, eds. G. M. Bolling and J. von Negelein, Leipzig, 1909.

Āśvalāyanagṛhyapariśiṣṭa, as an appendix to *The Grihyasutra of Aswalayana* ed. R. Vidyaratna and A. Vedantavagisa, The Asiatic Society, Culcutta, 1986.

Āśvalāyanagṛhyasūtrabhāṣya, (as an Appendix to the *Āávalāyanagṛhyasūtrabhāṣyam*, ed. K. P. Aithal, Adyar Library and Research Centre, 1980), pp.313–24.

Bṛhadyātrā, ed. D. Pingree, Government of Tamil Nadu, 1972.

Baudhāyanagṛhyaśeṣasūtra, ed. R. S. Shastri, reprinted New Delhi, 1982.

Bhaviṣyapurāṇa-Uttara, ed. Venkatesvara, reprinted Nag Publishers, Delhi, 1984(?).

Garuḍapurāṇa, ed. Venkatesvara, reprinted Nag Publishers, Delhi, 1984.

Hiraṇiyakeśigṛhyaśeṣasūtra, ed. Śaṃkaraśāstrī, Mārūlakara and Vināyaka Gaṇeśa Āpaṭe, (= Satyāṣāḍhaviracitaṃ Śrautasūtram, Ānandāśramasasaṃskṛtagranthāvaliḥ 53, Pt.8, 1929)

Jaiminigṛhyasūtra, ed. W. Caland, Lahore, 1922, reprinted Delhi, 1984.

Kāthakasaṃkaraṇa, ed. Sūryakānta, reprinted New Delhi, 1981.

Matsyapurāṇa, Text in devanagari, translation and notes in English, forwarded by H.H. Wilson, arranged by Nag Sharan Singh, Nag Publishers, 1983.

Nīlamata or Teachings of Nīla, Sanskrit text with critical notes, ed. K. De Vreese, Leiden, 1931.

Padmapurāṇa, ed. Viśvanātha Nārayaṇa Maṇḍalīka, Ānandāśrama Sanskrit Series Extra No. 1, 3 parts, Pune, 1893-94.

Śāntikalpa of the Atharvaveda, by G. M. Bolling *et. al.*, JAOS 33 (1913), pp.265-278.

Viṣṇudharmottarapurāṇa, ed. Venkatesvara, reprinted Nag Publishers, Delhi, 1985.

Vaikhānasagṛhyasūtra, Vaikhānasasmārtasūtram, The domestic rules of the Vaikhaanasa school belonging to the Black Yajurveda, ed. W. Caland, Bibliotheca Indica, Calcutta, 1927.

Vṛddhayavanajātaka, ed. D. Pingree. Gaekward's Oriental Series, Nos. 162 and 163, Baroda, 1976.

Yavanajātaka of Sphujidhvaja, ed. D. Pingree, 2 vols., Harvard Oriental Series, Vol. 48, Cambridge Ma., 1978.

Yājñavalkyasmṛti with the *Mitākṣara* commentary, ed. N. R. Acharya, Bombay, 1949.

Yogayātrā, ed. Ramacandra Jha, Darbanga, 1986.

References

S. Einoo, 'Who is the performer of the *Saṃdhyopāsana?*', *Wiener Zeitschrift für Kunde Süd-Asiens — Supplementband* (1993), pp. 59–67.

K. F. Geldner, *Der Rig-Veda*, 4 Vols., Cambrdige, Ma., 1951–5.

H. Jacobi, 'Beiträge zur indischen Chronologie', *Zeitschritf der Deutschen Morgenländischen Gesellschaft* 30 (1876), p. 302–307. Reprinted in his *Kleine Schriften*, Wiesbaden, 1970.

P. V. Kane (HDhS), *History of Dharmaśāstra*, II-1, Poona, 1974.

D. Pingree, *Jyotiḥśāstra*, Wiesbaden, 1981.

W. D. Whitney, *Atharvaveda-Saṃhitā*, 2 vols. Reprinted by Motilal Banarsidass, Delhi, 1962.

M. Yano, 'Calendar and Related Subjects in the Nīlamatapurāṇa', *A Study of the Nīlamata — Aspects of Hinduism in Ancient Kashmir —* , edited by Yasuke Ikari, Institute for Reseach in Humanities, Kyoto University, Kyoto, 1994, pp. 223–36.

——. 'Calendar, Astrology, and Astronomy', in *Blackwell Companion to Hinduism* ed. F. Gavin, London, 2003, pp. 276–92.

Competing Cosmologies in Early Modern Indian Astronomy

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1 *Introduction*

In an article on 'The Purāṇas and Jyotiḥśāstra' [Pingree 'Purāṇas'], David Pingree[1] sketched the history of the ways in which the cosmological account found in the Sanskrit astronomical Siddhāntas took shape in relation to the standard cosmology of the Sanskrit Purāṇas. While sharing sources of influence, and despite some attempts at accommodation, the astronomers' model was in fundamental respects not reconcilable with the Purāṇic model of the earth or of the heavens. As Pingree showed, the contradictions were already known to the authors of the earliest Siddhāntas, but it was Lallācārya in the early ninth century who formulated the view of the problem that came to be accepted as standard, and that was repeated in the influential Siddhāntas of the twelfth century, the *Siddhāntaśekhara* of Śrīpati and the *Siddhāntaśiromaṇi* of Bhāskara.[2] Lalla's solution maintained all the features of the astronomers' model that were necessary for supporting their calculations, and rejected those parts of the Purāṇic model that contradicted them. At the same time he

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[1] It seems fitting, in a volume dedicated to David Pingree (DEP), to continue something from a point where he left off. Many scholars will have the opportunity to do just that, given the nature of his work, which is in itself comprehensive at the same time that it has opened the way for further research. As will become clear in what follows, this work is greatly indebted to DEP's help on many levels—for the invaluable published work, for giving me access to copies of manuscripts, for patiently explaining and re-explaining many points, and for general encouragement of my studies in this field of research. Over the course of two decades I have learned more from him than from anyone else, and I am honored to be able to count him my teacher and friend.

[2] Pingree mentions the *Siddhāntasundara* of Jñānarāja as following this standard model, but it does not do so entirely, as the following will show.

sought to accommodate his cosmology to Purāṇic doctrines where these did not interfere with the astronomy.

At the end of his article Pingree referred to two Sanskrit works of the early modern period that awaited further study, both of which sought to reconcile the Siddhāntic and Purāṇic model in a way different from the standard accommodation of Lalla/Śrīpati/Bhāskara.

He closed by posing a question about the larger intellectual context in which those attempts at creating consistency were written. This article is therefore devoted to the works of several Sanskrit authors in the early modern period on the problem of cosmological reconciliation, or what I will call here the 'virodha problem'. It is now possible to fill in more of the picture of what form the discussions took in the early modern period, and what was at stake in them.[3] One finds discussion on this subject by astronomers in the Siddhāntic traditions, and also by adherents of the Purāṇic doctrines. The most interesting texts were written by authors who wished to consider themselves both.

Here I will primarily be interested in the views on the virodha problem advanced by three astronomers of Pārthapura, and in the reception of their views by two astronomers in Banaras. The two titles that Pingree mentioned in his work were the *Saurapaurāṇikamatasamarthana* of Nīlakaṇṭha Caturdhara, and the *Bhāgavatajyotiṣayoḥ Bhūgolakhagolavirodhaparihāra* of Kevalarāma Jyotiṣarāya. An edition and study of each of these texts is now in preparation, and I will refer to their contents only briefly here. The present essay provides a context in which to understand those works, and a number of other similar independent works on the virodha problem, which I shall mention at the end.

In the course of examining the arguments of these astronomers, the following survey will, I hope, contribute to the analysis of two larger historical problems—how to place the Siddhāntic astronomers in the wider intellectual history of Sanskrit literature, and how to place their discussion of cosmology in the larger history of early modernity in India.[4]

The astronomers present special problems for the history of

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[3] A richer body of manuscript material has become available, much of it accumulated by DEP himself, in this genre of reconciliation literature.

[4] The latter problem I hope to discuss elsewhere, and here will limit myself to making note of some pertinent facts in preparation.

Sanskrit literature in early modernity, as their intellectual currents seem (at least at first glance) to have run in isolation from the other śāstras. As a result the astronomers are often studied as independent of the Brahminical intellectual traditions in which they operated. And yet the Siddhāntic astronomers are of special interest because of the 'practical' nature of the knowledge they generate, and the potential points of contact they therefore present with non-Sanskritic knowledge traditions.

2 *Virodhaparihāra*

A way into the problem lies in asking a question: in what would a satisfying 'reconciliation' of Purāṇas and Jyotiṣ consist? The Sanskrit term here is virodha-parihāra, the 'removal of contradiction'. Yet removal of contradiction can conceivably be done in a variety of ways. Some of our authors simply refute one or the other of the contradictory views as untruth or illogic, hence leaving the field to the other view and in that way removing any contradiction. Others accord different levels of truth to the two positions, saying that one describes practical, the other, ultimate reality. A related removal of inconsistency is done by putting the two positions on different planes of reality by appeal to their purpose, one being for timings of mundane affairs, the other for soteriological aims. Some adopt an exegetical strategy, according one of the two views a higher truth, but reading its texts so that they only confirm what the other view already maintains. Others assume that both views have the same subject and purpose on the same plane, and actually adjust and revise doctrines so that they conform to each other. Some of our authors resort to combining several of these theoretical approaches.

A point of contact with the larger Sanskrit intellectual tradition is found in the epistemological theory that the astronomers rely on to achieve their 'removals'. One finds in the astronomers of this period evidence of a growing interest in integrating their scientific method with the fully elaborated pramāṇa system of proof developed in the principal śāstras, especially in logic. Here it is the status of textual authority (śabdapramāṇa) in particular that is a matter of urgency—the canonical status of the Purāṇas and Siddhāntas, their hierarchical relation, and the degree of autonomy accorded to the other pramāṇas, that is, logic (anumāna)

and experience (pratyakṣa).

Fixing the Limits. As for the limits of the period, it is easy to assign the beginning to the appearance of the *Siddhāntasundara* (SSJ) in 1503.[5] This work by Jñānarāja was the first astronomical Siddhānta to appear in approximately three and a half centuries, following the impact of the hugely influential *Siddhāntaśiromaṇi* (ŚSB). The return to writing a Siddhānta after such a long time marks a renewal of the past, but also a break with it, and Jñānarāja challenged the ŚSB on some cosmological points, as we shall see. The close of the period does not present itself so neatly. The last of the dated works I will consider here appeared in 1763, and this seems an appropriate terminus, as a lull ensued before the Siddhāntins confronted the challenge of the new Copernican model introduced by the Europeans.

The 'Standard' Accommodation. Several studies of the cosmological debates in the nineteenth century have recently appeared, and a new essay by Plofker gives an elegant historical survey of the whole trajectory of the problem.[6] Nevertheless, before beginning with Jñānarāja it will be useful to give a brief summary of the state of play when Jñānarāja began to write; that is, a summary of Lalla's/Bhāskara's 'standard' solution, both its accommodations and its criticisms, and the arguments for them. In some texts of the early modern period, the conclusions stay the same but are supported by different reasons.

In the 'standard' astronomical accommodation, Mt. Meru, the central axial mountain of the Purāṇas, becomes the polar axis inside the earth. The gods, who live atop Mt. Meru, are placed at the North Pole, while the Asuras, their demon counterparts, are placed at the South Pole. The seven underworlds (pātālas) are placed inside the earth. The (flat) Jambūdvīpa and its surrounding salt ocean become the northern hemisphere and the sea surrounding it, while the six concentric continents and oceans of the Purāṇas (the 'seas of treacle and seas of butter' of Macaulay's notorious 'Minute on Education'), are made into annular continents and oceans in the southern hemisphere, where they lie beyond our local experience. The motions of the stars and

[5] All dates in the following are given in their A.D. or Common Era form.

[6] [Young 'Receding from Antiquity']; [Minkowski 'Nīlakaṇṭha's Comments']; [Minkowski 'The Paṇḍit']; [Plofker 'Derivation and revelation'].

planets, though predicted according to a geocentric and epicyclic model, are still caused by the Pravaha and other winds named in the Purāṇas.

On the other hand, criticisms of the Purāṇic model are unavoidable. If the earth is of the huge Puraṇic size (50 koṭi yojanas, or 100,000 times larger than the size given in the Siddhāntas of ca. 5000 yojanas circumference), then the calculation of the rising and setting of planets, of the conjunctions of planets, of eclipses, and of other celestial phenomena, which are all geared, through the size of the orbits of planets and stars, to the Siddhāntic size of the earth, and which are seen to compute well in that system, will no longer produce useful results.

If the earth is flat, then why is a tall tree at a distance not visible to an observer? (Bhāskara provided another argument which became more widely used: why does the North pole star rise in the heavens as one travels northward, while other stars at midheaven correspondingly decline, if the earth is flat?) The earth is only assumed by the unthinking to be flat, because a hundredth part of its surface appears flat to us. And (perhaps not a very good argument) why does water flow downhill if the earth is flat?

If the earth is flat, moreover, how can night be explained, for then the sun would always be visible, circling overhead? Nor can it be said that the shadow of Meru causes night, because we do not see Meru in the heavens, and the gods, atop Meru, would never know night. Furthermore, if the earth is flat, and the sun is always overhead, why would days and nights get longer and shorter in the course of a year?

The earth has no support—not on water, and not on an embodied deity such as Śeṣa or Kūrma. For if it did have a support, as the Purāṇas say, then one would have to assume a support for the supporter. In that way one would either come to the problem of an infinite regress, or one would have to assume a supporter that innately had the power to support itself. Why not let the earth be what has this power?

The moon cannot be higher than the sun, despite what the Purāṇas say, for then it would be always illuminated, and neither eclipses of the sun nor of the moon could occur. Nor can Rāhu be the cause of eclipses.

This, then, was the position of astronomers about the Purāṇas and their possible consistency with astronomical treatises. Some of these arguments were already made by earlier astronomers, and Bhāskara differs from Lalla in places, but those differences will not concern us here. It is enough to say that the received viewpoint is a substantially standardized, if not quite a stable and uniform one.[7] For some time after Bhāskara, this was a topic that did not elicit new thinking. The stability of the ideas is made clear enough in a commentary on the SŚB by Lakṣmīdāsa, the *Gaṇitatattvacintāmaṇi*, written in 1501, two years before Jñānarāja's work.[8] Lakṣmīdāsa explicates verse by verse, but makes no attempt to expand Bhāskara's viewpoints or depart from them when it comes to the virodha problem. As we shall see, the situation had changed for the two great commentators on the SŚB living a century later.

3 *Jñānarāja*

It is against this background that we should read Jñānarāja's chapter on cosmology. Jñānarāja belonged to an eminent family of astronomers and śāstrīs.[9] An ancestor had served at the court of the Devagiri Yādavas. In Jñānarāja's day the family was settled in Pārthapura, along the Godāvarī river in Vidarbha. Jñānarāja's sons Cintāmaṇi and Sūryadāsa both wrote works on astronomy (more on both of them below), as did other members of the family.[10] Pārthapura was something of a center for astronomy in these centuries, Jñānarāja's being only one of the families based there. According to his son Sūryadāsa, Jñānarāja also wrote works on astrology, poetics, and music, but these texts are not extant.[11]

Siddhāntasundara. The text of the *Siddhāntasundara* (or

[7] See [Plofker 'Derivation and revelation'] on early differences on the treatment of eclipses, and also on the extent to which invoking Purāṇic authority might have been secondary to astronomical argument.

[8] [Pingree CESS A5, 523–5]. He is also named Lakṣmīdhṛta. He was the son of the great Vācaspati Miśra. His commentary is to date unpublished. Thanks to DEP I had access to a MS from Bombay—AS Bombay 287.

[9] [Pingree CESS A3, 75–6; A4, 100; A5, 122–3.]

[10] For a family tree see [Pingree *Jyotiḥśāstra*, 124].

[11] [Dikshit *History*, 2, 143], citing Sūryadāsa's commentary on the Bījagaṇita.

'Treatise Beautiful') was widely read over the north of the subcontinent, as can be seen from the number of manuscripts extant.[12] One finds in the work a much stronger Purāṇicizing tendency, coupled with a greater concern with artistic expression. Jñānarāja regularly vies with Bhāskara's SŚB.[13] The closing chapter of the *Siddhāntasundara* comprises a ṛtuvarṇana, that is, a poetic evocation of the seasons of the year, after the style of Kālidāsa's *Ṛtusaṃhāra.* It was Bhāskara who was the first to devote a brief section of his Siddhānta to the cycle of seasons.[14] Among later authors of Siddhāntas, it is characteristic that only Jñānarāja produced such a chapter, and that he made his chapter 34 elegant verses long, more than twice the length of Bhāskara's mere 15![15]

In the chapter on cosmology (Bhuvanakośa), Jñānarāja displays a much greater concern with textual authority, of the śruti, and, especially, of the Purāṇas.[16] He breaks with Bhāskara's understanding of several points so that he can make the Purāṇic statements about the earth's support be true. On the other hand, he revises the interpretation of the Purāṇas, taking it out of the hands of the Paurāṇikas, as it were, so that the Purāṇas can be in agreement with the Siddhāntic model. Jñānarāja in attempting his new accommodations limits himself to the size, shape and support of the earth. When it comes to the planets and stars above, he resorts to what might have been an old argument, namely that the Purāṇas and Siddhāntas are both true, but describe the world in different creations (kalpabheda), the Jyotiḥśāstra being pertinent to our experience today.

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[12] [Pingree CESS] lists more than 50 extant MSS. The text is as yet unpublished, but thanks to DEP I have been able to examine 3 MSS—BM 14, 365; RORI (Kotah) 981; and SB 35318, the latter including Cintāmaṇi's commentary.

[13] Indeed, Sudhākara Dvivedī's brief characterization of the SSJ [Dvivedī *Gaṇakataraṅgiṇī*, 57] mentions these three features: atra siddhānte bahutra purāṇamatamaṇḍanam bhāskarācāryamatakhaṇḍanaṃ kṛtam asti — kutra cit kalpanā yuktiyuktā, 'paratra ... kalpanā golayuktibahirbhūtā jalpanāpūrṇā kavitāparā ca.

[14] Chapter 12 of the Golādhyāya.

[15] Sūryadāsa, Jñānarāja's son, has a chapter of his *Siddhāntasaṃhitāsārasamuccaya* called the Ṛtucaryā. This work is not exactly a full Siddhāntic treatise. See below.

[16] The Bhuvanakośa appears as the first chapter in the second half (Golādhyāya) of the SSJ.

Jñānarāja's greater concern with textual authority is made clear in the unusually lengthy account of the origins of the world that begins the chapter. This treatment is also unusual for a Siddhānta in that it blends two accounts of the genesis of the worlds, one from the Purāṇic Sāṃkhya philosophy, the other from the *Śatapatha Brāhmaṇa*.[17]

The chapter concludes with a statement of the purpose of cosmological description, which is to enable appreciation of the creation in all its multiplicity as nothing other than the Supreme Being. The one who knows the world as Vyāsa describes it achieves communion with the Deity. This purpose of cosmology—which is knowledge of the Creator's Creation—effectively eradicates the distinct purpose of Jyotiṣ, since what is salvific is the *Bhāgavata*'s account and not the Siddhānta's.[18] The chapter ends with astronomy no longer in sight, as the creation's purpose is said to be an extensive preparation by the creator Brahmā for staging the worship of Śiva as the Deity in his form as cosmic Śivaliṅga.[19]

Jñānarāja writes in response not just to the astronomers he names, but also to Paurāṇikas who criticize the astronomers' view. No texts are mentioned, and I have not come upon any texts in which such criticisms are made, so these remain for the moment offstage voices. We learn, though, that the proof that the astronomer Pṛthūdaka gave for the sphericity of the earth was not accepted by those with base minds (khalamatayaḥ).[20] Moreover there are those who cite the authority of the Purāṇas to show that the earth is flat. Mere prating, says Jñānarāja.[21]

Support of the Earth. Jñānarāja proposes that the earth is indeed supported by various incarnated divine beings as depicted

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[17] SSJ II.1.9–19. ŚB 6.1.1.1–6.1.2.10 is referred to in SSJ II.1.19.

[18] SSJ II.1.77. Except, says the commentator Cintāmaṇi, insofar as the Siddhāntasundara's explication might more quickly facilitate an ability to understand the *Bhāgavata*'s cosmology.

[19] SSJ II.1.78. The chapter also includes the account of all seven dvīpas and samudras, with geographical and ethnographical comments, which had become *de rigeur* for astronomical Siddhāntas (SSJ II.1.39–60). Jñānarāja makes a point of coordinating the four Purāṇic sub-continents of Jambūdvīpa with the four quadrant cities of the astronomers (SSJ II.1.43–9). See [Pingree 'Purāṇas and Jyotiḥśāstra', 278].

[20] SSJ II.1.23. This means Paurāṇikas, the commentator Cintāmaṇi tells us.

[21] SSJ II.1.27. More on this below.

in the Purāṇas, that is, the great serpent Śeṣa, the boar incarnation Varāha, the tortoise incarnation Kūrma, and so on. This doctrine can be reconciled with a model of a spherical earth if we suppose that the supporters are stationed inside the globe, in the Pātālas, supporting the earth from its interior.[22]

The implication of this proposal entails two further arguments with Bhāskara, however, since Bhāskara has proposed that the earth remains in space because it has an inherent power to hold itself there.[23] Jñānarāja must first of all criticize Bhāskara's theory that the power to remain fixed is a natural property of the earth element, just as fluidity is a natural property of the water element, or heat is a natural property of the fire element. Jñānarāja makes the telling point that if fixity is a natural property of the earth element, it should also be seen in small bits of earth, in the way that a bit of water is fluid, and a bit of fire is hot.[24]

After all, Jñānarāja argues, if attraction power were what drew things to earth, we would see lighter things 'fall' faster to earth, and heavier things slower, since the attraction would work more powerfully on the former; yet our experience is the opposite. Why not rather suppose a supporter?[25] A vulture carrying a huge snake in its beak can remain soaring in air for a long time. And a vulture is a weak creature, by comparison with the high deities, who could easily stay aloft, carrying the earth, for the entire duration of a creation, that is, a kalpa of Brahmā, such is their power.[26]

The second part of the argument has to do with the people and other creatures living on the sides and bottom of the globe. Bhāskara has argued that they are held there by an attraction power inherent in the earth (ākṛṣṭaśakti), that is, an additional power that the earth possesses to draw things to it. Jñānarāja

[22] SSJ II.1.30.

[23] SSJ II.1.29. ŚSB Bhuvanakośa vs. 2. Lalla had said this power belonged to earth but that it was god-given (ŚDVT 20.46). This occasions a discussion of earthquakes in the commentary of Cintāmaṇi, which is continued in Nṛsimha and Munīśvara. For if the earth has an inherent fixity, how can it move as it does in an earthquake?

[24] SSJ II.1.30. Cintāmaṇi, the commentator, points out that our common experience of clods released in mid-air is that they fall down, after all, and do not remain fixed in the air.

[25] SSJ II.1.32.

[26] SSJ II.1.31.

denies that the earth has this attraction power. He proposes instead that there is a property of things on the sides and bottom of the globe to stay on the earth and not fall off into space. Just as we all perceive that certain substances can exhibit unusual properties in special contexts—the sunstone, for example, which becomes fiery in contact with the sun's rays, or the moonstone which turns to liquid when exposed to moonlight—so people and other things on the sides and bottom of the earth, by virtue of being located there, exhibit the property of being able to stay on and not fall down into space.[27]

Jñānarāja presents a second, slightly alternative argument for the same point. Just as people in different regions of our own subcontinent are seen to be different in their speech, appearance, behavior and so on, Jñānarāja reasons, how much more different must people and other things be in the other distant continents? Why not suppose that one different feature they possess is the power to stay on the earth, though they are positioned on its sides and bottom?[28]

There is a more general principle that informs a number of these arguments, having to do with the 'downness of down', that is, with whether directionality is absolute or relational. Most of the authors of the Siddhāntas include in their discussion of the sphericity of the earth a statement of relational directionality, that wherever one is on the globe, 'up' is over one's head, and 'down' is below one's feet. People opposite each other on the globe are mutually 'under' each other. There is no such statement in Jñānarāja, who instead cites the Purāṇas as supporting his Siddhānta's view that the gods' position on the North pole of Meru is above, while the demons' position on the South pole of Meru is below.[29] Cintāmaṇi, the son and commentator, does not fully agree with his father on this point.

Jñānarāja and Purāṇic Revision. Jñānarāja makes efforts to argue against the Paurāṇikas, especially concerning their contention that the earth is flat. Jñānarāja reuses some old arguments against the earth's flatness, and introduces new ones. A traveller going north sees the pole star's elevation in the night

[27] SSJ II.1.35–6.
[28] SSJ II.1.37.
[29] SSJ II.1.51.

sky increase. A traveller east, on the other hand, sees the pole star stay at the same elevation, however far he travels.[30] In fact Jñānarāja introduces some figures here—a traveller will observe one degree of elevation of the pole star for every 14 yojanas that he travels north.[31] Furthermore, a swift-moving astronomer who observes local sunrise, and then, carrying a sand-clock with him, travels east for 10 yojanas, will find that the next day's sunrise occurs there $7\frac{1}{5}$ palas earlier.[32] This is a new argument, and not a very strong one, since it is only true for an observer who happens to be on the equator.

Jñānarāja introduces the old argument that the earth has been mistakenly thought to be flat because a hundredth part of its surface appears to be flat, but he uses this specifically as a strategy of accommodation. That is, Purāṇic statements of flatness must be interpreted to refer only to an apparently flat hundredth part of the world around us.[33] Those who argue that the Purāṇas propound the whole earth as flat, says Jñānarāja, do not know the true meaning of the Purāṇas, which is made clear in passages where the term 'bhūgola', the 'earth's sphere', appears. In other words, the author of the Purāṇas is showing that he knows the earth is spherical by using this term. He also shows it when he asserts that Mt. Meru is to the north of all continents and oceans, which could only be true if the earth were spherical.[34] In these cases Jñānarāja argues against received Purāṇic opinion, in order to adjust Purāṇic meaning to the Siddhāntic view.

Compartmentalization. I mentioned above that when it comes to the phenomena in the heavens Jñānarāja resorts to a different mode of reconciliation, the two models having different truths and purposes. The Purāṇic description confers liberating knowledge of the Deity and His universal emanation, and in this task astronomy is ancillary at best. This is a form of what I would consider an older, less literal-minded approach to reconciliation,

[30] SSJ II.1.24.

[31] SSJ II.1.25. This supposes a circumference of the earth of 5040 yojanas, which is approximately the value accepted by the astronomers of the Saura pakṣa.

[32] SSJ II.1.26.

[33] SSJ II.1.28.

[34] SSJ II.1.27. See below, where Sūryadāsa cites the passage from the *Bhāgavata* and the *Viṣṇu Purāṇas* (note 64).

one that in a variety of ways compartmentalizes truth claims.

Excursus: Vyāsasiddhānta and Untroubled Blending. A curious variant of this compartmentalizing strategy is evident in the *Vyāsasiddhānta*, a cosmological work whose date is unspecified, but which appears to have been written in the period before Jñānarāja.[35] This text juxtaposes the conflicting models in an impossible and yet blithely untroubled, hybridized cosmology.[36] The account of the earth follows the Purāṇic cosmology—huge, flat earth, ring continents and oceans, supporting Śeṣa below. On the other hand, the universe above the earth's surface is described according to the astronomer's cosmology, with the order of planets and distances just as one would find them in the Bhuvana chapter of an astronomical Siddhānta.[37] The orbits and distances imply a size of the earth's radius, but this is passed over in silence. Clearly even though there are mathematical and kinematic problems, they are not felt to be so, or are willed not to be so simply by the juxtaposition of the two.[38]

4 *Cintāmaṇi*

Cintāmaṇi was Jñānarāja's son. He also lived in Pārthapura. He has no known works other than a commentary on the *Siddhāntasundara*, entitled the *Grahagaṇitacintāmaṇi.* This lengthy work was circulated widely in Northern India, though not as widely as the SSJ. It was known to the seventeenth-century astronomers in Benares, as we shall see, and survives in about

[35] [Pingree CESS A5, 754].

[36] This text is as yet unedited, but thanks to DEP I had access to a copy of a MS from Berlin—Berlin 1738 (or. fol. 981a).

[37] The orbit sizes are those of the modern Sūryasiddhānta. In its second half the text goes so far as to offer proofs of the sphericity of the earth that echo the arguments of the SŚB. A person traveling north and south will see the pole star rise and decline in the sky respectively (vss. 213–4). The earth only appears to be flat and circular because humans are small and the earth appears flat locally. (vss. 195–6). (Compare SŚB Bhuvanakośa 13, 48–9.)

[38] Nṛsiṃha in his *Vāsanāvārttika* on the SŚB cites the *Vyāsasiddhānta* as giving the circumference of the earth as 5000 yojanas [Caturvedī *Vāsanāvārttika*, 358, l. 3]. That figure is not found in the Berlin MS. Note however that the *Vyāsasiddhānta* I examined echoes the SŚB in giving the earth the power to support itself in vs. 8; on the other hand, it has Śeṣa supporting the earth in vss. 171–6.

twenty known manuscripts.[39]

Virodha. Cintāmaṇi, as one would expect, expands and secures the position of Jñānarāja concerning the Purāṇas and Jyotiḥśāstra, with some notable additions and differences. Cintāmaṇi is the earliest author I have found who proposes to use an alternative meaning of the word for crore (koṭi) as part of the virodha discussion, i.e. as meaning not 10 million, but rather 100.[40] Cintāmaṇi appeals to this revised meaning in order to resolve the discrepancy in the earth's size. If the earth's size of 50 koṭis of yojanas in the Purāṇas actually means 50 hundreds, then one has quickly arrived at consistency with the Siddhāntic size of 5000 yojanas.

Cintāmaṇi does not always agree with Jñānarāja. He does not fully support his father's implied directional absolutism. Instead he concedes that, wherever one stands on the globe, 'up' must be over one's head, and 'down' must be beneath one's feet. Otherwise how would rain fall 'down' to earth? How would grain grow properly, and how would fire burn properly, i.e. upward? Without grain or the fire to cook it, how would someone on the side or bottom of the earth be able to eat, and therefore live?[41]

Cintāmaṇi and Pramāṇaśāstra. The most distinctive feature of Cintāmaṇi's work is his preoccupation with integrating the arguments and demonstrations of astronomy with the pramāṇa system of the philosophical śāstras. The concern is especially with Nyāya, but Mīmāṃsā and Vyākaraṇa are also introduced.[42]

Cintāmaṇi categorizes and reformulates Jñānarāja's arguments in the vocabulary and format of the mainstream śāstraic method—are the arguments inferences from positive evidence (anumāna), or presumptive conclusions (arthāpatti), or some other mode of argument? Do the inferences fulfill the criteria of the logical syllogism, having the five elements of proof (pakṣa, sādhya, hetu, sapakṣa and vipakṣa) in their proper re-

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[39] [Pingree CESS A3, 49; A4, 94]. The biographers Dvivedi and Dikshit give very little information about Cintāmaṇi. The *Grahagaṇitacintāmaṇi* (GGC) is unpublished, but I was able to examine a partial copy from Banaras thanks to DEP—SB 35318. All references to the GGC are drawn from this MS.

[40] GGC on SSJ II.1.26, f. 16v. ll. 1–2.

[41] GGC on SSJ II.1.37, ff. 23v, l. 12–24r, l. 3.

[42] Thus the standard śastraic triad (in reverse order) of pada, vākya, and pramāṇa.

lationships to each other? Or are there rather faulty arguments (hetvābhāsa), defective in their concomitance relations by departure (vyabhicāra), or misidentification of cause (upādhi), or aptness of demonstration (sādhāraṇatā)? The vocabulary is not incidental, but is basic to the commentary's agenda.

Even the mathematical results are reformulated as syllogisms: for example, in the case of the swift astronomer who goes east 10 yojanas with a sand-clock, and measures a time difference in local sunrise of $7\frac{1}{5}$ palas. The solution to the 'rule of three' problem that this presents—$7\frac{1}{5}$ is to 3600 as 10 is to what?—becomes a syllogism of this form: Earth is possessed of the property of being 5000 yojanas in overall measure (pratijñā); because there is the property of generating $7\frac{1}{5}$ palas time difference at sunrise by an interval of 10 yojanas (hetu); for wherever there is the generation of $7\frac{1}{5}$ palas time difference in an interval of 10 yojanas, there is for a time difference of 60 ghaṭikās an interval of this size (udāharaṇa).[43]

Cintāmaṇi diplomatically signals when Jñānarāja has not made a good argument by these standards. When arguments are not very complete, or beg the question, or assume what has to be proved, he will buttress the proof by referring to arguments in later verses, or he will simply add a more śāstraic argument. Cintāmaṇi sometimes uses the dialectical style of the more argumentative philosophers.[44] He especially exerts himself to support his father's criticisms of Bhāskara, arguing out the virtues of Jñānarāja's position, and the vices of Bhāskara's, at some considerable length. Such a concern with regularizing the argumentation of the astronomers is not found in earlier astronomical commentaries, and constitutes Cintāmaṇi's main contribution. One finds the repercussions of Cintāmaṇi's innovation in the commentaries of the next century.

Sources. Jñānarāja's sources are amplified and sometimes identified in the *Grahagaṇitacintāmaṇi.* Cintāmaṇi fills in the passages from the SŚB in most cases. He adds support from

[43] $C/10 = 3600/7\frac{1}{5}$. $C = 10 \cdot 3600/\frac{36}{5} = 5000$ yojanas. GGC on SSJ II.1.26, f. 16r ll. 8–10. Cintāmaṇi acknowledges that there is no sapakṣa for this inference.

[44] A style of critique inherited, many polemical generations earlier, from Nāgārjuna. Examples are GGC on SSJ II.1.22, f. 14v ll. 1–4; GGC on SSJ II.1.30, ff. 18v, l. 8–19r, l. 1; GGC on SSJ II.1.30, f. 19v. ll. 5–9.

śruti for the notion that the earth is supported by Śeṣa and other incarnate divine beings, citing ṚV 10.121.1c.[45] On the other hand, one is disappointed in several places with sources that he does not cite. Where in the Purāṇas is the term 'bhūgola' used?[46] Where does Pṛthūdaka make his proof of the earth's sphericity?[47] Who are the critics of Pṛthūdaka in particular, and of the Siddhāntas generally? Because of Cintāmaṇi's preoccupation with Pramāṇaśastra, one does encounter citations from sources that are not typical for astronomical texts. There is, for example, a lengthy discussion of the logician Udayana's *Kiraṇāvalī* on the physical properties of air (or vāyu) and the mechanics of flight.[48]

Cintāmaṇi and Experiments/Realia. Cintāmaṇi is especially interesting when it comes to the examples and the realia they presuppose, especially in his section on the properties of objects under certain conditions, and in his description of people in different parts of the world. He discusses the properties of diamonds and loadstones as known to alchemical science. He knows the special excellences of Persian horses.[49] He discusses at length possible explanations for the tides, and the extent to which these can be caused by lunar activity, given their variant timings. He investigates the causes of earthquakes, the reasons for which he takes from Varāhamihira and the other Siddhāntas. There is a long discussion of how vultures manage to stay aloft.[50]

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[45] GGC on SSJ II.1.30, f. 18v. 1.1. sá dādhāra pṛthivī́ṃ dyā́m utémā́ṃ, from the 'Ka' hymn.

[46] Sūryadāsa (see below) does provide a citation from the Viṣṇu Purāṇa. Kevalarāma picks this argument up, and uses it in a different way, and provides citation.

[47] The argument appears in Pṛthūdaka's commentary on Brāhmasphuṭasiddhānta 21.3. My thanks to Setsuro Ikeyama for sharing with me a draft of his forthcoming edition of this text. The argument, roughly, is that for each of the four quadrant cities on the equator, there are four other cities on the horizon; two different ones always lie relatively to the East and West, while the same two always lie to the North and South. Hence the earth is a globe.

[48] GGC on SSJ II.1.31, f. 20v, l. 12ff; f. 21r, l. 10ff.

[49] His list of other people on the subcontinent includes not just Southerners and Karṇātakans, but also 'Persians', pārasīka—GGC on SSJ II.1.37, f. 23v, l. 4.

[50] This is where the *Kiraṇāvalī* enters into the discussion. Note that the Indian vulture does famously soar on thermal updrafts, but does not eat serpents, while Jñānarāja's point in invoking the vulture is his ability to carry a serpent in mid-air for a full muhūrta. The serpent eagle does both,

Most entertaining are his references to experiments, which are perhaps only thought experiments. The swift astronomer of Jñānarāja's verse, who travels east with his sand-clock, is given a fully satisfying touristic experience in Cintāmaṇi's account, even if the details of his experiment, or of its mathematical difficulties as a proof of anything, are left unexamined. 'Travelling west 10 yojanas (exactly), he comes upon a fortress stronghold atop a hill, and sees there the gateway to a royal city, decorated with masses of gems brought from all over the king's domain and from the domains of his conquered enemies. Seeing the reddish reflection of the rising sun's rays on the walls of the gateway decorated with silk banners and with stone from Kashmir, and seeing that the sun has risen, his mind is filled with wonder, and with a broad smile on his face, he reflects, "what has happened is amazing".'[51]

There is reference to a quasi-Galilean experiment on the behavior of objects of different weight. An iron ball and an āmalaka fruit, of equal size, each threaded onto a string, are pulled toward the observer with equal force at the same moment. The āmalaka reaches him more quickly.[52] Hence lighter things are attracted more quickly, and yet we see that, in nature, heavy things fall to the earth faster. Thus it can't be the earth's attraction that makes them fall. Galileo's result was somewhat different.

5 *Sūryadāsa*

Sūryadāsa (born 1508), son of Jñānarāja and brother of Cintāmaṇi, also lived in Pārthapura. Sūryadāsa was a prolific writer in a variety of genres, especially in mathematics and poetry. He made some essays at mathematics as poetry, as it were.[53] Perhaps the most widely known of his works today is a work of ingenious

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but it does not seem to receive the name of gṛdhra or its reflex in modern Indian languages [Ali *Indian Birds*, 108–13].

[51] GGC on SSJ II.1.26, f. 16r, ll. 2–4.

[52] GGC on SSJ II.1.32 f. 22r, ll. 6–7; and GGC on SSJ II.1.37, f. 23v, ll. 8–10. In the first case the experiment is done with a slab of rock and an areca nut.

[53] For the most complete list of his works, see [Sarma 'Siddhānta-Saṃhitā-Sāra-Samuccaya']; see also [Dikshit *History*, 144–5], [Dvivedi *Gaṇakataraṅginī*, 65–7].

poetry, the *Rāmakṛṣṇavilomakāvya*.[54]

Siddhāntasaṃhitāsārasamuccaya. Sūryadāsa was the author of a *Siddhāntasaṃhitāsārasamuccaya* (SSSS), a 'compendium of essential points about the astronomical Siddhāntas and Saṃhitās'.[55] A reference in the last chapter appears to indicate a date of composition in 1583.[56] The SSSS concludes with a twelfth chapter entitled Jyotiḥśāstrapurāṇavirodhaparihāra, or the 'removal of contradiction(s) of the Purāṇas and Jyotiḥśāstra'. This is the earliest and perhaps only independent chapter in a Siddhāntic work devoted to the virodha problem.[57] Sūryadāsa discusses the topic in other chapters as well, however, for example in the previous chapter, where he attempts to show the Vedic authority for astronomical ideas.[58] The text was not as widely known as the work of Jñānarāja and Cintāmaṇi among astronomers.

In the virodha-parihāra chapter the textual authority of the Purāṇas is stated to be infinitely greater than that of the Siddhāntas. As a result Sūryadāsa's purpose in the chapter is to use a sort of exegetical apologetics to show where there is Purāṇic authority for the cosmology of the Siddhāntas. If a passage from the Purāṇas can be shown to express, either directly or indirectly,

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[54] There is a commentary on the ŚSB attributed to him, which unfortunately was not available to me at the time of this writing. Sūryadāsa is a figure worthy of a great deal more study, especially for the particular nature of his polymathia, which combined the fields of kāvya and astronomy.

[55] The text is as yet unpublished; the last two chapters were available to me (courtesy of DEP) only from a single, flawed manuscript. (Jaipur Khasmohor 5026). My synopsis in the following is provisional, pending the publication of a proper edition.

[56] Ff. 48r, l. 9–48v, l. 1. evaṃ varttamāne śālivāhanaśaka 1505 haste saptarṣayas tiṣṭhanti || There are no bhūtasaṃkhyā numbers to secure the date. If correct this would make it a work of Sūryadāsa's 75th year, as currently reckoned. If the text was written this late, it might explain why Sūryadāsa does not mention the work in some of the lists he made of his works, which appear at the end of texts completed earlier than this.

[57] There are six sections to Sūryadāsa's virodha chapter, the first untitled, then bhūparidhipramāṇa, ākāravipratipatti, samudradvīpasaṃsthā, grahakakṣākrama, grahakakṣākramasya abhiprāyaḥ.

[58] The eleventh chapter, on jyotiḥśāstrāmnāyamūlatva. This chapter provides a basis in śruti for some of the astronomers' cosmological and calendrical ideas. See [Sarma 'Siddhānta-Saṃhitā-Sāra-Samuccaya', 225], which cites a verse elsewhere in the text that is about the earth's support and attraction-power, in keeping with the views of Jñānarāja and Cintāmaṇi listed above.

an idea consistent with the Siddhāntas, then this constitutes a removal of contradiction. In this sense, although Sūryadāsa accords the Purāṇas a much higher canonical status and authority, all the 'leeway' (as it were) for removing contradiction is found in readings of the Purāṇic text, and not in any adjustments to astronomy. The chapter is not written in commentarial form, but in the form of a series of independent verses (in lengthy, difficult meters) followed by citations of Purāṇic passages that serve to demonstrate the claims.

Sūryadāsa's work very much shows the stamp of Jñānarāja's and Cintāmaṇi's ideas; the same arguments with the same examples appear, making this another work of the 'Pārthapura school', with its pronounced tilt toward the Purāṇas.[59] At the same time Sūryadāsa makes his own expansions to the argument.[60] Sūryadāsa adds a variant of the 'different creations' argument, whereby the current creation started out much larger in absolute size, and has subsequently grown smaller. The main evidence he adduces comes from the Mucukunda story of the *Bhāgavata Purāṇa*, in which Mucukunda, a sort of Hindu Rip Van Winkle, awakens from a long sleep to find that everything and everyone has gotten smaller. Thus the Purāṇas and Siddhāntas are both correct, but describe earlier and later stages in the history of creation.[61]

In places Sūryadāsa simply rediscovers the accommodations of

[59] The term 'koṭi' means not crore, but rather 20, or 100 (SSSS f. 46r l. 2); the flatness described in the Purāṇas refers only to a hundredth part of the globe of the earth (SSSS f. 46r, ll. 7–8). As for the problem of the 'downness of down', and why people on the sides and bottom do not fall off, Sūryadāsa follows Cintāmaṇi in adducing the example of insects that can walk on the bottom of a roof beam without falling (SSSS f. 46v, ll. 1–6). Cintāmaṇi proposes this idea in GGC on SSJ II.1.21, f. 13r, ll. 8–10.

[60] Sūryadāsa adds an argument not found in the work of J and C: that the Purāṇa's statements of uniform planetary distance only make sense if the earth is a globe (SSSS f. 46v, ll. 1–2). Note that this is treated not as an astronomical or kinematic problem, but as a problem for the authority of the Purāṇas if they are supposed not to be always true.

[61] On the annular oceans and continents Sūryadāsa discovers a problem in the old Siddhāntic accommodation: Although the oceans and continents are placed in the same order in both models, their sizes must be different, as the oceans farther down in the southern hemisphere must be smaller, given the smaller circumference of the earth closer to the poles (SSSS f. 46v, l. 10–f. 47r, l. 1). Sūryadāsa makes no attempt to work out this irregularity, having been the first to notice it.

earlier Siddhāntins, showing how Purāṇas and Siddhāntas have the same model in their direct meaning.[62] In other places he resorts to the secondary or implied meanings of Purāṇic passages where these support the proposals already made by Jñānarāja and Cintāmaṇi, or support new ones of his own. For example, on the differences in the order and distance of the planets and stars, Sūryadāsa wishes to show that all the planets and stars are described in the Purāṇas as having two motions, both the diurnal rotation to the west, and an independent motion to the east. This he accomplishes by referring to passages that have the seven ṛṣis, or Ursa Major, rotating with respect to the nakṣatras.[63] Since the stars move (very slowly) east, so by implication do all the other planets. Other Purāṇic passages are invoked that show the sun and moon moving eastward, at different speeds. Even though the Purāṇic model appears to place the moon above the sun, Sūryadāsa argues, the Purāṇas also know that moon moves faster than any of the planets. The implication Sūryadāsa finds here is that the Purāṇic sages know the moon must be below the sun.[64]

6 *Nṛsiṃha*

So far we have considered the Purāṇicizing tendencies of three members of one family of astronomers. To trace the impact of their ideas we must move from Pārthapura to Banaras, and consider the work of two astronomers belonging to two rival families. Both of their families had made such a move, from the Deccan to Banaras.

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[62] The first of the chapter's six sections is devoted to pointing out that there are transparent uniformities between the Purāṇas and Jyotiṣ when it comes to the duration of the yugas. The sixth shows that units of time and the names for powers of ten are consistently described.

[63] On this see H. H. Wilson's notes to his tr. of VP 4.24.33 [Wilson VP, 2, 666] and *Bṛhatsaṃhitā* 13.1–11.

[64] This is clearer in the eleventh chaper's exposition (SSSS f. 44v, ll. 4-6). The eleventh chapter also fills out the understanding of Sūryadāsa's position on the Purāṇas. On the question of the earth's shape, Sūryadāsa adduces a passage from the *Bhāgavata Purāṇa* where the term 'bhūgola', the 'earth's globe', is used (BhP 10.8.37). He cites the passage from the *Viṣṇu Purāṇa* that places Meru to the north of all continents—sarveṣāṃ dvīpavarṣāṇāṃ merur uttarataḥ sthitaḥ (VP 2.8.20c). Jñānarāja and Cintāmaṇi had alluded to these passages, but had not cited them.

The family of Nṛsiṃha Daivajña (born 1586) had been based in Golagrāma, a village on the Godāvarī only a few miles from Pārthapura.[65] They were a family of vaidikas, śāstrīs, and astronomers. Nṛsiṃha's grandfather, Divākara—and later his uncle, Viśvanātha—were pupils of the highly innovative and widely read astronomer of the era, Gaṇeśa Daivajña. Divākara eventually moved the family from Golagrāma to Banaras. Four of his sons became astronomers and/or astrologers, most notably Nṛsiṃha's uncle Viśvanātha, with whom Nṛsiṃha studied. Nṛsiṃha in turn had three sons, all literary astronomers: Divākara, Kamalākara and Raṅganātha.[66] This family appears to have been the main force disseminating Gaṇeśa's work around the subcontinent from Banaras.

The move of this family is indicative of a shift in the institutional structures of Sanskrit intellectuals in north India in this period. After a period of dormancy, the 16th and 17th centuries in Banaras are marked by an influx of śāstrīs, especially from the Deccan. In Banaras, the structure of patronage appears to have operated rather differently from the sort of land-grant patronage that Jñānarāja's family probably enjoyed in the Deccan.[67] Support was provided in Banaras for the śāstrīs from many and far-flung princes; support, however, that appears to have been more piecemeal than what local patrons had provided their local pundits. In this period, then, Banaras re-emerged as an 'international' or 'cosmopolitan' center, and Sanskrit works that achieved success there were circulated quickly and widely on the subcontinent.

Nṛsiṃha wrote a huge commentary on the *Sūryasiddhānta* (*Saurabhāṣya*), as yet unpublished.[68] The work I will consider here is his commentary on the ŚSB, the *Vāsanāvārttika*, completed in 1621. Copies circulated widely in north and south India,

[65] [Pingree CESS A3, 204; A4, 162–3; A5, 202–3] with further bibliography noted. See also [Pingree *Jyotiḥśāstra*, 125]. The village's name translates approximately as 'Globeville' or 'Sphereton'.

[66] [Pingree *Jyotiḥśāstra*, 125], and the [Pingree CESS] entries for these three authors. The family produced many other śāstris as well.

[67] See [Minkowski 'Nīlakaṇṭha Caturdhara', n. 99].

[68] This work was completed in 1611. He also wrote a commentary on the *Tithicintāmaṇi* of Gaṇeśa Daivajña, and a 'vast nibandha on astrology' called the *Jātakasāradīpa* (ca. 1625) [Pingree *Jyotiḥśāstra*, 95].

as did copies of his *Saurabhāṣya*.[69] Unlike earlier commentaries on the ŚSB such as Bhāskara's own, or Lakṣmīdāsa's, Nṛsiṃha's does not proceed verse by verse, but presents, rather, an essay on each section, in which he arranges the verses in the order most convenient to his own discussion.

Nṛsiṃha and the 'Pārthapura School'. Though Nṛsiṃha defends Bhāskara in general, he does in places accept the revisions of Jñānarāja and Cintāmaṇi. He knows their work in detail and is influenced by them. He does not appear to know Sūryadāsa's SSSS. He reuses the arguments of Cintāmaṇi, sometimes with attribution, and sometimes without.

Nṛsiṃha accepts Jñānarāja's theory about the Purāṇic supporters of the earth; he agrees that we should assume that Śeṣa and the others perform this role, but that they must be located inside the earth. For the two entailed arguments, however, he remains with the ŚSB. He argues that the earth does have an inherent power to support itself, and that 'down' is relative to where one stands. Nṛsiṃha rejects Jñānarāja's refutation of Bhāskara on the first point saying that attraction can be a special property of earth as a whole, and not of earth the simple element.[70] And supposing that the supporters have an unlimited number of powers is a needless proliferation of suppositions, since giving the earth the one power of fixity solves all problems.[71] On the second point he takes up Cintāmaṇi's argument for the relative 'downness of down'—how could fire burn or rain fall or crops grow—nearly verbatim.[72]

Nṛsiṃha ridicules Cintāmaṇi's (and also Sūryadāsa's) argument about the meaning of the term 'koṭi', pointing out that, after all, if a Purāṇic crore of yojanas is equal to one hundred 'actual' yojanas, then the Purāṇic Jambūdvīpa of 100,000 yojanas, our entire continent, that is, must be understood to be only one yojana across![73]

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[69] Aside from Bhāskara's own, the *Vāsanāvārttika* is one of only two premodern commentaries on the ŚSB to have been published. The other is by Munīśvara (see below). Citations from Nṛsiṃha in the following are drawn from the edition of the *Vāsanāvārttika* by Muralīdhara Caturvedī.

[70] [Caturvedī, 340, ll. 27–9]. In fact Cintāmaṇi anticipates this explanation in a pūrvapakṣa.

[71] [Caturvedī, 341, l. 8].

[72] [Caturvedī, 338, ll. 11–14. See above, note 41.

[73] [Caturvedī, 358, ll. 5–9]. Nrsiṃha further rejected the argument that

Nṛsiṃha rejects a series of Jñānarāja's and Cintāmaṇi's proposals without argument. Jñānarāja's claim (SSJ vs. 29) that Bhāskara is not logical is to be ignored (upekṣaṇīyam). His argument (SSJ vs. 30) by way of the the variety of customs and appearances of people in our own continent is not logical (tad ayuktam). Cintāmaṇi's argument that many passages of śruti refer to Śeṣa et al. as the supporters of the earth is also to be doubted (sāpi cintyā).[74]

Elsewhere Nṛsiṃha accepts Cintamaṇi's arguments and examples cheerfully (not always acknowledging his source). He likes Cintāmaṇi's proof of the earth's sphericity by appeal to the example of the insect circling a lamp, which, he finds, is 'well-said'.[75] He borrows the two specified arguments about the size of the earth that Sūryadāsa and Cintāmaṇi have made, about the astronomer travelling north 14 yojanas, and travelling east 10 yojanas.[76] He refers to 'self-styled' Paurāṇikas, using the phrasing of Cintāmaṇi.[77] He makes use of a dialectical argument found in Cintāmaṇi phrased in the same structure and wording.[78]

Attitude to Pramāṇa. Although he has accepted at least one of Jñānarāja's Purāṇicizing proposals, Nṛsiṃha does not value the authority of the Purāṇas as highly as do the members of the 'Pārthapura school'. He considers them to be of an authoritative weight equal to the Siddhāntas, but not greater.[79] For Nṛsiṃha,

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the Purāṇic measurement is of an earth of a different era for two different reasons.

[74] All of these statements are found on [Caturvedī, 341, ll. 5–12].

[75] iti cintāmaṇinā kandukākārakatve 'numānaṃ saṃyag upanyastam [Caturvedī, 353, l. 15]. Cintāmaṇi uses this analogy in the commentary on vs. 23—f. 14r, l. 10.

[76] [Caturvedī, 357, ll. 22–30]. Note that Nṛsiṃha reuses the same argument for the first, but notices the problem that the second example only holds true for the equator, and adjusts it to make it more general. Cintāmaṇi is however not mentioned by Nṛsiṃha here, but see above, notes 31, 32, and 43.

[77] ye tu purāṇārtham ajānantaḥ paurāṇikaṃ manyā ... [Caturvedī, 355, l. 27]. See SSJ vs. 27 with GGC.

[78] [Caturvedī, 340, ll. 10–16], echoing a pūrvapakṣa in GGC on SSJ 2.1.30—f. 18v, l. 8ff.

[79] This is probably because of the argument made by astronomers in later periods that the Siddhāntas derive their authority from the status of their authors, who are, after all, various gods. For example, the author of the *Sūryasiddhānta* is the deity Sūrya. Even works attributed to human authors are, in this logic, revealed to them by deities; and this includes the works of some foreign astronomers. See [Pingree 'Indian Reception', 476–80].

therefore, the contradiction of the text of the Purāṇas by the text of the Siddhāntas results in an impasse, which is to be resolved by experience and reasoning, in which astronomy principally consists. A greater scope is thus allowed for astronomical argument.

Nṛsimha's Sources. Sūryadāsa and Cintāmaṇi are not Nṛsiṃha's only sources. His work takes note of a variety of views, some of which are not extant,[80] and provides something of a doxography of the virodha problem.[81] Most noteworthy is Nṛsiṃha's account of the view of the 'Yavanas', namely that the majority of the earth is underwater, and that the earth floats in the water like a tumba gourd.[82] This constitutes a reference (even if an incorrect one) to the presence in India in this period of models that belong to Arabic and Persian cosmology and geography, models that attracted some attention among Sanskrit astronomers in the 17th and 18th centuries.[83] Nṛsiṃha rejects this 'floating earth' model, since it is our common experience that the earth supports water and is not supported by it; and that water thrown into air falls until it rests on earth. Hence it cannot be the fixed support of earth.[84]

Kalpabheda and Virodha Again. Nṛsiṃha rejects the proposals of Jñānarāja and Cintāmaṇi (and Sūryadāsa) to reconcile the cosmological models of the Siddhāntas and Purāṇas by making them representations of different creations. Nṛsiṃha points out,

[80] Or not quite. He has the *Vyāsasiddhānta* give an earth's circumference of 5000 yojanas, but I do not find this reference in the Berlin MS (see above notes 36, 37, and 38). He also cites a passage of the *Śākalyasaṃhitā* other than what is available in the extant *Brahmasiddhānta* [Caturvedī, 338, ll. 23–5].

[81] Some of the views are old—the views of Bauddhas and Jainas, for example, which Bhāskara rejects. The Paurāṇikas again make an appearance, and there are some more arguments attributed to them, but still they are offstage voices. No other post-Bhāskara astronomers are mentioned by name other than Jñānarāja and Cintāmaṇi.

[82] I.e., it floats approximately in the way that an iceberg does [Caturvedī, 354, ll. 14-18]. The tumba is identified in the lexicons as *Lagenaria Vulgaris*, the common gourd. Apparently this is a garbled reference to the Aristotelian theory that most of the earth is under the seas, presumably brought into India through an Arabic source.

[83] For a discussion of some aspects of the phenomenon see [Pingree "Indian Reception'] and [Pingree 'Islamic Astronomy'].

[84] There is also the problem that we commonly see clods of earth dissolve in water, so why would the Earth, if floating in the ocean, not similarly dissolve? [Caturvedī, 354, l. 22].

first of all, that the *Mahābhārata*'s cosmology, which is about the same as the Purāṇic one, is clearly presented as a description relevant to our own age. He also notes that the Siddhāntas acknowledge a change in the size of the earth of only one yojana during the creation.[85] The implication is that this sort of accommodation puts the Siddhāntas and Purāṇas too much on the same plane. Nṛsiṃha proposes instead that we settle for a compartmentalizing solution, something like the conclusion of Jñānarāja, that the two models constitute different visions, having different uses and purposes.[86]

7 *Munīśvara*

Nṛsiṃha's sons encountered a formidable rival in Munīśvara Viśvarūpa (born 1603). He was the son of Raṅganātha, who wrote a widely disseminated commentary on the *Sūryasiddhānta*, the *Gūḍhārthaprakāśa*, in 1603.[87] Raṅganātha's father had moved the family to Banaras from the village of Dadhigrāma, a village on the Payoṣṇī river in Berar.[88] In Banaras, Raṅganātha's brother Kṛṣṇa had received patronage from Jahangir, and some evidence suggests that Munīśvara was at one time patronized by Shah Jahan or figures in his court.[89] Munīśvara (born 1603) was the most prolific writer in the family. He produced a lengthy commentary on the ŚSB, entitled the *Marīci*, sometime before 1638, and completed a Siddhānta of his own, the *Siddhāntasārvabhauma* (SSB), in 1646.[90]

[85] The growth is the result of the accumulated detritus of previously existing creatures. ŚSB Bhuvanakośa 62–3.

[86] [Caturvedī, 358–9]. Jñānarāja's proposal was found in SSJ 2.1.77. For Nṛsiṃha, the Purāṇic accounts are concerned with a different earth, the earth which forms part of the gross body of the cosmic Virāṭ, which is knowable only through the revelation experienced by sages who behold the Viśvarūpa of the Deity, such as Mārkaṇḍeya.

[87] [Pingree CESS A4, 436–41; A5, 314]; [Dikshit *History*, 161–2]; [Dvivedi *Gaṇakataraṅginī*, 61–4]. For a diagram of the family tree with list of works, see [Pingree *Jyotiḥśāstra*, 126].

[88] Thus this was another family of Deccani Brahmins, from a village North and West of Pārthapura and Golagrāma. The village's name translates approximately as 'Curdston'.

[89] [Pingree CESS A4, 440].

[90] Munīśvara completed his own commentary on the SSB, the *Āśayaprakāśinī*, in 1650. The SSB is, to date, not fully in print. The chapters

Munīśvara knows the work of both Jñānarāja and Cintāmaṇi, and cites both of them at length in the Bhuvana section of the *Marīci*.[91] On cosmological questions, Munīśvara rejects the innovations of the Pārthapura school to a greater extent than Nṛsiṃha does. He cites Cintāmaṇi and in a few cases re-uses his arguments and examples (though to a much lesser extent than Nṛsiṃha does). Of most interest are the places where Munīśvara uses Cintāmaṇi's own doubts about the *Siddhāntasundara* to argue against Jñānarāja.

Support of the Earth. When it comes to the support of the earth, Munīśvara relies on another part of Bhāskara's exposition to render most of this discussion peripheral. This is Bhāskara's statement that the earth is at the center of the Brahmāṇḍa.[92] Munīśvara takes this to mean that there is no 'down', except in the direction of the center of the earth. Where, then, would the earth fall? Where would other things fall except toward the center of the earth? Hence there is no real need for arguing about support of the earth, or its attraction powers, except as these are occasioned by arguments with others whose theories are more elaborate or faulty.

Unlike Nṛsiṃha, Munīśvara does not accept Jñānarāja's Purāṇicizing adjustment concerning the incarnated supporters of the earth, i.e. that Śeṣa et al. can do their supporting from inside the spherical earth. He cites Cintāmaṇi's discussion of this topic at length and argues against it, saying that Cintāmaṇi only shows his own ignorance in making such statements.[93]

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on gola are not published, and in any case are not very extensive. Munīśvara does refer to the SSJ in his SSB commentary, in a section on the size of the earth and its sphericity; he welcomes the 'swift astronomer' argument, and the vss. are cited from the SSJ. In the following I will use primarily the commentary on the Bhuvana sections of the ŚŚB. All citations are drawn from the edition of the *Marīci* on ŚŚB's Golādhyāya by Kedāradatta Jośī.

[91] Like Nṛsiṃha, he does not appear to know Sūryadāsa's work. He does not usually cite J and C or their work by name, but the passages are those of the *Siddhāntasundara* and *Grahagaṇitacintāmaṇi*. For a place where he cites the *Siddhāntasundara* by name see [Jośī, 80, l. 18].

[92] ŚŚB Bhuvanakośa vs. 1.

[93] Cintāmaṇi is cited from [Jośī, 33, l. 22] to [Jośī, 35, l. 17]. He is not named but identified as an astronomer who follows the Paurāṇikas—paurāṇikānuyāyino jyotirvidaḥ. The relevant sections of Cintāmaṇi are found in his commentary on SSJ vss. 30 and 31. Munīśvara's closing comment is worth citing for a flavor of his polemical style: ...granthadūṣaṇajalpanaṃ

The earth's inherent attraction power, proposed by Bhāskara, was rejected by Jñānarāja and Cintāmaṇi, arguing that the earth cannot have a power to attract, since we don't see such a power in small clumps of earth. Munīśvara turns this argument on its head. In fact, even small bits of earth do have the power to attract, argues Munīśvara; it is just that the spherical Earth's power is much greater, and overpowers the attraction power of smaller things.

As for the Pārthapurian argument that creatures on the sides and bottom of the earth have special properties that enable them to stay on, Munīśvara asserts that this amounts to an overly complex theory of three-fold attraction powers (one power each for things on the side, bottom, and top) which nevertheless suffers from the same fault it set out to solve, i.e. that heavy things are seen to fall faster than light things.[94]

Futhermore, Munīśvara invokes Cintāmaṇi's misgivings about Jñānarāja's 'downness' theory (though without attributing them to Cintāmaṇi)—how will rain fall down, and grain grow up, and fire burn upward?[95] Elsewhere, he invokes Cintāmaṇi explicitly. Arguing with Jñānarāja in the second person, he closes an argument about the earth's attraction power by appeal to the standpoint of 'your own son'.[96]

Elsewhere the reconciling arguments of the Pārthapura school are rejected without identification. The relevant passage of the Purāṇas about Meru, says Munīśvara, cannot serve to show the Purāṇas in comformity with the Siddhāntic model of the spherical earth.[97] The term 'koṭi' cannot mean 100, since one ends up with overly small continents and oceans.[98]

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svājñānasūcakamatimad anādarayogyaṃ siddhāntakartṝṇāṃ bhavādṛśāṃ anucitam eva.

[94] [Jośī, 37, ll. 11ff.]

[95] [Jośī, 37, ll. 14-17]. See above, notes 41 and 72.

[96] [Jośī, 37, ll. 21–24]. Cintāmaṇi's argument that the power of the earth to attract is adventitious (āgantuka) can be used against Jñānarāja. Your own son, he says, 'accepts that the earth's attraction power is not independently argued but only argued for the purpose of proving the primary power of the earth, to stay fixed in space'. Cintāmaṇi's comments appear on SSJ 2.1.38. In fact the passage in Cintāmaṇi is not about the simple claim, but its hierarchical positioning within a larger structure of arguments.

[97] [Jośī, 46, ll. 1–4]. See above, note 64.

[98] [Jośī, 54, l. 9]. M adds here that one would be able to cross an ocean

Munīśvara does briefly notice the Yavana view that the earth floats in the ocean like a gourd (tumba), but rejects it because of the basic arguments of the SŚB against any support.[99]

Munīśvara assumes, with Nṛsiṃha, that the Siddhāntas and Purāṇas are of equal weight as textual authorities, but that the Siddhāntas have the additional advantage that they are supported by experience, and by reason. In general Munīśvara is not very concerned about adjusting astronomical views to the Purāṇic model, simply preferring the older Siddhāntic standpoint. He saves his polemical strength for refutations of other astronomers, especially in their non-cosmological sections.

8 *Aftermath among the Paurāṇikas*

By way of closing this study I will turn to several Paurāṇika works that reflect the aftermath of the cosmological controversies awakened by Jñānarāja and Cintāmaṇi. This is not intended as a full study or description of these works, which will have to come elsewhere. The works discussed below were not governed by the same constraints by which the astronomers' accommodations were limited. It is only certain that one of them was written by an astronomer, and his abandonment of astronomical necessities is all the more striking. The survey will also serve as a conclusion, by showing the variety, not just in cosmological doctrines, but in what virodhaparihara, the removal of contradiction, comprises.

Saurapaurāṇikamatasamarthana. Nīlakaṇṭha Caturdhara flourished in Banaras in the mid-seventeenth century. A contemporary of Munīśvara and Kamalākara, he was also a Marāṭhī-speaking Brahmin who had moved from the upper Godāvarī to Banaras. He is best known for his commentary on the *Mahābhārata*, the *Bhāratabhāvadīpa*, but he was also the author of about a dozen other works. The work of interest to us here is the independent treatise on the virodha problem entitled *Saurapaurāṇikamatasamarthana* (SPMS), 'allaying the disputes in the

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only one yojana broad, and hence visit the fabulous outer continents no mortal from Bhārata has seen. Elsewhere [Jośī, 80, l. 18ff.] Munīśvara rejects the SSJ's championing of Pṛthūdaka (2.1.23), since it assumes things not in evidence to make its proof.

[99] [Jośī, 32 ll. 15ff.] He also cites the reasons that Nṛsiṃha gives.

doctrines of the Purāṇas and the *Sūryasiddhānta*'.[100] The earliest manuscript is dated 1679, and the text was probably not written much before then.[101] Nīlakaṇṭha had already made some forays into this topic in his *Bhāratabhāvadīpa* (BhBhD).[102]

For Nīlakaṇṭha the 'allaying of disputes' consists in arguing wholeheartedly for the Purāṇic model, in a number of ways. For the model of the earth (bhūgola), he attempts a numerical reconciliation, proposing to adjust the conflicting sizes of the earth found in the Purāṇas and Siddhāntas by making the yojana variable, and by an ingenious rereading of certain crucial passages in the *Mahābhārata*.[103] For the model of the heavens (khagola), Nīlakaṇṭha rejects outright the spherical model, and argues that Jyotiṣ cannot account for a variety of astronomical phenomena.[104] He attempts his own explanation of how several problematic phenomena work in the Purāṇic model—the changing length of days in the course of the year, and the waxing and waning of the moon.

It is clear that in making these arguments Nīlakaṇṭha does not have much comprehension of the Siddhāntic cosmology or what motivates it; in fact he has the least understanding of any of the authors surveyed here.[105] Instead he attempts his own novel diagrammatic explanations of the earth's shape and what causes the sun to be visible longer when north of the ecliptic, even though in the Purāṇic model this should imply that the sun is overhead a shorter time.[106] Nīlakaṇṭha is not very interested in preserving

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[100] The work has the alternative title *Paurāṇikajyotiṣam*. See [Minkowski 'Nīlakaṇṭha's Comments', 35–7] and bibliography cited there. An edition based on two manuscripts in the SB was produced by Indunātha Śarma. A fresh edition based on more manuscripts is forthcoming.

[101] SB 37122, ff. 1–16. [Minkowski 'Nīlakaṇṭha's Comments', 37, n. 44].

[102] See [Minkowski 'Nīlakaṇṭha's Comments', 28–35].

[103] These are the proposals already found in the BhBhD. The yojana changes size in different ages.

[104] The 'downness of down', the reason rivers flow only in one direction between two points, the length of days, the waxing and waning moon.

[105] Indeed, there is no evidence in his œuvre or in his educational biography that he had studied any branch of Jyotiḥśāstra. For bibliography, see [Minkowski 'Nīlakaṇṭha's Comments', 27–8]. Indunātha Śarma, the editor of the published SPMS, and a Lecturer in Jyotiṣ at the Sanskrit University in Banaras, spends several pages of his introduction [Śarma, ga–cha] deploring NC's lack of astronomical knowledge.

[106] Since the circle on which it revolves overhead is smaller, it ought to travel over us more quickly. Nīlakaṇṭha argues that it becomes visible sooner,

the integrity of the Siddhāntic model. He compares its status in relation to that of the Purāṇas to the status of vyāvahārika, or mundane, knowledge by comparison to ultimate Vedāntic knowledge.

Nīlakaṇṭha has picked up some of these ideas from his predecessors in the discussion. We have seen the variable yojana and kalpabheda ideas before, though Nīlakaṇṭha blends them. Ascribing the two models to different layers of philosophical truth is also an old idea, though Nīlakaṇṭha gives it a more explicitly Vedāntist rendering.

'Bhūgolavicāra'. Though the date, author, and actual title of this text are not known, I will discuss it here since it devotes a prominent section to rejecting Nīlakaṇṭha's ideas. This text is, nevertheless, also in favor of the Purāṇic model.[107] The *Bhūgolavicāra* is composed in the form of a dialogue, with a Siddhāntic point of view, Bhāskara's specifically, holding sway for the opening pages; then there is an argumentative turn, and the Purāṇic point of view is presented, apparently as the final opinion.[108] The author's mode of 'reconciliation' is really to support the Purāṇas as more authoritative, reasonable, and in keeping with our common experience. The Siddhāntas are not wicked, and have some provisional value, but depend on a contingent, unreliable sort of epistemology. The author dislikes attempts at reconciliation that assume the two models describe the same reality. For example, he rejects the 'different creations' (kalpabheda) argument out of hand.

By way of transition to the Purāṇic view, the author argues that the Purāṇic sages know the Siddhāntic model and have accounted for it. He adduces a verse from the *Mahābhārata* which he understands to mean that the earth, though actually flat, is reflected in the watery surface of the spherical moon, and that this

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however.

[107] It is preserved in only one manuscript, which lacks its first page, and an indeterminable number of final pages (Jaipur (Khasmohor) 5032, ff. 2–11). There is no colophon, no title, no author, no date. The text must be later than Nīlakaṇṭha, however. The cataloguer gives it a descriptive title, '*Bhūgolavicāra*', 'consideration of the earth's model'.

[108] As the MS ends, however, the Purāṇic point of view is beginning to be interrogated. It is possible that the dialogue takes another turn in its later pages.

reflection makes the earth look spherical.[109] It is for this reason that the astronomers think the earth is a sphere.[110] After all, the astronomers cannot measure the earth, and must depend for their understanding on an informal sort of practical reasoning based on their observations of the heavens. Thus the astronomers' knowledge is only a sort of customary lore (vyavahāra) supported by none of the philosophical Pramāṇas—not textual authority, not direct observation (since what they rely on is a scattering of indirect observations), and not logic (calculation not being reliably provable).

Since the verse that makes this argument possible is taken from the cosmological passage in the Bhīṣmaparvan of the *Mahābhārata*, the author takes up the commentary of Nīlakaṇṭha, in which an attempt at reconciling Siddhāntas and Purāṇas is discussed.[111] All of Nīlakaṇṭha's ideas are rejected as contradictory to scripture, to logic, and to common practical experience. To accept them would amount to saying that the sages are crazy (unmatta), and therein would the Vedic teachings perish.[112]

This text is also interesting for the sources it cites. The author knows the work of Bhāskara well, both the ŚSB and the *Karaṇakutūhala.* He challenges Bhāskara on the support of the earth and on the earth's attraction power with strong arguments, though he does not refer to the ideas of Jñānarāja and Cintāmaṇi. He knows a commentary on the ŚSB that seems to take in some of Munīśvara's arguments on these topics, but that has other ones as well. The author's father (tātacaraṇāḥ) has also apparently written on the virodha problem. It is hoped that a complete copy of the text will come to light.

Bhāgavatajyotiṣayor Bhūgolakhagolavirodhaparihāra. The astronomer Kevalarāma, who served as Jyotiṣarāya at the court of Sawāi Jaisingh in Jaipur, was commissioned to compose 'a work that removes the contradiction between the models of the earth

[109] MBh 6.6.15 Crit. Ed. (= 6.5.12 Bombay Ed.).

[110] This argument explains the astronomers' error as resulting from their basing their model of the earth on observing its reflection in the moon. This etiology is a sort of mirror image of the astronomers' explanation for the Purāṇic flat earth model, i.e. that it is predicated on a partial observation of the earth, i.e. of only one hundredth part of its surface. See above, note 33.

[111] The author is aware only of the BhBhD commentary, and shows no awareness of the SPMS of Nīlakaṇṭha.

[112] F. 8r, ll. 8–11.

and heavens found in the *Bhāgavata* (*Purāṇa*) and Jyotiḥśāstra'. Kevalarāma had written works on astronomy before, but had to ignore much of what he knew as an astronomer in order to compose this work.[113]

Kevalarāma proposes to reconcile the two models in a very unusual way. He argues that there are in reality two different earths, both the large, flat Purāṇic earth and also a small, spherical earth floating above the large Purāṇic one. The spherical earth is surrounded by the planets, whose orbits are the large ones posited in the Siddhāntas. The planets, therefore, must pass through the surface of the flat Purāṇic earth as they revolve.

In this (two) earth model, the Purāṇic doctrine holds the upper hand, with the *Bhāgavata* serving as the main source for establishing it. In the model of the heavens, it is the orbit distances established by Bhāskara that serve as the starting point for explaining the Purāṇic distances. Kevalarāma rejects the 'different creations' (kalpabheda) argument, but points to evidence in the *Bhāgavata* itself to show that the yojana is indeed of variable size.

Kevalarāma's model of 'removing contradiction' is thus an odd blend of keeping the two models distinct, of refuting one or the other model, and of actually proposing to adjust both models so that they conform to one another. In the end Kevalarāma argues (as Nṛsiṃha did and others before him) that the two models really represent two distinct visions of two distinct realities, with different purposes.[114]

Vyāsamatoddhāra. In 1753, Śaṅkara Mallimaya wrote a work which attempts a 'retrieval of the viewpoint of Vyāsa'.[115] Since the Purāṇas can satisfactorily explain all necessary astronomical

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[113] Several MSS of the work are extant. See [Pingree CESS A2, 63; A5, 54]. An edition, translation and study of this work are forthcoming. See also a brief mention in [Pingree 'Paurāṇic versus Siddhāntic Astronomy'].

[114] Kevalarāma's work was expanded and revised by Nandarāma Miśra in a text entitled *Goladarpaṇa*, completed in 1763. [Pingree CESS A3, 128–9; A5, 156]. The arguments are similar enough that I will omit reference to this work here. Nandarāma's date does serve as the terminus for the period I am investigating.

[115] This work is unpublished. To date I have had available only a partial MS, Nagpur 977 (I had this MS again thanks to DEP), and only some folios—ff. 6–14, 30–46, but the general implications of the work are clear from the sections at hand.

phenomena, argues Śaṅkara, a sense of duty and obligation commands us to prefer Vyāsa's, that is the Purāṇas', view. As the title would suggest, the bulk of the argumentation is defensive. It is designed to reply to Siddhāntic criticisms of the Purāṇic model. The arguments for the flatness of the earth incorporate some older Paurāṇika arguments (i.e. that water can't be made to flow properly on a sphere, and more generally, downness cannot be relative), as well as a new one to do with arcs and chords of arcs.[116] There is an ingenious argument to show that Bhāskara's theory of the earth's stability and of its gravitational power are mutually dependent and hence unprovable. Śankara reuses one of Kevalarāma's arguments for the Paurāṇika planetary model,[117] and even uses one of Nṛsiṃha's reasons to argue against Bhāskara.[118] The idea that the sun's distance from the observer is what causes the appearance of sunrise and sunset resurfaces in this text.[119]

In general the authority of the Purāṇas is emphasized. Correspondingly the authority of the Siddhāntas is belittled. To those who assert that the Siddhāntas are authored by gods,[120] Śaṅkara raises the question of whether these works are really the compositions of the deities to whom they are attributed, or are rather impostures created by wicked men.[121] In any case, the method of

[116] If an ant starts from the top and travels 18 inches down the circumference of a ball 12 inches in diameter, he reaches (approximately) the bottom. By that logic of 3/2, if we travel south 60 yojanas, we should be 40 yojanas farther 'down', and yet we experience ourselves no more 'down' than before. (In fact Śaṅkara has the relationship of circumference and diameter a little unclear in this; the evidence he invokes seems to me a better proof of sphericity.)

[117] I.e., that the relative coolness of the sun at dawn and its heat at midday is much better explained by the Paurāṇika model, in which the sun gets proportionately much closer to us at noon than at dawn, while in the Siddhāntic spherical model the difference in proximity of the sun is negligible. Kevalarāma makes this argument as well.

[118] I.e., that the fixed earth cannot support the revolving planetary spheres, as the earth's fixity would prevent the planets and stars from revolving [Caturvedī, 338, ll. 2–8].

[119] F. 46r. It is a matter of curiosity to note that Śaṅkara denies any Paurāṇika has ever asserted that it is the sun's disappearance behind Meru that causes sunset, though this is seen repeatedly as an argument; as for example in the next author, Harideva.

[120] As the Siddhāntic authors were increasingly tempted to argue in this period; see [Pingree 'Indian Reception', 476–80].

[121] Ff. 41r-42v. Of course Śaṅkara does not pursue the implications of this line of questioning for the provenance of the Purāṇas.

the Siddhāntas cannot be said to be based on any pramāṇa, even direct perception, as the '*Bhūgolavicāra*' also declares. Meanwhile we are better off to be obedient to Vyāsa's view.

Mataikyacandra. Harideva Bhaṭṭa composed a text on the virodha problem called the *Mataikyacandra*, 'moon of the oneness of views' together with a commentary.[122] The date of this work is unknown, but I include it here because of its reference to the astronomers of Pārthapura, and because its general approach to reconciliation brings us, I think, full circle. For the *Mataikyacandra* resorts to a strategy of perspectivism—both models are true depending on the standpoint from which the world is viewed (yathāyogyāt). Thus descriptions of the earth as spherical and flat are both true—the former refers to the uneven appearance of the earth in the days before the legendary figure Pṛthu leveled it out with the tip of his bow; the latter refers to the earth's even, smooth appearance afterwards.

Similarly the various sizes for the earth and the heavens that one encounters in the different Siddhāntas and then again in the Purāṇas can be explained by kalpabheda, that is, as true for different creations. Furthermore, the Purāṇic account of the earth's layout is true from the gods' perspective at the top of Mt. Meru; the Siddhāntic account is true from the human's perspective from the city of Laṅkā. And even if the order and sizes of the oceans and continents are not the same exactly, both accounts agree that the regions are circular. Varying accounts of the 'downness of down' are also acceptable and assimilable, because all of them have an explanation of 'down'.

Harideva knows all of the authors of the Pārthapura school. He agrees with Jñānarāja in rejecting Bhāskara's theory of the earth's lack of support. On the other hand he accepts the idea that the earth has 'gravity', against Jñānarāja. Nor does he like the idea of putting Śeṣa and the other supporters inside the earth. In general he does not like the critiques of the Purāṇic view that are found in the Siddhāntas and argues against them, but he is much more favorable toward the Siddhāntas than the authors de-

[122] Also known as the *Purāṇajyotiḥśāstramataikyavākyatā*. The commentary is called the *Mataikyārthaprakāśa*. This work is known from only one manuscript, RORI (Kota) 639, which I was able to examine through the courtesy of Richard F. Young via DEP. It is as yet unpublished, and I know of no extant studies of its contents.

scribed in the immediately preceding sections. Harideva is not keen on the strategy of directly adjusting or revising the models to make them conform to a single picture, and is more in favor of finding similarity within existing models, by selective interpretation of passages, and by making the terms of agreement loose enough that they can incorporate both views.

Of the types of removal of contradiction with which I began, I have suggested that the strategy of compartmentalization was an old one, that probably preceded the time when Jñānarāja stirred up the cosmological controversy again. Harideva has given it a new, much looser, grounding in perspectivism that does not attempt to establish an explicit hierarchy of views.

9 *Conclusion*

The *Mataikyacandra* throws up a question with which I might conclude. Suppose Harideva's solution is a new version of compartmentalization, this perspectivism which is not mere relativism but which enables several irreconcilable viewpoints to co-inhabit the same intellectual 'living space'. Why could not an even less forgiving scientific viewpoint also be so incorporated? If any viewpoint can be held together with the viewpoint of the śruti and smṛti through a perspectivist or compartmentalist strategy, why do the authors of the Siddhāntas need to seek intellectual accommodation with the Purāṇas at all? If the science was practically useful (which it was, in enabling a confident timing of ritual practices and casting of horoscopes), would that not be enough to guarantee the creation of a niche within the 'ecosphere' of canonical literature?

Apparently not. There can be little doubt that the astronomers in every period felt a pressure, in varying degrees, to make their science acceptable to doctrines found in the Purāṇas and in other religious literature. In the early modern period, this pressure appears to have significantly increased. The history of the avirodha problem in this period does not lend itself to a single-stranded or developmental explanation, but historical change there nevertheless was. For some of these works, the number of manuscripts is very small, but their existence serves to show the recurrent concern with 'reconciliation'. One can also see that there were differentials in institutional support, as it were,

the astronomers knowing and citing each other's work in detail, the Paurāṇikas working much more on their own, in apparent ignorance of other individual efforts.

Thus it will not do to appeal simply to an inveterate complacency about intellectual inconsistency among the Sanskrit intellectuals, for in this period at least, there is an awareness that the cosmology is problematic, and there are attempts, however accidental, to sort the problem out.

Nevertheless, the appeal to a 'layers of truth' mode of accounting for intellectual difference turns up in many of our authors, in more or less limited roles. It is a strategy that is almost perfectly ambiguous in its historical intepretation. It could be explained as a failure of Indian knowledge systems to adapt, by hampering the ability to attend much to new scientific models that might have become available in this period.

On the other hand the 'levels of truth' appeal could be interpreted as the basis for a modernizing intellectual adjustment, one that separates the scope of different sources of knowledge. An accommodation between science and religion of this kind is sometimes claimed to be the trademark of the arrival of modernity in the contemporary cosmological debates in Florence and Rome.

Abbreviations

AS Bombay	Asiatic Society of Bombay
BhBhD	*Bhāratabhāvadīpa* of Nīlakaṇṭha Caturdhara
BhP	*Bhāgavata Purāṇa*
BM	British Museum
C	Cintāmaṇi
CESS	*Census of the Exact Sciences in Sanskrit*
DEP	David E. Pingree
GGC	*Grahagaṇitacintāmaṇi*
J	Jñānarāja
M	Munīśvara
MBh	*Mahābhārata*
NC	Nīlakaṇṭha Caturdhara
RORI	Rajasthan Oriental Research Institute
ṚV	*Ṛg Veda*
SB	Sarasvati Bhavan

ŚB	*Śatapatha Brāhmaṇa*
ŚDVT	*Śiṣyadhīvṛddhidatantra* of Lalla
SPMS	*Saurapaurāṇikamatasamarthana* of NC
SSB	*Siddhāntasarvabhauma* of Munīśvara
SŚB	*Siddhāntaśiromaṇi* of Bhāskara
SSJ	*Siddhāntasundara* of Jñānarāja
SSSS	*Siddhāntasaṃhitāsārasamuccaya* of Sūryadāsa
VP	*Viṣṇu Purāṇa*

Bibliography

Salim Ali, *Book of Indian Birds*, 12th ed., Mumbai, 1996.

Muralīdhara Caturvedī, *Vāsanāvārttika*, Library Rare Texts Publication Series 5, Vāranasī, 1998.

S. B. Dikshit, *History of Indian Astronomy*, tr. R. V. Vaidya, 2 vols, Delhi, 1969.

Sudhākara Dvivedī, *Gaṇakataraṅgiṇī*, Benares, 1933.

Kedāradatta Jośī, *Golādhyāya of Siddhāntaśiromaṇi, with Marīci and Bhāskara's Vāsanā*, Delhi, 1988.

C. Minkowski, 'Nīlakaṇṭha's Cosmographical Comments in the Bhīṣmaparvan', *Purāṇa*, 42, 2000, pp. 24–40.

——. 'The Paṇḍit as Public Intellectual: The Controversy over virodha or Inconsistency in the Astronomical Sciences', in *The Pandit: Traditional Sanskrit Scholarship in India (Festschrift P. Aithal)*, ed. Axel Michaels, Heidelberg South Asian Studies XXXIX, New Delhi, 2001, pp. 83–102.

——. 'Nīlakaṇṭha Caturdhara's *Mantrakāśīkhaṇḍa*', *Journal of the American Oriental Society*, 122, 2002, pp. 329–44.

David Pingree, *Census of the Exact Sciences in Sanskrit*, Series A, vols 1–5, Philadelphia, 1970–94.

——. 'Islamic Astronomy in Sanskrit', *Journal for the History of Arabic Science*, 2, 1978, pp. 315–30.

——. *Jyotiḥśāstra: Astral and Mathematical Literature*, Wiesbaden, 1981.

——. 'The Purāṇas and Jyotiḥśāstra: Astronomy', *Journal of the American Oriental Society* 110.2, 1990, pp. 274–80.

——. 'Indian Reception of Muslim Versions of Ptolemaic Astronomy', in *Tradition, Transmission, Translation*, ed. F. J. and S. P. Ragep, Leiden, 1996, pp. 471–85.

——. 'Paurāṇic versus Siddhāntic Astronomy', *English Abstracts, Xth World Sanskrit Conference*, Bangalore, 1997, pp. 318–19.

Kim Plofker, 'Derivation and revelation: the legitimacy of mathematical models in Indian cosmology', in *Mathematics and the Divine*, ed. Teun Koetsier, Amsterdam, forthcoming.

Indunātha Śarma, *Paurāṇikajyautiṣam*, Laghu Granthamālā 45, Varanasi, 1989.

K. Madhava Krishna Sarma , 'Siddhānta-saṃhitā-sāra-samuccaya of Sūrya Paṇḍita', *Viśva Bhāratī*, 2, 1950, pp. 222–5.

Horace Hayman Wilson, *The Vishnu Purána: a system of Hindu mythology and tradition*, London, 1840.

Richard F. Young, 'Receding from Antiquity: Indian Responses to Science and Christianity on the Margins of Empire', *Kokusaigaku-Kenkyu* 16 (Meiji Gakuin Ronso 595), 1997, pp. 241–74.

Two Benares Manuscripts of Nārāyaṇa Paṇḍita's *Bījagaṇitāvataṃsa*

TAKAO HAYASHI

Preface

The main aim of this article is to provide an edition of the available portion of Part II of the *Bījagaṇitāvataṃsa* together with an English translation of its verses and with a mathematical commentary, based on two Benares manuscripts including one newly discovered by Professor Pingree. I express my sincere gratitude to the authorities of the Benares Sanskrit College and to Professor Pingree, thanks to whom I have had access to xerox copies of these manuscripts.

1 *Introduction*

1.1 *Nārāyaṇa and his works*

Nārāyaṇa (usually called Nārāyaṇa Paṇḍita), son of Nṛsiṃha (or Narasiṃha), composed one book each in the two major fields of Indian mathematics: *Gaṇitakaumudī* ('Moonlight of Mathematics') in *pāṭī-gaṇita* or the mathematics of algorithm, and *Bījagaṇitāvataṃsa* ('Garland of Seed-Mathematics') in *bīja-gaṇita* or the mathematics of seeds (equations).

He flourished in the middle of the fourteenth century of the Christian era. This is known from the colophonic verse of the *Gaṇitakaumudī*, which states that it was completed on Thursday, the tithi (lunar day) called Dhātṛ of the dark half of Kārtika month in Śaka year 1278 called Durmukha (in the sixty-year cycle),[1] which corresponds to 10 November 1356. The sphere of his activities was presumably somewhere in North India. This is

[1] gajanagaravimitaśāke durmukhavarṣe ca bāhule māsi / dhātṛtithau kṛṣṇadale gurau samāptigataṃ gaṇitam // Cf. CESS A3, 156b–157a.

inferred from the distribution of the available manuscripts of the two works.[2]

The *Gaṇitakaumudī* is a full-scale work on *pāṭī-gaṇita*, comprising the 'basic operations' (*parikarman*) and the 'procedures' (*vyavahāra*). Besides the eight traditional topics of the 'procedures',[3] Nārāyaṇa included in it the pulveriser (*kuṭṭaka*), which is a solution method for linear indeterminate equations of the type $y = (ax + c)/b$, the square nature (*vargaprakṛti*) or quadratic indeterminate equations of the type $Px^2 + t = y^2$, the acquisition of parts (*bhāgādāna*) or factorization, the manifestation of fractions (*aṃśāvatāra*) or partitioning, chains of digits (*aṅkapāśa*) or combinatorics, and the mathematics of magic squares (*bhadragaṇita*). The last four topics deserve special attention. None of them was new but they were greatly developed by Nārāyaṇa.

The *Gaṇitakaumudī* has been edited by Padmākara Dvivedī based mainly on a single manuscript which he inherited from his father, Sudhākara Dvivedī (Dvivedī 1936/42). He also had a Nepal manuscript of the same work at hand, but has mentioned it only sporadically in his edition.[4] The last two chapters, on combinatorics and on magic squares, have been critically edited with an English translation and a mathematical commentary by Takanori Kusuba (Kusuba 1994). The entire work has been translated from P. Dvivedī's edition into English by Paramanand Singh (Singh 1998/2002).

The *Bījagaṇitāvataṃsa* is a work on *bījagaṇita* modeled on the *Bījagaṇita* (A. D. 1150) of Bhāskara II. It is divided into two major parts. Part I deals with operations involving positive and negative numbers, zero, unknown quantities, and surds, and with the pulveriser and the square nature. These are the basic mathematical means necessary for treating the 'quartet of seeds' (*bīja-catuṣṭaya*) or the four types of equations that are the subjects of Part II.

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[2] See Kusuba 1994, 1–3.

[3] They are: mixture (*miśra*), mathematical series (*śreḍhī*), plane figures (*kṣetra*), ditches (*khāta*), brick-piling (*citi*), timber-sawing (*krākaca*), heaped-up grain (*rāśi*), and shadows (*chāyā*).

[4] See Dvivedī 1936/42, Part I, Bhūmikā, lines 9–10, where he refers to 'a manuscript obtained from Nepal' (*nepālaprāptapustaka*), and Part II, p. 165, where he cites a passage comprising several verses from it. The variants (*pāṭhāntara*) cited by him in Part I (pp. 25, 27, 32, 33, 44, 45, 49, and 53), too, are most probably from the same manuscript.

The *Bījagaṇitāvataṃsa* has been edited by K. S. Shukla (Shukla 1970) based on a single incomplete manuscript, which is a copy of the Benares manuscript that B. Datta used for his article on the contents of Part I of the work (Datta 1933). These manuscripts extend only up to the middle of the first example for the first 'seed', that is, linear equations with one unknown quantity, stated at the outset of Part II. Recently, Professor Pingree found another manuscript in the Sanskrit collection of the Benares Sanskrit College, which is also incomplete but extends up to the middle of the commentary on the fortieth example for the first 'seed'. The new manuscript therefore reveals that the *Bījagaṇitāvataṃsa* contained more examples than Bhāskara II's *Bījagaṇita*, which has twenty-five examples for the first 'seed'.

Notation

BGA = *Bījagaṇitāvataṃsa.* **B1** = A manuscript of the BGA (see §1.3 below). **B2** = A manuscript of the BGA (see §1.3 below). **L** = A manuscript of the BGA (see §1.3 below). **S** = Shukla's edition of the BGA, Part I. **GK** = *Gaṇitakaumudī* edited by P. Dvivedī. **Singh** = Paramanand Singh's translation of the GK. **uv** = unnumbered verse in the GK. The GK contains a number of unnumbered verses. In order to specify a verse in it, therefore, I provide the page number of the Sanskrit text and, when appropriate, I supply the verse number together with the chapter number (in Roman numerals) in Singh's translation as well. For example, GK prakīrṇaka (uv) (Part I, p. *m*) = Singh I, Ex. *n*. **Verse numbers**: The first, second, etc. lines of verse number *n* are respectively denoted by *n*a, *n*b, etc.; verse numbers for examples are put in parentheses when examples are numbered serially independently of the rules.

1.2 *Chronology of Nārāyaṇa's works*

Shukla, in his Introduction to his edition of the *Bījagaṇitāvataṃsa*, has pointed out 'a reference in the *Gaṇitakaumudī* to the author's *Bījagaṇita*',[5] and concluded that 'the *Bījagaṇitāvataṃsa* which is mentioned in the *Gaṇitakaumudī* was evidently written prior to this date', that is, before A. D. 1356 (Shukla 1970, iii). Strictly speaking, the reference occurs in the prose commentary, and therefore it only proves that the *Bījagaṇitāvataṃsa* was writ-

[5] GK, Part I, p. 13, lines 15–17, where occurs the phrase, *asmadīye bījagaṇite* ('in our ⟨book on⟩ *bīja-gaṇita*').

ten before the commentary of the *Gaṇitakaumudī*. On the other hand, although Shukla has not pointed this out, the prose commentary of the *Bījagaṇitāvataṃsa* cites six verses, at least, of the *Gaṇitakaumudī*.[6] This fact proves that the *Gaṇitakaumudī* was written before the commentary of the *Bījagaṇitāvataṃsa*. I therefore conclude that both commentaries (in prose) were written after the *Gaṇitakaumudī* and *Bījagaṇitāvataṃsa* (in verse).

There are two sections common to the two works, namely those on the pulveriser and on the square nature, and both commentaries on these sections are almost the same although there are a number of minor variants which seem to be due to the copyists. I therefore conclude that the two commentaries (or the prose parts) were composed by one and the same person, who may or may not be Nārāyaṇa.

Kusuba, in his Introduction to his edition of the last two chapters of the *Gaṇitakaumudī*, has argued that Nārāyaṇa's authorship of the commentary of the *Gaṇitakaumudī* is doubtful at least for some part of it. This is deduced from several discrepancies between the metric part (usually called the *mūla* or 'root') and the prose commentary, and the commentator's misunderstanding of some of the examples (Kusuba 1994, 201–202). Our commentary too appears to misunderstand an example (see BGA II.(35)). It may therefore seem at first sight to follow that some person other than Nārāyaṇa wrote the two prose commentaries on Nārāyaṇa's metrical works.

It seems, however, to me to be also possible that the commentator was Nārāyaṇa himself, because the discrepancies and misunderstandings can be explained in other ways. Like other Sanskrit mathematical works, Nārāyaṇa's works contain rules and examples made by his predecessors (see, for example, BGA II.(30)). Even though he was a talented mathematician, we cannot completely deny the possibility that Nārāyaṇa misunderstood some of those rules and examples. At the same time, it is also probable that the available manuscripts of the two commentaries contain

[6] GK prakīrṇaka 17 is cited in the comm. on BGA I.(12) (S, p. 11, line 16), prakīrṇaka 35 in the comm. on BGA I.(50) (S, p. 25, lines 3–4), prakīrṇaka 60 in the comm. on BGA II.(1)–(2), śreḍhī 2 in the comm. on BGA II.(13), (14), (15), and (16), śreḍhī 1 in the comm. on BGA II.(19) and (29), kṣetra 53 in the comm. on BGA II.(41), and kṣetra 34 (?) in the comm. on BGA II.(42).

various textual changes, some of which may have caused the apparent discrepancies and misunderstandings.

1.3 *Manuscripts of the* Bījagaṇitāvataṃsa

B1: No. 35579 in the Catalogue of the year 1963 (= No. 94 in the Catalogue of the year 1878), Sarasvati Bhavana, Benares Sanskrit College. Ff. 1–13. Incomplete: Up to the first word of Verse (2) of Part II. Size: 10.8 × 4.4 inches. Script: Devanāgarī. 12 to 16 lines to a page. About 50 to 70 letters (*akṣaras*) to a line. Written roughly in letters of different sizes from line to line and from page to page. Most of the lines of writing are not straight. A dot is used for punctuation. The double *daṇḍa* is used for marking verses and sections. The *avagraha* is used only sporadically. In each of Parts I and II, the verses for rules (*sūtra*) have been numbered separately from those for examples (*udāharaṇa*).

This manuscript seems to be the one used by B. Datta for his article on the mathematical contents of Part I of the BGA. He refers to it as 'No. 2298 in the collection of the Sanskrit College, Benares' (Datta 1933, 475).

B2: No. 98699 (Acc. No. 104591), Sarasvati Bhavana, Benares Sanskrit College. Ff. 1–42. Incomplete: Up to the middle of the commentary on Verse (42) of Part II. Size: 9.9 × 4.5 inches. Script: Devanāgarī. 9 lines to a page. About 38 letters (*akṣaras*) to a line. Written neatly but not free from trivial errors such as loss of the vowel signs, of the *visarga* and of the *anusvāra*. They occur more often in B2 than in B1. No *daṇḍa* is used, but some of the verses are punctuated by the numerical sign (digit) for 1. The *avagraha* is only sporadically used. A gap one letter long is indicated by a short horizontal stroke like a *mātrā* line of a letter. The verses are numbered only at the beginning of each of the sections for the introduction, for the six kinds of operations involving positive and negative quantities, for those involving *karaṇīs*, for the *kuṭṭaka*, and for the *vargaprakṛti* in Part I, and at the beginning of Part II. Orthographic peculiarities: Spells cha for ccha and ṣṭha, tka for kta and vice versa, dya for gha, dhva for bdha, and bhda for dbha. The letters, ca and ya, often resemble va and pa, respectively. Phonological peculiarities: Reduplicates consonants after r (internally vivarjjita, varttate,

apavarttana, vargga, etc.; externally chidorllopaḥ), and before r (kkriyate). Does not clearly distinguish ja and ya (jāta and yāta), and ba and va. Tables and figures are usually written 'in line', but when the remaining space on the line is not enough for them, the space is filled with the next sentence and the table or figure is shifted to the beginning of the next line.

L: The manuscript used by Shukla for his edition of the BGA. This is said to have been acquired by A. N. Singh and deposited in the Lucknow University (Shukla 1970, i), but the details of the ms. are unknown. According to Shukla's edition, this manuscript, too, is incomplete and available only up to the end of Verse (1) of Part II. I have not had access to this manuscript, but most probably this is a copy of B1 for the reasons stated in the next section.

Another manuscript of the BGA exists (see CESS A3, 156b) but it has not so far been available to me.

1.4 *Relationships of the manuscripts*

L and B1:

The manuscript L is a copy of B1. This is proved by the fact that some of the corrections made in B1 have been overlooked by the scribe of L.

When he noticed a lacuna in the passage that he had just written down, the scribe of B1 added the letters to be inserted, with the delimiters $X \cdots X$ in the next line immediately below the proper place, which is indicated by the wedge-like symbol $\wedge$.[7] They occur twelve times on the extant 13 folios (once each on fols. 3b, 4a, 5a, 6a, 8a, 9a, 12b, and 13a, twice on fol. 4b, and three times on fol. 5b). The shortest one consists of only one letter but the longest one (on fol. 8a) comprises more than one line of writing (about 70 letters). Some of those lacunae corrected in B1 seem to have been made by the scribe of B1 himself.

Three of those corrections (on fols. 3b, 6a, and 12b) have been overlooked by the scribe of L. The first of the three instances, for example, takes place in a passage on the notation of unknown

[7] A similar method of correction is used in a Deccan College manuscript of Śrīpati's *Jyotiṣaratnamālā*. See Panse 1957, 47.

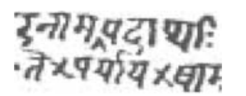


Figure 1: Insertion of *paryāya* between *-nāma* and *padārthāḥ* in B1.

quantities.[8] It reads as follows in Shukla's edition (p. 8, lines 3–6):

> *athavā asadṛśaprathamākṣaranāmapadārthāḥ kalpyante / eṣu samajātyorbahūnāṃ vā yogaviyogau kāryau / asadṛśajātyorbahūnāṃ vā varṇānāṃ pṛthaksthitiḥ syāt / teṣāṃ paryāyānāmapyuktānāmṛṇadhanayogādyupalakṣaṇaṃ rūpavadbhavatīti /*

The first underlined phrase seems to be exactly the same in L as Shukla gives no variant here, but in B1 (fol. 3b) it reads:

> *asadṛśaprathamākṣaranāma*$_{\wedge}$*padārthāḥ*,

where the wedge-like mark $\wedge$ indicates a lacuna and the letters to be inserted there are written down with the delimiters $X \cdots X$ in the next line of writing between *te* and *ṣā* of the second underlined phrase (see Fig. 1):

> *teXparyāyaXṣāmavyuktānāṃ.*

The scribe of L, having overlooked the wedge-like mark in the previous line, did not realize that this was a correction and wrote down the word *paryāya* as it was, ignoring the delimiters (S, p. 8, fn. 2):

> *teparyāyaṣāmadyuktānāṃ*,

which Shukla emended in order for the phrase to make sense. The correct readings of both phrases are preserved in B2 (fols. 4b–5a):

> 1st underline: *asadṛśaprathamākṣaranāmaparyāyapadārthāḥ*,
>
> 2nd underline: *teṣāmavyaktānāṃ.*

In two places in B1 (fols. 2a and 7a), missed words were added later in the margin either by the scribe himself or by someone else,

[8] For the context, see fn. 6 in Section 4.



Figure 2: Correction of "saḥnyā" with the digits, 1 and 2, in B1.

and one of the two instances has been overlooked by the scribe of L. The first line of the five verses that prescribe a method for the root-extraction of a number consisting of an integer and several surds reads in Shukla's edition as follows (p. 22, line 2):

karaṇīvarge niyamaḥ saṅkalitamitāni (khaṇḍakāni) syuḥ,

where the parentheses indicate that the word *khaṇḍakāni* has been added by the editor, meaning that L does not have that word. On the other hand, B1 (fol. 7a) reads:

karaṇīvarge niyamaḥ saṅkalitamitāni ∨ *syuḥ*,

where the inverted wedge indicates a lacuna and the word *khaṃḍāni* accompanied by the mark *X*3 (where '3' seems to mean that the line to be corrected is the 3rd from the bottom), is written in the right margin, although this correction itself is not exact. Shukla's restoration is supported by B2 (fol. 11b).

When he mistakenly interchanged two adjacent letters of a word, the scribe of B1 wrote down the digits 1 and 2 above the two letters in order to show their correct order.[9] Three instances of this type of correction occur in B1:

2 1
saḥnyā for *nyāsaḥ* (fol. 2b, see Fig. 2),

2 1
gukaṇakas (where the latter ka is superfluous) for *guṇakas* (fol. 10a),

2 1
kṛpratiḥ for *prakṛtiḥ* (fol. 12b).

L reads the first instance as seḥnyo (S, p. 5, fn. 3), regarding the digit 2 as the vowel sign -e and the digit 1 together with the sign for -ā (the vertical stroke) as the vowel sign -o.

Naturally L and B1 have a number of common variants and errors, and many (about 40) of the scribal errors in L can be

[9] This type of correction, too, occurs in the Deccan College manuscript of Śrīpati's *Jyotiṣaratnamālā* mentioned above. See Panse 1957, 47.

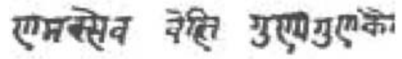


Figure 3: Indistinct writing of -ṇamastyeva, vetsi, and guṇyaguṇakau in B1.

explained as having been caused by the indistinct or peculiar writing of B1. For example, asseva (S, p. 1, fn. 3), vesi (p. 2, fn. 3), and guṇāguṇakau (p. 3, fn. 4) of L have no doubt been caused respectively by astyeva, vetsi, and guṇyaguṇakau written indistinctly in B1 (fols. 1b and 2a, see Fig. 3).

The scribe of L not only copied B1 but also in a sense edited it. Thus he sometimes rewrote words of B1, added new words, and changed the order of words in order to make the expressions of B1 clearer. Compare the readings of L and B1 in the following two cases.

Case 1:

B1 (fol. 3a, line 2):
nyāsaḥ 0 etatkhena yutaṃ jātaṃ / 0 varjitaṃ 0 ...
L (reconstructed from S, p. 7, line 6):
nyāsaḥ — rū 0 / etatkhena yutaṃ jātaṃ 0 / khena rahitaṃ jātaṃ 0 / ...

Case 2:

B1 (fol. 5b, line 8):
punaretāḥ ka 64 ka 36 ka 16 ka 4 dvikena chinnāḥ āsāṃ mūlayutiḥ 20 ...
L (reconstructed from S, p. 17. lines 2–3):
punaretāḥ dvikena chinnāḥ ka 64 ka 36 ka 16 ka 4 āsāṃ mūlayutiḥ 20 ...

In the first case, the scribe of L added the *rū*, the second *0*, and the second *jātaṃ*, and rewrote *0 varjitaṃ* as *khena rahitaṃ*. In the second case, he moved the phrase *dvikena chinnāḥ* ('divided by two'), before *ka 64 ka 36 ka 16 ka 4*, which is the result of the calculation indicated by that phrase.

B1 and B2:

B1 and B2 are akin to each other. They contain a number of common variants and errors in all senses, that is, mathematical,

(a) X — B2 — (Y) — (B1) — L — S

(b) X < (Y1) — B1 — L — S
(Y2) — B2

Figure 4: Two possible pedigrees of the three manuscripts and Shukla's edition.

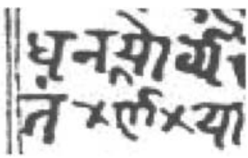


Figure 5: Addition of rṇa indicated by ∧ and *X* ... *X* in B1.

grammatical, metrical, syntactical, and orthographical. B2 is not a copy or a descendant of B1 since the former is longer than the latter. It is not certain whether B1 has descended from B2 (Fig. 4a) with or without an intermediary (Y) or B1 and B2 have descended from a common ancestor (Fig. 4b) with or without intermediaries (Y1, Y2).

Like the scribe of L, the scribe of B1 too seems to have edited the text in a sense by filling the lacunae he found in the parent manuscript with appropriate words.

In an example for the multiplication involving one unknown quantity, B1 (fol. 4a) reads:

guṇakasya dhana$_\wedge$*yorvyatyāse nyāsaḥ*,

where the wedge mark indicates a lacuna and the word to be inserted, *X*rṇa*X*, is written in the next line (see Fig. 5).[10] That the word *rṇa*, which is logically required in this passage,[11] did not exist in the ancestor of B1 and B2 (X in Fig. 4) is proved by the fact that the corresponding passage of B2 (fol. 5b) also lacks the same word. The scribe of B2 does not give any correction to

[10] This time, the scribe of L noticed the correction and restored the passage correctly (S, p. 10, line 10).

[11] The phrase, *dhanarṇayorvyatyāse*, refers to the inversion (*vyatyāsa*) of the positive (*dhana*) and the negative (*ṛṇa*) signs in the multiplier, '*yā* $\overset{\circ}{3}$ *rū* 5' (which corresponds to '$-3x+5$'); the result is '*yā* 3 *rū* $\overset{\circ}{5}$' (which corresponds to '$3x-5$'). The 'inversion of the two positive signs' is illogical in this case.

it. Moreover, he haplologically repeats the same incorrect phrase, *guṇakasya dhanayorvyatyāse*, at the end of the same paragraph.

In an example for the root-extraction of a number consisting of an integer and several surds, B1 (fol. 8a) reads:

> ... *guṇite karaṇīkhaṃḍe ka 24 ka 24* ∧ *yathākramaṃ nyaste* ...

where the wedge mark indicates a lacuna. The passage to be inserted,

> *khaṇḍanavake 'pi saṅkalitapadābhāvaḥ / ataḥ karaṇīyaṃ ka 108 ṣaḍvargeṇa bhaktā labdhaṃ 3 ṣaṇṇāmasame*[12] *khaṇḍe 4 / 2 anayorvargau 16 / 4 pūrvalabdhyā guṇitau jāte karaṇīkhaṇḍe ka 48 ka 12*,

is written in the next two lines with the delimiters, '$qX...Xq$', where 'q' stands for an unusual symbol consisting of a vertical stroke and a loop at its top. In this case too, B2 (fol. 13a) lacks exactly the same passage and therefore the lacuna goes back to the ancestor of B1 and B2 (X in Fig. 4). It may appear at first sight to be impossible for the scribe of B1 to insert such a long passage by himself. But in fact it was not difficult at all for one who understood the mathematical contents of the text he was copying, because the passage is logically required, and the mathematical procedure stated in the passage[13] has already been repeated twice immediately before this pasage. All that he had to do, therefore, was to replace the numerical figures in the previous passages with appropriate ones. In L and S (p. 24, lines 6–8) the passage has been embedded in the proper place.

In one place the scribe of B1 left a blank space about 13 letters long for a future addition of appropriate letters. Compare the following parallel passages in an example for the division of numbers consisting of several surds:

> B2 (fol. 10b):
> *anayā bhājye ka 2240 ka 210 ka 1920 ka 240 ka 120 guṇite* ...,
> B1 (fol. 7a):
> *anayā bhājye* [$\cdots$(blank space for about 13 letters)$\cdots$] *guṇite* ...,
> L (restored from S):
> *anayā bhājye* [$\cdots$(blank for a few letters?)$\cdots$] *guṇite* ...,

[12] L, followed by S, reads: *ṣaṇṇāṃ same*.

[13] The procedure is meant for rewriting a surd as the sum of two surds: $\sqrt{108} = \sqrt{6^2 \cdot 3} = 6\sqrt{3} = 4\sqrt{3} + 2\sqrt{3} = \sqrt{4^2 \cdot 3} + \sqrt{2^2 \cdot 3} = \sqrt{48} + \sqrt{12}$.

S (p. 20, ll. 19–20):
anayā bhājye (ka 2100 ka 1800 ka 1260 ka 1080) guṇite ....

In this case B2 does have a phrase corresponding to the blank in B1 but it is numerically wrong. Most probably the parent manuscript of B1 too had a numerically wrong phrase there and the scribe of B1, who noticed it, left enough blank space for the later insertion of the correct numerals, although he did not accomplish it (perhaps he simply forgot it). The numerals to be filled in there have been correctly restored in S within parentheses.

In another place the scribe of B1 'overcorrected' an expression of the parent manuscript by adding his own explanatory words. Compare the following parallel passages in an example for the multiplication of numbers consisting of several surds (*karaṇīs*, or numbers whose square-roots should be obtained):

B2 (fol. 8b):
pūrvachedena guṇitā jātāḥ karaṇīnāṃ yoga 800,
B1 (fol. 5b):
pūrvachedena 2 guṇitā jāto $_\wedge$*karaṇīnāṃ yogaḥ 800,*
and in the next line:
Xvargakaraṇīnāṃ yogaḥ 800 pūrvāvargaX,
L (restored from S):
pūrvacchedena 2 guṇitā jāte avargakaraṇīnāṃ yogaḥ ka 800,
S (p. 17, line 4):
pūrvacchedena 2 guṇitā jātaḥ avargakaraṇīnāṃ yogaḥ ka 800.

These passages refer to the last step of the following procedure: $\sqrt{128}+\sqrt{72}+\sqrt{32}+\sqrt{8}=\sqrt{64\cdot 2}+\sqrt{36\cdot 2}+\sqrt{16\cdot 2}+\sqrt{4\cdot 2}=8\sqrt{2}+6\sqrt{2}+4\sqrt{2}+2\sqrt{2}=20\sqrt{2}=\sqrt{400\cdot 2}=\sqrt{800}$. The above passage of B2 (with the minor corrections, *jātāḥ* to *jātaḥ* and *yoga* to *yogaḥ*) states:[14] '⟨The square of 20, that is 400, is⟩ multiplied by the previous divisor (2), and the sum, 800, of the *karaṇīs* (128, 72, 32, and 8) is produced.' This expression has no defect at all but the scribe of B1 seems to have felt the necessity of adding a few explanatory words for the *karaṇīs*. The passage of B1 with the correction suggested by the scribe states: '⟨The square of 20, that is 400, is⟩ multiplied by the previous divisor, 2, and the sum, 800,

[14] For the notation used in my English translations of Sanskrit texts, see the beginning of Section 3 below.

of the non-square[15] *karaṇīs*, that is, the sum, 800, of the previous non-square *karaṇīs* (128, 72, 32, and 8) is produced.' The scribe of B1 presumably meant to distinguish this sum from that of the 'square *karaṇīs*', that is, $\sqrt{64}+\sqrt{36}+\sqrt{16}+\sqrt{4}=20$, which has been obtained in the previous step, although the term 'square *karaṇīs*' does not occur there. The scribe of L adopted only the word *avarga* ('non-square') from that superfluous addition in B1.

From these we know that the scribe of B1, being interested in the mathematical contents of the text he was copying, corrected the text when he felt it necessary. The casual manner of writing of B1 (see under B1 in §1.3) may have been caused by the scribe's concentration upon the contents of the work he was copying. The scribe of B2, on the other hand, tried to copy the text faithfully. In either case of the two possible pedigrees (Fig. 4), therefore, B2 seems to preserve more older forms than B1 does.

1.5 *Table of contents of the* Bījagaṇitāvataṃsa

Part I			
Verse numbers[a]			
S(L)	B1	B2	Contents
1. Introduction.			
1	1	1	The author salutes Brahman and the seed-
·	·	·	mathematics (*bījagaṇita*) and lays stress on
·	·	·	the importance of numerical work (*sāṅkhya-*
·	·	·	*karman*), procedure (*karaṇa*), and seed-work
6	6	6	(*bīja-kriyā*) which treats unknown quantities.
2. Six kinds of operations involving positive and negative quantities.			
7	7	7	Rule for addition of positive and negative quantities.
8	8	8	(Continued)
(1)	(1)	9	Ex.[b]
9a	9a	–	Rule for subtraction of positive and negative quantities.

To be continued.

[15] *jāto varga-* for *jāto 'varga-*. The use of the avagraha in B1, as well as in B2, is not consistent.

[a] The symbol – in the first three columns indicates that the verse exists but is not numbered.

[b] All examples in Part I, except S(35), are purely numerical.

S(L)	B1	B2	Contents
(2)	(2)	–	Ex.
9b	9b	–	Rule for multiplication of positive and negative quantities.
(3)	(3)	–	Ex.
10a	10a	–	Rule for division of positive and negative quantities.
(4)	(4)	–	Ex.
10b	10b	–	Rule for square/square-root of positive and negative quantities.
10c	11a	–	(Continued)
(5)	(5)	–	Ex.
3. Six kinds of operations involving zero.			
11a	11b	–	Rule for addition and subtration involving zero.
11b	12a	–	(Continued)
(6)	(6)	–	Ex.
⟨12a⟩[a]			Rule for multiplication involving zero.
⟨–⟩[b]			Ex.
12b	12b	–	Rule for division involving zero.
13	13	–	(Continued)
(7)	(7)	–	Ex.
14	14	–	Operation involving both multiplication and division by zero.
15[c]	15	–	Nature of 'zero-divisor'.[d]
4. Six kinds of operations of zero (exclusively).			
15[e]	16	–	Rule for addition, etc. of zero.
(8)	(8)	–	Ex.
5. Six kinds of operations involving unknown quantities.			
17	17	–	Symbols for unknown quantities.
18	18	–	(Continued)
19	19	–	Rule for addition and subtraction involving unknowns.
20	20	–	(Continued)
(9)	(9)	–	Ex. (for one unknown)

To be continued.

[a] Supplied by the editor of S.
[b] Supplied by the editor of S.
[c] 15.2 omitted in L but supplied by the editor of S.
[d] Quoted from BG 20.
[e] The number 15 mistakenly repeated.

S(L)	B1	B2	Contents
(10)	(10)	–	Ex. (for one unknown)
21	21	–	Rule for multiplication involving unknowns.
22	22	–	(Continued)
(11)	(11)	–	Ex. (for one unknown)
23	23	–	Rule for division involving unknowns.
(12)	(12)	–	Ex. (for the square involving one unknown)
24	24	–	Rule for square-root involving unknowns.
(13)	(13)	–	Ex. (for more than one unknown)
(14)	(14)	–	Ex. (for more than one unknown)
6. Six kinds of operations involving *karaṇīs*.			
25	25	1	Rule for addition and subtraction of *karaṇīs*.
⋮	⋮	⋮	(Continued)
30	30	6	
(15)	(15)	–	Ex.
31	31	–	Rule for multiplication of *karaṇīs*.
32	32	–	(Continued)
(16)	(16)	–	Ex.
(17)	(17)	–	Ex.
33	33	–	Rule for division of *karaṇīs* (Method 1).
⋮	⋮	⋮	(Continued)
35	35	–	
36	36	–	Rule for partition of a *karaṇī*.
(18)	(18)	–	Ex.
37	37	–	Rule for division of *karaṇīs* (Method 2).
38	38	–	(Continued)
39	39	–	Rule for square of *karaṇīs*.
40	40	–	(Continued)
(19)	(19)	–	Ex.
(20)	(20)	–	Ex.
41	41	–	Rule for the square-root of *karaṇīs* (Method 1).
⋮	⋮	⋮	(Continued)
45	45	–	
46	46	–	Rule for the square-root of *karaṇīs* (Method 2).
⋮	⋮	⋮	(Continued)
49	49	–	
50	50	–	Rule (supplementary): Number of *karaṇī* terms in the root.
(21)	(20)[a]	–	Ex.

To be continued.

[a] The number (20) mistakenly repeated.

S(L)	B1	B2	Contents
51	51	–	Additional rule for the square-root of *karaṇīs*.
(22)	(21)	–	Ex.
(23)	(22)	–	Ex.
(24)	(23)	–	Ex.
(25)	(24)	–	Ex.
52	–	–	Additional rule for the square-root of *karaṇīs*.
7. *Kuṭṭaka* (pulveriser).			
53	53	–	Rule (main).
⋮	⋮	⋮	(Continued)
59	59	7[a]	
(26)	(25)	–	Ex.
60	60	–	Additional rule for *kuṭṭaka*.
(27)	(26)	–	Ex.
(28)	(27)	–	Ex.
61	61	–	Additional rule for *kuṭṭaka*.
62a	62[b]	–	(Continued)
(29)	(28)	–	Ex.
62b	63a	–	Additional rule for *kuṭṭaka*.
62c	63b	–	(Continued)
(30)	(29)	–	Ex.
(31)	(30)	–	Ex.
63	64	–	Additional rule for *kuṭṭaka*.
(32)	(31)	–	Ex.
(33)	(32)	–	Ex.
(34)	(33)	–	Ex.
64	65	–	Rule for the constant *kuṭṭaka*.
65	66	–	Rule (supplementary): *kuṭṭaka* and *trairāśika*.
⋮	⋮	⋮	(Continued)
68a	69a	–	
(35)	(34)	–	Ex. Distance traveled by a lame man and time for it.
68b	69b	–	Rule for the fractional *kuṭṭaka*.
68c	70a	–	(Continued)
(36)	(35)	–	Ex.
69a	70b	–	Additional rule for the fractional *kuṭṭaka*.
69b	71a	–	(Continued)

To be continued.

[a] The verses numbered 55, 57, and 59 in S and B1 are numbered in order 3, 5, and 7 in B2.

[b] Verse 62 in B1 is a half stanza.

S(L)	B1	B2	Contents
–[a]	71b	–	Notice.[b]
8. *Vargaprakṛti* (square nature).			
70	72	1[c]	Rule (main).
⋮	⋮	⋮	(Continued)
76	78	7	
(37)	(36)	–	Ex.
77	72	–	Rule for the *cakravāla* (cyclic method).
⋮	⋮	⋮	(Continued)
80	82	4[d]	
(38)	(37)	–	Ex.
81	83	–	Additional rule for *vargaprakṛti.*
(39)	(38)	1	Ex.
(40)	(39)	–	Ex.
82	84	–	Additional rule for *vargaprakṛti.*
(41)	(40)	–	Ex.
83	85	–	Additional rule for *vargaprakṛti.*
(42)	(41)	–	Ex.
(43)	(42)	–	Ex.
(44)	(43)	–	Ex.
84	86	–	Additional rule for *vargaprakṛti.*
85	86	–	Importance of intellect.
(45)	(44)	–	Ex.
86	87	–	Application of *vargaprakṛti* to the root approximation.
(46)	(45)	–	Ex.

End of Part I

Part II

1. Introduction.			
1	1	–	Salutation to Śiva and *bījagaṇita.*
2	2	–	Four seeds.
2. Seed 1: Equation procedure with one unknown.			
3	3	–	Rule 1: Main rule of Seed 1.

To be continued.

[a] This is regarded as a prose passage in S.

[b] Promises to discuss the 'skill of *kuṭṭaka*' (*kuṭṭaka-kauśalya*) in the chapter for the 'Equation procedure with more than one unknown' (Seed 2).

[c] The verses numbered 70–76 in S are numbered 1–7 in B2 except for S 72, which is left unnumbered.

[d] S 80 is numbered 4 in B2.

S(L)H[a]	B1	B2	Contents
⋮	⋮	⋮	(Continued)
7	7	–	
(1)	(1)	–	Ex. 1: Equation of properties (horses and money).

End of S(L) and B1

(2)		–	(Continued)
(3)		–	Ex. 2: Equation of properties (jewels and money).
(4)		–	Ex. 3: Yearly donations of money to a brāhmaṇa.[b]
(5)		–	Ex. 4: A lotus stalk sunken in layers of mud, moss, and water.
8		–	Rule 2 (supplementary): Division of fractions.
(6)		–	Ex. 5: The number of wasps.
(7)		–	Ex. 6: Equations after give and take.
(8)		–	Ex. 7: Capital and interest.
(9)		–	Ex. 8: Equation of capitals, and of capitals ± interests.[c]
(10)		–	Ex. 9: Equation after exchange of interests on two capitals.[d]
(11)		–	Ex. 10: Two related loans.
(12)		–	Ex. 11: Purchase of commodities in a given proportion.[e]
(13)		–	Ex. 12: Equation of journeys (chase of a mistress by a procuress).[f]
(14)		–	Ex. 13: Arithmetical progression (first term required).
(15)		–	Ex. 14: Arithmetical progression (common diff. required).

To be continued.

[a] H = Hayashi's edition of Part II of the BGA. See Section 2 below.

[b] Similar to the problem of GK miśra (uv) (Part I, p. 62) = Singh II, Ex. 15.

[c] Identical with the problem of GK miśra (uv) (Part I, p. 61) = Singh II, Ex. 14.

[d] Identical with the problem of the first of the two problems of GK miśra (uv) (Part I, p. 63) = Singh II, Ex. 16.

[e] Identical with the problem of GK miśra (uv) (Part I, p. 58) = Singh II, Ex. 8.

[f] Identical with the problem of GK śreḍhī (uv) (Part I, p. 112) = Singh III, Ex. 10.

H	B2	Contents
(16)	–	Ex. 15: Equation of journeys (meeting of two travelers).[a]
(17)	–	Ex. 16: Trade of musk and camphor.
(18)	–	Ex. 17: Arithmetical progression (first term and common diff. required).
(19)	–	Ex. 18: Equation of journeys (two meetings of two travelers).[b]
(20)	–	Ex. 19: Four purely numerical problems.
(21)	–	Ex. 20: Equation of the area of a right triangle and its sides.
(22)	–	Ex. 21: Purely numerical problem.
(23)	–	Ex. 22: Purely numerical problem.
(24)	–	Ex. 23: Purely numerical problem.
(25)	–	Ex. 24: Purely numerical problem.
(26)	–	Ex. 25: Purely numerical problem.
(27)	–	Ex. 26: Two purely numerical problems.[c]
9	–	Rule 3 (supplementary): Making the sum and difference of square numbers.
10	–	Rule 4 (supplementary): Square of cube and cube of square.[d]
11	–	Rule 5 (supplementary): Making the sum of cubes a square.[e]
(28)	–	Ex. 27: Purely numerical problem.
12	–	Rule 6: Comment on Seed 3 (the elimination of the middle terms).
(29)	–	Ex.28: Weights of nine gold bangles (an arithmetical progression).
(30)	–	Ex. 29: Equations of properties after exchange of part of them.
(31)	–	Ex. 30: Prices of musk and camphor.
(32)	–	(Continued)
(33)	–	Ex. 31: False number of horses of four horse dealers.[f]
(34)	–	Ex. 32: A hoard and the properties of four people.[g]
(35)	–	Ex. 33: Bet on a cock-fight.

To be continued.

[a] Identical with the verse of GK śreḍhī (uv) (Part I, p. 110) = Singh III, Ex. 8.

[b] Nearly identical with the verse of GK śreḍhī (uv) (Part I, p. 111) = Singh III, Ex. 9.

[c] Same problems as GK prakīrṇaka (uv)'s (Part I, pp. 36 and 38) = Singh I, Exs. 53 and 54.

[d] Nearly identical with the rule of GK prakīrṇaka 23b.

[e] First half is identical with the verse of GK śreḍhī 14b.

[f] Numerically equal to GK miśra (uv) (Part I, p. 85) = Singh II, Ex. 34.

[g] Identical with the verse of GK miśra (uv) (Part I, p. 87) = Singh II, Ex.

H	B2	Contents
(36)	–	Ex. 34: Property of a traveling merchant.
(37)	–	Ex. 35: Capital and interest.
(38)	–	Ex. 36: A pond filled with water from four springs.[a]
(39)	–	Ex. 37: Donation of one thousand cows belonging to three herds.
(40)	–	Ex. 38: Arithmetical progression.
(41)	–	Ex. 39: Series figures.
(42)	–	Ex. 40: Fractional number of terms of an arithmetical progression.

End of H and B2

2 *Text*

[Notation for vacant places and negative numbers: In the manuscripts B1 and B2, as in many other manuscripts, vacant (*śūnya*) places in the decimal notation (now indicated by zeros) and those in tables (*nyāsa*) are indicated indiscriminately by a small circle (∘) or by a dot (•) (both called *bindu* or 'dot'). A *bindu* placed above a number also serves to indicate that it is negative (*ṛṇa*). In the present edition, I use a small circle (∘) as a sign for a vacant place in a table and to mark a negative number, and 0 for zero.]

[1]ataḥprabhṛti[2] bījam //

yasmādetatsakalaṃ[3] viśvamanantaṃ prajāyate vyaktam /
avyaktādapi bījācchivaṃ ca gaṇitaṃ sadā[4] naumi // 1 //
avyaktasamīkaraṇaṃ[5] varṇasamatvaṃ ca madhyamāharaṇam /
bhāvitasamatvamasmin bījāni vadanti[6] catvāri // 2 //

tatrādau tāvadavyaktasāmye karaṇasūtramāryāpañcakam[7] /

35.
[a] Nearly identical with the verse of GK miśra (uv) (Part I, p. 94) = Singh II, Ex. 39.

[1] Fol. 13a, l. 6, B1; fol. 27b, l. 6, B2; p. 45, l. 1, S. Hereafter, I use the notation: 'om.' = omit(s). '–' = a gap one letter long, which is indicated by a short *mātrā* line in B2.

[2] ataḥ abhṛti L; om. S.

[3] yasmāttatsakalaṃ B1, yasmātatsakalaṃ B2, yasmāttatasakalaṃ L.

[4] ca sadā B1, B2, L; ca taṃ S.

[5] -samīkaraṇī B2.

[6] -asmiṃ chījāni dadeti B2.

[7] -paṃcake B2.

yāvattāvaccihnitamekaṃ vā bahumitaṃ tu parikalpya[8] /
rūpāḍhyaṃ vā rūponitamathavāvyaktamānamiti[9] // 3 //
māne[10] tasminnevodde[11]śālāpavatsamācaret[12]karma /
phalasiddhyai[13] tau[14] pakṣau tulyau kāryau prayatnena // 4 //
ekasmād[15]avyaktaṃ viśodhayedanyatastu rūpāṇi /
śeṣeṇāvyaktena[16] ca samuddharedrūpaśeṣamiha // 5 //
avyaktasya[17] ca rāśermānaṃ vyaktaṃ prajāyate nūnam[18] /
ajñāteṣu bahuṣu vā[19] yāvat[20]tāvaddvikādi[21]saṃguṇitam[22] // 6 //
bhaktaṃ rūpairyuktaṃ vivarjitaṃ vā prakalpayedevam /
nijabuddhyā vijñeyaṃ[23] kvacidavyaktasya ca mānam[24] // 7 //

yasminneko varṇo vartate tadavyaktasāmyaṃ[25] bījaṃ yatra bahavo varṇā yāvattāvatkālakādayo[26] vartante[27] tadanekavarṇasāmyaṃ[28] yatra ca varṇānāṃ vargādisamīkaraṇaṃ kriyate[29] tanmadhyamāharaṇaṃ yatrāvyaktavarṇānāṃ[30] bhāvitasamīkaraṇaṃ tadbhāvitasamatvamiti bījacatuṣṭayamādyāḥ prāhuḥ /

teṣu prathame yāvad[31]avyaktasāmye[32] yatrodāharaṇe yo 'jñāto rāśistanmānaṃ[33] yāvadekaṃ dvyādi vā[34] sarūpamarūpaṃ vā rūpairūnaṃ

[8] bahumita rū parī kalpyā B1, bahumita rū airī kalpyā B2, bahumitaṃ rūparīkalpya L.
[9] rūponitayathavāvyakta – lamiti B2.
[10] mānai B2.
[11] Fol. 28a, B2.
[12] tasminnuvadaṃśālā ekavatsamāharet B1, B2, L (om. t of -vat-).
[13] phalasirddhiddhyai B2, corrected.
[14] dvau S.
[15] ekaikasmād- B1, B2, L.
[16] śeṣeṇāvyaktaiti B2, śeṣeṇāvyakta L.
[17] sya om. L
[18] ṃ om. B2.
[19] ajñā – – – – vā B2.
[20] yavat- B2.
[21] -tāvadvikādi- B1.
[22] m om. B2.
[23] vijñerya B2.
[24] mānaṃ 5 B2.
[25] tadekavarṇasāmyaṃ B2.
[26] -kāla – dayo B2.
[27] varttate B2.
[28] tadaceka- L.
[29] kriyate om. B2.
[30] -vaṇānā B2.
[31] tāvat- L, S.
[32] -sāmyeḥ B1, -avyakta – ṣṭe B2.
[33] rāśistasya mānaṃ B2.
[34] dvyādikaṃ vā B2.

yutaṃ vā[35] prakalpya tasya rāśervāsanānusāreṇoddeśakālāpavad[36]yogaviyogaguṇanabhajanatrairāśikapañcarāśikaśreḍhīkṣetrakhātādikriyayā dvau pakṣau samau kāryau / yasminnālāpe pakṣayoḥ[37] samatvaṃ na vidyate tadekapakṣaḥ kenacitsaṃguṇito bhakto yuto[38] varjito [39]vā[40] nijabuddhyā pakṣau samau kāryau[41] / samapakṣayorekapakṣāvyaktamanyapakṣādviśodhyānyapakṣarūpāṇītarapakṣarūpebhyo viśodhya[42] tatra yadrūpaśeṣamavyaktaśeṣeṇa haret / yallakṣitaṃ tadekasyāvyaktasya[43] mānaṃ vyaktam / tena kalpitarāśimutthāpya[44] vyakto rāśiḥ kriyate[45] /

yatra dvyādayo 'vyaktarāśayo[46] bhavan[47]ti tatraikamavyaktaṃ śeṣaṃ dvyādibhirabhīṣṭairguṇitaṃ[48] bhaktaṃ veṣṭai[49] rūpairūnaṃ yutaṃ vā yāvatprakalpayet[50] / athavaikamavyaktaṃ śeṣāṇi vyaktāni mānāni kalpyānīti //

udāharaṇāni /

samānamaulyā vaṇijo[51] 'ṣṭaghoṭā
ekasya rūpāṇi śatāni ṣaṭca[52] /
ṛṇaṃ[53] śate 'nyasya ca vājino 'rka-
mitāḥ samasvau[54] ca kimaśva[55]maulyam[56] // (1) //[57]
ādyo[58] 'ṣṭayukto dalitaḥ pareṇa

[35] sarūpamarūpaṃ vā rūpaṃ vā rūpairūnaṃ vā B2.
[36] -oddeśaḥ kālyāyavad- L.
[37] ḥ om. B2.
[38] yuto om. B2.
[39] Fol. 28b, B2.
[40] varjito to vā B2.
[41] kāryauḥ B1.
[42] anyapakṣādviśodhya vyaktavargādikamapi itarapakṣarūpāṇītarapakṣarūpebhyo piśodhya B1, anyapakṣādviśodhya vyaktavargādikamapi itarapakṣarūpāṇi itarapakṣe rūpebhyo piśodhya B2, anyapakṣādviśodhya vyaktavargādikamapi itarepakṣarūpāṇi itarapakṣarūpebhyo piśodhya L.
[43] -ktasva L.
[44] rāśirutthāpya B1, B2, S; rāśirupāpya L.
[45] kriyato B2.
[46] vyaktarāyo B2.
[47] Fol. 13b, B1.
[48] śeṣadvyādirabhīṣṭairguṇitaṃ B2.
[49] vā iṣṭai B1, B2.
[50] yāvattāvatprakalpayet L, S.
[51] vaṇiyo B2.
[52] ṣaṭva B2.
[53] ṛṇe S.
[54] svasvau L, samau tau L.
[55] kimasva- B2.
[56] -mūlyaṃ B1.
[57] S and L end here.
[58] B1 ends here.

tulyo bhavedvāpi tathāparasya /
trighnasya[59] pūrvaścaturūnitasya
samo bhavenme[60] vada vājimaulyaṃ // (2) //

atrāśvamaulyamajñātaṃ[61] / tasya mānaṃ yāvattāvat yā 1[62] / atra trairāśikam / yadyekāśvasya[63] maulyaṃ yāvattāvanmānaṃ[64] tadāṣṭānāṃ kimiti / nyāsaḥ 1 yā 1 8[65] /

ādyantayoḥ pramāṇecche[66] samajātī phalaṃ tvitarajāti /
madhye tadantyatāḍitamā[67]dyahṛdicchāphalaṃ[68] bhavati //[69]

iti trairāśikena jātamaṣṭānāṃ maulyaṃ yā 8 / etadrūpaśataṣaṭke[70] prakṣipya jātamādyasya dhanaṃ[71] yā 8[72] rū 600 / punaryadyekasyāśvasya yā 1[73] tadā dvādaśānāṃ kimiti / 1 yā 1 12 / jātaṃ dvādaśānāmaśvānāṃ[74] mūlyaṃ yā 12 / etadṛṇe rūpaśatadvaye[75] prakṣipya jātamanyapuruṣasya dhanaṃ yā 12 rū $\overset{\circ}{2}00$[76] / pakṣāvetau samāviti śodhanārthanyāsaḥ

yā	8	rū	600
yā	12	rū	$\overset{\circ}{2}00$[77]

ekasmādavyaktaṃ viśodhayedanyatastu rūpāṇi /[78]

iti prathamapakṣāvyaktaṃ dvitīyapakṣāvyaktācchodhitaṃ śeṣaṃ yā 4 / dvitīyapakṣarūpāṇi prathamapakṣarūpebhyo[79] viśodhya śeṣaṃ 800 / punastrairāśikam / yadi yāvaccatuṣṭayasyāṣṭau śatāni maulyaṃ tadā

[59] trighasya B2.
[60] -bheveme B2.
[61] atrākhamaulyasajñātaṃ B2.
[62] -tāvat 5 yā 1 B2.
[63] yadekaśvāsya B2.
[64] yāvattāvanmūlyaṃ B2.
[65] nyāsa $\overset{\circ}{1}$ yā 1 8 B2.
[66] pramāṇe dalaṃ cche B2.
[67] Fol. 29a, B2.
[68] ādyaharitaṃ icchāphalaṃ B2.
[69] GK prakīrṇaka 60 (Part I, p. 47).
[70] -rūpaśate B2.
[71] dhaṃnaṃ B2.
[72] 5 (for 8) B2.
[73] yāva (for yā 1) B2.
[74] -aśvānā B2.
[75] etadrūpādrūpaśatadvaye B2.
[76] o om. B2.
[77] o om. B2.
[78] BGA II.5.
[79] -pakṣerūpebhyo B2, corrected.

yāvadekasya kimiti yā 4 rū 800 yā 1 / labdhaṃ yāvattāvanmānaṃ 200 / etadekasyāśvasya[80] mūlyam 200 / anena yāvadutthāpya[81] jātaṃ prathamasya dhanam[82] 2200 / dvitīyasya 2200 /

atha kalpitamaśvamūlyaṃ yā 1 rū 1 / prāgvattrairāśikena labdhamūlyayoḥ svasvadhanena[83] yathoktavadyuktayoḥ[84] samaśodhanārthanyāsaḥ

yā	8	rū	608
			o
yā	12	rū	188[85]

prāgvallabdhaṃ yāvattāvanmānaṃ 199 / anena yāvadutthāpya[86] jātamaśvamūlyaṃ [87]tadeva 200 //

dvitīyodāharaṇe prathamakalpitayostayoreva dhane[88]

yā	8	rū	600
			o
yā	12	rū	200[89]

atra prathamasyāṣṭayutasyārdhaṃ[90] dvitīyasya dhanena[91] samaṃ kāryam / athavā dvitīyadhanena dviguṇenāṣṭavarjitena prathamadhanaṃ samaṃ kāryam / tayorapi samatvaṃ bhavati / tathā kṛte śodhanārthanyāsaḥ

yā	4	rū	304
			o
yā	12	rū	200

samaśodhane kṛta ubhayato 'pi labdhaṃ[92] yāvattāvanmānaṃ 63 / utthāpite jāte tayordhane 1104 / 556 //

tṛtīyodāharaṇe tayoreva dhane

yā	8	rū	600
			o
yā	12	rū	200

[93]

80 -ekasyaśvasya B2.
81 yāvadusyāpya B2.
82 dha om. B2.
83 svasvadhanaṃ B2.
84 yathoktavaṃdyuktayoḥ B2.
85 o om. B2.
86 bāvadrutthāpya B2.
87 Fol. 29b, B2.
88 -kalpanayattayoreva dhaṇe B2 (the last letter, ṇe, corrected).
89 o om. B2.
90 ṃ om. B2.
91 dhanaṃ B2.
92 ṃ om. B2.
93 In B2, this table is written between 'prathamo' (prathamasya in B2) and

atra prathamo 'parasya[94] triguṇasya caturūnasya sama ityālāpe[95] kṛte śodhanārthanyāsaḥ

yā	8	rū	600
yā	36	rū	6o04[96]

labdhaṃ yāvattāvanmānaṃ 43 / tenotthāpite dhane 944 / 316 //
udāharaṇam /

prabālagomedakanīlavajrā
ekasya puṃso 'rkadigaṣṭatarkāḥ /
parasya saikāśca śatena ṣaṣṭyā[97]
yuktau samau sto vada[98] ratnamūlyam[99] // (3) //

atra prabālādīnāṃ[100] mūlyāni yāvattāvadekādīni[101] kalpitāni yā 1 yā 2 yā 3 yā 4 / prāgvattrairāśikeṇa[102] sādhitamūlyayogayo[103] rūpeṣu prakṣiptayoḥ samaśodhanārthanyāsaḥ

yā	90	rū	60
yā	80	rū	100[104]

samakriyayā labdhaṃ yāvattāvanmānaṃ 4[105] / anenotthāpitāni jātāni[106] ratnamūlyāni 4 / 8 / 12 / 16 / utthāpite jāte samadhane 420 / 420 /

athavā prabālagomedanīlamaulyāni vyaktā[107]ni kalpitāni 15 / 10 / 5 / vajramūlyaṃ yā 1[108] / yathoktakaraṇena jātaṃ yāvattāvanmānaṃ 10 / ratnamūlyāni 15[109] / 10 / 5 / 10 / samadhane[110] 480 / 480 //

dattaṃ dhanaṃ dhanavatā[111] kiyadādyavarṣe

'parasya' of the next sentence. See the next footnote.

[94] atra prathamasya

yā	8	rū	600
yā	12	rū	200

parasya B2.

[95] ityāktoye B2.

[96] o om. B2.

[97] ṣaṣṭya B2.

[98] vade B2.

[99] -mūlyaṃ 1 B2.

[100] pravālādīni B2.

[101] yāvattovad- B2.

[102] prā 'vatrairāśikeṇa B2.

[103] -yogeyo B2.

[104] yā 8 rū 500 B2.

[105] 4 om. B2.

[106] yātāni B2.

[107] Fol. 30a, B2.

[108] vajrammayāyāva 1 B2, first 'yā' crossed out.

[109] 15 om. B2.

[110] samadhanaṃ B2.

[111] dhaṃnavatā B2.

viprāya kenacidapi dviguṇaṃ sarūpam /
varṣaṃ[112] prati pravada pañcabhireva varṣaiḥ
pañcāśaduttaraśataṃ[113] kimadātsvamādyam // (4) //[114]

atra prathame varṣe dattaṃ dhanaṃ yā 1 / etatprativarṣaṃ[115] dviguṇamekādhikamiti[116] prativarṣadhanaṃ yā 1 / yā 2 rū 1 / yā 4 rū 3 / yā 8 rū 7 / yā 16 rū 15 / eṣāṃ yogaḥ yā 31 rū 26 / ayaṃ pañcāśaduttaraśatasamaḥ[117] / samaśodhanārthanyāsaḥ

yā	31	rū	26
yā	0	rū	150

samaśodhane kṛte labdhaṃ yāvattāvanmānaṃ 4 / anenotthāpitāni[118] jātāni prativarṣadhanāni[119] 4 / 9 / 21 / 49 / 113[120] //

padmākaraprabhavavārijanālikāyāḥ[121]
paṅke dalaṃ śaralavastvapi[122] śaivale[123] ca /
digbhāgako 'mbhasi tato 'parahastayugmaṃ
jānāsi cedjhaṭiti me vada nālamānam[124] // (5) //

atra nālapramāṇaṃ[125] yā 1 / ataḥ paṅkaśaivalajalasthitaṃ[126] nālapramāṇaṃ yā $\frac{4}{5}$ / etaddhastadvayayutaṃ[127] yāvatsamamiti śodhanārthanyāsaḥ

yā	$\frac{4}{5}$	rū	2
yā	1	rū	0

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[112] ṃ om. B2.
[113] -śate b1.
[114] ≈ GK miśra (uv) (Part I, p. 62), where ca vidvan for sarūpam, and jātaṃ śataṃ ca ṣaḍaśītyadhikaṃ kim for pañcāśaduttaraśataṃ kimadātsvam.
[115] ṃ om. B2.
[116] -ekādikamiti B2.
[117] samaḥ om. B2.
[118] ateno- B2.
[119] prativarṭhyadhanāni B2.
[120] 4 9 9 39 79 B2.
[121] -nānikāyāḥ B2.
[122] -lavaḥ stvapi B2.
[123] śaivame B2.
[124] nīlamānaṃ 4 B2.
[125] nīlapramāṇa B2.
[126] paṃkaḥ śaivalajalasthiṃta B2.
[127] etadaṣṭhahatsaṃdvayayutaṃ B2.

samacchedīkṛtya [128]śodhanārtha ādyapakṣe hriyamāṇe[129] harāṃśayorviparyāse kṛte cchedāṃśaśchedo[130] bhavati / yataḥ samakṣepa eva guṇakāraḥ syādguṇakāra[131]guṇyakṣepayorapavartane 'pyavaśyatvaṃ bhavati / ataḥ[132] samacchedapakṣayoḥ[133] chedagamaḥ kriyate / tathā paribhāṣitaṃ kṛtam[134] /

samānahārau[135] bhājakabhājyau tayoḥ[136] chidorlopaḥ /
tadbhājakena bhājye hṛte[137] phalaṃ syādvibhinnahṛtau[138] // 8 //

iti jātaṃ yāvanmānaṃ 10 / etatkamalanālamānam[139] //

yātaṃ bhṛṅgacayāddalaṃ[140] kamalinīṃ[141] śeṣatribhāgastathā[142]
cūtaṃ śeṣacaturthakaśca bakulaṃ śeṣasya[143] pañcāṃśakaḥ /
protphullatkuṭajaṃ[144] ca ṣaṭkusuminīṃ jātīṃ ca ṣaṭkastadā
vidvante[145] kati me drutaṃ vada sakhe jānāsi bījaṃ yadi // (6) //

atra bhramaracayamānaṃ[146] yā 1 / ata uddiṣṭāñ[147]śeṣabhāgānapāsya[148] jātaṃ dṛśyaṃ yā $\frac{1}{5}$ / etad[149]dṛśyenānena 12[150] samamiti[151] sāmyakaraṇena labdhaṃ yāvattāvanmānaṃ 60 //

athavā yāvattāvati bhāgāpavāhavidhinā[152] nyaste jātaṃ

[128] Fol. 30b, B2.
[129] hrīyamāṇe B2.
[130] chedāṃśachedo B2.
[131] guṇakāṃrā- B2.
[132] -apavartane thavaśyaṃ bhavati yatvaṃttaḥ B2.
[133] -pakeyoḥ B2.
[134] paribhāṣpitaṃ kṛta B2.
[135] samāsahārau B2.
[136] -bhājyau ca tayoḥ B2.
[137] hate B2.
[138] -hatau B2.
[139] -kamalamānamānaṃ B2.
[140] bhṛṃgavayād- B2.
[141] ṃ om. B2.
[142] śeṣastribhāgas- B2.
[143] śeṣyasya B2.
[144] praṃtyuttvatkuṭajaṃ B2.
[145] ṣaṭṣaḍvadādvaṃdete (for ṣaṭkas...te) B2.
[146] -bhānaṃ B2.
[147] uddiṣṭaṃ B2.
[148] apāśya B2.
[149] evaṃ B2.
[150] ca (for 12) B2.
[151] samamisi B2.
[152] yāvattāvattsīśāpavāhavidhinā B2.

1
1
o
1
2
o
1
3
o
1
4
o
1
5

[153]

savarṇite jātaṃ yā $\frac{1}{5}$ / etadanena 12[154] samaṃ kṛtvā labdhaṃ yāvattāvanmānaṃ[155] tadeva 60 / etadalikulamānam //

> ekaḥ [156]sakhāyamavadacchatamātmavittād-
> yacchasyahaṃ tava dhanāddviguṇo bhavāmi /
> anyastamāha mama yacchasi cettathāṣṭau
> tvadvittapañcaguṇito 'smi[157] tayordhane ke[158] // (7) //

atrādyaḥ paradhanācchate labdhe dviguṇaḥ syādityādyasya yāvattāvaddvayaṃ rūpaśatonaṃ[159] parikalpya dvitīyasya yāvattāvatādhike śate[160] kalpite tayordhane yā 2 rū 1o00[161] / yā 1 rū 100 / atrādyaparācchatena[162] dhanena gṛhītena paradhanādādyo dviguṇo bhavatītyeka ālāpo ghaṭate[163] / atha dvitīyālāpa ādyādaṣṭasu rūpeṣu parasmai datteṣu paraḥ[164] pañcaguṇa ityālāpe kṛta ādyādaṣṭau[165] rūpāṇyapāsya paradhane prakṣipyādyaṃ pañcaguṇaṃ kṛtvā jātaṃ yā 10 rū 5o40 / yā

[153] This table is written horizontally, rotated 90 degrees anticlockwise, in B2.
[154] 12 om. B2.
[155] labdhe pavittāvanmānaṃ B2.
[156] Fol. 31a, B2.
[157] -guṇito sthi B2.
[158] tayordhanaike 1 B2.
[159] -śatone B2.
[160] -tāvatādhikeceti B2.
[161] o om. B2.
[162] atrādyaparaśatena B2.
[163] -ekaḥ ālāpo haṭatai B2.
[164] ḥ om. B2.
[165] -aṣpau B2.

1 rū 108 / etau[166] samāviti prāgvallabdhayāvattāvanmānaṃ[167] 72 / anenotthāpite jāte dhane 44 / 172 //

pañcakaśataprayoge[168] savṛddhi[169] mūlaṃ prajāyate 'tīte /
varṣe dviguṇaṃ ṣoḍaśahīnaṃ[170] me kathaya dhanavṛddhī // (8) //[171]

atra mūladhanapramāṇaṃ yā 1 / pañcarāśike nyāsaḥ

1	12
100	yā 1
5	

[172]

adhunā pañcarāśikavidhinā kalāntaraṃ yā $\frac{3}{5}$ / etanmūladhanayutaṃ jātaṃ yā $\frac{8}{5}$ [173] / etaddviguṇamū[174]ladhanasya ṣoḍaśonasya[175] yā 2 rū $\overset{\circ}{16}$[176] samamiti samakriyayā labdhayāvattāvanmānenotthāpite[177] jāte mūlakalāntare 40 / 24 //

dattaṃ dvikatrikacatuṣkaśatena[178] māsāt-
khaṇḍaistribhiśca[179] śatamāśu vadāryavarya /
māseṣu pañcagajadikpramiteṣu tasmin
khaṇḍatraye 'pi hi phalaṃ sadṛśaṃ kathaṃ syāt /
yadvā phalena sahitaṃ nijakhaṇḍavittaṃ
tulyaṃ bhavedapi phalonadhanaṃ samaṃ vā // (9) //[180]

prathamodāharaṇe samaphalamālāpitam / tatpramāṇaṃ yāvattāvat / pañcarāśikena[181]

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[166] yutau B2.
[167] -yāvatovanmānaṃ B2.
[168] pañcaśatatrayayoge B2.
[169] samṛddhi B2,.
[170] ṣoṃḍaśa- B2.
[171] Cf. BG 99: pañcakaśatena dattaṃ mūm sakalāntaraṃ gate var dse / dviguṇaṃ ṣoḍaśahīnaṃ labdhaṃ kiṃ mūlamācakṣva //
[172] 2 (for 12) B2.
[173] 5 (for 8) B2.
[174] Fol. 31b, B2.
[175] ṣoḍaśāṃśonasya B2.
[176] o om. B2.
[177] notthāpyate B2.
[178] -catuśatenaṃ B2.
[179] vaṃḍaistribhiśca B2.
[180] = GK miśra (uv) (Part I, p. 61), where -phalena vittaṃ for -śatena māsāt, and ca for hi.
[181] -rāśikana B2.

1	5	1	8	1	10
100	o	100	o	100	o
2	yā 1	3	yā 1	4	yā 1

miśraṃ 100 /

prāgvatpañcarāśikena mūladhanāni yā $\genfrac{}{}{0pt}{}{10}{1}$ yā $\genfrac{}{}{0pt}{}{25}{6}$ yā $\genfrac{}{}{0pt}{}{5}{2}$ / eṣāṃ yogaḥ[182] yā $\genfrac{}{}{0pt}{}{50}{3}$ [183] ekaśatasama iti sāmyakaraṇena labdhaṃ yāvanmānaṃ 6 / anenotthāpitāni jātāni[184] mūladhanāni 60 / 25 / 15 / kalāntaraṃ samam //

dvitīyodāharaṇe saphalaṃ samamūladhanamālāpitam[185] / tasya pramāṇaṃ yā 1 / śatasya kalāntarāṇi 10 / 24 / 40 / śatayuktāni 110 / 124 / 140 / atra trairāśikam / yadi daśādhikaśatasakalāntarasya mūladhanaṃ śataṃ tadyāvattāvataḥ kimiti 110 100 yā 1 / jātaṃ [186]prathamamūladhanaṃ yā $\genfrac{}{}{0pt}{}{10}{11}$ [187] / evamanyayoḥ[188] yā $\genfrac{}{}{0pt}{}{25}{31}$ yā $\genfrac{}{}{0pt}{}{5}{7}$ / eṣāmanupātena kalāntarāṇi yā $\genfrac{}{}{0pt}{}{1}{11}$ yā $\genfrac{}{}{0pt}{}{6}{31}$ yā $\genfrac{}{}{0pt}{}{2}{7}$ / mūlānāṃ yogaḥ

yā $\genfrac{}{}{0pt}{}{5800}{2387}$

eṣa[189] śatasama iti sāmyakaraṇena labdha[190]yāvattāvanmānenotthāpitāni jātāni[191] mūladhanāni

2170	1925	1705
58	58	58

[192] kalāntarāṇi ca

217	462	682
58	58	58

[193]

tṛtīyodāharaṇe kalāntaronitamūladhanaṃ samamālāpitam / tataḥ śatasya kalāntaraṃ śatādapāsya jātāni śeṣāṇi /[194] 90 / 76 / 60 /

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182 yogena B2.
183 2 (for 3) B2.
184 yātāni B2.
185 samūladhanam- B2.
186 Fol. 32a, B2.
187 This fraction is framed in B2.
188 evamavayoḥ B2.
189 eṣaḥ B2.
190 labdhaṃ B2.
191 yātāni B2.
192 In B2, these three boxes are written separately and the middle term is framed together with 'eṣaḥ śa' in the above line,

eṣaḥ śa
1925
58

tasama ...
193 467 (for 462) B2.
194 A *daṇḍa* actually exists here in B2.

anupātena mūladhanāni[195] yā 10 [196] yā 25 [197] yā 5 [198] / anupā-
9 19 3
tena kalāntarāṇi yā 1 yā 6 [199] yā 2 / mūlānāṃ[200] yogaḥ yā
9 19 3
700 / eṣa śatasama[201] iti sāmyakaraṇena[202] labdhayāvattāvanmā-
171
nenotthāpitāni[203] mūladhanāni kalāntarāṇi ca

1710	4275	855
63	133	21

[204]

171	1026	342
63	133	21

[205]

māsena pañcakacatuṣkaśatena dattaṃ
mūlaṃ[206] ca saptadaśamāsasamudbhavābhyām[207] /
anyonyasaṅkalitameva kalāntarābhyāṃ
tulyaṃ kathaṃ vada sakhe yadi vetsi bījam // (10) //[208]

atraikaṃ mūladhanaṃ vyaktamiṣṭaṃ catvāriṃśanmitamanyaṃ[209] yāvattāvatprakalpya[210] pañcarāśikena ca

1	7	1	10
100	40	100	yā 1
5	o	4	o

[211]

[195] 90 / 76 / 60 / anupātena mūladhanāni om. B2.
[196] 9 om. B2.
[197] 16 (for 19) B2.
[198] yā 3 B2.
5
[199] yā 6 om. B2.
19
[200] kalāntaronitamūlaśeṣāṇāṃ (for mūlānāṃ) B2.
[201] śatasamā B2.
[202] -keraṇena B2, corrected.
[203] labdheyābat- B2.
[204] In B2, these three boxes are written in between 'mā' and 'se' of the first word of the next example. They are repeated in between 'ka' and 'li' of 'saṅkalita' in the next line and are immediately followed by the next three boxes for kalāntaras.
[205] 61 (for 63) B2.
[206] mūlaṃ om. B2.
[207] -samudbhāvābhyaṃ ga B2.
[208] ≈ GK miśra (uv) (Part I, p. 63), where dvaidhaṃ hi māsadaśasapta- for mūlaṃ ca saptadaśamāsa-, -sammilitam- for -saṅkalitam-, and dhanaṃ bhavati cāpi vivarjitābhyām (misprint for vivarjitaṃ vā?) for kathaṃ vada sakhe yadi vetsi bījam.
[209] catvāriśanmitamanya (two ṃ's om.) B2.
[210] -prakalpa B2.
[211] yā 1 and o, in the last column, om. B2.

jāte kalāntare 14 yā $\frac{2}{5}$ / anyonyakalān[212]tarayute mūladhane yā $\frac{2}{5}$ [213] rū 40 / yā 1 rū 14 / ete same[214] iti samacchedīkṛtya cchedagame[215] sāmyakaraṇena labdhaṃ yāvattāvanmānaṃ $\frac{130}{3}$ [216] //

prathamasya ṣaṣṭau mūladhane[217] kṛte labdhaṃ yāvattāvanmānaṃ 65 / etaddvitīyamūladhanaṃ 65 / kalāntaraṃ 26 / samadhane 86 / 86 //

pañcake ca phale yattatphalavargāvaśeṣakam /
saṃyuktaṃ daśakeneha samakālaḥ phalaṃ tayoḥ[218] // (11) //[219]

atra prayogadvaye 'pi phalamiṣṭaṃ rūpadvayam[220] kalpitaṃ 2 / kālapramāṇaṃ yāvattāvat / pañcarāśikena

1	yā 1
100	
5	2

labdhaṃ dhanaṃ

rū 40
yā 1

phalavargeṇonaṃ jātaṃ dvitīyamūladhanam / punaridaṃ[221] daśakena prayuktam / pañcarāśikena[222]

1	yā 1
100	yā 4̊ rū 40 che yā 1
10	

[223] labdhaṃ phalaṃ[224] yā $\frac{\mathring{2}}{5}$ [225] rū $\frac{20}{5}$ / etatpūrvaphalasyāsya 2[226] samamiti sāmyakaraṇena[227] labdhaṃ yāvattā-

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[212] Fol. 32b, B2.
[213] 8 (for 2) B2.
[214] sama B2.
[215] cchadagame B2.
[216] 3 (in the denominator) om. B2.
[217] ṣaṣṭhi mūlaṃ dhane B2.
[218] phalaṃ ca tayoḥ B2.
[219] Cf. BG 96: pañcakaśatadattadhanātphalasya vargaṃ viśodhya pariśiṣṭam / dattaṃ daśakaśatena tulyaḥ kālaḥ phalaṃ ca tayoḥ //
[220] One-*akṣara* space for 'pa' indicated by a short *mātrā* line.
[221] -mūladhavātpunaridaṃ B2.
[222] pacarāśikena (ṃ om.) B2.
[223]

1	yā 1
100	yā 4 rū 4
5	che yā 1

[224] pha rū 40 (for phalaṃ) B2.
[225] o om. B2.
[226] -pūvaṃphalasyā 12 B2.
[227] samya- B2.

vanmānaṃ 5 / anenotthāpya[228] mūle dhane[229] 8 / 4 //

athavā mūladhanaṃ yāvatkalpitaṃ yā 1 / tatphalamiṣṭaṃ[230] rūpadvayaṃ 2 / pañcarāśikena

1	
100	yā 1
5	2

labdhaḥ kālaḥ rūpa 40 / yā 1 [231]

phalavargonamūladhanaṃ dvitīyamūladhanamiti pañcarāśikena

1	
100	yā 1 rū $\overset{\circ}{4}$
10	2

[232] labdhaḥ kālaḥ rū 20 che yā 1 rū $\overset{\circ}{4}$[233] / ayaṃ

pūrvakālasyāsya

rūpa 40
yā 1

[234] sama iti[235] samacchedīkṛtya ccheda[236]-game sāmyakaraṇena labdhaṃ yāvattāvanmānaṃ 8 / anenotthāpite jāte mūladhane 8 / 4 / ubhayato 'pi[237] kālo māsāḥ 5 //

athavā kalpitapañcamāsasya[238] yāvanmūladhanasya pañcarāśikena / pañcarāśike nyāsaḥ

1	5
100	yā 1
5	

labdhaṃ phalaṃ yā 1/4 / asya vargonamūladhanaṃ[239]

dvitīyamūladhanamityālāpitamataḥ pañcarāśikena

1	5
100	yāva $\overset{\circ}{1}$/16 yā 1
10	o

[240] labdhaṃ kalāntaraṃ yāva $\overset{\circ}{1}$/32 [241] yā[242]

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[228]

laṃ yā 4
che yā 1

anenotthāpya la 4 B2.

[229] dhanai B2.

[230] taskalaṃ iṣṭaṃ B2.

[231] yā 1 om. B2.

[232]

1	yā 1 rū $\overset{\circ}{4}$
100	
10	22

B2.

[233] labdhaṃ kāleḥ che yā 1/0 B2.

[234]

rū 8 da 0
yā 1

B2.

[235] sama iti om. B2.

[236] Fol. 33a, B2.

[237] ubhavatopi B2.

[238] kalpita- B2.

[239] vargāna- B2.

$\frac{16}{32}$ / etatpūrvaphalasyāsya[243] yā $\frac{1}{4}$ samamiti pakṣau yāvattāvatāpavartya prāgvallabdhaṃ yāvattāvanmānaṃ 8 / anenotthāpite[244] mūladhane 8 / 4 / kalāntare ca 2 / 2 // evaṃ kalpanayodāharaṇaṃ samānīyate //

niṣkena candanapaladvayamāpyate[245] ca
dvābhyāṃ ca kuṅkumapalaṃ śaratulyamārya[246] /
bhadraśriyaṃ dviguṇitaṃ[247] daśabhiśca niṣkaiḥ
kāśmīrakaṃ[248] vitara śaṅkarapūjanāya // (12) //[249]

atra candanakuṅkumapramāṇe[250] yā 1 yā 2 / ihānupātaḥ[251] / yadi candanapaladvayasya[252] niṣko maulyaṃ tadā yāvattāvataḥ kimiti 2 1 yā 1 / labdhaṃ candanamaulyaṃ yā $\frac{1}{2}$ / punaryadi kuṅkumapalapañcakasya niṣkadvayaṃ mūlyaṃ tadā yāvattāvaddvayasya[253] kimiti 5 2 yā 2 / labdhaṃ[254] mūlyaṃ yā $\frac{4}{5}$ / anayoryo[255]gaḥ yā $\frac{13}{10}$ daśasama[256] iti sāmyakaraṇena labdhaṃ yāvattāvanmānaṃ $\frac{100}{13}$ / anenotthāpitaṃ candanakuṅkumamaulye[257] $\frac{50}{13}$ / $\frac{80}{13}$ / candanakuṅkumamāne $\frac{100}{13}$

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[240]

16		
100	yāva 16	
10	o	35

, written in between yāva $\frac{\overset{\circ}{1}}{32}$ and yā $\frac{16}{32}$, B2

[241] o om. B2.
[242] yāva B2.
[243] -phalāsta B2.
[244] ananotthāpite B2.
[245] niṣka candanadvayapalamāypate B2 (one-letter space left blank between 'ṣka' and 'ca').
[246] śarasīkhapamāryā B2.
[247] -śriyoddhiśuṇitaṃ B2.
[248] kāsmīkakaṃ B2.
[249] ≈ GK miśra (uv) (Part I, p. 58), where ced for the first ca, -sammitaṃ ca for -tulyamārya, and īśārcanāya mama dehi sucandanaṃ ca satkuṅkumaṃ dviguṇitaṃ daśabhiśca niṣkaiḥ for the latter half of the verse.
[250] kukumaṃ caṃdanapramāṇe B2 (three digits, 2, 1, 2, are written above the three *akṣaras*, ku, ku, maṃ, in order).
[251] ihānupāteḥ B2.
[252] caṃdana eladvayasya B2.
[253] yāvattāvaṣṭvayasya B2.
[254] lardhva B2.
[255] Fol. 33b, B2.
[256] yā 17 daśama B2.
[257] -maulyaṃ B2.

/ 200 [258] //
13

saṃsmṛtya kāntaṃ ramaṇī smarāturā
prayāti nityaṃ[259] daśa yojanāni tām[260] /
tryādidvivṛddhyānusasāra saṃbhalī
drutaṃ niṣeddhuṃ vada kairdinairyutiḥ[261] // (13) //[262]

atra samāgamadinapramāṇaṃ yā 1[263] / taddaśaguṇaṃ kāminīyojanāni yā 10 [264] / kuṭṭinyā tryādidvyuttare[265] yāvattāvati gacche ā 3 u 2 gaccha yā 1 / vyekapadārdhaghnacaya[266] ityādinā[267] kuṭṭanīyojanāni[268] yāva 1[269] yā 2 / etadanena yā 10 samamiti pakṣau yāvattāvatāpavartya[270] prāgvallabdhayāvattāvanmānaṃ 8 / anenotthāpite[271] samayojanāni 80 / 80 [272] //

śreḍhīphalaṃ śataṃ yatra caturgacche[273] ṣaḍuttare /
tatrādiṃ vada bījajña vetsi bījakriyāṃ yadi[274] // (14) //

atrādiryāvat / ā yā 1 u 6 ga 4 / idānīṃ

vyekapadārdhaghnacayaḥ[275]
sādiḥ[276] padasaṅguṇo bhavedgaṇitam //[277]

iti jātaṃ śreḍhīphalaṃ yā 4 rū 36 //

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[258] 23 (for the latter 13) B2.
[259] niṃtyaṃ B2.
[260] 3 (for tām) B2.
[261] kairdirnauryutiḥ B2.
[262] ≈ GK śreḍhī (uv) (Part I, p. 112), where anucacāra for anusasāra, śambhalī for saṃbhalī, and samāgamaḥ kairdivasaistayorbhaved for the last pāda.
[263] samaṇamapramāṇaṃ yā 12 B2.
[264] yā 1 B2.
10
[265] -dyuttare B2.
[266] -padārdhacaya B2.
[267] = GK śreḍhī 2a, which is fully cited in the commentary on the next example.
[268] -yo31nāni B2.
[269] yā 41 B2.
[270] yāvattāṃvatā- B2.
[271] 8 / anenotthāpite om. B2.
[272] 80 / 80 / 0 B2.
[273] śatatrayaṃ caturṇaccha B2.
[274] bījakiyāṃ yadvi B2.
[275] vyekapadārdhacayaḥ B2.
[276] ḥ om. B2.
[277] = GK śreḍhī 2a.

asyopapattiḥ / yāvattāvatādinā ṣaḍrūpottare gacche pṛthakpṛthagādyuttarāṇāṃ darśaṇam

yā 1	yā 1	yā 1	yā 1
	rū 6	rū 6	rū 6
		rū 6	rū 6
			rū 6

[278]

prathamapada uttaro nāsti / śeṣeṣu yathā kramamekadvitryādiguṇamuttaram / vyekapadaṃ [279]kuryāditi kṛte jātaṃ

yā 1	yā 1	yā 1	yā 1
3	3	3	3
3	3	3	3
3	3	3	3

[280]

idānīmavasthitottarārdhānāṃ vaiparītyenottarārdhadvayaṃ[281] saṃyojya jātaṃ

yā 1	yā 1	yā 1	yā 1
3	3	3	3
6	6	6	6

[282]

pṛthakpadeṣu caturṣu yāvattāvadekaṃ[283] rūpāṇāṃ[284] navakam / yāvattāvat 1 rū 9 / samamiti tena saṃguṇya[285] jātaṃ tadeva gaṇitaṃ yā[286] 4 rū 36 //

etadrūpaśatasamaṃ kṛtvā labdhaṃ[287] yāvattāvanmānaṃ 16 / ayamādyarāśiḥ[288] //

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[278]

yā 1	yā 1	yā 1	yā 1
rū 6	rū 6	rū 6	rū 6
	rū 6	rū 6	

B2.

[279] Fol. 34a, B2.

[280]

yā 1	yā 1	yā 1	yā 1
3	3	3	
	3		3
			3

B2.

[281] vaiparityenottarārddhadvaya B2.

[282]

yā 1	yā 1	yā 1	yā 1
6	3	6	3
3	6	3	6

B2.

[283] yāvattābattāvadekaṃ B2.

[284] ṃ om. B2.

[285] ṃ om. B2.

[286] yāva B2.

[287] laddhaṃ B2.

[288] aparāśiḥ B2.

śreḍhīphalaṃ śataṃ yatra pañca gacche caturmukhe /
tatra pracayamācakṣva dakṣo 'si gaṇite yadi // (15) //

atrottaraṃ yāvat / ā 4 u yā 1 ga 5 / vyekapadārdhaghnacaya ityādinā[289] jātaṃ gaṇitaṃ yā 10 rū 20[290] / etadrūpaśatasamaṃ[291] kṛtvā yāvanmānaṃ 8[292] / ayamuttaraḥ //

ādye dine nidhimitāni ca yojanāni
pañcottareṇa pathiko niyamena yāti /
anyaḥ[293] prayāti yugalaṃ divase tathādye
saptottareṇa[294] divasairvada kaiśca yogaḥ // (16) //[295]

atrobhayatra[296] gacchapramāṇaṃ yāvattāvat yā 1 / prathamasya ā 9 u 5 ga yā 1 / dvitīyasya ā 2 u 7 ga yā 1 / vyekapadārdhaghnacaya ityādinā[297] phale yāva[298] $\frac{5}{2}$ yā $\frac{13}{2}$ yāva[299] $\frac{7}{2}$ yā $\frac{\overset{\circ}{3}}{2}$ / ete same iti[300] yāvattāvatāpavartya prāgvallabdhaṃ yāvattāvanmānaṃ 8 / anenotthāpite[301] samagatiyojanamāne [302]212 / 212 //

kuraṅganābherhimavālukasya[303]
krītāni niṣkeṇa palāni cāṣṭau /
viparyayārgheṇa[304] ca vikritāni[305]
lābho daśa brūhi pṛthagvaṇikte[306] // (17) //

kastūrīkarpūrayormāne[307] vyakte kalpite $\frac{1}{2}$ $\frac{15}{2}$ / atra kastūrīmūlyaṃ yā 1 / anenonaṃ rūpamekaṃ karpūramūlyaṃ yā $\overset{\circ}{1}$ rū 1 /

[289] = GK śreḍhī 2a.
[290] guṇitaṃ 10 rū 20 B2.
[291] -śatesamaṃ B2, corrected.
[292] ṭa (for 8) B2.
[293] anpṛ B2.
[294] saptottaro B2.
[295] = GK śreḍhī (uv) (Part I, p. 110).
[296] avomapatra B2.
[297] = GK śreḍhī 2a.
[298] va om. B2.
[299] va om. B2.
[300] samā iti B2.
[301] -pitai B2.
[302] Fol. 34b, B2.
[303] kurayanābher- B2.
[304] viparyayorgheṇa B2.
[305] vakritāni B2. See GK Part I, p. 99, for the past passive participle, vikritāni, of vi-krī.
[306] -vaṇiki B2.
[307] -kapūrayor- B2.

idānīmarghaviparyāsena phalamānīyate / tadyathā / yadi kastūrikāsārdhasaptapalānāṃ[308] mūlyaṃ[309] yāvattāvattadā kastūrīpalārdhasya[310] kimiti $\frac{15}{2}$ yā 1 $\frac{1}{2}$ / labdhaṃ[311] kastūrikāmūlyaṃ yā $\frac{1}{15}$ [312] / punaḥ karpūrapalārdhasya[313] yāvadūnaṃ[314] rūpamekaṃ tadā sārdhasaptapalānāṃ[315] kimiti $\frac{1}{2}$ yā $\overset{\circ}{1}$ rū 1 $\frac{15}{2}$ / labdhaṃ[316] karpūramūlyaṃ[317] yā $\overset{\circ}{15}$ [318] rū 15 / anayoryogaḥ yā $\frac{\overset{\circ}{224}}{15}$ rū $\frac{225}{15}$ [319] / mūladhanenānena 1 ūno jāto lābhaḥ yā $\frac{\overset{\circ}{224}}{15}$ [320] rū $\frac{210}{15}$ / ayaṃ lābhena[321] 10 sama iti sāmyakaraṇena[322] labdhayāvattāvanmānaṃ $\boxed{\frac{15}{56}}$ anenotthāpite kastūrīkarpūramūlye $\boxed{\frac{15}{56} \mid \frac{41}{56}}$

athavā[323] kastūrīkarpūraphalamāne kalpanāvaśādanekadhā //

yatrādyuttaragacchānāṃ ghāte[324] ca da[325]śa kovida /
gaṇitaṃ tatra saptādi pracayaṃ ca vada drutam // (18) //

atrādau yāvattāvatkalpite kriyā na nirvahati / rūpadvayamiṣṭamā-

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[308] -sapūmalavānāṃ B2.
[309] ṃ om. B2.
[310] kapūrapalārdhasya B2.
[311] laddhaṃ B2.
[312] 10 (for 15) B2.
[313] kastūrīphalārddhasya B2.
[314] ūnaṃ om. B2.
[315] -saptamalavānāṃ B2.
[316] laddhaṃ B2.
[317] B2: Between 'ka' and 'rpū', the previous passage, 'kastūrikāmūlyaṃ ... kimiti $\frac{1}{2}$ yā $\overset{\circ}{1}$,' which occupies exactly one line of writing, is repeated and crossed out.
[318] yā $\frac{1}{5}$ B2.
[319] yā 224 rū 225 (o and both 15's om.) B2.
[320] 15 om. B2.
[321] An illegible letter between 'bhe' and 'na', crossed out, B2.
[322] samya- B2.
[323] athavo B2.
[324] chāte B2.
[325] Fol. 35a, B2.

diḥ kalpitaḥ 2 / uttaro[326] yāvattāvat yā 1 / ādyuttaraghātena[327] sarvaghāte bhakte jāto gacchaḥ[328]

rū 10
che yā 2

[329]

nyāsaḥ

ā 2	u yā 1	ga rū 10
		che yā 2

[330] ādyuttaragacchaghāte[331] daśa ityālāpo ghaṭate / prāgvacchreḍhīphalaṃ[332]

yā $\overset{\circ}{5}$ rū 45
che yā 2

[333] / etadgaṇitasya 7[334] samamiti samacchedīkṛtya chedagame sāmyakaraṇena labdhaṃ yāvattāvanmānaṃ

45
19

anenotthāpitau jātau cayagacchau

45	19
19	9

[335]

kenāpyādicayena yāti puratastveko naro 'ṣṭau dinā-[336]
nyanyaḥ kenacidādinā dvikacayenānuprasarpan[337] kramāt /
mārge[338] mitra tayordvivāramabhavatsaṅgo dinaiḥ kairvada[339]
śreḍhīvedikarīndra vāraṇaraṇaproḍho kharīndro[340]
'si cet // (19) //[341]

atrādyasyādyuttaragaccheṣu yāvattāvanmiteṣu kriyā na nirvahati yata ādyuttarau[342] vyaktau kalpitau ā 2[343] u 4 ga o purogatadināni

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[326] kalpitaḥ 23 ttaro B2.
[327] -chātena B2.
[328] ḥ om. B2.
[329] chayā 28 (for che yā 2) B2.
[330]

nyāsaḥ ā 2 u yā 1 ga rū 10
cha yā 5 rū 45 chi yā 2

B2.
[331] gaccha om. B2.
[332] -cchedhī- B2.
[333] B2 om. this box.
[334] 1 B2.
[335]

45	19
19	19

B2.
[336] ṣṭāṃdi B2.
[337] dvikacayenātuprasarpan B2.
[338] mātre B2.
[339] kairyadi B2.
[340] harīndro B2.
[341] = GK śreḍhī (uv) (Part I, p. 111), where ca puras- for puratas-, anūpasarpan for anuprasarpan, and prauḍho for proḍho.
[342] yataḥ ādyutterau B2.
[343] ra (for 2) B2.

8 / dvitīyasya ā o u[344] 2 ga o / atrāpyajñāte gaccha ādau yāvattāvati kalpite kriyā na nirvahati śreḍhīphalābhāvāt / ato[345] dvitīyasya kalpita iṣṭo gacchaḥ[346] 4 / dvitīyādādyo 'ṣṭadinādhika ityālāpe kṛte dvitīyasya[347] gaccho[348] 'ṣṭayuto jātaḥ pratha[349]masya gacchaḥ[350] 12 iti prathamasya ā 2 u[351] 4 ga 12 / dvitīyasya ā o u 2 ga 4 / prathamasya śreḍhīphalaṃ 288 / etaddvitīyasya śreḍhīphalaṃ prakalpyam / atrādiryāvattāvat / nyāsaḥ[352] ā yā 1 u 2 ga 4 / asya śreḍhīphalaṃ yā 4 rū 12 / etadanena 288 samamiti samakriyayā labdhaṃ yāvattāvanmānaṃ 69 / iti tāvadubhayoḥ[353] śreḍhyau

ā 2	u 4	ga 12	phalaṃ 288
ā 69	u 2	ga 4	phalaṃ 288

[354] ubhayoḥ saikagacchayoḥ[355]

vyekapadaghnacayo mukhayukto 'ntyadhanam /[356]

iti jāte 'ntyadhane[357] 50 / 77 / ete eva dvitīyasaṅgamasyādi / gaccho yāvattāvat / śreḍhyarthaṃ[358] nyāsaḥ

ā 50	u 4	ga yā 1
ā 77	u 2	ga yā 1

[359]

śreḍhīphale yāva 2 yā 48 / yāva 1 yā 76 / ete same iti pakṣau yāvattāvatāpavartya prāgvallabdhaṃ yāvattāvanmānaṃ 28 / ayamubhayordvitīyasamagacchaḥ[360] / pūrvagacchābhyāmābhyāṃ 12 / 4 yukto dvitīyasamāgamadivasasaṃkhye 40 / 32 / samāgamadvaye samagatiyojanāni 2912 / 2912 //

rāśīnāṃ taccaturṇāṃ[361] kṛtiyutisadṛśī[362] saṃyutiścātra keṣāṃ[363]
keṣāṃ[364] vargaikyatulyā ghanayutiratha tadyogaghātau sadṛśau /

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[344] 3 (for u) B2.
[345] atha B2.
[346] ḥ om. B2.
[347] dvitīsya B2.
[348] An illegible letter after 'gaccho' B2, crossed out.
[349] Fol. 35b, B2.
[350] maccha B2.
[351] 3 (for u) B2.
[352] prakalpagrādiryāvattāvannyāsaḥ B2.
[353] tāvaduptayoḥ B2.
[354] 3 (for the lower u) B2.
[355] saike gacchayoḥ B2.
[356] = GK śreḍhī 1a (Part I, p. 105).
[357] jāte'tyadhane B2.
[358] gacchau srāvattāvacchreḍhyarthaṃ B2.
[359] 3 (for the upper u), ra (for 2) B2.
[360] ayamuttayor- B2.
[361] ṛtaccaturṇāṃ B2.
[362] yu between 'ti' and 'yu' B2, crossed out.
[363] kepyāṃ B2.
[364] pheṣāṃ B2.

cedavyaktakriyāyāṃ[365] paṭurasi gaṇaka[366] brūhi tāneva śīghraṃ
rāśyoryogo 'ntarāḍhyaḥ[367] kṛtiyutisa[368]dṛśaḥ syātkayostau vadāśu // (20) //[369]

atra rāśayaḥ kalpitāḥ[370] yā 1 yā 2 yā 3 yā 4 / prathamodāharaṇe[371] eṣāṃ yutiḥ[372] yā 10 / ayaṃ kṛtiyuterasya yāva 30 sama iti[373] pakṣau yāvattāvatāpavartya prāgvadāptayāvattāvanmānenotthāpitā jātā rāśayaḥ[374]

1	2	3	4
3	3	3	3

dvitīyodāharaṇe ta eva rāśayaḥ yā 1 yā 2 yā 3 yā 4 / eṣāṃ[375] vargaikyaṃ yāva 30 / ghanaikyenānena yāgha 100 samamiti[376] pakṣau yāvadvargeṇāpavartya[377] prāgvallabdhayāvattāvanmānenotthāpitā jātā rāśayaḥ

3	6	9	12
10	10	10	10

[378]

tṛtīyodāharaṇe yogo ghātasama ityevamālāpitaṃ tatraiko rāśiravyaktaḥ śeṣā vyaktāḥ kalpitāḥ yā 1 2 3 4 / eṣāṃ yogaḥ yā[379] 1 rū 9 / ghātinānena[380] yā 24 samamiti samakriyayā labdhaṃ yāvattāvanmānaṃ 9/23 [381] / jātā[382] rāśayaḥ

9	2	3	4
23	1	1	1

[383]

caturthodāharaṇe kalpitau rāśī yā 1 yā 2 / anayoryogāntarasamāsaḥ yā 4 vargayogenānena yāva 5 sama iti pakṣau[384] yāvattāvatāpavartya

[365] ved- B2.
[366] gaṇaka 1 B2.
[367] -yogāṃtarāṃdyaḥ B2.
[368] Fol. 36a, B2.
[369] For the first two problems, cf. BG 106: asamānasamacchedānrāśīṃstāṃścaturo vada / yadaikyaṃ yadghanaikyaṃ vā yeṣāṃ vargaikyasammitam // (Variant: *asamānasamaprajña* for *asamānasamacchedān*)
[370] kalpiṃtāḥ B2.
[371] gho between 'ra' and 'ṇe' B2, crossed out.
[372] ḥ om. B2.
[373] kṛtiyuterapāsya pāba 30 sameti B2.
[374] jātāśayaḥ B2.
[375] ṃ om. B2.
[376] samamiti om. B2.
[377] yāvaddhargeṇā- B2.
[378] In B2, this table is written in between 'tī' and 'yo' of the next line.
[379] yo B2.
[380] ghatinānena B2.
[381] 23 om. B2.
[382] yātā B2.
[383] ra (for 2 in the 2nd column) B2.
[384] pakṣo B2.

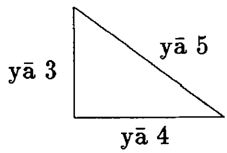
prāgvadāptayāvattāvanmānenotthāpitau[385] rāśī

4	8
5	5

koṭyā bhujena vā[386] śrutyā teṣāṃ[387] ghātena vā pṛthak[388] /
gaṇitaṃ[389] sadṛśaṃ jātaṃ gaṇaka brūhi tāni me // (21) //[390]

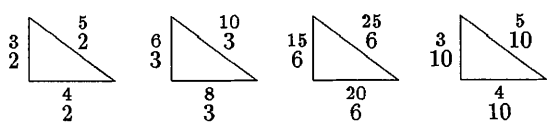
atrābhīṣṭaṃ tryasraṃ yāvattāvadguṇitam[391] / [392]nyāsaḥ

[393]



gaṇitaṃ yāva 6[394] / etatkoṭyā yā 3 bhujena ca[395] yā 4 karṇena ca yā 5 bhujakoṭikarṇānāṃ[396] ghātena[397] ca yāgha 60 samakriyayā labdhaiḥ svaiḥ svairyāvattāvanmānair[398]utthāpitāni jātāni tryasrāṇi[399] / darśanam

[400]



jātyajaneryau kāraṇamaṅkau[401] tau bījasaṃjñakau bhavataḥ /

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[385] -yāvattāvaneno- B2.
[386] bhujecadā B2.
[387] tepyāṃ B2.
[388] sa (for pṛthak) B2.
[389] gaṃṇitaṃ B2.
[390] For the last two problems, cf. BG 107: tryasrakṣetrasya yasya syātphalaṃ karṇena sammitam / doḥkoṭiśrutighātena samaṃ yasya ca tadvada //
[391] -gaṇitaṃ B2.
[392] Fol. 36b, B2.
[393] ba 4 (for yā 3) B2. In B2, this figure is put in a framed box.
[394] yavi 63 B2.
[395] va B2.
[396] bhujekoṭi- B2.
[397] dvātena B2.
[398] -yāvattāvanunonair- B2.
[399] jātāni (for tryasrāṇi) B2.
[400] 80 (for 20 of the third triangle) B2. In B2, these figures are put in a framed box having four compartments (the first of which lacks the left side), and placed at the beginning of a line of writing, in between 'yau' and 'kā' of the next quotation.
[401] jātyajananeyau kārasāmaṃkau B2.

tatkṛtyoryutiviyutī śrutikoṭī dorvadho dviguṇaḥ[402] //[403]

iti jāte jātye //

saṃyoge[404] jāyate vargo viyoge ca ghanaḥ kayoḥ /
hataṃ vāptavargamūlaṃ ghanamūlena mūladam[405] // (22) //

atra yāvattāvatkalpanayā kriyā na nirvahati ghanavargatvāt[406] / yatkalpitaṃ ghanamūlamiṣṭadvikavargārdhakaṃ 2 / etadiṣṭatrirūpavargeṇa guṇitaṃ 18[407] / etadvargamūlam / athaitayorghanavargau 8 / 324 / etāveva rāśyorantarayogau[408] / saṃkramaṇotpattyarthaṃ[409] kalpita eko rāśiḥ yā 1 / anenāntarayogau yutonitau samāviti samakriyayā labdhaṃ yāvattāvanmānaṃ 158 / anena yutamantaraṃ 166 / evaṃ[410] jātau rāśī 166 / 158[411] // anyeneṣṭenānyau rāśī bhavataḥ //

itaretaramūlena yutau vargau ca kaucana /
ādyātsa jāyate cānyastriguṇastau[412] drutaṃ vada // (23) //

atra kalpitau rāśī yāva 9 yāva 36 / anyonyamūlayutau yāva 9 yā 6 / yāva 36 yā 3 / ādyāttriguṇaḥ[413] para ityādyaṃ [414]triguṇaṃ parasamaṃ kṛtvā labdhaṃ yāvattāvanmānaṃ $\frac{5}{3}$ / anenotthāpitau jātau rāśī 25 / 100 //

analpālpakayo[415] rāśyorvadho 'lpakṛtisaṃyutaḥ[416] /
tulyaḥ syādalpayuktenānalpavargeṇa[417] tau vada // (24) //

atra kalpitau rāśī yā[418] 8 yā 5 / etau samāviti pakṣau yāvattāvatāpavartya prāgvallabdhayāvattāvanmānenotthāpitau jātau rāśī 40 / 25 //

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402 ḥ om. B2.
403 ≈ GK kṣetra 82 (Part II, pp. 99–100), where -saṃjñau staḥ for -saṃjñakau bhavataḥ, and dostayorvadho for dorvadho.
404 saṃyogo B2.
405 ghanamūlena mūlena mūlakṣau B2.
406 dyanavargatvāta B2.
407 guṇita $\overset{\circ}{1}8$ B2.
408 -antarayorvargau B2.
409 ṃ om. B2.
410 samāviti ... evaṃ om. B2.
411 1580 B2.
412 -taṃ B2.
413 ādyās- B2.
414 Fol. 37a, B2.
415 analpyālpakayo B2.
416 -vadho nyakṛti- B2.
417 syādālpa-...-vargaṇa B2.
418 yo B2, corrected.

rāśyoranalpavargo 'lpaghnaścālpakṛtivarjito[419] dalitaḥ /
alpaghanena samaḥ syāttau kathaya me gaṇaka vetsi cet[420]
// (25) //

atra kalpitau rāśī yā 3 yā 2 / analpakṛtiralpaguṇitā yāgha 18 / alpavargonā dalitā yāgha 9 yāva $\overset{\circ}{2}$ / iyamalparāśerghanasyāsya yāgha 8 sameti pakṣau yāvattāvadvargeṇāpavartya prāgvallabdha[421]yāvattāvanmānenotthāpitau jātau rāśī 6 / 4 //

yutighanayuktā yutikṛtirapi tulyā jāyate kayo rāśyoḥ /
dviguṇitaghanayogena ca brūhi sakhe tau pravetsi yadi
// (26) //[422]

atra kalpitau rāśī yā 1 yā 4 / anayoryutighanayuktā yutikṛtiḥ yāgha 125 yāva 25 / ghanayogena dviguṇena yāgha 130 sameti pakṣau yāvattāvatāpavartya prāgvadāptayāvattāvanmānenotthāpitau jātau rāśī 5 / 20 //

[423]rāśyoḥ kayoryogaviyogakau sto
vargau vadhaḥ[424] syāttu ghano vadāśu /
kayorghanaikyaṃ kṛtitāmupaiti
kṛtyoryuteḥ[425] syādghanatā ca kau tau // (27) //[426]

atra prathamodāharaṇe rāśyoryogaviyoge[427] vargaḥ syāttathā kalpitau rāśī yāva 5 yāva 4 / anayoryogaviyoge varga ityālāpadvayaṃ svayameva ghaṭate / atha tayo rāśyorvadhaḥ[428] yāvava 20 / ayamiṣṭayāvattāvaddaśakasya ghanena sama iti pakṣau yāvattāvadghanenāpavartya[429] prāgvadāptayāvattāvanmānenotthāpitau rāśī 12500 / 10000 //

[419] lpaghno'lpakṛti- B2.
[420] vetsi cet om. B2.
[421] bdha om. B2.
[422] Cf. BG 153: rāśiyogakṛtirmiśrā rāśyoryogaghanena ca / dvighnasya ghanayogasya sā tulyā gaṇakocyatām //
[423] Fol. 37b, B2.
[424] ḥ om. B2.
[425] -yutiḥ B2.
[426] $\approx$ GK prakīrṇaka (uv) (Part I, p. 36) + (uv) (Part I, p. 38): rāśyoryoge viyoge ca vargo ghāte ghano bhavet / sakhe yadi vijānāsi vada tau tvaritaṃ mama // (Part I, p. 36) ghanayoge kayorvargo vargayoge ghano bhavet / tau vadāśu sakhe vargakautuke kuśalo 'si cet // (Part I, p. 38) Cf. BG 108 and 109: yutau vargo 'ntare vargo yayorghāte ghano bhavet / tau rāśī śīghramācakṣva dakṣo 'si gaṇite yadi // 108 // ghanaikyaṃ jāyate vargo vargaikyaṃ ca yayorghanaḥ / tau cedvetsi tadāhaṃ tvāṃ manye bījavidāṃ varam // 109 //
[427] r before śyo B2, crossed out.
[428] 4 (for ḥ) B2.
[429] yyavattāvad- B2.

atra kalpanopāyaḥ paribhāṣitaḥ /

iṣṭayoḥ[430] kṛtiyogaḥ syādeko rāśistayorvadhaḥ /
dviguṇo[431] 'nyastayo[432] rāśyoryogaśeṣe kṛtirbhavet // 9 //

atreṣṭe 1 / 2 / anayorvargayogaḥ 5 / ghāto dviguṇaḥ 4 / etau yāvattāvadvargaguṇitau //

dvitīyodāharaṇe[433] rāśyorghanayoge yathā vargaḥ syāttathā kalpitau rāśī yāva 1 yāva 2 / anayorghanayogaḥ yāvagha[434] 9 / asya mūlaṃ[435] yāgha 3 //

yo hi vargaghanaḥ sa eva ghanasya vargaḥ / tadeva[436] paribhāṣitam /

rāśermūlasya ghanastadvargo rāśighanasamo bhavati // 10 //[437]

yathā trayāṇāṃ vargo nava 9 / a[438]sya ghanaḥ 728 / trayāṇāṃ ghanaḥ 27 / asya vargassa eva 729 / evamupapattiḥ //

atha tayo rāśyorvargayogaḥ yāvava 5 / ayamiṣṭayāvattāvatpañcakaghanena yāgha 125 sama iti pakṣau yāvattāvadghanenāpavartya prāgvadāptayāvattāvanmānenotthāpitau jātau rāśī 625 / 1250 //

yatrodāharaṇe[439] tricatuṣpañcādirāśaya ālāpitāstadarthaṃ[440] paribhāṣitam /

saṅkalitasya ca vargo rūpāderghanasamāsaḥ[441] syāt /
rūpādirūpacayena ca ghanayogo mūlado bhavati // 11 //[442]

viyoge jāyate vargo vargayoge ghanaḥ kayoḥ /
tau vadāśu tavābhyāso yadyasti samakarmaṇi // (28) //[443]

atrāntare vargaḥ syādyathā tathā kalpitau rāśī yāva 3 yāva 4 / anayorantare svayameva vargaḥ yāva[444] 1 / eka ālāpo ghaṭate[445] /

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430 4 (for ḥ) B2.
431 dinaghno B2.
432 dinaghnobhyas- B2.
433 tṛtīyo- B2.
434 yāvigha B2, corrected.
435 ṃ om. B2.
436 tadiva B2, corrected.
437 = GK prakīrṇaka 23b, where ghana- for rāśighana-.
438 Fol. 38a, B2.
439 atrodāharaṇe B2.
440 ālāpitā tad- B2.
441 -ghanasamaḥ sa B2.
442 First half = GK śreḍhī 14b (Part I, p. 117). Second half is metrically irregular.
443 Cf. BG 156: kayoḥ syādantare vargo vargayogo yayorghanaḥ / tau rāśī kathayābhinnau bahudhā bījavittama //
444 yāva om. B2.
445 gharatte B2.

tayorvargayogaḥ yāvava[446] 25 / eṣa iṣṭayāvattāvatpañcakaghanena sama iti pakṣau yāvattāvadghanenāpavartya labdhayāvattāvanmānenotthāpitau jātau rāśī 75 / 100 //

itthaṃ yadā kṛtighanādyasamakriyoktā[447]
vyutpattaye jaḍadhiyāmiha leśato 'pi /
sadvidyarañjanam[448]udāramatiprabodhaṃ
[449]vistāryate 'tha[450] matimeva tṛtīyabījam // 12 //

kaścinnijapriyatamākarabhūṣaṇāya
cakre suvarṇavalayāni navaiva teṣu /
ādyaṃ suvarṇayugalaṃ ca tadantyamarka-[451]
tulyaṃ ca tatpracayasarvadhane vadāśu // (29) //

atra cayapramāṇaṃ yāvattāvat / ā 2 ca yā 1 ga 9 / vyekapadaghnacayo mukhayukta[452] ityādinantyadhanaṃ yā 8 rū 2 / etadantyadhanasyāsya 12 samamiti sāmyakaraṇena labdhaṃ yāvattāvanmānaṃ

5	ayamuttaraṃ[453] dhanaṃ ca 63 //
4	

kasyāpyudāharaṇam[454] /

eṣāṃ ca mātaṅgaturaṅgago'jāḥ[455]
svamatra mātaṅgaturaṅgago'jāḥ[456] /
dattvaikamekaṃ[457] mitha eva jātāḥ
kathaṃ samārthāḥ[458] kati te samārthāḥ[459] // (30) //

atra yāvattāvadādayo[460] varṇā avyaktānāṃ[461] kalpyanta ityupalakṣaṇam / tannāmāṅkitaṃ kṛtvā samīkaraṇaṃ kāryam / anyonyamekaikaṃ dattvā[462] samadhanā jātāsteṣāṃ nyāsaḥ

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[446] yāva B2.
[447] -ghanādyaṃ samaṃ kriyoktā B2.
[448] sadvittaraṃjanam B2.
[449] Fol. 38b, B2.
[450] viṃstāryyatairtha B2.
[451] tadanyamarka- B2.
[452] = GK śreḍhī 1a.
[453] ṃ om. B2.
[454] kasyāthudāharaṇam B2.
[455] -turaṃgamagojāḥ B2.
[456] -turaṃgamagojāḥ B2.
[457] detvaikam- B2, de corrected.
[458] samarthāḥ B2.
[459] samarthaḥ B2.
[460] yāvattāvadayo B2.
[461] avyaktānī B2.
[462] datvā B2.

mā 5	tu 1	go 1	a 1
mā 1	tu 4	go 1	a 1
mā 1	tu 1	go 6	a 1
mā 1	tu 1	go 1	a 8

samānāṃ samakṣepe[463] samaśuddhau samataiva syāditi mātaṅgādyānāṃ śeṣebhyaḥ[464] pṛthagekaikaṃ[465] viśodhya śeṣāṇi samānyeva jātāni mā 4 tu 3 go 5 a 7 / atra yaccaturṇāṃ mātaṅgānāṃ maulyaṃ tadeva turagama[466]trayasya tadeva gopañcakasya tadevājāsaptakasyātassarvaśeṣairiṣṭe bhakte maulyāni syuḥ / tathā kalpitamiṣṭaṃ 100 / ato jātāni mūlyānīti nyāsaḥ 25 $\frac{100}{3}$ 20 $\frac{100}{7}$ samadhanaṃ 192 13 21 [467] /

atrāvaśeṣairbhakte śudhyati yathā[468] tathā kalpitaṃ śeṣasarvaghātasamamiṣṭaṃ 420 / ato jātāni maulyānyabhinnāni 105 / 140 / 84[469] / 60 / samadhanaṃ 809[470] //

> navānāmapi saptānāṃ kastūrīghanasārayoḥ /
> palānāṃ[471] saptasahitaṃ śataṃ jātaṃ[472] dhanaṃ punaḥ // (31) //
> saptānāṃ ca navānāṃ ca śatamekottaraṃ dhanam /
> tayorarghapramāṇaṃ me vada bījavidāṃ vara // (32) //

atra kastūrīmaulyaṃ yā 1 / yadyekasya palasya yāvattāvanmaulyaṃ tadā navānāṃ kimiti jātaṃ kastūrīmūlyaṃ yā 9 / etasmā 107 dapāsya karpūrasaptapalamaulyaṃ yā $\overset{\circ}{9}$ rū 107 / ata ekapalamaulyaṃ $\frac{\text{yā } \overset{\circ}{9} \quad \text{rū } 107}{7 \qquad 7}$ punarekasya kastūrīpalasyedaṃ yā 1 maulyaṃ tadā saptānāṃ[473] kimiti labdhaṃ kastūrīmūlyaṃ yā 7 / etadūnamekottaraṃ śataṃ karpūranavapalamūlyaṃ yā $\overset{\circ}{7}$ rū 101 / ata ekapalamūlyaṃ $\frac{\text{yā } \overset{\circ}{7} \quad \text{rū } 101}{9 \qquad 9}$ etatpūrvakarpūramūlyenānena $\frac{\text{yā } \overset{\circ}{9} \quad \text{rū } 107}{7 \qquad 7}$ samamiti pakṣau samacchedīkṛtya [474]chedagame[475] sāmyakaraṇena la-

[463] -kṣeye B2.
[464] mātaṃgānāṃ śevyetu B2.
[465] -aikaikaṃ B2.
[466] Fol. 39a, B2.
[467] 199 (for 192), 13 and 21 om. B2.
[468] tathā B2.
[469] 44 B2.
[470] 839 B2.
[471] pālānāṃ B2.
[472] jānaṃ B2.
[473] samānāṃ B2.
[474] Fol. 39b, B2.
[475] chadagame B2.

bdhayāvattāvanmānenotthāpite kastūrīkarpūramaulye[476] 8 / 5 //

catvāro hayavaṇijaḥ kati kati santi tu[477] pṛthakpṛthakturagāḥ /
iti śaulkikena[478] pṛṣṭā nijanijaturagānvigūhya[479] jaguḥ /
pañcadaśādidvyadhikāstadā sakhe kati hayāsteṣām // (33) //[480]

atra sarvaturagapramāṇaṃ yā 1 / asmātpañcadaśādidvyadhikāntu-ragānapāsya[481] jātāsteṣāṃ turagāḥ

yā 1	rū $\overset{\circ}{15}$
yā 1	rū $\overset{\circ}{17}$
yā 1	rū $\overset{\circ}{19}$
yā 1	rū $\overset{\circ}{21}$

[482] eṣāṃ yogaḥ yā 4 rū $\overset{\circ}{72}$ / eṣa yāvattāvatsama iti sāmyakaraṇena labdhaṃ yāvattāvanmānaṃ 24 / anenotthāpitā[483] jātāsturagāḥ 9 / 7 / 5 / 3 //

nidhiḥ prāptaḥ puṃbhiḥ kvacidapi caturbhiśca puruṣaḥ
samācaṣte caiko mama dhanayuto 'yaṃ yadi nidhiḥ /
caturghnaṃ yuṣmākaṃ dhanata iti caivaṃ[484] śararasā-
drisaṃnighnaṃ cānye jaguriha pṛthaṅme vada dhanam[485]
// (34) //[486]

guṇaḥ 4 / 5 / 6 / 7 / sarvadhanapramāṇaṃ yā 1 / kalpita iṣṭarāśiḥ 1[487] / ātmacaturgaṇayuteḥ 5 / atra trairāśikam / yadi rāśicaturgaṇasyāsya 5 eko rāśistadaikarūpasya kimiti jātaṃ prathamasya dhanaṃ $\frac{1}{5}$ / evaṃ sarveṣāṃ dhanāni

1	1	1	1
5	6	7	8

etāni pṛthagyāvattāvato 'pāsya jātāni caturṇāṃ dha[488]nāni

[476] -karthūramaulye B2.
[477] santīti B2.
[478] śulkāktena B2.
[479] -turagācijahya B2.
[480] Numerically equal to GK miśra (uv) (Part I, p. 85), which does not have the first line, for which the editor has supplied an Āryā verse composed probably by himself: hayayūthe ye militā nṛṇāṃ caturṇāṃ turaṅgamāsteṣām / śulkārthaṃ me saṃkhyāḥ pṛthakpṛthaktūrṇamācakṣva //
[481] -dvyadhikānvuragānāyāsya B2.
[482] 15 (for 17) B2. In B2, only the first row is written in the proper place without a frame and the remaining three, with a frame, in between 'tthā' and 'tā' of ananotthātā (for anenotthāpitā) in the next line.
[483] ananotthātā B2.
[484] caikaṃvaṃ B2.
[485] nijadhanaṃ B2.
[486] = GK miśra (uv) (Part I, p. 87), where śarassā- (misprint) for śararasā-.
[487] ḥ 1 om. B2.
[488] Fol. 40a, B2.

yā 1 rū $\overset{\circ}{1}$	yā 1 rū $\overset{\circ}{1}$	yā 1 rū $\overset{\circ}{1}$	yā 1 rū $\overset{\circ}{1}$
5	6	7	8

[489]

eṣāṃ yogaḥ

yā 4 rū $\overset{\circ}{533}$
840

[490] eṣa yāvattāvatā sama iti samacchedīkṛtya chedagame sāmyakaraṇāllabdhaṃ yāvanmānaṃ

533
2520

anenotthāpitāni jātāni

29	113	173	218
2520	2520	2520	2520

[491] yāvattāvanmānaṃ kalpitarūpādapāsya jātaṃ nidhimānaṃ

1987
2520

abhinnārthaṃ[492] samacchedānāṃ chedagame jātānyabhinnāni 29 / 113 / 173 / 218 / nidhānaṃ 1987 //

eko bravīti kṛkavākurayaṃ ca jetā
tvanmantrato jayati cetpradadāmi[493] te svam /
nocetpunardviguṇitaṃ svagajāṃśahīnaṃ
lābho bhavedrasakṛtirvada[494] dattavitte // (35) //

atra pratyarpaṇadhanapramāṇaṃ[495] yā 1 / rūpasya dviguṇasya svaṣaḍaṃśāṣṭāṃśakonasya

2	2
1	1
$\overset{\circ}{1}$	$\overset{\circ}{1}$
6	8

savarṇitaṃ

5	7
3	4

ete rūpasyaikasya tadyāvatāvataḥ ke iti jāte dhane

yā 5	yā 7
3	4

anayoryutiryāvattāvadūnā yā $\frac{6}{35}$ ṣaṭtriṃśatā sameti[496] samakriyayā[497] la-

[489] o (in the first cell) om. B2; 10 (for 1 above 8 in the last cell) B2. In B2, the first two results only are framed.
[490] o om. B2.
[491] In B2, this box is written at the beginning of a line of writing, between 'ka' and 'lpi' of the next sentence.
[492] abhi [533 / 2520] nnārthaṃ B2.
[493] cetapravadāmi B2.
[494] bhavaïsakṛtir- B2.
[495] pratyarpaṇaṃ dhana- B2.
[496] sama iti B2.
[497] sama yā 41 yā 35 kriyayā B2. The interpolation, yā 41 yā 35, must be a corruption of yā $\frac{41}{35}$ yā $\frac{\overset{\circ}{35}}{35}$, which symbolically expresses the calculation stated in the previous sentence, anayoryutiryāvattāvadūnā.

bdhaṃ yāvattāvanmānaṃ 210 / anenotthāpite jāte dhane 126[498] / 120 //

kaścitpurapraveśe dattvā[499] pañcābhavattato dviguṇam /
nirgamane pañcaivaṃ puratraye trigu[500]ṇamāsītkim // (36) //

atrādyadhanaṃ yā 1 / atra nagaratrayanirgame jātaṃ dhanaṃ yā 8 rū 105 / etadādyadhanasya triguṇasya yā 3 samamiti samakriyayā labdhaṃ yāvattāvanmānam 21 //

pañcakaśataprayoge varṣe yāte[501] dhanaṃ saphalam /
ṣaḍadhikamadhyardhaṃ ca[502] jātaṃ mūlaṃ phalaṃ[503]
kathaya // (37) //

atra mūladhanapramāṇaṃ[504] yā 1 / asya pañcarāśikena[505] kalāntaraṃ yā $\frac{3}{5}$ saphalaṃ yā[506] $\frac{8}{5}$ etatṣaḍadhikādhyardhasya[507] $\frac{\text{yā } 3 \quad \text{rū } 6}{2}$[508] samamiti sāmyakaraṇena labdhaṃ yāvattāvanmānaṃ 60 / anenotthāpite mūlakalāntare[509] 60 / 36 //

dinadinadalatribhāgāṃghribhiḥ[510] pṛthakpūrayanti ye vāpīm /
te nirjharāśca yugapanmuktā vada kena kālena // (38) //[511]

atra vāpījalapramāṇaṃ yā 1 / yadi dinadinadaladinatṛtīyāṃśadinacaturthāṃśairebhiḥ $\frac{1}{1}\ \frac{1}{2}\ \frac{1}{3}\ \frac{1}{4}$[512] pṛthakpṛthagyāvattāvaddroṇasaṃkhyamudakaṃ[513] pūrayanti tadaikena divasena kimiti pratyekaṃ nirjharāgatodakamānāni[514] yā 1 yā 2 yā[515] 3 yā 4 / eṣāṃ yogaḥ yā 10 /

498 1 om. B2.
499 datvā B2.
500 Fol. 40b, B2.
501 jāte B2.
502 ca om. B2.
503 phalaṃ om. B2.
504 -pramāṇaṃ pramāṇaṃ B2.
505 -rāśikeṇa B2.
506 yā om. B2.
507 etatpaüdhikādhyardhasya B2.
508 In B2, this box is written after 'samamiti'.
509 pūla- B2.
510 dinadinatribhāgāṃbhiḥ B2.
511 = GK miśra (uv) (Part I, p. 94), where bhāgena for kālena.
512 In B2, this table is written after the next word, 'pṛthak'.
513 -droṇasakhyam- B2.
514 bhirjharaugataudaka- B2 (tau corrected).
515 vā B2.

yadi[516] punaryāvattāvaddaśakaṃ dinaikena pūrayanti[517] tadā yāvataṃ keneti[518] yā 10 1 yā 1 / ādyantau bhāgahāraguṇakau yāvattāvatāpavartya trairāśikena labdho vāpīpūraṇadināṃ[519]śaḥ 1 //
10

atra yāvattāvatkalpanayā kim / trairāśikena sidhyati / tadyathā / vāpījalapramāṇaṃ droṇamiṣṭaṃ kalpitaṃ 60 / athānupātaḥ / yadi dinadinārdhadinatṛtīyāṃśadinacaturthāṃśāḥ pṛthakpṛthakṣaṣṭidroṇānpūrayanti[520] tadā dinenaikena kati pratyekadroṇānpūrayantīti[521] nirjharāgatadroṇamānāni 60 / 120 / 180 / 240 / eṣāṃ yogaḥ 600 / etāndroṇānekenāhnā sarve pūrayanti tadā ṣaṣṭiṃ droṇān[522]kena kāleneti[523] labdho dināṃśaḥ 1 //
10

anena nirjharāgatadroṇamānāni bhaktāni jātāḥ pṛthakpṛthagjaladroṇāḥ 6 / 12 / 18 / 24 //

goyūthataḥ[524] prathamato[525] dalamekahīnaṃ
tryaṃśaṃ[526] trigovirahitaṃ pradadau dvitīyāt /
tṛtīyatas[527]triyutamaṅghrimabhūtsahasraṃ
sūryagrahe dvijavarāya vibhuḥ pṛthakkim // (39) //

atra goyūthatrayasamapramāṇaṃ yā 1 / asya dalamekonaṃ [yā 1 rū 1̊ / 2] tryūnatṛtīyāṃśaḥ[528] [yā 1 rū 3̊ / 3][529] triyutaścaturthāṃśaḥ[530] [yā 1 rū 3 / 4][531] eṣāṃ yogaḥ [yā 13 rū 1̊ / 12] eṣa sahasrasama iti sāmyakaraṇena labdhaṃ yāvattāvanmānaṃ 924 / anenotthāpitā jātāḥ pṛthakpṛthaggāvaḥ 461 / 305 / 234 //

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516 yari B2.
517 pūrsyati B2.
518 yāvatā kimiti B2 (an illegible letter between 'va' and 'tā', crossed out).
519 Fol. 41a, B2.
520 -mūrayaṃti B2.
521 -pūrathaṃtīti B2.
522 ṣaṣṭyān B2.
523 koleneti B2, corrected.
524 e before go- B2, crossed out.
525 pṭhagater B2.
526 ṃ (latter) om. B2.
527 ? Tentative emendation of tāstīṣyaṃtas B2, but the first syllable should be long.
528 tryūnattṛ- B2.
529 [yā 1 / 4 | rū 3̊] B2.
530 ḥ om. B2.
531 Without a frame in B2 (the '4' is put in a semi-circular cup).

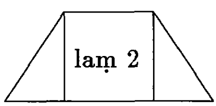
ādiścayena sadṛśaḥ [532]padena sadṛśaṃ phalam /
tadādyuttaragacchānāṃ kāḥ[533] saṃkhyā gaṇitasya ca // (40) //

atra[534] gacche yāvattāvatkalpite kriyā na nirvahati yataḥ kalpita iṣṭo gacchaḥ 7 / ādyuttarau yāvattāvatkalpitau ā yā 1 u yā 1 ga 7 / śreḍhīphalaṃ yā 28 [535] / etatpūrvaphalasyāsya 7 samamiti sāmyakaraṇena labdhayāvattāvanmānenotthāpitāv[536]ādyuttarau

1	1
4	4

ekādyekottaraśreḍhīkṣetre yatra dvilambake[537] /
tatra bhūvadane bījagaṇitajña vadāśu me // (41) //

nyāsaḥ



phalaṃ 3 / atra bhūmukhayogo yāvattāvatkalpitaḥ yā 1 / etatpūrvaphalasyāsya 3 samaṃ kṛtvā labdhaṃ yāvattāvanmānaṃ 3 / iyaṃ mukhabhūyutiḥ //

atha mukhapramāṇaṃ yā 1 / anenonā bhūmukhayutirjātā bhūḥ yā $\overset{\circ}{1}$ rū 3 [538] / idānīṃ

samalambakacaturasre[539]
vimukhā[540] bhūrbhūḥ prajāyate tryasre[541] /[542]

iti jātā tryasrabhūḥ yā $\overset{\circ}{2}$ rū 3 [543] / pārśvabhujau sadṛśau[544] yato 'syārdhamāvādhā[545]

yā $\overset{\circ}{2}$	rū 3
2	2

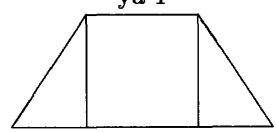
[546] atrānupātaḥ / yadi lambasyā-

[532] Fol. 41b, B2.
[533] ḥ om. B2.
[534] aṃtra B2.
[535] śreṣṭīphalaṃ 28 B2.
[536] -mānenauspāyitau B2.
[537] yatra-laṃbakau B2.
[538] yā 10 rū 3 B2.
[539] samalambacatudasre B2.
[540] vimukha B2.
[541] tryaṃsre B2.
[542] = GK kṣetra 53a (Part II, p. 63).
[543] jātāḥ yasrabhū yā 20 rū 3 B2.
[544] -bhujai saśau B2.
[545] -ālādhā B2.
[546] B2 om. the bottom line of the frame.

sya 2 [547] bhūkhaṇḍamidaṃ[548] yā $\frac{\overset{\circ}{2}}{2}$ rū $\frac{3}{2}$ [549] [550]taduparimakṣetra-syāsya kimiti[551] jātāvādhā

yā $\frac{\overset{\circ}{2}}{4}$	rū $\frac{3}{4}$

[552] yanmukhaṃ tallamba-nipātāntarabhūmānam[553] / darśanam[554]

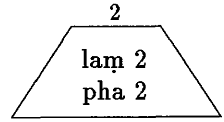


[555]

uparimakṣetrabhūkhaṇḍānāṃ[556] yoge bhū rū $\frac{3}{2}$ [557] / bhūmukha-dalamavalambaguṇitaṃ phalamiti[558]

yā $\frac{2}{4}$	rū $\frac{3}{4}$

etaduparimakṣe-trasyāsya[559] 1 samamiti sāmyakaraṇena labdhaṃ yāvattāvanmānaṃ $\frac{1}{2}$ [560] / anenotthāpya[561] jātaṃ śreḍhīkṣetram



[562]

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[547] In B2, this 2 is written above 'na' of -mānam two lines below.
[548] bhūkhaḍamidaṃ 6 B2.
[549] 2's (in the denominators) om. B2.
[550] Fol. 42a, B2.
[551] kiniti B2.
[552]

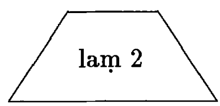
yā $\overset{\circ}{2}$ rū 3
44

B2.
[553] B2 has 2 above 'na' of -mānam.
[554] darśanaï B2.
[555] yā 2 (for yā 1) B2. In B2, this figure is put in a framed box and placed in between 'u' and 'pa' of the next word.
[556] -kṣe – sabhūkhaḍānāṃ B2.
[557] 2 om. B2.
[558] bhūkhaṃḍadalamava – – – – taphalami – – B2. Cf. GK kṣetra 34b (Part II, p. 42): samalambe bhūmukhayutidalahatalambaṃ phalaṃ caturbāhau //
[559] -upari – kṣetra- B2.
[560] yāvattā – nmā – – B2.
[561] anenotthātha B2.
[562] $\frac{5}{2}$ om. B2. In B2, the letters in the trapezium are not clear but the first line looks like 'lapha' and the second 'lāphara'.

vardhamānagatiḥ kaścid[563]dvyāditryuttarako bhavet[564] /
dinasyārdhena saṃjātopapattistatra[565] kathyatām[566] // (42) //

ā 2 u 3 ga 1/2 śreḍhīphalaṃ 5/8 [567] / asyopapattiṃ[568] kṣetra-dvāreṇāha / na rāśigatā[569] ceti / nirantaravardhamānagativaśādyasya mukhaṃ[570] vṛhadanekaṃ[571] gacchasaṃkhyalambakaṃ[572] tadutpannaṃ kṣetraphalaṃ śreḍhīphalasamaṃ[573] bhavati //

atha śreḍhīkṣetrasya dvikasaṃkhyagacchādvadanaṃ bhūmipramāṇaṃ ca[574] sidhyati / uttaraṃ gacchānna caikagacchād[575]upapattirdṛśyate yato 'trādirdvayamuttaraṃ trayam[576] / dvikasaṃkhye gacche śreḍhīphalaṃ 7 [577] / ataḥ śreḍhīkṣetrasya darśanam



asya phalaṃ sapta / lambo dvi[578]kaḥ / bhūvadane na jñāte / ato bhūmukhapramāṇaṃ yā 1 / bhūmukhayutidala[579]⟨hatalambaṃ phalamiti ...⟩[580]

3 *Translation*

[Notation: Angle brackets ⟨A⟩ indicate that A is not a translation of Sanskrit word(s) but has been supplied by me. Parentheses (A) indicate that A is an explanation, or an equivalent, of the immediately preceding word(s). Square brackets [A] indicate that A is a number expressed by word numerals or the so-called *bhūtasaṃkhyā*.[1] A double *daṇḍa* //

[563] vardha – – gati – ści B2.
[564] -uttarakaṃḥ – – – B2 (ḥ below the line of writing).
[565] saṃjātopattis- B2.
[566] kathya – – B2.
[567] śreḍhīpha – – B2.
[568] asyopapattiḥ B2.
[569] rāśigajā B2.
[570] vaśā – – – khaṃ B2.
[571] aneka B2.
[572] -laṃlakaṃ B2.
[573] śreṭīphala- B2.
[574] va B2.
[575] caikadāchād B2.
[576] yatotrādirhayamuttaratraya B2.
[577] treḍhīphā 37 B2.
[578] Fol. 42b, B2.
[579] B2 ends here abruptly, and the rest of fol. 42b is left blank.
[580] A tentative restoration. Cf. GK kṣetra 34b (Part II, p. 42): samalambe bhūmukhayutidalahatalambaṃ phalaṃ caturbāhau //

[1] For the word numerals, see Datta and Singh 1962, part 1, pp. 53–63.

separates two consecutive verses.]

1–2. Since this whole universe ⟨and mathematics of known quantities⟩, infinite and visible, are born from invisible seed⟨s⟩, I always bow down to Śiva and the ⟨seed⟩ mathematics ⟨which are their respective seeds⟩.// It is said that in this ⟨mathematics⟩ there are four seeds, namely, the equation procedure with an invisible (i.e., unknown) ⟨quantity⟩, equality with colors, the elimination of the middle, and equality with the product.

3–7. Having assumed the value of the unknown ⟨quantity⟩ to be marked with a singular or plural *yāvattāvat*, increased or decreased by numbers ⟨if necessary⟩,// one should perform computation upon that value according to the statements of the questioner. In order to obtain the result, the two sides should be made equal ⟨to each other⟩ carefully.// One should subtract the unknown from one ⟨side⟩ and the ⟨known⟩ number from the other, and divide the remainder of the number by the remaining unknown.// The value of the unknown quantity becomes known indeed ⟨in this way⟩. Or else, when there are many unknown ⟨quantities⟩, one should assume ⟨them⟩ to be *yāvattāvat* multiplied by two, etc.,// or divided ⟨by them⟩, or otherwise increased or decreased by numbers. So, the value of the unknown should be known by one's own intellect according to the case.

(1)–(2) A merchant has eight horses of the same price and six hundred *rūpas*, and another merchant has ⟨horses⟩ measured by [sun] (12) and a debt of two hundred ⟨*rūpas*⟩. The two ⟨merchants⟩ have equal properties. What is the price of a horse?// Or, let ⟨the property of⟩ the first increased by eight ⟨*rūpas*⟩ and halved be equal to ⟨the property of⟩ the other. Likewise, let ⟨the property of⟩ the former be equal to three times ⟨the property of⟩ the other decreased by four ⟨*rūpas*⟩. ⟨Then, in each case,⟩ tell me the price of a horse.

(3) A man has corals, zircons, sapphires, and diamonds ⟨measured respectively by⟩ [sun] (12), [directions] (10), eight (8), and [philosophical doctrines] (6), and another has ⟨the same, each⟩ increased by one ⟨in number⟩. The two ⟨properties⟩ increased by a hundred and by sixty ⟨*rūpas*, respectively⟩, are equal to each other. Say the price of each jewel.

(4)[2] A certain amount of money in the first year and twice

[2] Similar to GK miśra (uv) (Part I, p. 62) = Singh II, Ex. 15: 'A cer-

⟨the previous amount⟩ plus one in each ⟨succeeding⟩ year were donated by a certain wealthy person to a brāhmaṇa. In five years, one hundred and fifty ⟨*rūpas* were given in total⟩. Say, what amount of money did he give first?

(5) Of the stalk of a lotus which was born in a lotus pond, half is in ⟨a layer of⟩ mud, one [arrow]-th (1/5) in ⟨a layer of⟩ moss, and one [directions]-th (1/10) in water. There remains a pair of *hastas* ⟨seen above the water⟩. Tell me quickly the length of the stalk if you know.

8. ⟨When⟩ a divisor and a dividend have equal denominators, the denominators of the two ⟨quantities⟩ (i.e., the divisor and the dividend) are omitted. When the dividend is divided by that divisor, the quotient ⟨to be obtained⟩ in the case of the division of the fractions will be ⟨obtained⟩.

(6) From an assemblage of wasps, half has gone to a place abounding with lotuses, one third of the remainder to a mango tree, one fourth of the remainder to a bakula tree, one fifth of the remainder to a kuṭaja tree with opening blossoms, and six plus six to a jasmine tree in flower. O learned one, how many are they ⟨in total⟩? Tell me quickly, friend, if you know the seeds.

(7) One said to his friend, '⟨If⟩ you give ⟨me⟩ one hundred ⟨*rūpas*⟩ from your own property, then I will ⟨have⟩twice as much as your property.' The other (the latter) said to him (the former), 'If you give me eight ⟨*rūpas*⟩, then I will ⟨have⟩five times as much as your property.' What are their properties?

(8)[3] Being invested at ⟨the rate of⟩ five percent ⟨per month⟩, a capital together with ⟨its⟩ interest becomes twice ⟨the capital⟩ minus sixteen ⟨*rūpas*⟩ when one year has passed. Tell me the capital and the interest.

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tain amount of money in the first year and twice ⟨the previous amount⟩ in each ⟨succeeding⟩ year were donated by a certain wealthy person to a brāhmaṇa. In five years it amounted to one hundred and eighty-six ⟨*rūpas*⟩. What is the first ⟨donation⟩?' (The underlined text indicates the difference between the two problems.) Here and hereafter, I provide my own translations of the GK verses in order to make it easier to compare the corresponding passages. Refer also to Singh's translations.

[3] Numerically equal to BG 99: 'A capital loaned at ⟨the rate of⟩ five percent ⟨per month⟩ together with ⟨its⟩ interest is twice ⟨the capital⟩ minus sixteen when one year has passed. What is the gain (interest)? Say the capital.'

(9)[4] One hundred ⟨*rūpas*⟩ are loaned in three parts at ⟨the rates, respectively, of⟩ two, three, and four percent per month for months measured respectively by five (5), [elephants] (8), and [directions] (10). Say, O best of noble ones, how can the interests on all the three parts be the same? Or, ⟨how can⟩ the partial properties accompanied by their respective interests be equal ⟨to each other⟩? Or, ⟨how can⟩ the ⟨partial⟩ properties decreased by ⟨their respective⟩ interests be the same?

(10)[5] Capitals, ⟨two in number⟩, are loaned at ⟨the rates of⟩ five and four percent per month, ⟨respectively⟩. ⟨When they are⟩ increased by their reciprocal interests produced for seven and ten months, ⟨respectively⟩, how can ⟨the results⟩ be the same? Say, friend, if you know the seeds.

(11)[6] The difference between a certain ⟨capital⟩ and the square of its interest ⟨produced when it is loaned⟩ at ⟨the rate of⟩ five ⟨percent per month⟩ is invested at ⟨the rate of⟩ ten ⟨percent per month⟩. The ⟨two⟩ time periods are the same, ⟨and so also are⟩ their interests. ⟨What are they each?⟩

(12)[7] Two *palas* of sandalwood are obtained for one *niṣka*, and *palas* of saffron, equal to [arrows] (5) ⟨in number⟩, for two ⟨*niṣkas*⟩. O noble one, give ⟨me⟩ sandalwood (*bhadraśriya*) and saffron (*kāśmīraka*), which (latter) is twice as much ⟨as the former⟩, for ⟨a total of⟩ ten *niṣkas*, for worshipping Śaṅkara (= Śiva).

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[4] Numerically equal to GK miśra (uv) (Part I, p. 61) = Singh II, Ex. 14, whose first sentence reads: 'A property of one hundred ⟨*rūpas*⟩ is loaned in three parts by the interest⟨-rates⟩, of two, three, and four ⟨percent, respectively, per month⟩ for the months ...'

[5] Numerically equal to the first problem of GK miśra (uv) (Part I, p. 63) = Singh II, Ex. 16: '⟨Capitals⟩, in two parts, are loaned at ⟨the rates of⟩ five and four percent per month, ⟨respectively⟩. ⟨When they are⟩ increased <u>or decreased</u> by their reciprocal interests produced for ten and seven months, ⟨respectively⟩, the sums become equal ⟨to each other⟩. ⟨What are they each?⟩' (The underlined part is for the second problem.)

[6] Numerically equal to BG 96: 'When one has subtracted the square of the fruit (interest) from the capital loaned at ⟨the rate of⟩ five percent ⟨per month⟩, the remainder is loaned at ⟨the rate of⟩ ten percent ⟨per month⟩. The ⟨two⟩ time periods are the same and so also are their interests. ⟨What are they each?⟩'

[7] Numerically equal to GK miśra (uv) (Part I, p. 58) = Singh II, Ex. 8: 'If two *palas* of sandalwood are obtained for one *niṣka*, and *palas* of saffron, measured by [arrows] (5), for two ⟨*niṣkas*⟩, then, for worshipping Īśa (= Śiva), give me good sandalwood (*candana*) and excellent saffron (*kuṅkuma*), which (latter) is twice as much ⟨as the former⟩, for ⟨a total of⟩ ten *niṣkas*.'

(13)[8] A mistress, having remembered ⟨her⟩ lover and fallen lovesick, went (lit. goes) ⟨after him⟩ ten ⟨*yojanas*⟩ always (i.e., every day). A procuress immediately pursued her in order to forbid ⟨her to do so⟩ at the ⟨speed⟩ which is two ⟨*yojanas*⟩ on the first ⟨day⟩, increasing by three ⟨*yojanas* on every succeeding day⟩. Say, after how many days ⟨did⟩ the meeting ⟨of the two ladies take place⟩?

(14) When the sum of a series ⟨in arithmetic progression⟩whose steps are four ⟨in number⟩ and whose common difference is six, is one hundred, then say the first term, O expert in the seeds, if you know the seed computation.

(15) When the sum of a series ⟨in arithmetic progression⟩whose steps are five ⟨in number⟩ and whose first term is four, is one hundred, then say the common difference, if you are well versed in mathematics.

(16) A traveler goes *yojanas* measured by [treasures] (9) on the first day and increases ⟨his daily journey⟩ regularly by five ⟨*yojanas*⟩. Another goes likewise (in the same direction) a pair ⟨of *yojanas*⟩ on the first day and increases ⟨his daily journey⟩ by seven ⟨*yojanas*⟩. Say, after how many days ⟨does⟩ the meeting ⟨of the two travelers take place⟩?

(17) Eight *palas*, ⟨in total⟩, of the deer-navel (i.e., musk) and the snow-gravel (i.e., camphor) are bought for one *niṣka* and sold at the ⟨same⟩rate-quantities (*argha*) interchanged. The profit is ten ⟨*niṣkas*⟩. O merchant, say ⟨the purchase prices of⟩ the two separately.

(18) When, O learned one, the product of the first term, the common difference and the number of terms ⟨of an arithmetical progression⟩ is ten and ⟨its⟩ computed ⟨sum⟩ is seven, then say quickly the first term and the common difference.

(19) A man goes with certain initial ⟨speed⟩ and ⟨daily⟩ increase, ⟨starting⟩ eight days ahead of another, who proceeds after ⟨him⟩ with a certain initial ⟨speed⟩ and with a ⟨daily⟩ increase of two ⟨*yojanas*⟩. O friend, in how many days did the meeting on the road of the two ⟨persons⟩ take place twice? Say, O lordly elephant who knows mathematical series, if you are ⟨really⟩ a lordly

[8] Numerically equal to GK śreḍhī (uv) (Part I, p. 112) = Singh III, Ex. 10: '... A procuress pursued her at the ⟨speed⟩ which is two ⟨*yojanas*⟩ on the first ⟨day⟩, increasing by three ⟨*yojanas* on every succeeding day⟩. After how many days would there be the meeting of the two ⟨ladies⟩?'

elephant who is mighty at elephant war.

(20)[9] Of what quantities, four in number, is the sum equal to the sum of their squares? Of what ⟨quantities, four in number⟩, is the sum of the cubes equal to the sum of their squares, or are the sum and the product the same? Say them quickly, calculator, if you are proficient in the unknown work (i.e., the computation with unknown quantities). Of what quantities, two in number, will the sum increased by the difference be equal to the sum of the squares? Say the two immediately.

(21)[10] To the upright, to the side, to the ear (hypotenuse), or to their product, the area ⟨of a right-angled triangle⟩ happened to be equal. Tell me them, calculator.

(22) From what ⟨quantities⟩, two in number, is a square produced in the case of the sum and a cube in the case of the difference? The square root of the ⟨square⟩ obtained, multiplied by the cube root ⟨of the cube obtained⟩, has (lit. gives) a root.

(23) Two certain squares are increased by ⟨their⟩ reciprocal roots. ⟨Then⟩, the second ⟨of the two results⟩ becomes three times the first. Say the two quickly.

(24) The product of two quantities, one small and one greater, increased by the square of the smaller, is equal to the square of the greater increased by the smaller. Say the two.

(25) The square of the greater of two quantities, multiplied by the smaller, decreased by the square of the smaller and halved, will be equal to the cube of the smaller. Tell me the two, calculator, if you know.

(26)[11] Of what quantities, two in number, does the square of the sum increased by the cube of the sum become equal to to

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[9] The first half of this example is numerically equal to BG 106: 'Say four different quantities having equal denominators, whose sum, or the sum of whose cubes, is measured by the sum of their squares.' For the reading, 'different quantities having equal denominators', the commentator Kṛṣṇa mentions a variant, 'different quantities, O unrivalled learned', which he thinks is better because the 'state of having equal denominators' (*samacchedatva*) is not necessary for the problem.

[10] The second half of this example is numerically equal to BG 107: 'Tell such a ⟨right-angled⟩ trilateral field whose fruit (area) is measured by its ear (hypotenuse) or equal to the product of its side, upright and ear.'

[11] Numerically equal to BG 153: 'The square of the sum of two quantities combined with the cube of the sum of the ⟨same⟩ two quantities is equal to twice the sum of ⟨their⟩ cubes. O calculator, let it be told.' The commentator Kṛṣṇa introduces BG 153 as 'an example of predecessors'.

twice the sum of the cubes? Say the two, friend, if you know.

(27)[12] Of what quantities, two in number, will the sum and the difference be squares, and the product a cube? Say quickly. Of what ⟨quantities⟩, two in number, does the sum of the cubes have the state of being a square, and the sum of the squares the state of being a cube? What are the two?

9. The sum of the squares of two optional ⟨quantities⟩ shall be one quantity and twice the product of the two ⟨quantities⟩ another. When the sum or the difference of the two quantities ⟨is taken⟩, there will be a square.

10.[13] The cube of the ⟨square-⟩root of a quantity ⟨is made⟩. Its square is equal to the cube of the ⟨original⟩ quantity.

11.[14] The square of a *saṅkalita* (i.e., the sum of a natural series) will be ⟨equal to⟩ the sum of the cubes of the ⟨integers⟩ beginning with unity. ⟨Therefore⟩, the sum of the cubes ⟨of the numbers which make a progression⟩ with unity both for the first term and for the common difference becomes what has (lit. gives) a ⟨square-⟩root.

(28)[15] From what ⟨quantities⟩, two in number, is a square produced in the case of the difference and a cube in the case of

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[12] Numerically equal to GK prakīrṇaka (uv) (Part I, p. 36) + (uv) (Part I, p. 38) = Singh I, Exs. 53 and 54. First verse: 'There will be a square in the case of the sum or the difference of ⟨certain⟩ two quantities, and a cube in the case of their product. Tell me, friend, quickly such two ⟨quantities⟩ if you know.' Second verse: 'What ⟨quantities⟩, two in number, will make a square in the case of the sum of their cubes, and a cube in the case of the sum of their squares? Tell ⟨me⟩, friend, quickly such two ⟨quantities⟩ if you are well versed in the Wonder of Square.' (For the 'Wonder of Square', see Remarks under Verse (27) in Section 4.) Also numerically equal to BG 108 + 109. BG 108: 'Say quickly, if you are well versed in mathematics, such a pair of quantities, in the case of whose sum there will be a square, in the case of whose difference a square, and in the case of whose product a cube.' BG 109: 'I think you are the best of experts in the seeds if you know such a pair of quantities, the sum of whose cubes becomes a square and the sum of whose squares a cube.'

[13] Nearly identical with the verse of GK prakīrṇaka 23b: 'The cube of ... equal to the cube ⟨of the original quantity⟩.' (The underline indicates the difference.)

[14] First half is identical with the verse of GK śreḍhī 14b.

[15] Numerically equal to BG 156: 'Of what ⟨quantities⟩, two in numbers, will the difference be a square and the sum of the squares be a cube? O best of experts in the seeds, tell ⟨me⟩ many times such a pair of non-fractional quantities.'

the sum of the squares? Say the two quickly if you have trained well in the equation procedure.

12. When the equation procedure involving squares, cubes, etc. has been told here in this way for the sake of ⟨producing⟩ perfect proficiency in a dull-minded person even though it is a little, the third seed, which gratifies well-educated people and awakes proper intelligence, is expounded.[16]

(29) A certain man, in order to adorn the best of his dearest lovers, made nine gold rings. Out of them, the first weighs two *suvarṇas* and the last is equal ⟨in weight⟩ to [sun] (12) ⟨*suvarṇas*⟩. Say quickly the common difference and the total value.

(30)[17] They (four people) have elephants, horses, cows, and goats (*aja*), ⟨respectively⟩, which are ⟨measured in order by⟩ [elephants] (8), [horses] (7), [cows] (9), and [unborn (*aja*)] (11).[18] When they have given one from one's own property to each other, how do they become of equal properties and what are those equal properties[19]?

(31)–(32) The ⟨total⟩ value of nine and seven *palas*, ⟨respectively⟩, of musk and camphor (lit. 'hard essence') happens to be one hundred and seven ⟨*rūpas*⟩, while// the value of seven and nine ⟨*palas* of them⟩ is one hundred and one ⟨*rūpas*⟩. Tell me the prices of the two ⟨commodities⟩, O best of the experts in the seeds.

(33)[20] Four horse-dealers were asked by a custom officer, 'How many horses are there ⟨in possession of⟩ each ⟨of you⟩?' They answered ⟨in order the numbers⟩ beginning with fifteen and increasing by two ⟨as the total number of their horses⟩, hiding ⟨the number of⟩ one's own horses. O friend, how many horses do they have ⟨each⟩?

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[16] I do not understand the meaning of the phrase *atha matimeva* (or *arthamatimeva*).

[17] This example is introduced as 'an example of a certain person' (*kasyāpyudāharaṇam*).

[18] Here, exactly the same words for the four kinds of domestic animals are used as word numerals in order to indicate the number of each animal.

[19] In the last quarter of the stanza, the compound, *samārthāḥ* ('equal properties'), occurs twice, once as a Bahuvrīhi and once as a Karmadhāraya.

[20] Numerically equal to GK miśra (uv) (Part I, p. 85) = Singh II, Ex. 34: '⟨Four horse-dealers⟩ were asked by a custom officer, 'For customs, tell me quickly the number of the horses of four men, one by one, which are assembled together in a horse troop.' They answered ...'

(34) At a certain place a hoard was obtained by four men. One of them said, 'If this hoard is increased by my property, it is four times your ⟨total⟩ property.' Likewise, the others said ⟨severally that the sum of the hoard and each one's own property was⟩ [arrows] (5), [tastes] (6), and [mountains] (7) times as great ⟨as the total property of the others⟩. In this case, tell me each property.

(35) One says, 'If this victorious cock wins thanks to your spell, I give you ⟨a certain amount of⟩ money, but if not, ⟨you have to give me⟩ twice ⟨that amount⟩ decreased by one [elephants]-th part (1/8) of itself.' Let the gain ⟨of the gambler⟩ be the square of [tastes] (6). Say the two given ⟨amounts of⟩ money.

(36) A certain man, having paid five ⟨*rūpas* at a tollhouse⟩ when he entered a town, became twice ⟨as rich as before by business in the town⟩, ⟨and again paid⟩ five ⟨*rūpas* at a tollhouse⟩ when he went out ⟨of the town⟩. What ⟨original property⟩ became ⟨increased⟩three times after ⟨he went through⟩ three towns?

(37) When invested at ⟨the rate of⟩ five percent ⟨per month⟩, a property together with the interest ⟨on it⟩ after one year had elapsed became six ⟨*rūpas*⟩ plus one and a half times ⟨the capital⟩. Say the capital and the interest.

(38) There are four springs, which separately fill a pond in one day, in a half day, in a third, and in a quarter ⟨of a day⟩. Say how long it takes for them ⟨to fill the same pond⟩ when opened simultaneously.

(39) On the occasion of a solar eclipse, a king donated to the best of brāhmaṇas a half of the first herd of cows minus one ⟨cow⟩, a third of the second minus three cows, and a quarter of the third plus three ⟨cows⟩, which, ⟨in total⟩, became one thousand. What is ⟨the number of cows in⟩ each ⟨herd⟩?

(40) ⟨Of an arithmetical progression⟩, the first term is equal to the common difference, and the sum (lit. 'the fruit') is equal to the number of terms. What are the numerical values (lit. 'the numbers') of the first term, of the common difference, of the number of terms, and of the sum (lit. 'the computed one') of it?

(41) O expert in the seed mathematics, tell me quickly the base and the top of the figure for the series whose first term and common difference are both unity, a figure whose perpendicular is two ⟨in length⟩.

(42) A certain man with an ever-increasing speed would have two ⟨*yojanas*⟩ for the first term and three ⟨*yojanas*⟩ for the increase. A proof ⟨of the computational rule for the sum⟩ obtained there for half a day should be told.

4 *Commentary*

Verses 1–2: Introduction.

Nārāyaṇa first salutes Śiva and the 'seed mathematics' (*bīja-gaṇita*), alluding to the parallelism between the two. Here, the mathematics of invisible (unknown) quantities (*avyakta-gaṇita*) is regarded as the unique 'seed' of the mathematics of visible (known) quantities (*vyakta-gaṇita*) just as Śiva is the unique, invisible seed of this whole, visible universe. Nārāyaṇa makes a similar salutation in the first verse of Part I of the present work also.[1] The mathematics of unknown quantities or algebra is a seed of the mathematics of known quantities or of algorithm (*pāṭī-gaṇita*) because the former produces the computational rules or algorithms of the latter.

Then, Nārāyaṇa refers to the traditional concept of a 'quartet of seeds' (*bīja-catuṣṭaya*), which is the main theme of this work. Here, the 'seeds' are algebraic equations or the solution procedures that utilize algebraic equations (*samīkaraṇa/samatva*, which literally mean 'making equal'/'equality'[2]). They are:

Seed 1: Equation procedure with one 'color' or variable (*ekavarṇa-samīkaraṇa*),[3]
Seed 2: Equation procedure with more than one color (*anekavarṇa-samīkaraṇa*),
Seed 3: Elimination of the middle term (*madhyama-āharaṇa*),
Seed 4: Equation procedure with the product (*bhāvita-samīkaraṇa*).

[1] BGA I.1: ekamanekasyāsyāvyaktaṃ vyaktasya guṇavato jagataḥ / gaṇanāvidheśca bījaṃ brahma ca gaṇitaṃ ca tadvande // 1 // Cf. also a similar statement of Bhāskara II in his BG 1: utpādakaṃ yatpravadanti buddheradhiṣṭhitaṃ satpuruṣeṇa sāṃkhyāḥ / vyaktasya kṛtsnasya tadekabījamavyaktamīśaṃ gaṇitaṃ ca vande // 1 //

[2] The commentator also uses the synonym, *sāmya* ('equality').

[3] Also called 'equation procedure with an unknown ⟨quantity⟩' (*avyakta-samīkaraṇa*).

All these topics had been treated by Nārāyaṇa's predecessors such as Brahmagupta (in Chapter 18 of his *Brāhmasphuṭasiddhānta*, A.D. 628), Śrīpati (in Chapter 14 of his *Siddhāntaśekhara*, ca. A.D. 1040), and Bhāskara II (in his *Bījagaṇita*, A.D. 1150).[4] The following descriptions of the 'four seeds' are mostly based on these works as our text ends abruptly in the middle of the commentary on Example 40 for Seed 1.

Seed 1 produces the solution $x = (d - b)/(a - c)$ of the linear equation, $ax + b = cx + d$. See Verses 3–7 below.

The word 'color' (*varṇa*) used in the names of Seeds 1 and 2 stands for an unknown number. The word *varṇa* also means 'a syllable' or 'a syllabic letter'; the initial letters of the names not only of colors but also of any group of objects are used to denote unknown numbers in algebraic equations just as the initial letter, *rū*, of the word *rūpāṇi* ('units') is used to denote a known number.[5] Nārāyaṇa himself gives a long list of color names beginning with *yāvattāvat*,[6] which 'have been provided ⟨by predecessors⟩ as

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[4] Also refer to Chaps. 4 (*prakīrṇaka* or miscellaneous) and 6 (*miśraka* or mixture) of Mahāvīra's *Gaṇitasārasaṃgraha* (ca. A. D. 850), where he gives computational rules (or algorithms) for a number of problems involving various linear and quadratic equations with one or more unknown quantities, although he does not use any expressions pertaining to equations as the work is not for *avyakta-gaṇita* but for *vyakta-gaṇita*.

[5] BGA I.7: rūpāṇāmavyaktānāṃ nāmādyakṣarāṇi lekhyāni / upalakṣaṇāya teṣāmṛṇagānāmūrdhvabindūni // 7 // 'The initial letters of the names of *rūpāṇi* and of unknown (*avyakta*) ⟨numbers⟩ (i.e., of objects whose quantities are unknown) should be written down in order to designate them. Negative ⟨numbers⟩ have over-dots.' Cf. BG 5.

[6] BGA I.17–18a list the following color names: *yāvattāvat* ('as much as', 'as long as', etc.), *kālaka* ('black'), *nīlaka* ('blue'), *pīta* ('yellow'), *lohita* ('red'), *harita* ('green'), *śvetaka* ('white'), *citraka* ('bright'), *kapilaka* ('tawny'), *pāṭalaka* ('pink'), *pāṇḍu* ('pale-white'), *dhūmra* ('smokey grey'), *śabala* ('variegated'), *śyāmalaka* ('black'), *mecaka* ('black'), *dhavalaka* ('white'), *piśaṅga* ('tawny'), *śāraṅga* ('variegated'), *babhru* ('brown'), *gaura* ('white'), etc. The commentator remarks: yāvattāvatkālaka- ⋯ -gaurakā ityādyā varṇā athavā varṇāḥ kādayo 'thavā madhurādayo rasaparyāyā athavāsadṛśaprathamākṣaranāmaparyāyapadārthāḥ kalpyante // (I have corrected -nāmapadārthāḥ of Shukla's edition to -nāmaparyāyapadārthāḥ based on the two manuscripts, B1 and B2. See under 'L and B1' in §1.4) 'The colors such as *yāvattāvat*, *kālaka*, ..., and *gauraka*, or else the syllabic letters beginning with *ka*, or else the homogeneous words for the tastes beginning with *madhura* ('sweet'), or else ⟨a group of⟩ objects having homogeneous names whose intial letters are different from each other, are assumed ⟨as the designations of unknown quantities⟩.' The second choice mentioned here is the

the designations of the values of unknown ⟨quantities⟩ in order to produce (or enable) computation'.[7] The 'color' used for Seed 1 is always *yāvattāvat*, which is in fact not the name of a color but a combination of the correlative attributes or adverbs, *yāvat* + *tāvat*, meaning 'as much as', 'as long as', etc.

Seed 2 produces solutions to a system, determinate or indeterminate, of linear equations by reducing it to a single linear equation by means of substitution (*utthāpana*, lit. 'raising'). For example, a system of two linear equations with two unknown numbers, $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$, is first rewritten as $y = (c_1 - a_1x)/b_1$ and $y = (c_2 - a_2x)/b_2$. Then these are combined into a single linear equation, $(c_1 - a_1x)/b_1 = (c_2 - a_2x)/b_2$, which is rewritten as $b_2c_1 - a_1b_2x = b_1c_2 - a_2b_1x$, to which Seed 1 is applied. When the number (n) of unknown quantities exceeds that (m) of equations by one (that is, $n - m = 1$), the procedure called *kuṭṭaka* or 'pulverizer' is employed.[8] When n exceeds m by more than one (that is, $n - m \geq 2$), arbitrary numbers are assumed for $(n-m-1)$ unknown quantities, and then the *kuṭṭaka* is employed.[9]

Seed 3 produces a solution to the quadratic equation with one unknown, $ax^2 + bx = c$, by reducing it to a single linear equation by means of the 'elimination of the middle term', a procedure which is equivalent to the so-called 'completion of the square'. That is, both sides of the quadratic equation are first multiplied

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table of the letters for the consonants, ka, kha, ga, gha, ṅa, etc. The third one is: madhura (sweet), amla (sour), lavaṇa (salty), kaṭu (pungent), tikta (bitter), and kaṣāya (astringent). See Verse (30) for an example of the last (fourth) case.

[7] BGA I.18b: gaṇanotpattyai vihitāḥ saṃjñāścāvyaktamānānām // 18 // Cf. BG 21.

[8] The *kuṭṭaka* is a procedure for deriving integer solutions to linear indeterminate equations of the type, $y = (ax + c)/b$. This is one of the favorite topics of Indian mathematicians starting, at the latest, with Āryabhaṭa I (b. A.D. 476), who in his *Āryabhaṭīya* 2.32–33 (A.D. 499 or a little later) gave a general solution to a general form of the same problem, $y = a_ix_i + r_i$ $(0 \leq r_i < a_i; i = 1, 2, ..., n)$. The *kuṭṭaka* is also treated in: BSS 18.3–29, 46–50 (examples), and 52–59 (examples), MB 1.41–52, *Govindakṛti* (Shukla 1963, 103–114), GSS 6.115b–139a (rules with examples), MS 18.1–66, SS 14.22–31, and BGA I.52–69 (rules with examples). The references to Sanskrit texts here and hereafter are limited to those prior to Nārāyaṇa.

[9] Seed 2 is treated in: BSS 18.51 and 52–59 (examples), SS 14.15–16, and BG 134 and 135–148 (examples).

by $4a$ and increased by b^2: $4a^2x^2+4abx+b^2 = b^2+4ac$. Then, the square roots of both sides are extracted:[10] $2ax+b = \pm\sqrt{b^2+4ac}$, to which Seed 1 is applied. Due to the extraction of the square roots, the first and the last terms of the left-hand side, $4a^2x^2$ and b^2, become $2ax$ and b, respectively, and the 'middle term', $4abx$, disappears. Hence follows the name of the third seed, 'elimination of the middle term'.[11]

Seed 4 produces solutions to equations involving the product of two or more unknown numbers. The most typical case is $axy = bx + cy + d$, which is solved by factorizing the right-hand side.[12]

These four 'seeds', as well as the *kuṭṭaka* ('pulverizer') and *vargaprakṛti* ('square nature')[13] that are treated by Nārāyaṇa both in Part I of the present work and in the *Gaṇitakaumudī*, were tools for solving algebraic equations. The algebraic equations treated by Indian mathematicians included special types of cubic and bi-quadratic equations and even those of higher orders, with one or more unknown numbers, but they had to be reduced somehow to (at most) quadratic equations, so as to be solvable by means of these tools. In other words, Indian mathematicians

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[10] Brahmagupta (BSS 18.44–45) ignored the negative root of the right-hand side. Acceptance of the two solutions of a quadratic equation goes back at latest to Mahāvīra (GSS 4.57 and 61, ca. A.D. 850), although the negative root was rejected by Indian mathematicians even after him, and so also was a solution that made an element in the statement of the problem a negative number (BG 116).

[11] Seed 3 is treated in: BSS 18.43b–45 and 49–50 (examples), SS 14.17–19, TPG 87 and 92–93, and BG 115–133 (rules with examples) and 149–180 (rules with examples).

[12] Bhāskara II (BG 185), for example, reduces the equation to $xy = Bx + Cy + D$, where $B = b/a$, $C = c/a$, and $D = d/a$. Assuming $BC + D = pq$, he obtains the two types of solutions, (1) $x = C \pm p$ and $y = B \pm q$, and (2) $x = C \pm q$ and $y = B \pm p$. Seed 4 is treated also in: BSS 18.60–63, SS 14.20–21, and BG 181–187 (rules with examples). Cf. also BM N16 and Ex. 1 for it.

[13] The *vargaprakṛti* is a procedure for rational solutions to indeterminate equations of the second degree of the type $Px^2 + t = y^2$. Brahmagupta treated this problem in his BSS 18.64–70 and 75–82 (examples), and gave, among other rules, a method for arriving at a solution for $t = 1$ from a solution for $t = \pm 4$, ± 2, or -1. Jayadeva, cited in Udayadivākara's *Sundarī* (A.D. 1073), gave the so-called 'cyclic method' (*cakravāla*), which enabled one to obtain a solution for $t = \pm 4$, ± 2, or -1 from a solution for any t. See Shukla 1954. The *vargaprakṛti* is also treated in: SS 14.32–35, BG 70–88 (rules with examples), and BGA I.70–80 (with examples).

used these limited tools very skilfully in order to solve various equations including those of higher orders.

Verses 3–7: Rule[14] 1. In these verses Nārāyaṇa describes Seed 1.[15] It may be summarized as follows.

1. Express the unknown quantity in question by the first syllable, *yā*, of *yāvattāvat* accompanied by the digit for unity (yā 1, which corresponds to $1x$ or x in modern notation). Assume, if necessary, the unknown to be its multiple (yā 2, yā 3, etc., corresponding to $2x$, $3x$, etc.), perhaps increased or decreased by a number (yā 2 rū 3, yā 2 rū $\overset{\circ}{3}$, etc., corresponding to $2x + 3$, $2x - 3$, etc.). — Verse 3.

2. Perform computation upon that unknown quantity according to the statements (*ālāpa*) of the questioner (*uddeśaka*) and make two sides (*pakṣa*, which literally means 'a wing') equal to each other. All the 'statements' have to 'be fulfilled' ($\sqrt{}$*ghaṭ*) with these procedures. — Verse 4.

[The two sides of an equation are placed one above the other. No sign for equality is used. The most typical expression of a linear equation is in the form

$$\begin{array}{llll} \text{yā} & a & \text{rū} & b \\ \text{yā} & c & \text{rū} & d \end{array}$$

[14] *karaṇa-sūtra*, which literally means 'a procedural rule'. It is always versified, and often consists of algorithms in modern sense designed for a specific type of problem. Nārāyaṇa also uses the word, *paribhāṣita* (which literally means 'laid down as a convention or as a general rule'), when he introduces a supplementary rule to be employed in the course of an application of a procedural rule (*karaṇa-sūtra*) to an example (*udāharaṇa*). See Verses 8, 9, 10, and 11 below, which are introduced as a rule (or a means) that has been 'laid down as a general rule' (*paribhāṣita*). He gives a kind of definition of that word in Part I of the present work (BGA I.65): yasminyasmiṅkarmaṇi yadyatparibhāṣitaṃ samuditaṃ ca / tasmintasmiṅkarmaṇi tattatparibhāṣitaṃ bhavati // 65 // 'Whatever has been laid down conventionally and consented to in a work has been laid down conventionally for that work.' But *paribhāṣita* here, like *paribhāṣā* in other works, seems to mean 'terminology' including weights and measures. In fact, the word, *paribhāṣita-unmiti*, in the next verse (BGA I.66) means conversion ratios between time measures 'laid down conventionally' (or defined) as 60 *ghaṭikās* = 1 day, 30 days = 1 month, and 12 months = 1 year.

[15] The same topic is treated in: AB 2.30 and Bhāskara I's comm. on it (with five examples), BM 51 and Ex. 1 for it, BSS 18.43a and 46–48 (examples), SS 14.14, and BG 89 and 90–114 (examples).

This is called 'a setting-down (or table) for equal subtraction' (*samaśodhanārtha-nyāsa*—see the next step for the 'equal subtraction'). When the equation involves fractions, all the coefficients are reduced into fractions having equal denominators (*samacchedīkaraṇa*), and then the equal denominators are removed (*chedagamana*) from the equation.][16]

3. Subtract the unknown term of one side from the other (from both sides, to be precise) and the known term of the other from the first side (from both sides, to be precise). This procedure is called 'the equal subtraction' (*sama-śodhana*). — Verse 5a.

4. Divide the remainder of the known quantity by that of the unknown quantity. The quotient is the value of one *yāvattāvat* (x). [Substitute, if necessary, this value in the assumed expression of the unknown quantity ($2x$, $2x + 3$, etc.). This procedure is called 'raising' (*utthāpana* from *ud*-$\sqrt{}$*sthā-paya*, 'to raise').][16] The result is the value of the unknown quantity. — Verses 5b–6a.

5. When there are multiple unknown quantities in a problem, express them by *yāvattāvat* and *rūpa* only (yā 2, yā 3, yā 1 rū 1, etc.), or otherwise express one of them by *yāvattāvat* and assume the rest to be known numbers. Find the value of the unknown quantity in this way by means of one's own intellect (*nijabuddhyā*). — Verses 6b–7.

Remarks: The main part of Seed 1, which is also called 'the equation procedure' (*samakarman*/*samakriyā*/*sāmyakaraṇa*/etc.), consists of the 'equal subtraction' (in Step 3) and the subsequent division (in Step 4), which are preceded by 'the reduction to equal denominators' and 'the removal of the denominators' (in Step 2).

In Example 17, for example, the sum (7) of an arithmetical progression[17] and the product (10) of its three elements—the first term[18] a, the common difference[19] d, and the number of terms[20] n—are given, and each of the three elements is required. The commentator, observing that the assumption $a =$ yā 1 would

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[16] The bracketed portions are not explicitly written in the verses but known from the actual working processes given in the commentary on the extant examples for Seed 1.

[17] Called 'fruit of series' (*śreḍhī-phala*) or 'computed ⟨sum⟩' (*gaṇita*).

[18] Called 'first' (*ādi*, *ādya*) or 'mouth/face' (*mukha*, *vadana*).

[19] Called 'increase' (*caya*, *vṛddhi*).

[20] Called 'steps' (*gaccha*, *pada*).

make the computation by Seed 1 impossible,[21] assumes a and d to be 2 and yā 1 respectively. Then n $(= 10/ad)$ is expressed as: $\begin{array}{l}\text{rū } 10\\ \text{che yā } 2\end{array}$, where *che* is an abbreviation of *cheda* or divisor (or denominator). For this value of n, he obtains the sum of the progression by means of a formula for it (for which see the Remark under Verse (13)): $\begin{array}{l}\text{yā } \overset{\circ}{5} \text{ rū } 45\\ \text{che yā } 2\end{array}$. This is equal to the given sum, 7. Hence follows the equation[22] $\begin{array}{l}\text{yā } \overset{\circ}{5} \text{ rū } 45\\ \text{che yā } 2\\ \text{yā } 0 \text{ rū } 7\end{array}$, both sides of which are reduced to fractions having equal denominators, $\begin{array}{l}\text{yā } \overset{\circ}{5} \text{ rū } 45\\ \text{che yā } 2\\ \text{yā } 14 \text{ rū } 0\\ \text{che yā } 2\end{array}$. Then the denominators are removed, $\begin{array}{l}\text{yā } \overset{\circ}{5} \text{ rū } 45\\ \text{yā } 14 \text{ rū } 0\end{array}$. To this, the 'equation procedure' (*samakriyā*) is applied. That is, the 'equal subtraction' is first applied, $\begin{array}{l}\text{yā } 0 \text{ rū } 45\\ \text{yā } 19 \text{ rū } 0\end{array}$, and then the remainder of the known quantity, 45, is divided by that of the unknown quantity, 19: $\begin{array}{c}45\\ 19\end{array}$. This is the value of one *yāvattāvat.* When 'raised by this' (*anenotthāpitau*), the common difference and the number of terms, $\begin{array}{c}45\\ 19\end{array}$ and $\begin{array}{c}19\\ 9\end{array}$, are obtained.

In the solutions of mathematical problems in general, the three-quantity operation (*trairāśika,* usually called the rule of three) and its derivatives are used very often. The three-quantity operation is a computation of an unknown quantity x from three known quantities a, b, c when there is a proportional relationship among them, that is,

$$a : b = c : x \quad \longrightarrow \quad x = \frac{bc}{a}.$$

The a, b, c, and x are usually called in order the standard quantity

[21] atrādau yāvattāvatkalpite kriyā na nirvahati / 'Here, when the first term is assumed to be *yāvattāvat,* the computation does not work.' (Comm. on BGA II.(18).)

[22] This and the following three expressions of equation, which are omitted in the commentary, have been restored here by me.

(*pramāṇa-rāśi*), the fruit-of-standard quantity (*pramāṇa-phala-rāśi*) (or simply fruit quantity), the requisite quantity (*icchā-rāśi*), and the fruit-of-requisite quantity (*icchā-phala-rāśi*).

Nārāyaṇa's rule for the three-quantity operation (GK prakīrṇaka 60) is cited in full in the commentary on Verses (1)–(2): 'The standard and the requisite ⟨quantities put down⟩ in the first and the last (i.e., the third) ⟨places⟩ are of equal categories, but the fruit ⟨quantity put down⟩ in the middle ⟨place⟩ is of a different category. That ⟨middle term⟩ multiplied by the last and divided by the first is the fruit of the requisite.'[23]

The proportional relationship on which a three-quantity operation is based is expressed as 'When b is obtained from a, what is obtained from c?', or, 'If b is for a, what is for c?'. This is called 'a verbal expression of a three-quantity operation' (*trairāśika-vācoyukti*). In this Commentary, I will express it symbolically as $a : b = c : x$.

Likewise, the five-quantity operation (*pañcarāśika*), the seven-quantity operation (*saptarāśika*), etc. are employed for computing an unknown quantity from five known quantities, seven known quantities, etc., when there are proportional relationships among them. That is, when $(2n+1)$ quantities ($a_1, \ldots, a_n; b_1, \ldots, b_n$ and p) satisfy the conditions

$$\begin{aligned} a_1 : p &= b_1 : x_1, \\ a_2 : x_1 &= b_2 : x_2, \\ a_3 : x_2 &= b_3 : x_3, \\ \vdots \quad & \quad \vdots \quad \vdots \\ a_n : x_{n-1} &= b_n : x, \end{aligned}$$

we have

$$x_1 = \frac{pb_1}{a_1}, \quad x_2 = \frac{x_1 b_2}{a_2}, \quad x_3 = \frac{x_2 b_3}{a_3}, \quad \ldots, \quad x = \frac{x_{n-1} b_n}{a_n},$$

according to the three-quantity operation. Hence follows

$$x = \frac{pb_1 b_2 \cdots b_n}{a_1 a_2 \cdots a_n}.$$

This is $(2n+1)$-*rāśika* or the $(2n+1)$-quantity operation. It is, so to speak, a compound computation consisting of n three-quantity

[23] Cf. AB 2.26, BM C10, N19, and Q11, BSS 12.10, PG 43, Tr 29, GP 24, GSS 5.2, MS 15.24–25, GT 86, SS 13.14, L 73, and GS 1.63.

operations. Just as in the case of the three-quantity operation, the a_i's are called the standard quantities, p the fruit quantity, and b_i's the requisite quantities.

In our text, the five-quantity operation is often used for the computation of interests on capitals. See Verses (8), (9), (10), (11), and (37).

Verses (1)–(2): Example 1. Three independent equations of properties of two merchants having horses and money: When x_1 and x_2 stand respectively for their properties, and x for the unknown price of a horse, (1) $x_1 = x_2$, (2) $\frac{x_1+8}{2} = x_2$, (3) $x_1 = 3x_2 - 4$, where $x_1 = y_1 + 600$ and $x_2 = y_2 - 200$, where y_1 and y_2 denote the monetary values of the eight and twelve horses ($8x$, $12x$) of the first and the second merchants, respectively.[24]

Solution of (1): Let $x = s$ [= yā 1 in the Sanskrit text]. The commentator first calculates y_1 and y_2 by means of the three-quantity operation:

$$1 : s = 8 : y_1 \rightarrow y_1 = 8s; \quad 1 : s = 12 : y_2 \rightarrow y_2 = 12s.$$

Hence follows the equation

$$8s + 600 = 12s - 200$$

which he calls 'a setting-down (or table) for ⟨equal⟩ subtraction' (*śodhanārtha-nyāsa*). Then he applies the operation called 'equal subtraction' (*sama-śodhana*) to this equation and obtains $4s = 800$, which means four horses cost 800. Based on this, he performs the three-quantity operation,

$$4 : 800 = 1 : s \rightarrow s = 200.$$

That is, $x = 200$. 'Having raised by this' (*anenotthāpya*), he obtains the equal properties, $x_1 = x_2 = 2200$.

Another solution: The commentator assumes $x = s+1$. Then the equation is $8s + 608 = 12s - 188$, from which he obtains $s = 199$. Hence $x = 200$, and the equal properties are $x_1 = x_2 = 2200$.

Solution of (2): The 'setting-down for subtraction' is $4s + 304 = 12s - 200$. By the 'equal subtraction', $8s = 504$. Hence

[24] Cf. Ex. 1 for BM 51, BSS 18.46–48, Bhāskara I's Exs. 1–5 for AB 2.30, and BG 90–91.

$s = 63$. 'When raised by this' (*anenotthāpite*), $x_1 = 1104$ and $x_2 = 556$. The commentator, in passing, refers to another form of the same equation, $x_1 = 2x_2 - 8$.

Solution of (3): According to the 'statement' (*ālāpa*), the 'setting-down for subtraction' is $8s + 600 = 36s - 604$. Hence $s = 43$. When raised by this, $x_1 = 944$ and $x_2 = 316$.

Verse (3): Example 2. Equation of properties of two people, each having four kinds of jewels and money: When x_i ($i = 1, 2, 3, 4$) stand for the unit price of each jewel, $12x_1 + 10x_2 + 8x_3 + 6x_4 + 100 = 13x_1 + 11x_2 + 9x_3 + 7x_4 + 60$.[25]

Solution: Let $x_i = is$ [s = yā 1]. The 'statement' of the problem produces the equation, $80s+100 = 90s+60 \rightarrow 10s = 40 \rightarrow s = 4$. Hence $x_i = 4i$, and the equal properties are 420 each.

Another solution: Let $x_i = 15/i$ for $i = 1, 2, 3$ and $x_4 = s$ [= yā 1]. Then $6s + 420 = 7s + 410 \rightarrow s = 10$. Hence $x_4 = 10$ and the equal properties are 480 each.

Verse (4): Example 3. Five yearly donations of money to a brāhmaṇa: When x_i ($i = 1, 2, 3, 4, 5$) stand for the annual donations, $x_i = 2x_{i-1}+1$ ($i = 2, 3, 4, 5$) and $x_1+x_2+x_3+x_4+x_5 = 150$.

Solution: Let $x_1 = s$ [= yā 1]. Then $x_2 = 2s+1$, $x_3 = 4s+3$, $x_4 = 8s+7$, $x_5 = 16s+15$, and $x_1+x_2+x_3+x_4+x_5 = 31s+26$. Hence follows the equation, $31s + 26 = 0s + 150 \rightarrow 31s = 124 \rightarrow s = 4$. Hence the donations in five successive years are 4, 9, 21, 49, and 113.

Remarks: An example expressed in similar words occurs in GK miśra (uv) (Part I, p. 62) = Singh II, Ex. 15.[26] However, the problem treated in it is not identical with the present one. The problem in the GK is $x_i = 2x_{i-1}$ ($i = 2, 3, 4, 5$) and $x_1 + x_2 + x_3 + x_4 + x_5 = 186$.[27] The commentator of the GK solves it by means of the 'investment procedure' (*prakṣepa-karaṇa*), which is an algorithm for calculating the share of each person in proportion to the amount of each investment.[28] Assuming $x_1 = 1$, he first

[25] Cf. BG 92 = BG 135.

[26] See the fns. for Verse (4) in Sections 2 and 3 above.

[27] Cf. Ex. 1 for BM N9.

[28] Prescribed in GK miśra 1a (Part I, p. 54): prakṣepāstadyutihṛtamiśreṇa hatāḥ pṛthakphalāni syuḥ // 1a // 'The investments, each multiplied by the mixture (i.e, the total gain) divided by the sum of them (i.e, the investments), will be the results (i.e., shares).' Cf. BSS 12.16a, PG 59a, Tr 38a, GSS 6.79b–

obtains $x_2 = 2$, $x_3 = 4$, $x_4 = 8$, and $x_5 = 16$. (Hence $x_1 : x_2 : x_3 : x_4 : x_5 = 1 : 2 : 4 : 8 : 16$.) Then he applies the 'investment procedure': $x_1 = 186 \times 1/(1+2+4+8+16) = 6$, $x_2 = 186 \times 2/(1+2+4+8+16) = 12$, etc.

Verse (5): Example 4. A lotus stalk sunken in layers of mud, moss, and water: When x stands for the length of the lotus stalk, $\frac{x}{2} + \frac{x}{5} + \frac{x}{10} + 2 = x$.[29]

Solution: Let $x = s$ [$=$ yā 1]. Then the equation produced from the statement is: $\frac{4}{5}s + 2 = s + 0 \rightarrow \frac{4}{5}s + \frac{10}{5} = \frac{5}{5}s + \frac{0}{5} \rightarrow \frac{10}{5} = \frac{1}{5}s \rightarrow s = 10$. That is, $x = 10$.

Remarks: The identical denominators of the three fractions $\frac{10}{5}$, $\frac{5}{5}$, and $\frac{0}{5}$ in the second form of the equation are called 'the same throw (or addendum)' (*samakṣepa*) by the commentator probably because they are 'added' to the integer terms of the original equation in order to reduce them to fractions with the same denominators as that of the fractional term, $\frac{4}{5}s$. He explains the last step of this solution as $s = \frac{10}{5} \div \frac{1}{5} = \frac{10}{5} \times \frac{5}{1} = 10 \div 1 = 10$, and cites Verse 8, which supports this computation.

Verse 8: Rule 2 (supplementary). Division of a fraction by a fraction when they have equal denominators: $\frac{b}{a} \div \frac{c}{a} = b \div c$.

Verse (6): Example 5. The number of wasps: When x $(= x_0)$ stands for the number of the entire wasps, $x_i = x_{i-1} - \frac{x_{i-1}}{i+1}$ $(i = 1, 2, 3, 4)$, and $x_4 = 6 + 6$.[30]

80a, MS 15.36, SS 13.19a, L 94, and GS 3.7.

[29] This type of problem was called 'Pillar Problem' (*stambha-uddeśaka*) by Śrīdhara, 'Partial-Quantity Class' (*bhāga-jāti*) by Mahāvīra, 'Visible-Quantity Class' (*dṛśya-jāti*) by Śrīpati (and by Śrīdhara in GP), and 'Pillar-Part Class' (*stambha-aṃśaka-jāti*) by Ṭhakkura Pherū, all of whom gave algorithms for solving it. See PG 74a and (96), Tr 27b and (23)–(29), GP (5b), GSS 4.4a and 4.5–28, GT 55–57, and GS 2.14–17. Bhāskara II and Nārāyaṇa, in the field of *pāṭī-gaṇita*, solved it by means of the 'optional-number computation' (*iṣṭa-karman*). See L 53 and GK prakīrṇaka (18), (19) and (22).

[30] This type of problem was called 'Remainder Problem' (*śeṣa-uddeśaka*) by Śrīdhara and 'Remainder Class' (*śeṣa-jāti*) by Mahāvīra and others. Śrīdhara reduced this type to the Pillar Problem (see the fn. for Example 4 above). See PG (97). Mahāvīra, Āryabhaṭa II and Śrīpati gave algorithms for it. See GSS 4.4b and 4.29–32, MS 15.20, and GT pp. 44–45. Bhāskara II and Nārāyaṇa, in the field of *pāṭī-gaṇita*, solved it by means of the 'optional-number computation' (*iṣṭa-karman*). See L 54 and GK prakīrṇaka (18) and

Solution: Let $x = s$ [$=$ yā 1]. Calculating x_i both step by step and by the method called 'partial-subtraction procedure', the commentator obtains $x_4 = s/5$, which is equal to 12. Hence follows the equation $\frac{1}{5}s = 12 \rightarrow s = 60$. That is, $x = 60$.

Remark: The 'partial-subtraction procedure' (*bhāga-apavāha-vidhi*) is an algorithm for obtaining the result of $\frac{b}{a} - \frac{b}{a} \cdot \frac{d}{c}$, and is usually coupled with the 'partial-addition procedure' (*bhāga-anubandha-vidhi*) for $\frac{b}{a} + \frac{b}{a} \cdot \frac{d}{c}$. These procedures have been dealt with by Indian mathematicians since at least the time of Brahmagupta.[31] Nārāyaṇa too prescribes a rule for them in the GK: 'The denominator of the first ⟨fraction⟩ is multiplied by the additional denominator, and the numerator of the first is multiplied by the additional denominator increased or decreased by its own numerator.'[32] The two fractions are usually arranged in a vertical line and the above rule is applied to it:

$$\begin{array}{ccc} b & \longrightarrow & b(c \pm d) \\ a & & ac \\ \pm d & & \\ c & & \end{array}$$

In the solution of the present example, this procedure (for the minus sign) is applied four times repeatedly to the column of the five fractions, 1/1 (= 1), −1/2, −1/3, −1/4, and −1/5. See Fig. 1.

Verse (7): Example 6. Equations after give and take: When x_1 and x_2 stand for the properties of two people, $x_1 + 100 = 2(x_2 - 100)$ and $5(x_1 - 8) = x_2 + 8$.[33]

Solution: Let $x_1 = 2s - 100$ and $x_2 = s + 100$ [$s =$ yā 1]. Then, the first 'statement' (*ālāpa*) 'is fulfilled' (*ghaṭate*) automatically.

(20).

[31] See BM 25–26 and C4, BSS 12.9, PG 39–40, Tr 24, GP 18, GAR 2.5, GSS 3.113 and 126, MS 15.11–12, GT 48 (p. 34, lines 15–16) and 51 (p. 37, lines 2–5), L 34, and GS 2.6 and 8.

[32] GK prakīrṇaka 27b (Part I, p 9): ādyacchidadhicchidghnaḥ svāṃśayugūnādhiharahatādyaṃśaḥ // 27b // I have slightly emended Dvivedī's reading of the latter half of this passage, svāṃśayugūnaśca svaharahatādyaṃśāḥ, which does not make sense. A Nepal manuscript (4–1689, Jyotiṣa 125, National Archives, Kathmandu, folio 3a) reads: ādyachidadhichidghnaḥ svāṃśayugūnā harahatādyaṃśāḥ // 27b // This is also defective mathematically as well as metrically.

[33] Cf. GSS 6.253b–259a and BG 93 (= BG 136).

$$
\begin{array}{rcrcrcrcr}
1 & \rightarrow & 1 & \rightarrow & 1 & \rightarrow & 1 & \rightarrow & 1 \\
1 & & 2 & & 3 & & 4 & & 5 \\
-1 & & -1 & & -1 & & -1 & & \\
2 & & 3 & & 4 & & 5 & & \\
-1 & & -1 & & -1 & & & & \\
3 & & 4 & & 5 & & & & \\
-1 & & -1 & & & & & & \\
4 & & 5 & & & & & & \\
-1 & & & & & & & & \\
5 & & & & & & & &
\end{array}
$$

Figure 1: Repeated applications of the partial-subtraction procedure.

The second statement produces the equation $10s - 540 = s + 108 \rightarrow 9s = 648 \rightarrow s = 72$. When raised by this, $x_1 = 44$ and $x_2 = 172$.

Remark: In the GK Nārāyaṇa gives a computational rule for a general case of this type of problem, where the number of people is extended to any integer n (> 1), that is,

$$x_i + \sum_{j \neq i} c_{ij} = a_i \sum_{j \neq i} (x_j - c_{ij}),$$

for every i ($i = 1, 2, ..., n$), where c_{ij} ($j \neq i$) is the property that the i-th person receives from the j-th person.[34]

He reduces this problem to a problem of 'different ⟨parts of a⟩ hoard or packet' (*viṣama-nidhāna/poṭala*), that is,

$$x_i + b_i y = a_i \sum_{j \neq i} x_j,$$

for every i ($i = 1, 2, ..., n$), where b_i is a fraction.[35]

[34] Cf. BM 10 (where $n = 2$) and GSS 6.251b–253a.

[35] For similar problems with a hoard, see the Remarks under Verse (34) below.

His rule for the latter problem can be expressed as[36]

$$x_i = \frac{\left(\sum_{j\neq i} \frac{a_i b_j + b_i}{a_j+1}\right) - b_i(n-2)}{a_i+1}, \quad y = \frac{\left(a_i \sum_{j\neq i} x_j\right) - x_i}{b_i};$$

and his rule for the former problem may be expressed as follows.[37] Calculate:

$$b_i = (a_i + 1) \sum_{j\neq i} c_{ij},$$

and apply the above rule for the 'different ⟨parts of the⟩ packet' to the given a_i and the b_i obtained thus (y being assumed to be unity). Let the results be x_i'. Then,

$$x_i = \frac{x_i'}{\left(\sum_{i=1}^{n} \frac{a_i}{a_i+1}\right) - 1}.$$

[36] GK miśra 30–32 (Part I, pp. 87–88): parabhāgāḥ svaguṇaghnā nijabhāgayutāḥ paraiḥ sarūpaistaiḥ / guṇakairvihṛto yogo netronanarāhataiḥ svāṃśaiḥ // 30 // (Read *vihṛtā* instead of *vihṛto*.) hīno nijaguṇakena ca saikena hṛto muhurdhanāni syuḥ / paradhanayogo nijaguṇakena hataḥ svasvavarjito vibhajet // 31 // (Read *bhaktaḥ* instead of *vibhajet*.) nijabhāgena nidhānaṃ prajāyate viṣamapūrvaṃ tat / kṛtasamaharaluptaharāste 'ṃśāstebhyastu pūrvavadvāpi // 32 // 'The fractions of the others (b_j) are multiplied ⟨severally⟩ by one's own multiplier (a_i), increased by one'e own fraction (b_i), and divided by the other ⟨corresponding⟩ multipliers (a_j). The sum ⟨of the quotients⟩ is decreased by one's own fraction multiplied by the number of people minus [eyes] (2) and divided by one's own multilpier plus one. ⟨The same procedure is⟩ repeated many times ⟨for each person⟩, and there will be the properties ⟨possessed by them⟩. The sum of the properties of the others is multiplied by one's own multiplier, decreased by one's own property, and divided by one's own fraction. That hoard modified by ⟨the word⟩ 'different' is produced. Alternatively, those fractions are reduced to the same denomitators and the denominators are removed. From them, ⟨solutions are obtained⟩ as before.' Cf. GSS 6.238–251a (rules with examples).

[37] GK miśra 33–34a (Part I, pp. 90–91): prāptānvitāḥ sarūpairguṇakairnihitāstu viṣamapoṭalavat / nijanijaguṇakāḥ saikairguṇakairvihṛtāśca tadyogaḥ // 33 // (Read *nihatās* instead of *nihitās*.) rūponena hṛtāḥ syurdhanāni teṣāṃ pṛthakpṛthaktāni // 34a // 'The gains (c_{ij}) summed up and multiplied by ⟨one's own⟩ multiplier (a_i) plus one ⟨are treated⟩ like ⟨the fractions (b_i) in the rule for⟩ the 'different packet'. Each multiplier (a_i) is divided by ⟨the same⟩ multiplier plus one and their sum ⟨is calculated⟩. The properties (x_i') ⟨obtained by means of the rule for the 'different packet'⟩ divided by ⟨the sum⟩ minus one will be those (properties) of them.'

Verse (8): Example 7. Capital and interest:[38] When the interest on one hundred for one month is five, the sum of a certain amount of money (x) plus the interest (y) on it for one year becomes twice the capital less sixteen. That is to say, $y = x \cdot \frac{5}{100} \cdot \frac{12}{1}$ and $x + y = 2x - 16$.[39]

Solution: Let $x = s$ [$=$ yā 1]. Then, by means of a five-quantity operation, $y = \frac{3}{5}s$. Hence follows the equation $s + \frac{3}{5}s = 2s - 16 \rightarrow s = 40$. When raised by this, $x = 40$ and $y = 24$.

Verse (9): Example 8. Three independent systems of equations of interests, of capitals plus interests, and of capitals minus interests: A sum of one hundred is divided in three (x_1, x_2, x_3), and they are loaned separately at two, three, and four percent interests per month. When y_1, y_2 and y_3 stand for the interests they produce for five, eight and ten months, respectively, (1) $y_1 = y_2 = y_3$; (2) $x_1 + y_1 = x_2 + y_2 = x_3 + y_3$; and (3) $x_1 - y_1 = x_2 - y_2 = x_3 - y_3$. The equation $x_1 + x_2 + x_3 = 100$ is common to the three cases.

Solution of (1): Let $y_1 = y_2 = y_3 = s$ [$=$ yā 1]. Then, by means of five-quantity operations, $x_1 = \frac{10}{1}s$, $x_2 = \frac{25}{6}s$, $x_3 = \frac{5}{2}s$. Therefore, $x_1 + x_2 + x_3 = \frac{50}{3}s$. Hence follows the equation $\frac{50}{3}s = 100 \rightarrow s = 6$. When raised by this, $x_1 = 60$, $x_2 = 25$, $x_3 = 15$, and $y_1 = y_2 = y_3 = 6$.

Solution of (2): Let $x_1 + y_1 = x_2 + y_2 = x_3 + y_3 = s$ [$=$ yā 1]. The commentator first obtains the interests, 10, 24, and 40, on one hundred for the given periods at the given rates. The corresponding sums are 110, 124, and 140. Then, by means of three-quantity operations, he calculates the three capitals for the equal sums (s): $110 : 100 = s : x_1 \rightarrow x_1 = \frac{10}{11}s$, $124 : 100 = s : x_2 \rightarrow x_2 = \frac{25}{31}s$, $140 : 100 = s : x_3 \rightarrow x_3 = \frac{5}{7}s$; and then the three interests: $100 : 10 = x_1 : y_1 \rightarrow y_1 = \frac{1}{11}s$, $100 : 24 = x_2 : y_2 \rightarrow y_2 = \frac{6}{31}s$, $100 : 40 = x_3 : y_3 \rightarrow y_3 = \frac{2}{7}s$. Therefore, $x_1 + x_2 + x_3 = \frac{5800}{2387}s$. Hence follows the equation $\frac{5800}{2387}s = 100 \rightarrow s = \frac{2387}{58}$. When raised by this, $x_1 = \frac{2170}{58}$, $x_2 = \frac{1925}{58}$, $x_3 = \frac{1705}{58}$; $y_1 = \frac{217}{58}$, $y_2 = \frac{462}{58}$, $y_3 = \frac{682}{58}$.

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[38] For various problems involving capital and interest, see AB 2.25, BSS 12.14–15, PG 47–52a and (52)–(60), Tr 33–35 and (57)–(62), GSS 6.21–79a, MS 15.31–33, GT 111–125, SS 13.17–18, L 90–93, and GK Part I, pp. 60–76.

[39] This example is numerically equal to BG 99. See the fns. for Verse (8) in Sections 2 and 3 above.

Solution of (3): Let $x_1 - y_1 = x_2 - y_2 = x_3 - y_3 = s$ [= yā 1]. The commentator subtracts the three interests on one hundred obtained above from one hundred; the results are 90, 76, and 60. By proportion, $x_1 = \frac{10}{9}s$, $x_2 = \frac{25}{19}s$, $x_3 = \frac{5}{3}s$; and also $y_1 = \frac{1}{9}s$, $y_2 = \frac{6}{19}s$, $y_3 = \frac{2}{3}s$. Therefore, $x_1 + x_2 + x_3 = \frac{700}{171}s$. Hence follows the equation $\frac{700}{171}s = 100 \to s = \frac{171}{7}$. When raised by this, $x_1 = \frac{1710}{63}$, $x_2 = \frac{4275}{133}$, $x_3 = \frac{855}{21}$, $y_1 = \frac{171}{63}$, $y_2 = \frac{1026}{133}$, $y_3 = \frac{342}{21}$.

Remark: This example is identical with the problem of GK miśra (uv) (Part I, p. 61) = Singh II, Ex. 14, which the commentator of the GK solves by means of the 'investment procedure' (*prakṣepa-karaṇa*), that is, the rule of proportional distribution (see the Remark under Verse (4)). In each of (1), (2), and (3), he first obtains the ratio, $x_1 : x_2 : x_3$, by assuming an appropriate unknown quantity to be unity, and then applies the rule (note that $x_1 + x_2 + x_3 = 100$).

Verse (10): Example 9. Equation after exchange of interests on two capitals: Two unknown capitals x_1, x_2 are loaned at five percent and four percent interest respectively per month, and the sum of the first capital and the interest y_2 on the second for ten months is equal to the sum of the second capital and the interest y_1 on the first for seven months. That is to say, $y_1 = x_1 \cdot \frac{5}{100} \cdot \frac{7}{1}$, $y_2 = x_2 \cdot \frac{4}{100} \cdot \frac{10}{1}$, and $x_1 + y_2 = x_2 + y_1$.

Solution: Let $x_1 = 40$ and $x_2 = s$ [= yā 1]. Then, by means of five-quantity operations, $y_1 = 14$ and $y_2 = \frac{2}{5}s$. Hence follows the equation, $\frac{2}{5}s + 40 = s + 14 \to 2s + 200 = 5s + 70 \to s = \frac{130}{3}$. When raised by this, $x_2 = \frac{130}{3}$ and $y_2 = \frac{52}{3}$. The equal sums are $\frac{172}{3}$.

Another solution: Let $x_1 = 60$ and $x_2 = s$ [= yā 1]. Then, in exactly the same way, $y_1 = 21$ and $s = 65$. When raised by this, $x_2 = 65$ and $y_2 = 26$. The equal sums are 86.

Remarks: This example is identical with one of the two problems given in GK miśra (uv) (Part I, p. 63) = Singh II, Ex. 16; the other problem replaces the condition, $x_1 + y_2 = x_2 + y_1$, with $x_1 - y_2 = x_2 - y_1$.

Nārāyaṇa prescribes an algorithm for this type of problem in the GK. Type of problem: Two unknown capitals (x_i; $i = 1, 2$) are loaned individually at certain rates (b_i per a_i monetary units for periods c_i), and the sum (or difference) of the first capital and the interest y_2 on the second for the period t_2 is equal to the sum

(or difference) of the second capital and the interest y_1 on the first for the period t_1. Computational rule:[40]

$$x_i = \frac{k}{k \mp y_i(k)},$$

where k is an optional number (the minus sign for the sum), and

$$y_i(x_i) = \frac{b_i}{a_i} \cdot \frac{t_i}{c_i} \cdot x_i.$$

Verse (11): Example 10. Two related loans: A capital x_1 is loaned for a certain period (say, t months) at five percent per month. Another capital x_2, which is equal to the first capital minus the square of the interest y_1 on it, is loaned for another period of the same length (t) at ten percent per month, and produces an interest (y_2) equal to the first interest. That is to say, $y_1 = x_1 \cdot \frac{5}{100} \cdot \frac{t}{1}$ and $y_2 = x_2 \cdot \frac{10}{100} \cdot \frac{t}{1}$, where $y_1 = y_2$ and $x_2 = x_1 - y_1^2$.[41]

Solution: Let $y_1 = y_2 = 2$ and $t = s$ [= yā 1]. Then, by means of a five-quantity operation, $x_1 = \frac{40}{s}$. Therefore, $x_2 = \frac{40}{s} - 2^2 = \frac{-4s+40}{s}$. Again by means of a five-quantity operation, $y_2 = -\frac{2}{5}s + \frac{20}{5}$. Hence follows the equation $-\frac{2}{5}s + \frac{20}{5} = 2 \rightarrow -2s + 20 = 10 \rightarrow s = 5$. When raised by this, $t = 5$, $x_1 = 8$ and $x_2 = 4$.

Second solution: Let $y_1 = y_2 = 2$ and $x_1 = s$ [= yā 1]. Then, by means of a five-quantity operation, $t = \frac{40}{s}$. On the other hand, $x_2 = x_1 - y_1^2 = s - 4$. Again by means of a five-quantity operation, $t = \frac{20}{s-4}$. Hence follows the equation $\frac{40}{s} = \frac{20}{s-4} \rightarrow \frac{40(s-4)}{s(s-4)} = \frac{20s}{s(s-4)} \rightarrow 40(s-4) = 20s \rightarrow 20s = 160 \rightarrow s = 8$. When raised by this, $x_1 = 8$, $x_2 = 4$, and $t = 5$.

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[40] GK miśra 4–5a (Part I, pp. 62–63): iṣṭakalāntarahīnādhikeṣṭabhakteṣṭamiha bhavennīvī / iṣṭāntaritā nīvī kalāntaraṃ jāyate niyatam // 4 // anyonyakalāntarayutahīne nīvyau same bhavataḥ // 5a // 'Any optional number, divided by the optional number decreased or increased by the interest on the optional number, will be the capital in this case. ⟨When⟩ the capital is substituted for the optional number ⟨in the calculation of the interest⟩, the interest ⟨for that capital⟩ is surely produced. The two capitals ⟨thus obtained⟩, increased or decreased by the reciprocal interests, will be the same.'

[41] This example is numerically equal to BG 96, which the commentator Kṛṣṇa introduces as 'an example told by others'.

Third solution: Let $t = 5$ and $x_1 = s$ [= yā 1]. Then, by means of a five-quantity operation, $y_1 = \frac{1}{4}s$. Therefore, $x_2 = x_1 - y_1^2 = -\frac{1}{16}s^2 + s$. Again by means of a five-quantity operation, $y_2 = -\frac{1}{32}s^2 + \frac{16}{32}s$. Hence follows the equation $\frac{1}{4}s = -\frac{1}{32}s^2 + \frac{16}{32}s \rightarrow 8 = -s + 16 \rightarrow s = 8$. When raised by this, $x_1 = 8$, $x_2 = 4$, and $y_1 = y_2 = 2$.

Remark: The commentator concludes his solution with the remark: 'In this way, the example is solved on ⟨various⟩ assumptions (*kalpanā*).'

Verse (12): Example 11. Purchase of commodities in a given proportion: When two *palas* of sandalwood cost one *niṣka* and five *palas* of saffron two *niṣkas*, one purchases them in the ratio, 1 : 2, for a total of ten *niṣkas*. When x_1 and x_2 stand for their quantities, and y_1 and y_2 their prices, $x_1 : x_2 = 1 : 2$, $y_1 + y_2 = 10$, $x_1/y_1 = 2/1$, and $x_2/y_2 = 5/2$.[42]

Solution: Let $x_1 = s$ and $x_2 = 2s$ [s = yā 1]. Then the first condition is satisfied automatically. By means of three-quantity operations, $y_1 = \frac{1}{2}s$ and $y_2 = \frac{4}{5}s$. Hence follows the equation $\frac{1}{2}s + \frac{4}{5}s = 10 \rightarrow s = \frac{100}{13}$. When raised by this, $x_1 = \frac{100}{13}$, $x_2 = \frac{200}{13}$, $y_1 = \frac{50}{13}$, $y_2 = \frac{80}{13}$.

Remark: This example is identical with the problem of GK miśra (uv) (Part I, p. 58) = Singh II, Ex. 8, for which Nārāyaṇa prescribes a computational rule: 'When the price is multiplied by one's own part and divided by ⟨the quantity of⟩ one's own commodity, the procedure ⟨for investment is applied⟩ as before.'[43] That is, when

$$x_1 : x_2 : \dots : x_n = m_1 : m_2 : \cdots : m_n,$$

$$y_1 + y_2 + \cdots + y_n = p,$$

$$x_i : y_i = a_i : b_i \text{ (for every } i\text{)},$$

calculate the ratio

$$\frac{b_1 m_1}{a_1} : \frac{b_2 m_2}{a_2} : \cdots : \frac{b_n m_n}{a_n} \quad (= y_1 : y_2 : \cdots : y_n),$$

[42] Cf. Ex. 1 for BM N3, PG (73)–(75), Tr (70)–(72), GSS 6.90b–97a, and L 99.

[43] GK miśra 2b (Part I, p. 57): nijabhāgahate mūlye svapaṇyabhakte vidhiḥ prāgvat // 2b // Cf. BM N3, PG 59b, Tr 38b, GSS 6.87b–90a, MS 15.37–38a, SS 13.19b, and L 98.

and then apply the 'investment procedure' to this ratio and p. See the Remark under Verse (4) for the 'investment procedure'.

Verse (13): Example 12. Equation of two journeys (chase of a mistress by a procuress): A mistress goes constantly ten *yojanas* a day, and a procuress who runs after her goes three *yojanas* on the first day and increases her daily journey by two *yojanas* every day. How many days (t) does it take for the latter to catch up with the former? That is to say, $x_1 = x_2$, where x_1 and x_2 are the distances traveled by each for t days.[44]

Solution: Let $t = s$ [= yā 1]. Then $x_1 = 10s$ and, according to a formula for the sum of an arithmetical progression, $x_2 = s^2 + 2s$. Hence follows the equation, $10s = s^2 + 2s \rightarrow 10 = s + 2 \rightarrow s = 8$. When raised by this, $t = 8$ and $x_1 = x_2 = 80$.

Remarks: The formula used for the calculation of x_2 has been prescribed in GK śreḍhī 2a, which is cited partially under the present example and in full under the next example. It reads: 'The increase (d), multiplied by half the number of terms (n) less one, increased by the first term (a) and multiplied by the number of terms, will be the sum.'[45] That is to say,

$$A = A(n) = n\left(a + \frac{n-1}{2} \cdot d\right),$$

where $A(n)$ is the sum of the first n terms of an arithmetical progression whose first term and common difference are a and d, respectively, that is,

$$A(n) = a + (a + d) + (a + 2d) + \cdots + \{a + (n-1)d\}.$$

This example is identical with the problem of GK śreḍhī (uv) (Part I, p. 112) = Singh III, Ex. 10, for which Nārāyaṇa prescribes a computational rule immediately before it.[46] That is, when $vn =$

[44] Cf. PG (111), GSS 6.320, and GS 4.15 and 4 prakīrṇaka 10.

[45] Cf. AB 2.19, BSS 12.17, PG 85a and 95b, Tr 39, GSS 2.61–64, 3.22, 3.45, and 6.290, MS 15.47, SS 13.20, L 121, GS 3.26, and GK śreḍhī 1.

[46] GK śreḍhī 9a (Part I, p. 112): niyatagatirvadanonā cayadalahṛdrūpasaṃyuktā gacchaḥ // 'The constant speed, decreased by the first term, divided by half the increase and increased by unity, is the number of terms.' Cf. PG 96, GSS 6.319, and GS 4.16 and 4 prakīrṇaka 9. Cf. also BM 20, 21 and 23, which treat the same equation, although the situations of the problems are different.

$A(n)$,

$$n = \frac{v-a}{d/2} + 1.$$

Verse (14): Example 13. An arithmetical progression: When the sum of the first four terms of an arithmetical progression whose common difference is six is one hundred, what is the first term a? That is to say, when $A(4) = 100$ and $d = 6$, a is to be obtained.[47]

Solution: Let $a = s$ [= yā 1]. According to the formula for $A(n)$, which is cited here again, $A(4) = 4s + 36$. Hence follows the equation, $4s + 36 = 100 \rightarrow s = 16$. Hence $a = 16$.

Remarks: In the GK Nārāyaṇa provides a computational rule for this type of problem: 'The first term is ⟨obtained⟩ when the sum divided by the number of terms is decreased by half the increase multiplied by the number of terms less one.'[48] That is,

$$a = \frac{A}{n} - \frac{d}{2} \cdot (n-1).$$

The commentator provides a kind of proof (*upapatti*) of the formula for $A(n)$. That is, assuming $a = s$ [= yā 1] and $d = 6$, he diagrammatically shows that $A(4) = 4s + 36$. His expression is only partly symbolical insofar as he uses yā 1 for a, but I use modern notation in order to show his idea more clearly. His proof demonstrates the two identities,

$$a + (a+d) + \cdots + \{a + (n-1)d\} = \left\{a + (n-1) \times \frac{d}{2}\right\} \times n,$$

$$\left\{a + (n-1) \times \frac{d}{2}\right\} \times n = \left\{a + \frac{n-1}{2} \times d\right\} \times n,$$

by transforming a triangular expression of $A(n)$ as shown in Fig. 2. First, each d in the triangular expression is divided into two halves and one of each pair is put in one of the empty cells constituting the imaginary triangle located below the first triangle, so that the rectangular array of $d/2$, which consists of n columns

[47] Cf. GSS 2.77b, 3.37, and 6.293, L 125, and GK śreḍhī (uv) (Part I, p. 106) = Singh III, Ex. 3.

[48] GK śreḍhī 2b (Part I, p. 106): vadanaṃ padabhaktaphale vyekapadaghnottarārdhone // 2b // Cf. PG 86a and 88, Tr 40a, GSS 2.73b–74a, 2.76, 3.36a, and 6.292a, MS 15.48, SS 13.23a, L 124, and GS 3.28.

$$\begin{array}{cccc} a & a & a & a \\ & d & d & d \\ & & d & d \\ & & & d \end{array} \rightarrow \begin{array}{cccc} a & a & a & a \\ \frac{d}{2} & \frac{d}{2} & \frac{d}{2} & \frac{d}{2} \\ \frac{d}{2} & \frac{d}{2} & \frac{d}{2} & \frac{d}{2} \\ \frac{d}{2} & \frac{d}{2} & \frac{d}{2} & \frac{d}{2} \end{array} \rightarrow \begin{array}{cccc} a & a & a & a \\ \frac{d}{2} & \frac{d}{2} & \frac{d}{2} & \frac{d}{2} \\ d & d & d & d \end{array}$$

Figure 2: Transformation of a triangular expression of $A(n)$ when $n = 4$.

and $(n-1)$ rows, may be produced under the row of a's. Then, as many pairs of rows of $d/2$'s as possible, from the bottom upward, are replaced with rows of d's. Thus, $(n-2)/2$ rows of d's and one row of $d/2$'s are produced under the a's if n is even, and $(n-1)/2$ rows of d's if n is odd. Hence follows the formula.

Verse (15): Example 14. An arithmetical progression: When the sum of the first five terms of an arithmetical progression whose first term is four is one hundred, what is the common difference (d)? That is to say, when $A(5) = 100$ and $a = 4$, d is to be obtained.[49]

Solution: Let $d = s$ [= yā 1]. According to the formula for $A(n)$, which is cited here again, $A(5) = 10s + 20$. Hence follows the equation $10s + 20 = 100 \rightarrow s = 8$. Hence $d = 8$.

Remark: The published edition of the GK lacks a rule for the common difference but the original work must have contained one because the published edition contains an example for it (see the fn. for Example 14 in Verse (15) above). In fact, some of the manuscripts of the GK have the rule:[50]

$$d = \frac{A - an}{\frac{n(n-1)}{2}}.$$

Verse (16): Example 15. Equation of journeys (meeting of

[49] Cf. GSS 2.77a, 3.37, 6.293, L 127, and GK śreḍhī (uv) (Part I, p. 106) = Singh III, Ex. 4.

[50] Kathmandu Ms. (National Archives 4–1689, fol. 23a), Jodhpur Ms. (RO RI 37782, fol. 35b), and Benares Ms. (Sarasvati Bhavana 98703, fol. 36a): mukhapadaghātonaphale vyekapadaghnapadadalahṛte pracayaḥ // 'When the sum decreased by the product of the first term and the number of terms is divided by half the number of terms multiplied by the number of terms less one, the increase ⟨is obtained⟩.' Cf. PG 86b, Tr 40b, GSS 2.73a, 2.74b–75, 3.36b, and 6.292b, MS 15.49. SS 13.23b, L 126, and GS 3.29.

two travelers): One traveler goes nine *yojanas* on the first day and increases his daily journey by five *yojanas* every day, while another goes two *yojanas* on the first day and increases his daily journey by seven *yojanas* every day. After how many days (t) do they meet? That is to say, when $x_1 = x_2$, where x_1 and x_2 are the distances traveled by each, t is to be obtained.[51]

Solution: Let $t = s$ [= yā 1]. According to the formula for $A(n)$, which is cited here again, $x_1 = \frac{5}{2}s^2 + \frac{13}{2}s$ and $x_2 = \frac{7}{2}s^2 - \frac{3}{2}s$. Hence follows the equation $\frac{5}{2}s^2 + \frac{13}{2}s = \frac{7}{2}s^2 - \frac{3}{2}s \rightarrow \frac{5}{2}s + \frac{13}{2} = \frac{7}{2}s - \frac{3}{2}$ $\rightarrow 5s + 13 = 7s - 3 \rightarrow 2s = 16 \rightarrow s = 8$. When raised by this, $t = 8$ and $x_1 = x_2 = 212$.

Remark: This example is identical with GK śreḍhī (uv) (Part I, p. 110) = Singh III, Ex. 8, for which Nārāyaṇa prescribes a computational rule immediately before it.[52] That is, when $A_i(n)$ ($i = 1, 2$) stand for the distances traveled in n days by two travelers who increase their daily journeys according to arithmetical progressions whose first terms are $a_{i,1}$ and whose common differences are d_i (where $a_{1,1} > a_{2,1}$ and $d_1 < d_2$), and if $A_1(n) = A_2(n)$, then n is obtained by:

$$n = \frac{a_{1,1} - a_{2,1}}{d_2/2 - d_1/2} + 1.$$

Verse (17): Example 16. Trade of musk and camphor: One bought, in total, eight *palas* of musk and camphor for one *niṣka*, and sold them at the rate-quantities (*argha*) interchanged. The profit was ten *niṣkas*. How much were their purchase prices (x_1, x_2)? That is, $x_1 + x_2 = 1$, $y_1 + y_2 = 8$, $y_2 : x_1 = y_1 : z_1$, $y_1 : x_2 = y_2 : z_2$, $(z_1 + z_2) - (x_1 + x_2) = 10$, where y_1 and y_2 stand for their quantities, and z_1 and z_2 for their selling prices.

Solution: Let $y_1 = \frac{1}{2}$, $y_2 = \frac{15}{2}$, and $x_1 = s$ [= yā 1]. It follows that $x_2 = -s + 1$. By means of three-quantity operations,

[51] Cf. Exs. for BM 16–18 and GSS 6.323b–324a.

[52] GK śreḍhī 6 (Part I, p. 109): eko bṛhadādyalpapracayastvaparo mukhaṃ bṛhatpracayaḥ / tanmukhavivare cayadalaviyogabhakte sarūpake gacchaḥ // 6 // (Read -pracayo 'lpamukho 'paro bṛhat-) 'One ⟨traveler⟩ has a greater initial ⟨velocity⟩ and a smaller ⟨daily⟩ increase, and the other has a smaller initial ⟨velocity⟩ and a greater ⟨daily⟩ increase. When the difference of their initial ⟨velocities⟩ is divided by the difference of halves of the increases and increased by unity, the number of terms (i.e., the days) ⟨will be obtained⟩.' Cf. BM 16–18 and GSS 6.322b–323a.

$z_1 = \frac{x_1}{y_2}\cdot y_1 = \frac{1}{15}s$ and $z_2 = \frac{x_2}{y_1}\cdot y_2 = -15s+15$. Therefore, $z_1+z_2 = -\frac{224}{15}s + \frac{225}{15}$. Hence follows the equation $-\frac{224}{15}s + \frac{225}{15} - 1 = 10 \rightarrow -\frac{224}{15}s + \frac{210}{15} = 10 \rightarrow -224s + 210 = 150 \rightarrow s = \frac{15}{56}$. When raised by this, $x_1 = \frac{15}{56}$ and $x_2 = \frac{41}{56}$.

Remark: The commentator concludes the above solution with the remark: 'Or, the results (i.e., prices) of the musk and camphor are various owing to the assumption (*kalpanā*).'

Verse (18): Example 17. An arithmetical progression: When $A(n) = 7$ and $adn = 10$, a and d are to be obtained (see the Remark under Verse (13) for the notation).

Solution: Let $a = 2$ and $d = s$ [= yā 1].[53] Then $n = \frac{10}{ad} = \frac{10}{2s}$. With this, the 'statement' (*ālāpa*), $adn = 10$, 'is fulfilled' (*ghaṭate*). According to the formula cited above, $A\left(\frac{10}{2s}\right) = \frac{-5s+45}{2s}$. Hence follows the equation $\frac{-5s+45}{2s} = 7 \rightarrow \frac{-5s+45}{2s} = \frac{14s}{2s} \rightarrow -5s + 45 = 14s \rightarrow s = \frac{45}{19}$. When raised by this, $d = \frac{45}{19}$ and $n = \frac{19}{9}$.

Remark: The number n is not required in the problem but is calculated in the solution. See the Remarks under Verse (42) for the fractional period (or number of terms) of an arithmetical progression.

Verse (19): Example 18. Equation of journeys (two meetings of two travelers): One traveler increases his daily journey according to an arithmetical progression. Eight days after his departure, another traveler starts for the same direction and increases his daily journey according to another arithmetical progression, whose common difference is two. They meet twice. When does the second meeting take place? That is to say, when $a_{i,1}$, d_i, n_i, and $A_i(n_i)$ stand for the first term, the common difference, the number of terms, and the sum of each arithmetical progression, $n_1 = n_2 + 8$, $d_2 = 2$, and $A_1(n_1) = A_2(n_2)$.[54]

Solution: Let $a_{1,1} = 2$, $d_1 = 4$, and $a_{2,1} = s$ [= yā 1].[55] Assume that the first meeting takes place four days after the departure of the second person, that is, $A_1(12) = A_2(4)$. Now,

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[53] As the commentator remarks, the assumption, $a = s$ [= yā 1], would make the computation by Seed 1 impossible.

[54] Cf. PG (112).

[55] As the commentator remarks, when $a_{1,1}$, d_1, or n_1 is expressed by (a multiple of) s (*yāvattāvat*), 'the computation ⟨by Seed 1⟩ does not work.'

$A_1(12) = 288$ and $A_2(4) = 4s + 12$. Hence follows the equation $4s + 12 = 288 \to s = 69$. That is, $a_{2,1} = 69$. The results so far obtained are as follows. (The quantity given in the problem is underlined and the quantities assumed in the solution process are italicized.)

i	$a_{i,1}$	d_i	n_i	A_i
1	*2*	*4*	12	288
2	69	$\underline{2}$	*4*	288

For the calculation of the second meeting, let $B_1(n)$ and $B_2(n)$ be the sums of the arithmetical progressions whose first terms are respectively the thirteenth and the fifth terms of the above progressions, that is, $a_{1,13}$ (= 50) and $a_{2,5}$ (= 77),[56] and whose common differences are the same as those of A_1 and A_2. In other words, $B_1(n) = A_1(n + 12) - A_1(12)$ and $B_2(n) = A_2(n + 4) - A_2(4)$. Assuming $B_1(n) = B_2(n)$ when $n = s$ [= yā 1], the commentator obtains the equation $2s^2 + 48s = s^2 + 76s \to 2s + 48 = s + 76 \to s = 28$. Hence, for the second meeting, $n_1 = 40$ and $n_2 = 32$, and the equal distance traveled by each is 2912.

Remarks: The measure of distance, *yojana*, is specified only with the last result, 2912.

This example is identical with the problem of GK śreḍhī (uv) (Part I, p. 111) = Singh III, Ex. 9, for which Nārāyaṇa prescribes a computational rule immediately before it.[57] A scheme of the

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[56] In order to calculate these two values, the commentator cites GK śreḍhī 1a, which reads: 'The common difference, multiplied by the number of terms less one and increased by the first term, is the last value.' That is, $a_n = a + (n-1)d$. Cf. BSS 12.17a, Tr 39a, SS 13.20a, L 121a, and GS 3.26a.

[57] GK śreḍhī 7–8 (Part I, pp. 110–111): prathamasyānalpacayo mukhamiṣṭaṃ padamitirdvitīyasya / iṣṭādyasya phalādiḥ kalpyo vadanaṃ dvitīyasya // 7 // padacayaghātau samukhau dvitīyayoge ca mukhamitī bhavataḥ / tābhyāṃ ca pūrvavidhinā dvitīyayoge ca yutidivasāḥ // 8 // 'The increase which is greater ⟨than that of the second progression, which is given⟩, the first term, and the number of terms of the first ⟨progression⟩ are assumed optionally. [From the sum of the first ⟨progression⟩ for an assumed ⟨period for the second progression increased by the difference of the departure days, which is given⟩], the first term of the second ⟨progression, whose sum for the assumed period is equal to that sum of the first⟩ should be calculated. The products of ⟨each⟩ period and common difference, increased by ⟨each⟩ first term, become the values of the first terms ⟨of the progressions⟩ for the second meeting. And, from these two ⟨pairs of quantities⟩, the number of days for

Table 1: Procedures of *Gaṇitakaumudī*, śreḍhī 7–8.

d_2 and m $(= n_1 - n_2)$	Given.
↓	
$a_{1,1}$, d_1 $(> d_2)$, and n_2	Assumed optionally (*iṣṭa*).
↓	
n_1	From $n_1 - n_2 = m$.
↓	
$A_1(n_1)$	From GK śreḍhī 2a.[58]
↓	
$A_2(n_2)$	From $A_1(n_1) = A_2(n_2)$.
↓	
$a_{2,1}$	From GK śreḍhī 2b.[59]
↓	
Two progressions (B_i) for the 2nd meeting: first term $= a_{i,1} + n_i d_i$ common difference $= d_i$.	
↓	
n for $B_1(n) = B_2(n)$	From GK śreḍhī 6.[60]
↓	
$n_1 + n$ $(= n_2 + m + n)$	Answer.

procedures of that rule is shown in Table 1.

These procedures are almost the same as those of the commentator of the BGA, which have been described in the Solution above. The only difference lies in the last but one step of the procedures. That is, n for $B_1(n) = B_2(n)$ is obtained by means

the second meeting is ⟨calculated⟩ by means of the previous procedure (i.e., the rule given in śreḍhī 6).' The meaning of the passage *iṣṭādyasya phalādiḥ kalpyo* in the second half of Verse 7 is not clear. The editor suggests *phalādeḥ* for *phalādiḥ* and *kalpyaṃ* for *kalpyo*, but still it is not understandable. My translation of that passege, which is bracketed, is tentative. Singh's translation of the passage (Singh III, R. 7–8) is: 'Supposing the first term (etc.) of the first (progression), obtain the sum (of the series in A.P.).' Cf. PG 97–98.

[58] See the Remark under Verse (13).

[59] GK śreḍhī 2b (Part I, p. 106): vadanaṃ padabhaktaphale vyekapadaghnottarārdhone // 2b // 'The first term is ⟨obtained⟩ when the sum divided by the number of terms is decreased by half the common difference multiplied by the number of terms minus one.' That is to say, $a = A/n - (n-1)d/2$.

[60] See the Remark under Example 16.

of a prescribed algorithm in the GK while it is obtained by means of Seed 1 or the equation procedure with one unknown (*avyakta-samīkaraṇa*) in the commentary of BGA.

Verse (20): Example 19. Four purely numerical problems:[61] (1) $x_1 + x_2 + x_3 + x_4 = x_1^2 + x_2^2 + x_3^2 + x_4^2$, (2) $x_1^2 + x_2^2 + x_3^2 + x_4^2 = x_1^3 + x_2^3 + x_3^3 + x_4^3$, (3) $x_1 + x_2 + x_3 + x_4 = x_1x_2x_3x_4$, (4) $(x_1 + x_2) + |x_1 - x_2| = x_1^2 + x_2^2$.

Solution of (1): Let $x_i = is$ $(i = 1, 2, 3, 4)$ [$s =$ yā 1]. Then $x_1 + x_2 + x_3 + x_4 = 10s$ and $x_1^2 + x_2^2 + x_3^2 + x_4^2 = 30s^2$. Hence follows the equation $10s = 30s^2 \to 10 = 30s \to s = \frac{1}{3}$. When raised by this, $x_1 = \frac{1}{3}$, $x_2 = \frac{2}{3}$, $x_3 = \frac{3}{3}$, $x_4 = \frac{4}{3}$.

Solution of (2): Let $x_i = is$ $(i = 1, 2, 3, 4)$ [$s =$ yā 1]. Then $x_1^2 + x_2^2 + x_3^2 + x_4^2 = 30s^2$ and $x_1^3 + x_2^3 + x_3^3 + x_4^3 = 100s^3$. Hence follows the equation $30s^2 = 100s^3 \to 30 = 100s \to s = \frac{3}{10}$. When raised by this, $x_1 = \frac{3}{10}$, $x_2 = \frac{6}{10}$, $x_3 = \frac{9}{10}$, $x_4 = \frac{12}{10}$.

Solution of (3): Let $x_1 = s$ [$=$ yā 1] and $x_i = i$ $(i = 2, 3, 4)$. Then $x_1 + x_2 + x_3 + x_4 = s + 9$ and $x_1x_2x_3x_4 = 24s$. Hence follows the equation $s + 9 = 24s \to s = \frac{9}{23}$. That is, $x_1 = \frac{9}{23}$, and $x_2 = \frac{2}{1}$, $x_3 = \frac{3}{1}$, $x_4 = \frac{4}{1}$.

Solution of (4): Let $x_i = is$ $(i = 1, 2)$ [$s =$ yā 1]. Then $(x_1 + x_2) + |x_1 - x_2| = 4s$ and $x_1^2 + x_2^2 = 5s^2$. Hence follows the equation $4s = 5s^2 \to 4 = 5s \to s = \frac{4}{5}$. When raised by this, $x_1 = \frac{4}{5}$ and $x_2 = \frac{8}{5}$.

Verse (21): Example 20. Four independent equations of the area of a right-angled triangle and its three sides:[62] When x, y, and z stand for the three sides of a right-angled triangle (z for the hypotenuse) and A for its area, (1) $x = A$, (2) $y = A$, (3) $z = A$, (4) $xyz = A$.

Solution of (1): Let $x = 3s$, $y = 4s$, and $z = 5s$ [$s =$ yā 1]. Then $A = xy/2 = 6s^2$. Hence follows the equation $3s = 6s^2 \to 3 = 6s \to s = \frac{1}{2}$. When raised by this, $x = \frac{3}{2}$, $y = \frac{4}{2}$, $z = \frac{5}{2}$.

Solution of (2): On the same assumption as in (1), the commentator obtains the equation $4s = 6s^2 \to 4 = 6s \to s = \frac{2}{3}$. When raised by this, $x = \frac{6}{3}$, $y = \frac{8}{3}$, $z = \frac{10}{3}$.

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[61] Problems (1) and (2) are identical with the problems of BG 106. See the fns. for Verse (20) in Sections 2 and 3 above.

[62] Problems (3) and (4) are identical with the problems of BG 107. See the fns. for Verse (21) in Sections 2 and 3 above.

Solution of (3): On the same assumption as in (1), the commentator obtains the equation $5s = 6s^2 \to 5 = 6s \to s = \frac{5}{6}$. When raised by this, $x = \frac{15}{6}$, $y = \frac{20}{6}$, $z = \frac{25}{6}$.

Solution of (4): On the same assumption as in (1), $xyz = 60s^3$. Hence follows the equation $60s^3 = 6s^2 \to 60s = 6 \to s = \frac{1}{10}$. When raised by this, $x = \frac{3}{10}$, $y = \frac{4}{10}$, $z = \frac{5}{10}$.

Remark: At the end of the above solution, the commentator cites GK kṣetra 82 (Part II, pp. 99–100), which prescribes a general method for producing (*janana*) the three sides of a right-angled triangle (*jātya-tryasra*, lit. a noble trilateral): 'Two integers (lit. digits) which are a means for producing a noble one (right-angled triangle) are called 'seeds'. The sum and the difference of their squares are the ear (hypotenuse) and the upright, ⟨respectively⟩, and twice the product is the side.'[63] That is to say, when

$$x = a^2 - b^2, \qquad y = 2ab, \qquad z = a^2 + b^2,$$

where a and b are integers called 'seeds' (*bīja*), we have the relationship, $x^2 + y^2 = z^2$. In the above solution, the right-angled triangle produced from the seeds, $a = 2$ and $b = 1$, is used as a basic triangle.

Verse (22): Example 21. Purely numerical problem: $x + y = z_1^2$, $x - y = z_2^3$, $z_1 z_2 = z_3^2$, where z_1, z_2, and z_3 are not required.

Solution: The commentator first assumes

$$z_2 = \frac{a^2}{2}, \qquad z_1 = \frac{a^2}{2} \cdot b^2,$$

and substitutes 2 and 3 for the optional numbers, a and b, respectively, to obtain $z_2 = 2$ and $z_1 = 18$, or $z_2^3 = 8$ and $z_1^2 = 324$. Let $y = s$ [= yā 1]. Then $x = s + 8$ and also $x = -s + 324$. Hence follows the equation $s + 8 = -s + 324 \to s = 158$. Hence $x = 166$ and $y = 158$.

Remarks: The commentator remarks at the end of his solution: 'Other pairs of quantities (x and y) would be ⟨obtained⟩ by means of other optional numbers' (*anyeneṣṭenānyau rāśī bhavataḥ*). The 'optional numbers' mentioned here refer to a and b employed for the assumption of z_1 and z_2.

[63] Cf. BSS 12.23, GSS 7.90b–93a, and L 147–148.

The z_1 and z_2 assumed above certainly satisfy the condition $z_1 z_2 = z_3^2$, since $z_1 z_2 = \left(\frac{a^2 b}{2}\right)^2$, although the commentator does not mention it in his solution.

On the same assumption, in passing, one can make an algorithm for a pair of solutions in terms of a and b,

$$x = \frac{2a^4b^4 + a^6}{16}, \qquad y = \frac{2a^4b^4 - a^6}{16},$$

which will enable one to obtain the same result without using an equation, but it is out of the range of a book on algebra.

Verse (23): Example 22. Purely numerical problem: $3(x + \sqrt{y}) = \sqrt{x} + y$.

Solution: Let $x = 9s^2$ and $y = 36s^2$ [s = yā 1]. Then the equation is $3 \times (9s^2 + 6s) = 3s + 36s^2 \rightarrow 9s^2 = 15s \rightarrow 9s = 15 \rightarrow s = \frac{5}{3}$. When raised by this, $x = 25$ and $y = 100$.

Verse (24): Example 23. Purely numerical problem: $xy + x^2 = x + y^2$ $(x < y)$.

Solution: Let $x = 5s$ and $y = 8s$ [s = yā 1]. Then the equation is $40s^2 + 25s^2 = 5s + 64s^2 \rightarrow 40s + 25s = 5 + 64s \rightarrow s = 5$. When raised by this, $x = 25$ and $y = 40$.

Verse (25): Example 24. Purely numerical problem: $\frac{xy^2 - x^2}{2} = x^3$ $(x < y)$.

Solution: Let $x = 2s$ and $y = 3s$ [s = yā 1]. Then $\frac{xy^2 - x^2}{2} = 9s^3 - 2s^2$ and $x^3 = 8s^3$. Hence follows the equation $9s^3 - 2s^2 = 8s^3 \rightarrow 9s - 2 = 8s \rightarrow s = 2$. When raised by this, $x = 4$ and $y = 6$.

Verse (26): Example 25. Purely numerical problem: $(x + y)^3 + (x + y)^2 = 2(x^3 + y^3)$, where $x < y$.[64]

Solution: Let $x = s$ and $y = 4s$ [s = yā 1]. Then $(x + y)^3 + (x + y)^2 = 125s^3 + 25s^2$ and $2(x^3 + y^3) = 130s^2$. Hence follows the equation $25s^2 + 125s^3 = 130s^3 \rightarrow 25 + 125s = 130s \rightarrow s = 5$. When raised by this, $x = 5$ and $y = 20$.

Verse (27): Example 26. Two purely numerical problems: (1) $x + y = z_1^2$, $x - y = z_2^2$, $xy = z_3^3$; and (2) $x^3 + y^3 = z_1^2$, $x^2 + y^2 = z_2^3$,

[64] This problem is identical with that of BG 153. See the fns. for Verse (26) in Sections 2 and 3 above.

where z_1, z_2, and z_3 are not required.[65]

Solution of (1): Let $x = 5s^2$ and $y = 4s^2$ [s = yā 1]. Then the first 'two statements are fulfilled automatically' (*ālāpadvayaṃ svayameva ghaṭate*) since $x + y = (3s)^2$ and $x - y = s^2$. Now, $xy = 20s^4$. Assuming $z_3 = 10s$, the commentator calculates $z_3^3 = 1000s^3$. Hence follows the equation $20s^4 = 1000s^3 \rightarrow 20s = 1000 \rightarrow s = 50$. When raised by this, $x = 12500$ and $y = 10000$.

Solution of (2): Let $x = s^2$ and $y = 2s^2$ [s = yā 1]. Then the first statement is fulfilled automatically since $x^3 + y^3 = (3s^3)^2$. Now, $x^2 + y^2 = 5s^4$. Assuming $z_2 = 5s$, the commentator calculates $z_2^3 = 125s^3$. Hence follows the equation $5s^4 = 125s^3 \rightarrow 5s = 125 \rightarrow s = 25$. When raised by this, $x = 625$ and $y = 1250$.

Remarks: For the assumption, $x = 5s^2$ and $y = 4s^2$, made in the solution of (1), Rule 3 in Verse 9, with $a = 2$ and $b = 1$, is employed. For the assumption, $x = s^2$ and $y = 2s^2$, made in the solution of (2), Rule 5 in Verse 11, with $n = 2$, is employed. For the computation of $x^3 + y^3$, Rule 4 in Verse 10 is employed.

These problems, (1) and (2), are treated by Nārāyaṇa in the section called 'Wonder of Square' (*kṛti-kautūhala*) of the GK as well.[66] He prescribes a computational rule for each case. The rule for (1) is[67]

$$x = \left(a^2 + b^2\right) \cdot \frac{\left(c^3\right)^2}{\left\{2ab\left(a^2 + b^2\right)\right\}^2},$$

$$y = 2ab \cdot \frac{\left(c^3\right)^2}{\left\{2ab\left(a^2 + b^2\right)\right\}^2},$$

[65] These problems, (1) and (2), are identical with those of BG 108 and 109, respectively. See the fns. for Verse (27) in Sections 2 and 3 above.

[66] See the fns. for Verse (27) in Sections 2 and 3 above.

[67] GK prakīrṇaka 49 (Part I, pp. 35–36): prāguktau yau ca tayorvadhakṛtibhakteṣṭaghanakṛtihatau tau / rāśyoryoge vivare vargo ghāte ghano bhavati // 49 // 'The two ⟨quantities⟩ told above (in prakīrṇaka 48, for which see the fn. for Rule 3 in Verse 9 below) are ⟨each⟩ multiplied by the square of the cube of any optional number divided by the square of the product of the two ⟨quantities⟩. There will be a square in the case of the sum or the difference of the two quantities, and a cube in the case of the product ⟨of the two⟩.'

where a, b, and c are any optional numbers. The rule for (2) is[68]

$$x = \frac{(a^3)^2}{5^2}, \qquad y = \frac{2\,(a^3)^2}{5^2},$$

where a is any optional number.

Verse 9: Rule 3 (supplementary). Let $x = a^2+b^2$ and $y = 2ab$. Then $x + y = (a + b)^2$ and $x - y = (a - b)^2$.

Remark: This rule is employed when the sum and the difference of two unknown quantities have to be equal to square numbers. See the Remarks under Verse (27). Nārāyaṇa gives the same rule in a slightly different expression in the GK.[69]

Verse 10: Rule 4 (supplementary). $((\sqrt{x})^3)^2 = x^3$ or, by assuming $\sqrt{x} = t$, $(t^3)^2 = (t^2)^3$.

Remark: For example, $(3^2)^3 = 9^3 = 729$ and $(3^3)^2 = 27^2 = 729$. See the Remarks under Verse (27).

Nārāyaṇa gives the same rule in almost the same expression in the GK.[70]

Verse 11: Rule 5 (supplementary). $1^3 + 2^3 + 3^3 + \cdots + n^3 = (S(n))^2$, where $S(n)$ is the sum of the natural series (*saṅkalita*) up to n, that is, $S(n) = 1 + 2 + 3 + \cdots + n = n(n + 1)/2$. The sum, $1^3 + 2^3 + 3^3 + \cdots + n^3$, is therefore '⟨an integer⟩ which gives a square-root' (*mūlada*), i.e., a square number.

Remark: This rule is employed when the sum of two or more cubic quantities has to be a square number. See the Remarks under Verse (27). The first half of this rule is identical with GK śreḍhī 14b (Part I, p. 117).[71]

[68] GK prakīrṇaka 50 (Part I, p. 37): iṣṭaghanavarga eko dvighno 'nyaḥ pañcakṛtihṛtau rāśī / vargayutau ca ghanaḥ syāttayorbhavedghanayutau vargaḥ // 50 // 'The square of the cube of an optional number is one ⟨quantity⟩, and twice ⟨that⟩ is the other. The two quantities are divided by the square of five. There will be a cube in the case of the sum of the squares of the two ⟨quantities⟩, and a square in the case of the sum of the cubes ⟨of the two⟩.'

[69] GK prakīrṇaka 48 (Part I, p. 35): vargayutiḥ prathamā syādabhīṣṭayorāhatirdviguṇitānyaḥ / saṃyoge ca viyoge pṛthaktayorjāyate vargaḥ // 48 // 'The sum of the squares of two arbitrary ⟨numbers⟩ will be the first, and their product multiplied by two will be the second. When the sum and the difference of the two ⟨are taken⟩ separately, a square is produced.' Cf. L 20.

[70] GK prakīrṇaka 23b. See the fns. for Verse 10 in Sections 2 and 3. Cf. L 26b in the Ānandāśrama edition.

[71] Cf. AB 2.22b, BSS 12.20b, PG 103a, GSS 301, SS 13.22b, L 119, and GS

Verse (28): Example 27. Purely numerical problem: $x - y = z_1^2$, $x^2 + y^2 = z_2^3$, where z_1 and z_2 are not required.[72]

Solution: Let $x = 4s^2$ and $y = 3s^2$ [s = yā 1]. Then the first statement is fulfilled automatically since $x - y = s^2$. Now, $x^2 + y^2 = 25s^4$. Assuming $z_2 = 5s$, the commentator calculates $z_2^3 = 125s^3$. Hence follows the equation $25s^4 = 125s^3 \rightarrow 25s = 125 \rightarrow s = 5$. When raised by this, $x = 100$ and $y = 75$.

Verse 12: Rule 6. Not understood clearly.

Remark: This seems to be not a computational rule but a statement of the importance (?) of Seed 3 or 'the elimination of the middle term', which is useful for problems involving squares, cubes, etc. of unknown quantities such as those which have just been treated in the preceeding examples. For Seed 3 see under Verses 1–2 of this section.

Verse (29): Example 28. Nine gold bangles (*valaya*) whose weights make an arithmetical progression: When $a = 2$, $n = 9$, and $a_9 = 12$, d and $A(9)$ are to be obtained.[73]

Solution: Let $d = s$ [= yā 1]. Then, according to the rule for the n-th term of an arithmetical progression, $a_n = a + (n-1)d$, prescribed in GK śreḍhī 1a,[74] the commentator calculates $a_9 = 8s + 2$. Hence follows the equation $8s + 2 = 12 \rightarrow s = \frac{5}{4}$ or $d = \frac{5}{4}$. Hence $A(9) = 63$.

Remark: For the rule for the sum of an arithmetical progression used at the end of the solution, see Verses (13), (14), (15), and (16). For computational rules for the common difference, see Verse (15) above in this section.

Verse (30): Example 29. Equation of properties after exchange of part of them: The first person has eight elephants, the second seven horses, the third nine cows, and the fourth eleven goats. After exchange of one each among them, their properties become equal to one another. What are the unit prices (x_1, x_2,

3.33b.

[72] This problem is identical with that of BG 156. See the fns. for Verse (28) in Sections 2 and 3 above.

[73] A similar example of a set of bangles whose prices make an arithmetical progression occurs in PG (110), where $a = 8$ *paṇas*, $n = 24$, and $a_{24} = 13$ *paṇas* are given and $A(24)$ is required, for which problem Śrīdhara prescribes a computational rule (PG 95b), namely, $A(n) = n \cdot (a + a_n)/2$.

[74] See the Solution of Example 18 in Verse (19).

x_3, x_4) of those animals? That is to say, $5x_1 + x_2 + x_3 + x_4 = x_1 + 4x_2 + x_3 + x_4 = x_1 + x_2 + 6x_3 + x_4 = x_1 + x_2 + x_3 + 8x_4$.[75]

Solution: The commentator first reduces the above equations to: $4x_1 = 3x_2 = 5x_3 = 7x_4 = k$, and, assuming $k = 100$, obtains $x_1 = 25$, $x_2 = \frac{100}{3}$, $x_3 = 20$, $x_4 = \frac{100}{7}$, and the equal properties $192\frac{13}{21}$.

Another solution: Assuming $k = 4 \times 3 \times 5 \times 7 = 420$ in the equation $4x_1 = 3x_2 = 5x_3 = 7x_4 = k$, he obtains the integer solutions, $x_1 = 105$, $x_2 = 140$, $x_3 = 84$, $x_4 = 60$, and the equal properties 809.

Remarks: The commentator introduces this example as 'an example of a certain person' and solves it as a problem of an indeterminate system of three equations with four unknown numbers. This is noteworthy because, in most of the other extant examples for Seed 1 (excepting Example 36 in Verse (38), where no equation is employed), he reduces the given problems to problems of one equation with one unknown number.

In order to express the four unknowns, he uses not the usual symbols, yā, kā, nī, and pī, but the initial letters, mā, tu, go, and a, of the words meaning the animals in question (*mātaṅga*, *turaṅgama*, *go*, *ajā*). He says at the beginning of his solution: '⟨The rule⟩ that the ⟨words expressing⟩ colors beginning with *yāvattāvat* are assumed for unknowns has implication (*upalakṣaṇa*). That is to say, not only the traditional symbols, yā, kā, etc., but also other symbols may be used for unknown numbers according to the commentator.[76]

In the GK Nārāyaṇa gives a computational rule for this type of problem: 'When an optional number is divided by the ⟨number of one kind of⟩ jewels (a_i) decreased by the ⟨number of⟩ gifts (m) multiplied by the ⟨number of⟩ people (n), the price ⟨is obtained⟩.'[77] That is, when

$$\{a_i - m(n-1)\}\, x_i + \sum_{j \neq i} x_j = y,$$

[75] Cf. Exs. 1 and 2 for BM 13, GSS 6.164–166, L 102, and BG 98

[76] Cf. fn. 6 under Verses 1–2 of this section.

[77] GK miśra 27a (Part I, p. 83): narahatadānavihīnai ratnairiṣṭe vibhājite maulyam // 27a // Cf. BM 13, GSS 6.163, and L 101.

for every i $(i = 1, 2, ..., n)$, a set of solutions is obtained by:

$$x_i = \frac{k}{a_i - mn},$$

where k is an optional number. In the example that Nārāyaṇa provides for this rule,[78] the initial properties of four people are, in order, five sapphires, six zircons, seven beryls, and eight diamonds, and they exchange one piece each among themselves.

Verses (31)–(32): Example 30. Prices of musk and camphor: Nine *palas* of musk and seven *palas* of camphor cost one hundred and seven monetary units in total, whereas seven *palas* of musk and nine *palas* of camphor cost one hundred and one monetary units in total. What is the unit price of each (x_1, x_2)? That is to say, $9x_1 + 7x_2 = 107$ and $7x_1 + 9x_2 = 101$.[79]

Solution: Let $x_1 = s$ [= yā 1]. Then $x_2 = -\frac{9}{7}s + \frac{107}{7}$ and also $x_2 = -\frac{7}{9}s + \frac{101}{9}$. Hence follows the equation $-\frac{9}{7}s + \frac{107}{7} = -\frac{7}{9}s + \frac{101}{9}$ $\rightarrow -\frac{81}{63}s + \frac{963}{63} = -\frac{49}{63}s + \frac{707}{63} \rightarrow -81s + 963 = -49s + 707 \rightarrow$ $32s = 256 \rightarrow s = 8$. When raised by this, $x_1 = 8$ and $x_2 = 5$.

Verse (33): Example 31. False number of horses of four horse-dealers: The four dealers say, in order, to a custom-officer that the total number (x) of the horses they possess is fifteen, seventeen, nineteen, and twenty-one, each person hiding his own (x_i; $i = 1, 2, 3, 4$). That is to say, $x_1 + x_2 + x_3 + x_4 = x$, and $x - x_1 = 15$, $x - x_2 = 17$, $x - x_3 = 19$, $x - x_4 = 21$.[80]

Solution: Let $x = s$ [= yā 1]. Then $x_1 + x_2 + x_3 + x_4 = 4s - 72$. Hence follows the equation $4s - 72 = s \rightarrow s = 24$. When raised by this, $x = 24$, $x_1 = 9$, $x_2 = 7$, $x_3 = 5$, $x_4 = 3$.

Remark: This example is identical with the problem of GK miśra (uv) (Part I, p. 85) = Singh II, Ex. 34, for which Nārāyaṇa prescribes a computational rule immediately before it.[81] That is,

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[78] GK miśra (uv) (Part I, p. 83) = Singh II, Ex. 32.

[79] Mahāvīra gives a numerically identical example (GSS 6.140b–143a), in which 9 citrons (*mātuluṅgas*) and 7 wood-apples (*kapitthas*) cost 107 in total and 7 citrons and 9 wood-apples cost 101 in total. For this type of problem he prescribes a computational rule (GSS 6.139b–140a): When $ax_1 + bx_2 = c$ and $bx_1 + ax_2 = d$, the solution is obtained by: $x_1 = (ac - bd)/(a^2 - b^2)$ and $x_2 = (ad - bc)/(a^2 - b^2)$.

[80] Cf. Bhāskara I's Exs. 1 and 2 for AB 2.29 and GSS 6.160–162.

[81] GK miśra 28 (Part I, p. 85): vañcitabhāṇḍasamāse vyekanarāpte prajā-

when $x_1 + x_2 + \cdots + x_n = x$ and $x - x_i = a_i$ $(i = 1, 2, \ldots, n)$,

$$x = \frac{a_1 + a_2 + \cdots + a_n}{n - 1}, \qquad x_i = x - a_i.$$

Verse (34): Example 32. A hoard and the properties of four people: Each of the four people says that his property (x_i; $i = 1, 2, 3, 4$), when added to the hoard (y), becomes severally four, five, six, and seven times the sum of the properties of the other people. That is to say, $x_1 + y = 4(x_2 + x_3 + x_4)$, $x_2 + y = 5(x_1 + x_3 + x_4)$, $x_3 + y = 6(x_1 + x_2 + x_4)$, $x_4 + y = 7(x_1 + x_2 + x_3)$.[82]

Solution: Let $x_1 + x_2 + x_3 + x_4 = s$ [= yā 1], and $s + y = t$, that is, $t = x_1 + x_2 + x_3 + x_4 + y$. Then $t = 5(x_2 + x_3 + x_4) = 6(x_1 + x_3 + x_4) = 7(x_1 + x_2 + x_4) = 8(x_1 + x_2 + x_3)$. Assuming $t = 1$, the commentator obtains: $x_2 + x_3 + x_4 = \frac{1}{5}$, $x_1 + x_3 + x_4 = \frac{1}{6}$, $x_1 + x_2 + x_4 = \frac{1}{7}$, $x_1 + x_2 + x_3 = \frac{1}{8}$. Therefore, $x_1 = s - \frac{1}{5}$, $x_2 = s - \frac{1}{6}$, $x_3 = s - \frac{1}{7}$, $x_4 = s - \frac{1}{8}$, and $x_1 + x_2 + x_3 + x_4 = 4s - \frac{533}{840}$. Hence follows the equation $4s - \frac{533}{840} = s \to s = \frac{533}{2520}$. When raised by this, $x_1 = \frac{29}{2520}$, $x_2 = \frac{113}{2520}$, $x_3 = \frac{173}{2520}$, $x_4 = \frac{218}{2520}$, and $y = t - s = \frac{1987}{2520}$.

Another solution: Removing the common denominators 2520 from the fractional solutions obtained above, the commentator obtains the integer solutions, 29, 113, 173, 218, and 1987.

Remarks: In the first solution the commentator calls t 'an optional quantity' (*iṣṭa-rāśi*) and assumes $t = 1$, but in the second solution he does not refer to it although the removal of the denominators 2520 is equivalent to assuming $t = 2520$.

This example is identical with GK miśra (uv) (Part I, p. 87) = Singh II, Ex. 35, for which Nārāyaṇa prescribes a computational

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yate yogaḥ / tasminnuktavihīne ⟨pṛthagdhanāni prajāyante⟩ // 28 // 'When the sum of the false ⟨totals of⟩ commodities is divided by the number of people less one, the ⟨true⟩ sum is obtained. When it is decreased by the ⟨false total⟩ told ⟨by each person⟩, their properties are produced.' Cf. AB 2.29 and GSS 6.159.

[82] Cf. GSS 6.236–237 and 244–247.

rule immediately before it.[83] That is, when

$$x_i + y = a_i \sum_{j \neq i} x_j,$$

for every i $(i = 1, 2, \ldots, n)$, let $x_1 + x_2 + \cdots + x_n = x$ and $x + y = t$. Then

$$x - x_i = \frac{t}{a_i + 1}.$$

Assume any optional number for t and apply the previous rule (GK miśra 28) to obtain x and x_i. The value of the hoard is: $y = t - x$.

Verse (35): Example 33. Bet on a cock-fight: An owner of fighting cocks will give a certain amount of money x_1 to a counselor if the former's cock wins thanks to the latter's magical charm (*mantra*), but will take an amount of money x_2, which is equal to twice that less one eighth of itself, from the latter if he loses. When the former wins and loses once each and gains the square of six in total, 'say the two given amounts of money' (*vada dattavitte*). That is to say, $x_2 = 2\left(1 - \frac{1}{8}\right) x_1$ and $x_2 - x_1 = 36$.

Solution: The commentator assumes that the owner of fighting cocks gives money to the counselor once, say x_1, and takes from him twice, say x_2 and x_3, where $x_2 = x_1/2\left(1 - \frac{1}{6}\right)$ and $x_3 = x_1/2\left(1 - \frac{1}{8}\right)$, and that $(x_2 + x_3) - x_1 = 36$. He probably regarded the words, 'the two given amounts of money', as meaning not 'once in each direction' but 'twice from the counselor to the owner of cocks in addition to once in the other direction'. He therefore had to introduce the factor, $2\left(1 - \frac{1}{6}\right)$, into his solution, a factor which does not occur in the statement of Example 33 itself. His solution proceeds as follows.

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[83] GK miśra 29 (Part I, p. 86): saikaguṇāptamabhīṣṭaṃ vicintya vidhinā nṛṇāṃ dhanāni syuḥ / tadyogonamabhīṣṭaṃ nidhimānaṃ jāyate nūnam // 29 // 'When one has considered any optional number divided by ⟨each⟩ multiplier plus one ⟨as the false totals⟩, the properties of the men will be ⟨obtained⟩ by means of the ⟨previous⟩ procedure ⟨told in GK miśra 28⟩. The optional number decreased by their sum becomes indeed the value of the hoard.' For GK miśra 28, see the Remark under Verse (33). Cf. GSS 6.233–235. For Nārāyaṇa's rules for similar types of problems, see the Remark under Verse (7).

Let $x_1 = s$ [= yā 1]. He first reduces the two factors into simple fractions: $2\left(1-\frac{1}{6}\right) = \frac{5}{3}$ and $2\left(1-\frac{1}{8}\right) = \frac{7}{4}$, and then performs a couple of three-quantity operations:[84]

$$1 : \frac{5}{3} = s : x_2 \to x_2 = \frac{5}{3}s; \quad 1 : \frac{7}{4} = s : x_3 \to x_3 = \frac{7}{4}s.$$

These results, however, do not fit the rest of his computation. A slight change of the text[85] will bring us another pair of three-quantity operations:

$$\frac{5}{3} : 1 = s : x_2 \to x_2 = \frac{3}{5}s; \quad \frac{7}{4} : 1 = s : x_3 \to x_3 = \frac{4}{7}s;$$

which would produce the results actually employed in the next step, although the meaning of this computation is not clear to me. Then the commentator forms and solves the equation $\frac{3}{5}s+\frac{4}{7}s-s = 36 \to \frac{6}{35}s = 36 \to s = 210$. When raised by this, $x_2 = 126$ and $x_3 = 120$.

Remarks: The correct solution of the problem in the present example, which is complete by itself both mathematically and metrically, seems to be as follows. Let $x_1 = s$ [= yā 1]. Then $x_2 = \frac{7}{4}s$. Hence follows the equation $\frac{7}{4}s - s = 36 \to s = 48$. When raised by this, $x_1 = 48$ and $x_2 = 84$.

Note that the equation solved by the commentator is

$$\frac{x}{2\left(1-\frac{1}{6}\right)} + \frac{x}{2\left(1-\frac{1}{8}\right)} - x = 36,$$

the meaning of which is open to question.

The commentator's curious interpretation of Verse (35) may have been influenced by a similar problem given by Mahāvīra, in which 'a great man having powers of magical charm (*mantra*) and medicine (*auṣadha*)' separately and secretly promises each of two cock-fighters: 'If you lose, I will give you a certain part (a_i/b_i, where $i = 1, 2$) of the money (x_i) that you should pay to me when

[84] ete rūpasyaikasya tadyāvat⟨t⟩āvataḥ ke iti jāte dhane yā $\frac{5}{3}$ yā $\frac{7}{4}$.

[85] I suggest the following emendations of the text: ete rūpasyaikasya → etayo rūpa ekas; yā $\frac{5}{3}$ yā $\frac{7}{4}$ → yā $\frac{3}{5}$ yā $\frac{4}{7}$.

you win'; and in either case he gains the same amount of money (c). That is,

$$x_1 - \frac{a_2}{b_2} \cdot x_2 = c = x_2 - \frac{a_1}{b_1} \cdot x_1,$$

for which Mahāvīra provides the computational rule (GSS 6.268b–270a):

$$x_1 = \frac{b_1(a_2+b_2)}{|b_1(a_2+b_2) - a_2(a_1+b_1)|} \cdot c,$$

$$x_2 = \frac{b_2(a_1+b_1)}{|b_2(a_1+b_1) - a_1(a_2+b_2)|} \cdot c.$$

In Mahāvīra's example (GSS 6.270b–273a), $a_1/b_1 = 2/3$, $a_2/b_2 = 3/4$, and $c = 12$.

Verse (36): Example 34. Property of a traveling merchant: A merchant visits three towns. He pays five monetary units for tax each time he enters and leaves a town, but makes his property double through his transactions in the town. His final property is three times his original property (x). That is to say, $x_i = 2(x_{i-1} - 5) - 5$, where $i = 1, 2, 3$ and $x_0 = x$, and $x_3 = 3x$.[86]

Solution: Let $x = s$ [$=$ yā 1]. Then $x_1 = 2s - 15$, $x_2 = 4s - 45$, $x_3 = 8s - 105$. Hence follows the equation $8s - 105 = 3s \rightarrow s = 21$. That is, $x = 21$.

Verse (37): Example 35. Capital and interest: When a capital (x) is loaned for one year at the rate of five percent per month, the total sum of the capital and the interest (y) on it is greater than the capital by one half of itself and by six monetary units. That is to say, $x + y = x + \frac{x}{2} + 6$, where $y = x \cdot \frac{5}{100} \cdot \frac{12}{1}$.

Solution: Let $x = s$ [$=$ yā 1]. Then, by means of a five-quantity operation, $y = \frac{3}{5}s$. Hence follows the equation $s + \frac{3}{5}s = \frac{3}{2}s + 6 \rightarrow s = 60$. When rasied by this, $x = 60$ and $y = 36$.

Verse (38): Example 36. A pond filled with water from four springs: The pond is filled with water in one day, in a half day, in one third of a day, and in a quarter of a day respectively when the gate of each spring is opened individually. How long (t) does

[86] Cf. Ex. 1 for BM C1, GSS 6.112b–115a, and BG 101. The BM and Mahāvīra (GSS 6.111b–112a) solve the examples by means of the 'rule of inversion'.

it take for the four springs, opened at the same time, to fill the pond?[87]

Solution: Let the capacity of the pond be s [= yā 1]. Then the quantity of water supplied by each spring in one day (x_i; $i = 1, 2, 3, 4$) is calculated by means of the three-quantity operations:

$$\frac{1}{i} : s = 1 : x_i \rightarrow x_i = is \quad (i = 1, 2, 3, 4).$$

The sum of these is $10s$. Therefore, the four springs, opened altogether for one day, can fill a pond whose capacity is $10s$. Hence follows another three-quantity operation:

$$10s : 1 = s : t \rightarrow t = \frac{1}{10} \text{ day}.$$

Another solution: The commentator gives another solution by assuming $s = 60$ *droṇas*. By means of three-quantity operations simiar to the above, he first obtains $x_i = 60i$ *droṇas* ($i = 1, 2, 3, 4$), whose sum is 600 *droṇas*, and then performs another three-quantity operation, $600 : 1 = 60 : t$, from which he obtains $t = \frac{1}{10}$ day again. Finally, he calculates the quantity of water supplied by each spring, $60i/10$, that is, 6, 12, 18, and 24 *droṇas*.

Remarks: In the first solution, the symbol yā 1 is employed but the equation procedure is not. In answer to the imaginary question, 'What is the use of assuming *yāvattāvat* here?' (*atra yāvattāvatkalpanayā kim*), the commentator gives the second solution, which does not use any algebraic symbolism.

This example is identical with the problem of GK miśra (uv) (Part I, p. 94) = Singh II, Ex. 39, for which Nārāyaṇa prescribes a computational rule immediately before it.[88] That is, if the i–th spring ($i = 1, 2, \ldots, n$) can fill a pond in b_i/a_i of a day, when opened together, they fill it in

$$T = \frac{1}{\frac{a_1}{b_1} + \frac{a_2}{b_2} + \cdots + \frac{a_n}{b_n}} \text{ day}.$$

---

[87] Cf. PG (91), GSS 8.34, and L 97.

[88] GK miśra 36a (Part I, p. 94): aṃśahṛtarūpasaṃyutibhakte rūpe prapūrtikālaḥ syāt // 36a // 'When unity is divided by the sum of ⟨the quotients of⟩ the divisions of unity by the fractions ⟨of a day given⟩, the time for filling ⟨a pond⟩ will be ⟨obtained⟩.' Cf. PG 69, GSS 8.32b–33, MS 15.43, and L 96.

Verse (39): Example 37. Donation of one thousand cows from three herds: On the occasion of a solar eclipse, a king donates to the best of brāhmaṇas a half of the first herd of cows minus one, one third of the second herd minus three, and a quarter of the third herd plus three. All the three herds contain the same number of cows (x), and the total number of cows donated is one thousand. That is to say, $x_1 = \frac{x}{2} - 1$, $x_2 = \frac{x}{3} - 3$, $x_3 = \frac{x}{4} + 3$, and $x_1 + x_2 + x_3 = 1000$.[89]

Solution: Let $x = s$ [= yā 1]. Then $x_1 = \frac{1}{2}s - 1$, $x_2 = \frac{1}{3}s - 3$, $x_3 = \frac{1}{4}s + 3$, and $x_1 + x_2 + x_3 = \frac{13}{12}s - 1$. Hence follows the equation $\frac{13}{12}s - 1 = 1000 \rightarrow s = 924$. When raised by this, $x_1 = 461$, $x_2 = 305$, $x_3 = 234$.

Verse (40): Example 38. An arithmetical progression: An arithmetical progression is to be determined when its first term is equal to its common difference and its sum is equal to the number of terms, that is, $a = d$ and $A(n) = n$.[90]

Solution: Let $n = 7$, $A(7) = 7$, and $a = d = s$ [= yā 1].[91] Then, according to the formula for $A(n)$ cited before (see under Verses (13)–(16)), $A(7) = 28s$. Hence follows the equation $28s = 7 \rightarrow s = \frac{1}{4}$. When raised by this, $a = d = \frac{1}{4}$.

Verse (41): Example 39. Series figures (*śreḍhī-kṣetra*): When the perpendicular (h) of a series figure for the progression whose first term and common difference are both unity is two, the figure (trapezium) is to be determined. That is to say, when $a = d = 1$ and $n = h = 2$, the top (x) and the base (y) of the trapezium are required.[92]

Solution: Let $x + y = s$ [= yā 1]. According to the formula, $A(2) = 3$. Now, the area of the series figure is $\frac{x+y}{2} \cdot h = s$, which is equal to $A(2)$. Hence follows the equation $s = 3$. That is, $x + y = 3$.

Let $x = s$ [= yā 1]. Then $y = -s + 3$; see Fig. 3. The projection (*āvādhā*, lit. 'depression') on the base corresponding

[89] Cf. Exs. 1–9 for BM N6 (rule) and GSS 6.97b–98a (rule) and 6.98b–102a (examples).

[90] Cf. GSS 2.80–83 and 6.317–318 for equations involving a, d, n, and $A(n)$. See the Remark under Verse (13) for the notation.

[91] As the commentator remarks, when n is assumed to be s [= yā 1], 'the computation does not work.'

[92] Cf. GK kṣetra (60)–(62).

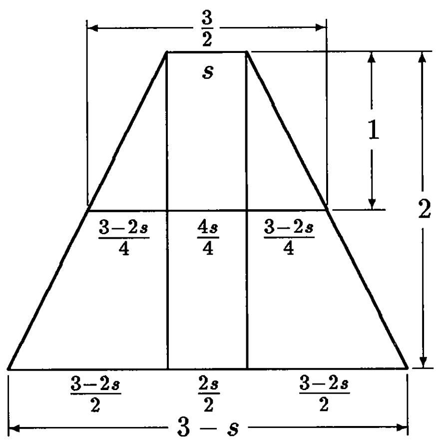


Figure 3: Series figures in Example 39 (rearranged).

to the left (or right) flank of the trapezium is $\frac{y-x}{2} = -\frac{2}{2}s + \frac{3}{2}$. Now, the series figure for the case $n = 1$ of the same progression must be a small trapezium cut out of the first trapezium by a line parallel to its top and base at its middle. The small trapezium thus obtained has the height 1 the top s and the base $\left(-\frac{2}{4}s + \frac{3}{4}\right) \times 2 + s = \frac{3}{2}$, where $-\frac{2}{4}s + \frac{3}{4}$ is half of the projection obtained above (the property of similar triangles is implicitly used here). Its area is $\frac{s+3/2}{2} \times 1 = \frac{2}{4}s + \frac{3}{4}$, which is equal to $A(1)$ $(= 1)$. Hence follows the equation $\frac{2}{4}s + \frac{3}{4} = 1 \rightarrow s = \frac{1}{2}$. When raised by this, $x = \frac{1}{2}$ and $y = \frac{5}{2}$.

Remarks: A 'series figure' is a diagrammatic expression of an arithmetical progression by an isoceles trapezium whose area and height represent the sum and the number of terms, respectively, of the progression. The series figure has been discussed by Śrīdhara in PG 79–85 and by Nārāyaṇa himself in GK kṣetra 73–77 (Part II, pp. 85–95).[93]

Verse (42): Example 40. Fractional number of terms of an arithmetical progession: When a traveler has 'an ever-increasing

[93] For the series figure see Sarasvati 1979, 238–250 and Hayashi, Kusuba, Yano 1997, 269–273.

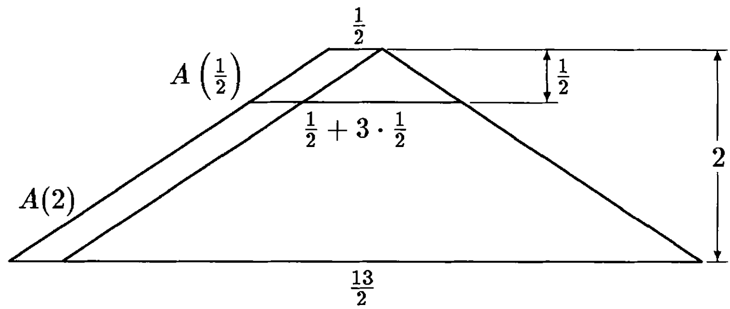


Figure 4: Series figures in Example 40 (reconstructed).

speed' (*vardhamāna-gati*) determined by an arithmetical progression, where $a = 2$, $d = 3$, and $n = \frac{1}{2}$, 'a proof should be told' (*upapattistatra kathyatām*) that the distance traveled by him can be calculated by means of the usual formula for $A(n)$ even though n is a fraction.[94]

Solution (proof): The commentator first gives $A\left(\frac{1}{2}\right) = \frac{5}{8}$. The formula he used must be:

$$A(n) = n\left(a + \frac{n-1}{2}\cdot d\right),$$

although he does not mention it (cf. the Remarks under Verse (13)). In the next few lines, if my restoration is correct, he seems to be declaring that, even though n is a fraction, this result can be expressed by a series figure since the traveler in the present example is asssumed to have 'a continuously increasing speed' (*nirantara-vardhamāna-gati*). See the Remarks below.

Then, he is going to obtain the top x and the base y of the series figure for the same progression when $n = 2$. The area of the figure is equal to $A(2)$ $(= 7)$, and its height to n $(= 2)$. Let $x + y = s$ [= yā 1].

[The manuscript B2 suddenly ends here in the middle of the quotation of a formula for the area of an isoceles trapezium. The following is my tentative restoration of the rest of the solution.

Proceeding just as in the case of the solution to Example 39, one obtains $s = 7$, that is, $x + y = 7$. Assuming $x = s$ [= yā

[94] A similar example with exactly the same arithmetical progression occurs in PG (103a). For fractional number of terms of an arithmetical progression, see BM 18.Ex.1, N18 and Q12, PG 89–93, (9), and (103)–(107), GSS 3.22–53, and GK śreḍhī (7) and kṣetra (60)–(63), and (66).

1], one obtains $s = \frac{1}{2}$, that is, $x = \frac{1}{2}$ and $y = \frac{13}{2}$. When one cuts a small trapezium out of this series figure with a horizontal line at the distance 1/2 from the top (see Fig. 4), its base line is calculated according to GK kṣetra 74a:[95] $3 \cdot \frac{1}{2} + \frac{1}{2} = 2$. The area of this small trapezium is: $\frac{1/2+2}{2} \cdot \frac{1}{2} = \frac{5}{8}$. This is equal to $A\left(\frac{1}{2}\right)$. The small trapezium is therefore the series figure for the progression when $n = \frac{1}{2}$, and it has been thus verified that $A(n)$ is meaningful when n is a fraction.]

Remarks. In the GK, Nārāyaṇa prescribes a rule for obtaining the base and the face directly.[96]

The commentator refers negatively to the 'numerical' (*rāśigatā*) proof as against the 'geometrical method' (*kṣetradvāra*) that is to be adopted in the solution of the present example. The 'geometrical method' no doubt applies the ordinary sum of an arithmetical progression to a fractional number of terms:

$$A(n+\epsilon) = (n+\epsilon)\left\{a + \frac{(n+\epsilon)-1}{2} \cdot d\right\},$$

where n is a non-negative integer and ϵ is a positive fraction smaller than unity or zero. The 'numerical' method, on the other hand, was presumably based on the 'numerical sum' (*rāśigata-yuti*) that has been defined by Śrīdhara (PG 89) as:

$$A'(n+\epsilon) = \{(a+nd) + a - d\} \cdot \frac{n}{2} + \epsilon \cdot (a+nd),$$

or,

$$A'(n+\epsilon) = A(n) + \epsilon \cdot a_{n+1},$$

where a_{n+1} is the $(n+1)$-th term of the progression. We have the relationship, $A(n+\epsilon) = A'(n+\epsilon)$ if $\epsilon = 0$, but $A(n+\epsilon) < A'(n+\epsilon)$ if $0 < \epsilon < 1$.

---

[95] GK kṣetra 74a (Part II, p. 85): avalambakhaṇḍaguṇitaścayaḥ svavadanena saṃyutastadbhūḥ // 74a // 'The increase multiplied by a part of the perpendicular and increased by its own face is its base.'

[96] GK kṣetra 73 (Part II, p. 85): ādiścayadalahīno vadanaṃ padacayavadhaḥ savadano bhūḥ / gaccho lambo gaṇitaṃ śreḍhīgaṇitena tulyaṃ syāt // 73 // 'The first term decreased by half the increase is the face and the product of the ⟨number of⟩ steps and the increase accompanied by the face ⟨just obtained⟩ is the base. The ⟨number of⟩ steps is the perpendicular and the computed ⟨area⟩ shall be equal to the computed ⟨sum⟩ of the series.'

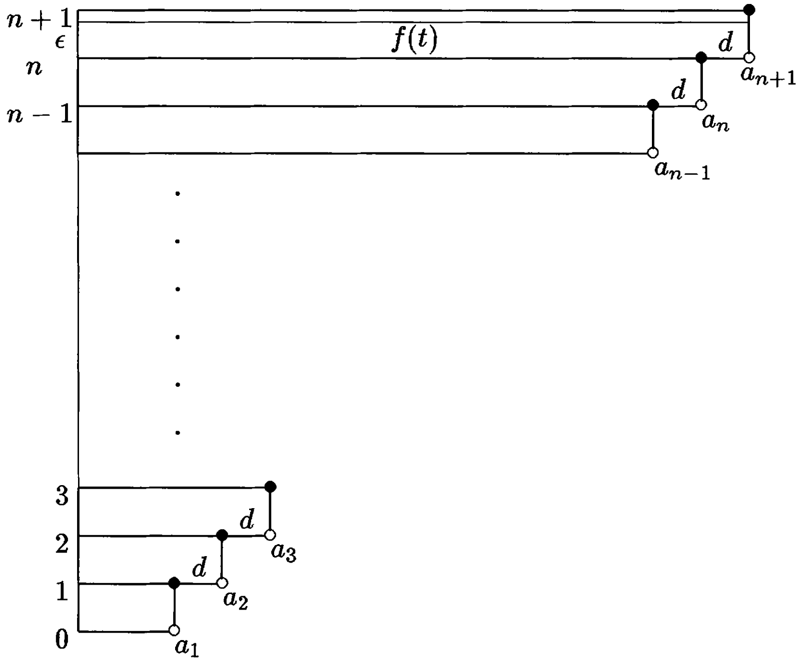


Figure 5: Increase of the speed by the step function $f(t)$.

When a traveler 'discretely' increases (or decreases) his daily journey according to an arithmetical progression, his 'speed' can be expressed by the step function (see Fig. 5):

$$f(t) = f(n+\epsilon) = \begin{cases} a_n & \text{if } \epsilon = 0 \\ a_{n+1} & \text{if } 0 < \epsilon < 1 \end{cases}$$

and his total journey in the first $(n+\epsilon)$ days is expressed by the 'numerical sum', $A'(n+\epsilon)$, which is, in Fig. 5, the sum of the areas of the first n strips and of an ϵ-th part of the $(n+1)$-th strip.[97] This is the most natural way of treating a fractional number of terms. But, how can one treat it with the ordinary sum $A(n+\epsilon)$? In other words, how can one interpret the ordinary sum with a fractional number of terms? Śrīdhara's 'series figure' seems to have been his answer to this question. Since the ordinary sum can be expressed as

$$A(n) = \frac{(a - d/2) + \{(a - d/2) + nd\}}{2} \cdot n,$$

$A(n)$ is equal to the area of a trapezium whose base, face, and height are $(a - d/2)$, $(a - d/2) + nd$, and n, respectively (PG 81–85), and this holds good for a fractional number of terms, too, as the base is constant and the face is a linear function of the height:

$$g(t) = g(n+\epsilon) = \left(a - \frac{d}{2}\right) + (n+\epsilon)d.$$

Or, in other words, the 'speed' of a traveler, $g(t)$, expressed by the face of the trapezium increases (or decreases) linearly with regard to the time (t) expressed by its height (see Fig. 6). This seems to be what our commentator meant by the phrase, 'due to the continuously increasing speed' (*nirantara-vardhamāna-gati-vaśāt*).

Nārāyaṇa made the trapezium upside-down (see GK kṣetra 73 cited above), and applied it even to negative numbers of terms ($n < 0$) in two of his seven examples for the series figure (GK kṣetra (64) and (65)).

[97] A similar geometrical expression, with an integral number of terms, is employed by Nīlakaṇṭha in his proofs of Āryabhaṭa's rules for arithmetical progressions (NAB 2.19–20, pp. 133–138). Cf. Sarasvati 1979, 243–245 and Hayashi, Kusuba, Yano 1997, 208–247.

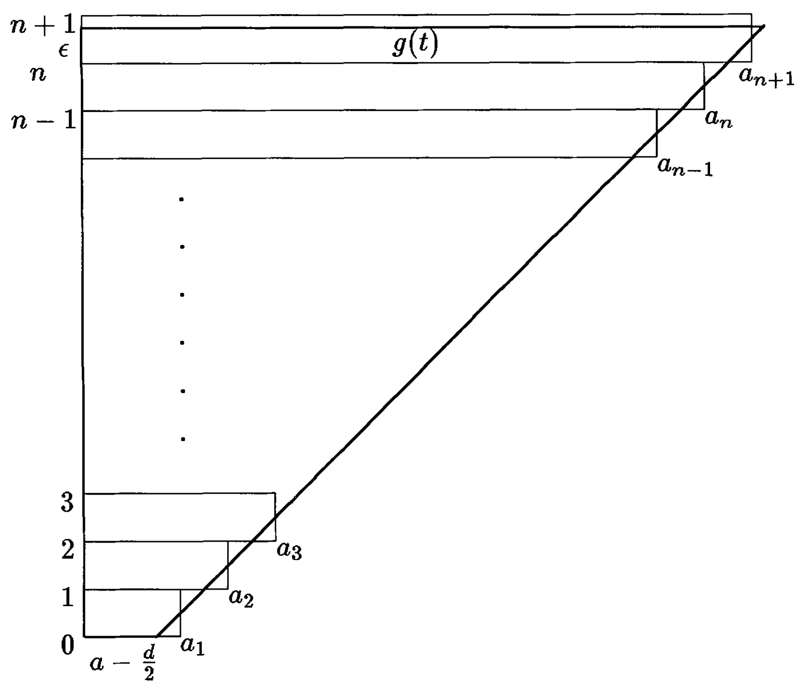


Figure 6: Increase of the speed by the linear function $g(t)$.

Abbreviations of the titles of works

AB = Āryabhaṭa I's *Āryabhaṭīya*, Kern 1973, Sāmbaśiva Śāstrī 1930, Shukla 1976.

BG = Bhāskara II's *Bījagaṇita*, Āpaṭe 1930.

BGA = Nārāyaṇa's *Bījagaṇitāvataṃsa*, Shukla 1970.

BM = *Bakhshālī Manuscript*, Hayashi 1995.

BSS = Brahmagupta's *Brāhmasphuṭasiddhānta*, Dvivedī 1902, Sharma 1966.

GAR = Govindasvāmin's Arithmetic Rules, Hayashi 2000.

GK = Nārāyaṇa's *Gaṇitakaumudī*, Dvivedī 1936/42.

GP = Śrīdhara's *Gaṇitapañcaviṃśī*, Pingree 1979.

GS = Ṭhakkura Pherū's *Gaṇitasāra*, Agaracanda and Nāhaṭā 1961.

GSS = Mahāvīra's *Gaṇitasārasaṃgraha*, Raṅgācārya 1912, Jain 1963.

GT = Śrīpati's *Gaṇitatilaka*, Kāpadīā 1937.

L = Bhāskara II's *Līlāvatī*, Āpaṭe 1937, Sarma 1975.

MB = Bhāskara I's *Mahābhāskarīya*, Kuppanna Sastri 1957, Shukla 1960.

MS = Āryabhaṭa II's *Mahāsiddhānata*, Dvivedī 1995.

NAB = Nīlakaṇṭha's commentary on the AB, Sāmbaśiva Śāstrī 1930.

PG = Śrīdhara's *Pāṭīgaṇita*, Shukla 1959.

SS = Śrīpati's *Siddhāntaśekhara*, Miśra 1932/47.

TPG = Anonymous commentary (*ṭīkā*) on the PG, Shukla 1959.

Tr = Śrīdhara's *Triśatikā*, Dvivedī 1899.

References

Agaracanda and B. Nāhaṭā (eds.), 1961: *Gaṇitasāra* of Ṭhakkura Pherū, *Ratnaparīkṣādisaptagranthasaṅgraha*, Rājasthāna Purātana Granthamālā 44, Part 2, pp. 41–74, Jodhpur.

Āpaṭe, D., et al. (eds.), 1930: *Bījagaṇita* of Bhāskara II, with the commentary, *Navāṅkura*, of Kṛṣṇa, Ānandāśrama Sanskrit Series 99, Poona.

——. (eds.), 1937: *Līlāvatī* of Bhāskara II, with the commentaries, *Buddhivilāsinī* of Gaṇeśa I and *Līlāvatīvivaraṇa* of Mahīdhara, Ānandāśrama Sanskrit Series 107. Poona.

CESS. See Pingree 1970/94.

Colebrooke, H. T., 1817: *Algebra with Arithmetic and Mensuration from the Sanscrit of Brahmegupta and Bhāscara*, London 1817; reprinted, Wiesbaden 1973.

Datta, B., 1933: 'The Algebra of Nārāyaṇa,' *Isis* 19, 472–85.

Datta, B. and A. N. Singh, 1962: *History of Hindu Mathematics: A Source Book*, 2 vols., Lahore 1935/38; reprinted in one vol., Bombay 1962; reprinted in two vols., Delhi 2001.

Dvivedī, P. (ed.), 1936/42: *Gaṇitakaumudī* of Nārāyaṇa, 2 vols., Princess of Wales Sarasvati Bhavana Texts 57, Benares.

——. (ed.), 1899: *Triśatikā* of Śrīdhara, Benares 1899.

——. (ed.), 1902: *Brāhmasphuṭasiddhānta* of Brahmagupta, with the editor's commentary in Sanskrit, Benares.

Dvivedī, S. (ed.), 1995: *Mahāsiddhānta* of Āryabhaṭa II, with the editor's commentary in Sanskrit, Benares Sanskrit Series 148, 149 and 150, Benares 1910; reprinted, Vrajajīvana Prācya Granthamālā 81, Delhi 1995.

Hayashi, T. (ed. & tr.), 1995: *The Bakhshālī Manuscript: An Ancient Indian Mathematical Treatise*, Groningen Oriental Studies 11, Groningen.

——. 2000: 'Govindasvāmin's Arithmetic Rules Cited in the *Kriyākramakarī* of Śaṅkara and Nārāyaṇa,' *Indian Journal of History of Science* 35, 189–231.

Hayashi, T., T. Kusuba, M. Yano, 1997: *Studies in Indian Mathematics: Series, Pi and Trigonometry*, Tokyo.

Jain, L. C. (ed. & tr.), 1963: *Gaṇitasārasaṃgraha* of Mahāvīra, with Hindi translation, Sholapur.

Kāpadīā, H. R. (ed.), 1937: *Gaṇitatilaka*, with the commentary, *vṛtti*, of Siṃhatilaka Sūri, Gaekwad's Oriental Series 78, Baroda.

Kern, H. (ed.), 1973: *Āryabhaṭīya* of Āryabhaṭa I, with the commentary, *Bhaṭadīpikā*, of Parameśvara, Leiden 1874; reprinted, Osnabrück 1973.

Kuppanna Sastri, T. S. (ed.), 1957: *Mahābhāskarīya* of Bhāskara I, with the commentaries, *bhāṣya* of Govindasvāmin and *Siddhāntadīpikā* of Parameśvara, Madras Government Oriental Series 130, Madras.

Kusuba, T. (ed. & tr.), 1994: 'Combinatorics and Magic Squares in India: A Study of Nārāyaṇa Paṇḍita's *Gaṇitakaumudī*, Chapters 13–14,' Ann Arbor 1994 (Doctoral dissertation, Brown University 1993).

Miśra, B. (ed.), 1932/47: *Siddhāntaśekhara* of Śrīpati, with the commentaries, *Gaṇitabhūṣaṇa* of Makkibhaṭṭa and *vivaraṇa* of the editor, 2 vols., Calcutta.

Panse, M. G. (ed.), 1957: *Jyotiṣa-Ratna-Mālā of Śrīpati Bhaṭṭa*, Deccan College Monograph Series 20, Poona.

Pingree D., 1970/94: *Census of the Exact Sciences in Sanskrit, Series A*, 5 vols., Memoirs of the American Philosophical Society 81, 86, 111, 146, and 213, Philadelphia 1970, 1971, 1976, 1981, and 1994.

——. (ed.), 1979: '*Gaṇitapañcaviṃśī* of Śrīdhara,' *Ṛtam: Ludwik Sternbach Felicitation Volume*, pp. 887–909, Lucknow.

Raṅgācārya, M. (ed. & tr.), 1912: *Gaṇitasārasaṃgraha* of Mahāvīra, Madras.

Sāmbaśiva Śāstrī, K. (ed.), 1930: *Āryabhaṭīya*, Part I (gaṇitapāda), of Āryabhaṭa I, with the commentary, *bhāṣya*, of Nīlakaṇṭha Somasutvan, Trivandrum Sanskrit Series 101, Trivandrum.

Sarasvati Amma, T. A., 1979: *Geometry in Ancient & Medieval India*, Delhi.

Sarma, K. V. (ed.), 1975: *Līlāvatī* of Bhāskara II, with the commentary, *Kriyākramakarī*, of Śaṅkara and Nārāyaṇa, Vishveshvaranand Indological Series 66, Hoshiarpur.

Sharma, R. S., et al. (eds.), 1966: *Brāhmasphuṭasiddhānta* of Brahmagupta, with Sanskrit and Hindi commentaries, New Delhi.

Shukla, K. S., 1954: 'Ācārya Jayadeva, the Mathematician,' *Gaṇita* 5, 1–20.

——. (ed. & tr.), 1959: *Pāṭīgaṇita*, with an anonymous commentary, *ṭīkā*, Lucknow.

——. (ed. & tr.), 1960: *Bhāskara I and His Works, Part II: Mahābhāskarīya*, Lucknow.

——. (ed. & tr.), 1963: *Bhāskara I and His Works, Part III: Laghubhāskarīya*, Lucknow.

——. (ed.), 1970: *Bījagaṇitāvataṃsa* of Nārāyaṇa, Lucknow.

——. (ed.), 1976: *Āryabhaṭīya* of Āryabhaṭa I, with the commentaries, *bhāṣyas*, of Bhāskara I and Someśvara, Āryabhaṭīya Critical Edition Series 2, New Delhi.

Singh, P. (tr.), 1998/2002: 'The Gaṇita Kaumudī of Nārāyaṇa Paṇḍita (Translation with Notes),' *Gaṇita Bhāratī* 20, 1998, 25–82 (Chaps. I–III); 21, 1999, 10–73 (Chap. IV); 22, 2000, 19–85 (Chaps. V–XII); 23, 2001, 18–82 (Chap. XIII); and 24, 2002, 35–98 (Chap. XIV).

Indian Rules for the Decomposition of Fractions

TAKANORI KUSUBA

1 *Introduction*

Datta and Singh [1935. 185–203] discuss the rules for arithmetic operations with fractions in Sanskrit mathematical texts and explain the rules for the reduction of fractions to a common denominator, called *kalāsavarṇana*, meaning literally 'making [fractions] have the same color'. This reduction, common to all the Sanskrit mathematical texts available, is treated as part of the topic called *parikarman* (basic operations), and is usually classified into the following four categories:[1]

bhāgajāti (fraction class) $\frac{b}{a} \pm \frac{d}{c} = \frac{bc}{ac} \pm \frac{ad}{ac}$.
prabhāgajāti (sub-fraction class) $\frac{b}{a}\frac{d}{c} = \frac{bd}{ac}$.
bhāgānubandhajāti (partial-addition class) $a + \frac{c}{b} = \frac{ab+c}{b}$ or $\frac{b}{a} + \frac{b}{a}\frac{d}{c} = \frac{b(c+d)}{ac}$.
bhāgāpavāhajāti (partial-subtraction class) $a - \frac{c}{b} = \frac{ab-c}{b}$ or $\frac{b}{a} - \frac{b}{a}\frac{d}{c} = \frac{b(c-d)}{ac}$.

At the end of their explanation Datta and Singh discuss some rules for the decomposition of fractions in the *Gaṇitasārasaṅgraha* of Mahāvīra, and remark that 'Mahāvīra has given a number of rules for expressing any fraction as the sum of a number of unit fractions. These rules do not occur in any other work.' However, we find similar rules to decompose fractions and unity into unit fractions or ordinary fractions[2] in the *Gaṇitakaumudī* of

[1] Cf. *Brāhmasphuṭasiddhānta* 12. 2, 8–9; *Pāṭīgaṇita* 36–40; *Gaṇitasārasaṅgraha* 3. 54–137; *Mahāsiddhānta* 15. 12–13; *Siddhāntaśekhara* 13. 11–12; *Gaṇitatilaka* pp. 30–39; *Līlāvatī* 30–38; *Gaṇitakaumudī* Part 1. pp. 9–10.

[2] Indians were familiar with unit fractions. The *śulbasūtra* texts, compilations of the geometric knowledge needed for the construction of altars for Vedic sacrifice, give as an approximation of $\sqrt{2}$

$$1 + \frac{1}{3} + \frac{1}{3 \cdot 4} - \frac{1}{3 \cdot 4 \cdot 34}.$$

Nārāyaṇa. In this paper I summarize the rules which Datta and Singh discussed, with examples given in the *Gaṇitasārasaṅgraha*, compare the corresponding rules in the *Gaṇitakaumudī*, and discuss the implications.

2 *Mahāvīra*

Mahāvīra wrote the *Gaṇitasārasaṅgraha* (*The Essence of Mathematics*, hereafter GSS) in about 850 A.D. and gave the rules and examples for fractions in its section dealing with the topic considered the second *vyavahāra* (practical operation) in arithmetic, namely *kalāsavarṇavyavahāra* (the operation of reduction of fractions). The text includes some numerical examples, but not the solutions to them.[3]

Mahāvīra gives the first of these rules in the *bhāgajāti* section, namely GSS *kalāsavarṇa* 55–98. This section includes the rules which Datta and Singh discussed; I summarize them as follows.
(1) To express 1 as the sum of any number (n) of unit fractions.

$$1 = \frac{1}{2} + \frac{1}{3} + \frac{1}{3^2} + \cdots + \frac{1}{3^{n-2}} + \frac{1}{2 \cdot 3^{n-2}}.$$

This rule is given in GSS *kalāsavarṇa* 75. Based on a literal translation of the versified rule,[4] the denominator of the first term is to be written as $1 \cdot 2$, and that of the last term as $3^{n-1} \cdot \frac{2}{3}$. Following the rule GSS *kalāsavarṇa* 76 gives examples when $n = 5, 6, 7$.
(2) To express 1 as the sum of an odd number of unit fractions.

$$1 = \frac{1}{2 \cdot 3 \cdot 1/2} + \frac{1}{3 \cdot 4 \cdot 1/2} + \cdots + \frac{1}{(2n-1) \cdot 2n \cdot 1/2} + \frac{1}{2n \cdot 1/2}.$$

Here $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{34}$ are expressed by ordinal numbers such as third, *tṛtīya*, fourth, *caturtha*, and thirty-fourth, *catustriṃśa*, respectively.

pramāṇaṃ tṛtīyena vardhayet taccaturthenātmacatustriṃśonena saviśeṣaḥ/

[3] The *Gaṇitasārasaṅgraha* was commented on in Kannaḍa and in Sanskrit. However, none of the commentaries has been published. See Pingree [1981. 60].

[4] *rūpāṃśakarāśīnāṃ rūpādyās triguṇitā harāḥ kramaśaḥ/*
dvidvitryaṃśābhyastāv ādimacaramau phale rūpe//

Translation

When the result is one, the denominators of the quantities having one as numerators are [the numbers] beginning with one and multiplied by three, in order. The first and the last are multiplied by two and two-thirds [respectively].

This rule is given in GSS *kalāsavarṇa* 77.
(3) To express a unit fraction $1/q$ as the sum of a number of other fractions, the numerators being given.

$$\begin{aligned}\frac{1}{q} &= \frac{a_1}{q(q+a_1)} + \frac{a_2}{(q+a_1)(q+a_1+a_2)} + \cdots + \\ &\quad \frac{a_{n-1}}{(q+a_1+a_2+\cdots+a_{n-2})(q+a_1+a_2+\cdots+a_{n-1})} + \\ &\quad \frac{a_n}{a_n(q+a_1+a_2+\cdots+a_{n-1})},\end{aligned}$$

where $a_1, a_2, \cdots, a_n$ and q are given. This is given in GSS *kalāsavarṇa* 78. GSS *kalāsavarṇa* 79 provides examples where $a_1 = 7, a_2 = 9, a_3 = 3, a_4 = 13$ for $q = 1$, 4, and 6.
(4) To express any fraction p/q as the sum of unit fractions.

Let the number i be so chosen that $\frac{q+i}{p}$ is an integer r; then

$$\frac{p}{q} = \frac{1}{r} + \frac{i}{r \cdot q}.$$

For the second term the same procedure is performed. This is given in GSS *kalāsavarṇa* 80. Gupta [1993] explains this rule with some examples. GSS 81 contains an example requiring the denominators of three unit fractions the sum of which is $\frac{1}{4}$, and those of four unit fractions whose sum is $\frac{3}{7}$.
(5) To express a unit fraction as the sum of two other unit fractions.

$$\frac{1}{n} = \frac{1}{p \cdot n} + \frac{1}{\frac{p \cdot n}{p-1}}.$$

$$\frac{1}{a \cdot b} = \frac{1}{a(a+b)} + \frac{1}{b(a+b)}.$$

This is given in GSS *kalāsavarṇa* 85. GSS *kalāsavarṇa* 86 is an example thereof where n or $a \cdot b$ equals 6 or 10.
(6) To express any fraction as the sum of two other fractions whose numerators are given.

$$\frac{p}{q} = \frac{a}{\frac{ai+b}{p} \cdot \frac{q}{i}} + \frac{b}{\frac{ai+b}{p} \cdot \frac{q}{i} \cdot i}$$

when p, q, a, b are given and i, such that $ai + b$ is to be divided by p without remainder, is to be found. This is stated in GSS *kalāsavarṇa* 87. The example in GSS *kalāsavarṇa* 88 seeks the denominators of the two unit fractions whose sum is $\frac{1}{4}$, and also those of the two fractions whose numerators are 7 and 9 respectively and whose sum is $\frac{2}{5}$.

Pṛthūdaka (*fl.* 864), a contemporary of Mahāvīra, in his commentary on 'the first *jāti*', that is the *bhāgajāti*, in the *Brāhmasphuṭasiddhānta* (12. 8) written by Brahmagupta in 628, offers an example[5] requesting the sum of

$$\frac{5}{22} + \frac{7}{66} + \frac{9}{38} + \frac{1}{39} + \frac{4}{13} + \frac{11}{114}.$$

When these quantities are rearranged in the order

$$\frac{5}{22} + \frac{7}{66} + \frac{9}{38} + \frac{11}{114} + \frac{4}{13} + \frac{1}{39},$$

there occur three consecutive pairs. The first pair (with 22 and 66 as denominators) can be produced from GSS Rule 6 with $p = 1, q = 3, a = 5, b = 7$ and $i = 3$. The second pair is also obtainable from the same rule when $p = 1, q = 3, a = 9, b = 11$ and $i = 3$. The third and last pair results when $p = 1, q = 3, a = 4, b = 1, i = 3$. Pṛthūdaka might have known this rule and used it to construct his sample problem.

(7) Datta and Singh mention a particular case of Rule 6, described in GSS *kalāsavarṇa* 89:

$$\frac{p}{q} = \frac{a}{\frac{aq+b}{p}} + \frac{b}{\frac{aq+b}{p} \cdot q},$$

when p, q, a, b are given, provided that $(aq + b)$ is divisible by p. Sample problems in GSS *kalāsavarṇa* 90–92 require the denominators of the two unit fractions whose sum is $\frac{2}{7}$; the denominators of fractions which have 6 and 8 respectively as numerators and whose sum is also $\frac{2}{7}$; the two unit fractions that sum to $\frac{1}{15}$ when

[5] *dviyamā rasaṣaṭkāś ca vasulokā navāgnayaḥ/*
trīndavaḥ kṛtarudrāś ca chedasthāne prakalpitāḥ//
pañcāgā nava rūpaṃ ca vedā rudrās tadaṃśakāḥ/
militā yatra dṛśyante kas tatra dhanasañcayaḥ//
Quoted by Dvivedī in his edition of the *Brāhmasphuṭasiddhānta* p. 176. This stanza is found in folio 48a.

$1 = \frac{1}{2} + \frac{1}{3} + \frac{1}{10} + \frac{1}{15}$ is given; and the two fractions whose sum is $\frac{1}{20}$ and whose denominators are 7 and 11 respectively, when $1 = \frac{1}{2} + \frac{1}{4} + \frac{1}{5} + \frac{1}{20}$ is given.

Toward the end of their discussion Datta and Singh mention a rule 'to express a given fraction as the sum of an even number of fractions whose numberators are previously assigned'. This is their translation:[6] 'After splitting up the sum into as many parts, having one for each of their numerators, as there are pairs (among the given numerators), these parts are taken as the sum of the pairs, and (then) the denominators are found according to the rule for finding two fractions equal to a given unit fraction.'

We add two remarks not mentioned by Datta and Singh.

1) The examples to find the sum of the given fractions in GSS *kalāsavarṇa* 60–62 can be expressed as follows:

$$\frac{a}{a(a+1)} + \frac{a}{(a+1)(a+2)} + \frac{a}{(a+2)(a+3)} + \cdots + \frac{a}{\{(a+(n-1)\}(a+n)} + \frac{a}{a+n}.$$

2) The examples in GSS *kalāsavarṇa* 83–84 require the denominators of three unit fractions whose sum is $\frac{23}{12}$, and the three denominators whose numerators are given as 3, 7 and 9 when the sum of those three fractions is $\frac{73}{46}$. As the solution for these problems GSS *kalāsavarṇa* 82 gives the following rule:

$$\frac{a_1}{b_1} + \frac{a_2}{b_2} + \cdots + \frac{a_n}{b_n} = \frac{p}{q},$$

where $a_1, a_2, \cdots, a_n$ and p, q are given. One has to find $i_1, i_2, \cdots, i_n$ such that $a_1 i_1 + a_2 i_2 + \cdots + a_n i_n = p$, then $b_n = q/i_n$.

3 *Nārāyaṇa Paṇḍita*

Nārāyaṇa Paṇḍita wrote the *Gaṇitakaumudī (Moonlight of Mathematics)* in 1356.[7] The *Gaṇitakaumudī* (hereafter GK) consists of the *mūla* (root or original), that is, versified rules (*sūtra*) and examples (*udāharaṇa*), and of a prose commentary (*vāsanā*) thereon. The answers to the worked examples are given in the

[6] In the footnote they identify this stanza as GSS 89, but the correct stanza number is 93.

[7] For the author Nārāyaṇa Paṇḍita, see Kusuba [1994. 1–5].

vāsanā. Nārāyaṇa gave the four simple rules for reduction of fractions (discussed above in the Introduction) in the *parikarman*. However, he devoted the twelfth chapter, named *aṃśāvatāra-vyavahāra* (the operation of the appearance of fractions) to additional rules for fractions. The eight rules in the section called *bhāgajāti* in that chapter are of five sorts:

1) to decompose 1 to a sum of unit fractions (Rules 1–2)
   to decompose a given fraction to the sum of unit fractions (Rule 3)
2) to decompose 1 to a sum of arbitrary fractions (Rule 4)
3) to decompose 1 to the sum of fractions whose numerators are given (Rules 5–6)
4) to find denominators of fractions with given numerators, summing to a given result (Rule 7)
5) to find numerators when denominators and the result (sum) are given (Rule 8)

I give a critical edition of the rules with the English translation and explain the rules and some of examples thereof.[8]

Rule 1

ekādyekacayānāṃ dvayor dvayor nikaṭayor vadhāś chedāḥ/
yo 'ntyaḥ so 'ntyaharaḥ syād yoge rūpaṃ tad iṣṭaphalaguṇitam//1//

The products of two successive [numbers] beginning with one and increasing by one are denominators. The one which is last is the last divisor (i.e., denominator). When [they are] added together, [the result is] one. That is multiplied by any result.

$$1 = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{(n-1)n} + \frac{1}{n}.$$

Example thereof

The number of terms is given as 6. This is set out thus in the *vāsanā*.

1	1	1	1	1	1	phalam 1/
0	0	0	0	0	0	

[8] I have used three manuscripts. For their sigla, see Bibliography.

Here the zero signs indicate unknown numbers which are required. The word *phalam*, which literally means fruit, indicates the result. The answer given is

$$1 = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{6}.^{9}$$

Nārāyaṇa's rule seems to be more general than GSS Rule 2, a similar rule for an odd number of terms. When in GSS Rule 3 $a_1 = a_2 = \cdots = a_n=1$ and $q = 1$, it reduces to GK Rule 1. Pṛthūdaka, whom we mentioned above, might have known GK Rule 1 or a similar application of GSS Rule 3, because in his commentary he includes an example requiring the sum of

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{4}.^{10}$$

Rule 2

ekāditriguṇottaravṛddhyāṅkasthānasammitāś chedāḥ/
ādyantau ca dviguṇāv antyas trihato 'ṃśake rūpam//2//
2c ca] vada; 2d aṃśaka NRV

When there is unity in numerator the denominators are measured by the [number of] places of the numbers beginning with one and increasing by [their] triples. The first and the last are multiplied by two. The last is multiplied by three.

$$1 = \frac{1}{2} + \frac{1}{3} + \frac{1}{3^2} + \cdots + \frac{1}{3^{n-2}} + \frac{1}{2 \cdot 3^{n-2}}.$$

Rule 2 is an alternative rule for the decomposition of 1 to the sum of unit fractions. The word *trihato* (multiplied by three) should be emended to *trihṛto* (divided by three); otherwise the denominator of the last term becomes $2 \cdot 3^n$.

[9] The *vāsanā* gives the answer to another problem whose solution is half that of the previous one:

1	1	1	1	1	1
4	12	24	40	60	12

[10] *rūpārdhaṃ rūpaṣaḍbhāgo rūpāṃśo dvādaśas tathā/*
rūpasya ca turīyāṃśaḥ ko 'rthaḥ sampiṇḍite bhavet//
Quoted by Dvivedī p. 176. This stanza is found on folio 48a.

There is no example for Rule 2, but the *vāsanā* gives an answer to the case where the number of the terms is 6 as in the previous example.

$$1 = \frac{1}{2} + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \frac{1}{162}.$$ [11]

Rule 3

phalahāro 'bhīṣṭayutaḥ phalāṃśabhakto yathā bhavec chuddhiḥ/
labdhiś chedo bhāgaṃ phalataḥ saṃśodhayec ca taccheṣam//3//
tasmād utpādyānyaṃ śeṣam upāntyāṅkaśeṣaṃ ca/
ekaikeṣv aṃśeṣu kramo 'yam āryoditaḥ spaṣṭaḥ//4//
3b -bhaktau NV, 3c labdhi- N, chedo] kṣepo NRV, 3d read yaccheṣam, 3] numbered 5 R, not numbered NV, 4ab om.V, 4a utpādyāntyaṃ N, 4b upāntyakāśeṣam N, upāntyakaḥ śeṣam R, ca om.NR, 4d yam] cam NV, tvam R, āryoditaspaṣṭā NRV, numbered 5 R, not numbered NV

[One should suppose an arbitrary number] such that the divisor of the result added to an arbitrary number and [then] divided by the numerator of the result leaves no remainder. The quotient is the denominator. One should subtract the fraction from the result. Having produced from what remains another remainder and the remainder from the penultimate number, [one should operate in the same way] for each fraction. This procedure which was told by the noble man is evident.

A fraction p/q, which is the "result," is given, and one is to find denominators of unit fractions that sum to the "result." An "arbitrary number" i is to be determined so that the quotient $(q+i)/p$ is an integer; this quotient is the first desired denominator, and the numerator is always 1. Therefore the next "result" is $\frac{p}{q} - \frac{1}{\frac{q+i}{p}}$, and the next denominator is found in the same way by assuming a new i.

An example is given in the *vāsanā* for an alternative solution to the previous problem, in which the result is equal to 1, that is 1/1, and the number of terms is six. In this case the *vāsanā* gives

[11] Here also the *vāsanā* gives a solution to another problem which halves the previous one:

1	1	1	1	1	1
4	6	18	54	162	324

the numbers $1, 1, 1, 1, 1$ as the arbitrary numbers. The operation is as follows:

$\frac{1+1}{1} = 2, 1 - \frac{1}{2} = \frac{1}{2}; \frac{2+1}{1} = 3, \frac{1}{2} - \frac{1}{3} = \frac{1}{6};$
$\frac{6+1}{1} = 7, \frac{1}{6} - \frac{1}{7} = \frac{1}{42}; \frac{42+1}{1} = 43, \frac{1}{42} - \frac{1}{43} = \frac{1}{1806};$
$\frac{1806+1}{1} = 1807; \frac{1}{1806} - \frac{1}{1807} = \frac{1}{3263442}.$

Therefore

$$1 = \frac{1}{2} + \frac{1}{3} + \frac{1}{7} + \frac{1}{43} + \frac{1}{1807} + \frac{1}{3263442}.$$

Two problems with multiple solutions are given. In both cases the number of terms is four. In the first case the given fraction is 5/6.

$\frac{5}{6} = \frac{1}{2} + \frac{1}{4} + \frac{1}{13} + \frac{1}{156}$ (the arbitrary numbers are 4,1,1 in order.)
$= \frac{1}{2} + \frac{1}{5} + \frac{1}{8} + \frac{1}{120}$ (the arbitrary numbers are 4,2,1 in order.)[12]
$= \frac{1}{3} + \frac{1}{5} + \frac{1}{4} + \frac{1}{20}$ (the arbitrary numbers are 9,3,2 in order.)

In the second case, on the other hand, the given fraction is 7/9.
$\frac{7}{9} = \frac{1}{2} + \frac{1}{4} + \frac{1}{37} + \frac{1}{1332}$ (the arbitrary numbers are 5,2,1 in order);
$= \frac{1}{4} + \frac{1}{2} + \frac{1}{38} + \frac{1}{684}$ (the arbitrary numbers are 19,2,2 in order.).
According to the Egyptian-Greek method discussed by Knorr [1982], each of these fractions may be written in one and only one way: $\frac{5}{6} = \frac{2+3}{6} = \frac{2}{6} + \frac{3}{6} = \frac{1}{3} + \frac{1}{2}$; $\frac{7}{9} = \frac{1}{3} + \frac{1}{4} + \frac{1}{9} + \frac{1}{12}$. On the other hand the GK does not yield unique solutions, but rather allows many answers according to the consistent use of a particular computational procedure. After stating the answers the *vāsanā* reads: *evam iṣṭavaśād bahudhā* (Thus there are many ways according to the [choice of] arbitrary [numbers].)

The procedure in GK Rule 3 is equivalent to GSS Rule 4; after stating it, the GK comments '*kramo 'yam āryoditaḥ spaṣṭaḥ* (this procedure which was told by the noble man is evident).' It is not certain whether Nārāyaṇa is referring to Mahāvīra or someone else.

Rule 4

parikalpyeṣṭān aṅkān ādyaḥ kandābhidho 'ntimo 'grākhyaḥ/
nijapūrvaghno hi paro 'ntaraṃ harāṃśau kramāt syātām//5//

[12] The last two numbers, 2,1, are missing in the edition and all the manuscripts.

antyāgracchedaḥ syād rūpaṃ cāṃśo 'tha te 'ṃśakāḥ sarve/
kandavinighnās teṣāṃ saṃyogo jāyate rūpam//6//
5c ntaraṃ om. NRV, 5 numbered 6 NRV, 6a read antye 'graś chedaḥ, 6b cāṃśo 'tha] cāṃśātha NRV, 6d saṃyogo P, 6 numbered 7 NRV

Supposing arbitrary numbers, [one] calls the first [number] *kanda*, and the last *agra*. [Each] one multiplied by its previous one, and the difference [between them], are the divisor and the numerator, in order. For the last [term] the denominator is the *agra* and the numerator is unity. All these numerators are multiplied by the *kanda*. Their sum is unity.

The arbitrary numbers are $k_1, k_2, k_3, k_4, \cdots, k_n$, where k_1 is called *kanda* (root), and k_n *agra* (tip).

$$1 = \frac{(k_2 - k_1)k_1}{k_2 \cdot k_1} + \frac{(k_3 - k_2)k_1}{k_3 \cdot k_2} + \cdots + \frac{(k_n - k_{n-1})k_1}{k_n \cdot k_{n-1}} + \frac{1 \cdot k_1}{k_n}.$$

Example thereof

The number of terms is equal to 6; the successive k_i are $1, 2, 3, 4, 5, 6$ in order.

$$1 = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{6}.$$

In this case the result is same as what was derived from Rule 1 (see example above). Another example given in the *vāsanā* is:

$$1 = \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \frac{1}{21} + \frac{2}{7} \, (k_i = 2, 3, 4, 5, 6, 7).$$

The *vāsanā* enumerates the following results.

$$1 = \frac{1}{4} + \frac{3}{20} + \frac{1}{10} + \frac{1}{14} + \frac{3}{56} + \frac{3}{8} \, (k_i = 3, 4, 5, 6, 7, 8).$$

$$1 = \frac{2}{3} + \frac{2}{15} + \frac{2}{35} + \frac{2}{63} + \frac{2}{99} + \frac{1}{11} \, (k_i = 1, 3, 5, 7, 9, 11).$$ [13]

$$1 = \frac{2}{3} + \frac{5}{24} - \frac{3}{40} - \frac{3}{10} + \frac{5}{14} + \frac{1}{7} \, (k_i = 1, 3, 8, 5, 2, 7).$$ [14]

[13] The published edition reads 2/11 for 1/11.

[14] The published edition and the manuscripts read

$$1 = \frac{2}{3} + \frac{5}{24} - \frac{3}{40} - \frac{3}{10} + \frac{51}{147}.$$

And the edition reads $k_i = 1, 3, 8, 5, 2, \frac{98}{83}$.

Negative numbers are usually indicated by a dot placed above them.

Rule 5

> *parikalpyādau rūpaṃ sāṃśaṃ parataḥ paraṃ tad eva syāt/*
> *nikaṭavadhas tacchedāḥ prāntyo yo 'ṅkaḥ sa eva tacchedaḥ//7//*
> sāṃśaḥ NRV, 7d tacchedāḥ NRV, numbered 18 NRV.
>
> Assuming unity first [one should] add to the [given] numerators successively. The product of [two] successive [added numbers] [gives] their denominators. The number which is last is itself its denominator.

This is a case where the numerators of fractions summing to 1 are given. If these numerators are indicated by a_i, and it is required to calculate $i_1, i_2, i_3, i_4, \cdots, i_n$ such that

$i_1 = a_1 + 1$,
$i_2 = a_2 + i_1$, which is equal to $a_2 + (a_1 + 1)$,
$i_3 = a_3 + i_2$,
$i_4 = a_4 + i_3 \cdots$

This is equivalent to the following formula:

$$1 = \frac{a_1}{1 \cdot i_1} + \frac{a_2}{i_1 \cdot i_2} + \frac{a_3}{i_2 \cdot i_3} + \cdots + \frac{a_n}{i_{n-1} \cdot i_n} + \frac{1}{i_n}.$$

The *sūtra* does not explicitly state that the last numerator is 1. This rule can be derived from GSS Rule 3 when $q = 1$. Example thereof in the GK gives a problem in which the numerators are the integers beginning with 3 and increasing by 2 in four places.[15] The setting for this problem is as follows:

3	5	7	9	1	phalam 1/
0	0	0	0	0	

Answer
$1 = \frac{3}{4} + \frac{5}{36} + \frac{7}{144} + \frac{9}{400} + \frac{1}{25}$ because $i_1 = 4, i_2 = 9, i_3 = 16$ and $i_4 = 25$. The *vāsanā* gives five denominators in the answer to the problem although the number of places is given as four.

[15] *aṃśās trikādidvicayāś caturṣu sthāneṣu tacchedanakāś ca kaiścit/ saṃyojitā yena lavena rūpaṃ bhaved dhi tatrātha harān vadāśu//*

Rule 6

> *utpādayoś ca bhāgān yugmamite tadyutau yathā rūpam/*
> *tacchedahatoddiṣṭāṃśakaḥ parāṃśādhikas tu pūrvaharaḥ//8//*
> *so 'pi haraghnas tu paro hara evaṃ nikhilayugmeṣu/*
> *viṣamapadeṣu tathā prāntyaharaghnoddiṣṭabhāgaś ca//9//*
> *chedaḥ syād antyastho nijayugmalavair hṛtāś chedāḥ/*
> 8a read utpādayec 8b yugmamites NRV, 8] numbered 9 NRV, 9b evaṃ] evā NRV, 9d -bhāgaghnaḥ (ca om.) NRV, 9] numbered 10 NRV, 10a antyascho R, 10b -lavo hṛtau NRV.
>
> When [the numbers of the fractions] are taken in pairs, one should produce fractions in such a way that their sum is unity. The indicated numerator multiplied by that denominator and increased by the other numerator is the first divisor. That multiplied by the divisor is the other divisor. [One should operate] thus for all the pairs. For an odd [number of] terms [one should operate] thus, [but] the indicated numerator multiplied by the last divisor is the denominator placed last. The denominators are divided by the numerators of [the fractions for] their own pairs.

This is another case where the numerators $a_1, a_2, \cdots, a_n$ of the fractions whose sum is equal to one are given and one has to find their denominators.

When n is an even number, let $n = 2m$. First, from the previous rule one must find $b_1, b_2, \cdots, b_m$ such that

$$1 = \frac{1}{b_1} + \frac{1}{b_2} + \frac{1}{b_3} + \frac{1}{b_4} + \cdots + \frac{1}{b_m}.$$

Then the GK gives the following formula for expressing each of the $1/b_i$ in terms of two of the given numerators:

$$\frac{1}{b_i} = \frac{a_{2i-1}}{a_{2i-1} \cdot b_i + a_{2i}} + \frac{a_{2i}}{b_i(a_{2i-1} \cdot b_i + a_{2i})}.$$

This rule can be deduced from GSS Rule 6 when $p = 1, q = b_i, a = a_{2i-1}, b = a_{2i}$.

When n is an odd number, on the other hand, let $n = 2m-1$. Then

$$1 = \frac{1}{b_1} + \frac{1}{b_2} + \frac{1}{b_3} + \frac{1}{b_4} + \cdots + \frac{1}{b_{m-1}} + \frac{1}{b_m}.$$

For the first $m-1$ terms one proceeds as above; the last term is rewritten as

$$\frac{1}{b_m} = \frac{a_n}{a_n \cdot b_m}.$$

If the numerator of any b_i is not unity, one has to divide each of its denominators by that numerator.
Example thereof

Six numerators 3,5,7,9,11,13 and the result 1 are given. The *vāsanā* runs:

Text

nyāsaḥ

3	5	7	9	11	13
0	0	0	0	0	0

phalam 1/ ṣaṭsu padeṣu yugatrayaṃ vartate/ yugamite rūpotpannabhāgāḥ

1	1	1
2	6	3

Translation

The setting.

3	5	7	9	11	13
0	0	0	0	0	0

The result is 1. In six terms there occur three pairs. The fractions produced from one in [terms] taken in pairs are

1	1	1
2	6	3

.

This shows that 1 is divided into the sum of three fractions such as $\frac{1}{2}, \frac{1}{6}, \frac{1}{3}$. Then for the first fraction, $\frac{1}{2}$, one applies the rule with the numerators 3,5.

$$\frac{1}{2} = \frac{3}{11} + \frac{5}{22}.$$

In a similar way one finds: $\frac{1}{6} = \frac{7}{51} + \frac{9}{306}$ and $\frac{1}{3} = \frac{11}{46} + \frac{13}{138}$. Therefore

$$1 = \frac{3}{11} + \frac{5}{22} + \frac{7}{51} + \frac{9}{306} + \frac{11}{46} + \frac{13}{138}.$$

Alternative solution: from the equation $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$, the rule produces another answer as follows:

$$1 = \frac{3}{14} + \frac{5}{42} + \frac{7}{30} + \frac{9}{90} + \frac{11}{46} + \frac{13}{138}.$$[16]

In the former case $\frac{9}{306}$ can be reduced to $\frac{1}{34}$. In the latter case also $\frac{9}{90}$ can be reduced to $\frac{1}{10}$. However, the *vāsanā* says as follows

[16] The published edition reads 1/10 instead of 9/90.

at the end of the next solution: "When the reduction of the indicated numerator and the denominator [obtained] is made there would occur change of the indicated numerators. Therefore the reduction is not to be made."[17]

The *vāsanā* gives another alternative solution illustrating the technique for dealing with non-unity numerators, by using Rule 4 with the arbitrary numbers $k_i = 1$, 3, and 5 to obtain the equation $1 = \frac{2}{3} + \frac{2}{15} + \frac{1}{5}$. Then Rule 6 is employed to produce $\frac{1}{3} = \frac{3}{14} + \frac{5}{42}, \frac{1}{15} = \frac{7}{114} + \frac{9}{1710}, \frac{1}{5} = \frac{11}{68} + \frac{13}{340}$.

Because the numerator for the denominator 3 is not unity we have to divide both its derived denominators, 14 and 42, by its numerator, 2. Therefore $\frac{2}{3} = \frac{3}{7} + \frac{5}{21}$. In the same way $\frac{2}{15} = \frac{7}{57} + \frac{9}{855}$ is obtained. Hence

$$1 = \frac{3}{7} + \frac{5}{21} + \frac{7}{57} + \frac{9}{855} + \frac{11}{68} + \frac{13}{340}.$$

The *vāsanā* gives the following answer. From the equation $1 = \frac{4}{5} + \frac{4}{45} + \frac{1}{9}$[18]

$$1 = \frac{3}{5} + \frac{5}{25} + \frac{7}{81} + \frac{9}{3645} + \frac{11}{112} + \frac{13}{1008}.$$

When the number of numerators is odd, such as when the numerators are $3, 5, 7, 9, 11, 13, 15$, one has an expression consisting of four terms such as $1 = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{4}$, which might be obtained by means of Rule 1. According to the procedure in Rule 6 for odd n,

$$1 = \frac{3}{11} + \frac{5}{22} + \frac{7}{51} + \frac{9}{306} + \frac{11}{145} + \frac{13}{1740} + \frac{15}{60}.$$

Other answers are given:

$$1 = \frac{3}{17} + \frac{5}{68} + \frac{7}{37} + \frac{9}{148} + \frac{11}{57} + \frac{13}{228} + \frac{15}{60}$$ [19]

is derived from $1 = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$;

$$1 = \frac{3}{7} + \frac{5}{21} + \frac{7}{57} + \frac{9}{855} + \frac{11}{199} + \frac{13}{6965} + \frac{15}{105}$$

[17] *yady uddiṣṭāṃśachedayor apavartane kṛte taduddiṣṭāṃśānāṃ vikṛtir bhavati yato 'pavartanaṃ na kartavyaṃ/* taduddiṣṭāṃśānāṃ] uddiṣṭānāṃ; yato] tadā tayor in the published edition.

[18] $\frac{1}{9}$ is omitted in both the printed edition and the manuscripts.

[19] The term 7/37 is missing in both the printed edition and the manuscripts.

is derived from $1 = \frac{2}{3} + \frac{2}{15} + \frac{2}{35} + \frac{1}{7}$ obtained by Rule 4 with the arbitrary numbers $1, 3, 5, 7$.

Rule 7

uddiṣṭāṃśe prathame phalahāraghne parāṃśasaṃyukte/
phalabhāgāpte vyagre hāraḥ syāt phalaharaghno 'ntyaḥ//10//
śuddhir na bhaved yadi vālpo 'ṃśo bhājyaṃ tathetaraḥ kṣepam/
hāraḥ phalāṃśa iti vā kuṭṭakena sakṣepakā labdhiḥ//11//
chedaḥ syāt phalahārād alpo 'nalpaḥ phalacchedam/
kramaśo vibhajed guṇayed yatra na śuddhis tad eva khilam//12//
10 numbered 11 NRV, 11c iti vā om. NRV, 11 numbered 12 NRV, 12c guṇayed vibhajed NRV, 12 numbered 13 NRV.

When the former numerator indicated is multiplied by the divisor of the result, and added to the other numerator, and divided by the numerator of the result without any remaninder, [the quotient is] the divisor. [That quotient] multiplied by the divisor of the result is the latter [denominator]. When it is not divisible, the quotient with the addendum (i.e., the general solution) [is obtained] by means of the indeterminate equation such that the smaller numerator is the dividend, the other [numerator] is the addendum, and the numerator of the result is the divisor. According to whether the denominator [obtained] is smaller or greater than the divisor of the result, one should divide or multiply the denominator of the result respectively. If it is not divisible it is insoluble.

When two numerators a_1 and a_2 and the result a/b are given, then the corresponding denominators x and y are to be found such that $\frac{a_1}{x} + \frac{a_2}{y} = \frac{a}{b}$, as follows:

$$x = \frac{a_1 b + a_2}{a}, y = bx.$$

This rule, which implies that x is to be an integer, is equal to GSS Rule 7.

If $a_1 b + a_2$ is not divisible by a, Nārāyaṇa uses an indeterminate equation, which he discusses in chapter 10 of the GK.

Example

when $\frac{1}{x} + \frac{1}{y} = \frac{1}{20}$, $x = \frac{1\times 20+1}{1} = 21, y = 20 \times 21 = 420$;

when $\frac{3}{x} + \frac{7}{y} = \frac{2}{5}$, $x = \frac{3\times 5+7}{2} = 11, y = 11 \times 5 = 55$;

when $\frac{3}{x} + \frac{5}{y} = \frac{71}{70}$, $x = \frac{3\times 70+5}{71} = \frac{215}{71}$, which is not an integer. In this case one has to find x by means of an indeterminate

equation: According to the rule, the smaller numerator (3) is the dividend, the greater numerator (5) the addendum, and the numerator of the result (71) is the divisor. Therefore in this case it is required to find some integers x and z satisfying

$x = \frac{3z+5}{71}$.

When z is equal to any positive integer t multiplied by 71 and increased by 22, x will be an integer of the form $3t + 1$; so for $t = 0, 1, 2, 3, \cdots$ we find $x = 1, 4, 7, 10, \cdots$. No such solution for z appears in the *vāsanā*, but the value of x is chosen to be 10. Again according to the rule, since this x is smaller than the "divisor of the result" b, then $y = b/x$ or $70/10 = 7$. Therefore $\frac{3}{10} + \frac{5}{7} = \frac{71}{70}$.

Rule 8

The last rule of the *bhāgajāti* is for a case where denominators are given and the numerators are to be found.

> *ajñāteṣv aṃśeṣu prakalpya rūpaṃ pṛthak pṛthak cāṃśān/*
> *kṛtvā tulyacchedān phalahāreṇa cchido lopyāḥ//13//*
> *teṣu dvayoḥ kayościd hāras tv ekaḥ paraś ca ṛṇabhājyaḥ/*
> *iṣṭāṃśahatānyonitaphalaṃ bhavet kṣepako 'tha dṛḍhakuṭṭāt//14//*
> *guṇalabdhī sakṣepe vibhājyaharayor lavau syātām/*
> *harabhājyakṣepāṇāṃ yathāpavartas tathāṃśakā kalpyāḥ//15//*
> 13 numbered 14 NRV, 14a dvayor dvayościd NV, 14b ṛṇabhājyaḥ] bhājyor ṇam NRV, 14c -hato nyonita- NRV, 14d kuṭṭān V, 14 numbered 15 NRV, 15a -labdhi V, 15c hara- V, 15 numbered 16 NRV.
>
> When the numerators are unknown, assuming each numerator to be unity and making the denominators equal to divisors of the result, one should remove the denominators [newly obtained]. One of a certain pair among them is the divisor, and the other the negative dividend. The result decreased by the assumed numerators multiplied by the other [numerators] is the addendum. Then the multiplier and the quotient accompanied by the addendum (i.e, general solutions) [obtained] from the fixed indeterminate equation will be numerators for [the two denominators chosen as] the dividend and the divisor. The numerators should be assumed in such a way that reduction of the divisor, the dividend, and the addendum is possible.

I will explain this rule by means of an example given as Example 9. The *vāsanā* writes

0	0	0	0	phalam	3
5	8	9	12		1
					40

One has to find the numerators of four fractions whose denominators are 5, 8, 9, and 12 respectively. The sum of these fractions is $3\frac{1}{40}$.[20]

First one has to suppose all the numerators to be 1 and reduce them to the same denominator; $\frac{1}{5} = \frac{72}{360}, \frac{1}{8} = \frac{45}{360}, \frac{1}{9} = \frac{40}{360}, \frac{1}{12} = \frac{30}{360}$. $3\frac{1}{40}$ is equal to $\frac{1089}{360}$. Then one supposes two arbitrary numbers as the numerators for two of them. The *vāsanā* supposes 2 as the numerator of the denominator 5, and 1 as the numerator of the denominator 8. Then one has to find corresponding numerators for the two remaining denominators 9, 12. The *vāsanā* first calculates the combined value of the remaining numerators: $1089 - (72 \times 2 + 45 \times 1) = 900$.[21] Suppose this to be the addendum, one of the numerators with the denominator 360 to be the negative dividend, and the other numerator to be the positive divisor of the indeterminate equation that satisfies $900/360 = 30y/360 + 40x/360$:

$$\frac{-40x+900}{30} = y, \text{ which is reduced to } \frac{-4x+90}{3} = y.$$

The general solution is $y = -4t + 30, x = 3t$. The list of answers given is:

[20] Read khābdhy- for svābdhy- in *udāharaṇa* 9b.

[21] In the footnote on this operation, the printed text reads:

> *atra truṭir asti pustakadvaye 'pi* (p.272)
>
> (Here there occurs an omission in both the manuscripts.)

In the preface to the printed text, Padmākara Dvivedī says that he discovered a manuscript of the *Gaṇitakaumudī* in the collection of his father, Sudhākara, after the father's death. Padmākara published his edition based on this manuscript only. In the footnote at page 286, Padmākara says again, '*atrobhayatra truṭiḥ/* (here there occurs an omission in both.)' In Part 1 among the two volume published edition he refers to other readings (*pāṭhāntara*) at pages 25, 27, 32, 33, 44, 49, 53. At the Sanskrit introduction and at footnote at page 165 of the second volume he mentions the Nepal manuscript (*nepālaprāptapusṭa*) and says that its reading is similar to (*pratilipisamāna*) that of the manuscript in his hand.

2	1	3	26
2	1	6	22
2	1	9	18
2	1	12	14
2	1	15	10
2	1	18	6
2	1	21	2

This means: when one takes the case $t = 1$, the answer to the problem is $\frac{2}{5} + \frac{1}{8} + \frac{3}{9} + \frac{26}{12} = 3\frac{1}{40}$. In the same way we can obtain equivalent solutions for successive values of t: $3\frac{1}{40} = \frac{2}{5}+\frac{1}{8}+\frac{6}{9}+\frac{22}{12} = \frac{2}{5}+\frac{1}{8}+\frac{9}{9}+\frac{18}{12} = \frac{2}{5}+\frac{1}{8}+\frac{12}{9}+\frac{14}{12} = \frac{2}{5}+\frac{1}{8}+\frac{15}{9}+\frac{10}{12} = \frac{2}{5}+\frac{1}{8}+\frac{18}{9}+\frac{6}{12} = \frac{2}{5}+\frac{1}{8}+\frac{21}{9}+\frac{2}{12}$. In this way the *vāsanā* merely enumerates a great number of possible solutions, some of which are reproduced below:

2	3	3	23
2	3	6	19
2	3	9	15
2	3	12	11
2	3	15	7
2	3	18	3

2	5	3	20
2	5	6	16
2	5	9	12
2	5	12	8
2	5	15	4

2	7	3	17
2	7	6	13
2	7	9	9
2	7	12	5
2	7	15	1

2	9	3	14
2	9	6	10
2	9	9	6
2	9	12	2

2	11	3	11
2	11	6	7
2	11	9	3

4 *Conclusions*

This survey attests to a remarkable continuity of computational tradition from Mahāvīra to Nārāyaṇa despite the five centuries for which we know of no representatives of that tradition. Some of Nārāyaṇa's rules are equivalent to or can be deduced from Mahāvīra's as the table below shows. The use of indeterminate equations seems to be characteristic of Nārāyaṇa.

GSS Rule 1 = GK Rule 2

GSS Rule 3 ⇒ GK Rules 1 and 5

GSS Rule 4 = GK Rule 3

GSS Rule 6 ⇒ GK Rule 6

GSS Rule 7 = GK Rule 7

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Bibliography

Original Sources

Nārāyaṇa. *Gaṇitakaumudī*
P: ed. Padmākara Dvivedī, Saraswati Bhavana Text 57, Benares 1936 (vol.1) and 1942 (vol.2)
N: National Archives, Kathmandu, no. 4–2700, Jyotiṣa 124.
R: Rājasthāna Oriental Research Institute (Jodhpur) 37782.
V: Vārānaseya Saṃskṛta Viśvavidyālaya Srasvatībhavana (Benares) 904595.

Brahmagupta. *Brāhmasphuṭasiddhānta*
1) ed. S. Dvivedī with his own commentary, Benares 1902
2) ed. R. S. Sharma with three comentaries, New Delhi 1966.
3) Manuscript India Office Library, Eggeling 2769.

Mahāvīra. *Gaṇitasārasaṅgraha*
1) ed. M. Raṅgācārya with English Translation and Notes, Madras 1912
2) ed. L. C. Jain, Jīvarāja Granthamālā 12, Sholapur 1963.

Secondary Sources

Datta, Bibhutibhusan and Singh, Avadesh Narayan [1935]
History of Hindu Mathematics, A Source Book, Lahore.

Gupta, R. D. [1993]
"The Mahāvīra-Fibonacci Device to Reduce p/q to Unit Fractions" *International Study Group on the Relations between History and Pedagogy of Mathematics, Newsletter* no. 29, 10–12.

Knorr, Wilbur [1982]
"Techniques of Fractions in Ancient Egypt and Greece" *Historia Mathematica* 9, 133–171.

Kusuba, Takanori [1994] *Combinatorics and Magic Squares in India, A Study of Nārāyaṇa Paṇḍita's Gaṇitakaumudī, Chapters 13–14*, Ph.D. Dissertation Brown University, 1993.

Pingree, David [1981]
Jyotiḥśāstra A History of Indian Literature vol. 6 fasc 4, Wiesbaden.

Area of a Bow-Figure in India

R. C. GUPTA

1 *Introduction*

In Fig. 1, $PNQP$ is a segment of a circle (i.e., circular disc) whose center is at O and whose radius is $OP = OQ = r$. Due to the figure's resemblance to an archer's bow, the arc PNQ ($= s$ in length) was called *cāpa* ('bow'), the chord PQ ($= c$) was called *jyā* or *jīvā* ('bow-string'), and the segment's height MN ($= h$) was called *bāṇa* or *śara* ('arrow') in ancient India. The *cāpa-kṣetra* ('bow-figure') or segment of a circle had great importance in Indian cosmography and geography, especially in the Jaina school. The Bharata-kṣetra (= Bhārata-varṣa or 'land of India') of those times was in the shape of a bow-figure which formed the southernmost part of the central continent or Jambūdvīpa ('Jambū Island') which is stated to be circular and of diameter one lac (100,000) *yojanas*. This cartographic description may be taken to represent the oldest map of India as part of Asia. The maximum north-south breadth of the country was $526\frac{6}{19}$ *yojanas*.

The exact relation between c and h for any segment of a circle of diameter d ($= 2r$) is

$$c = \sqrt{4h(d-h)} \qquad (1)$$

which easily follows by applying the so-called Pythagorean theorem to the right-angled triangle OPM. An explicit verbal statement of (1) is found in the *Bhāṣya* on the *Tattvārthādigamasūtra* (III, 11) of Umāsvāti.[1]

The usual method for finding the exact area A of the circular segment takes

$$A = \text{sector } OPNQ - \text{triangle } OPQ \qquad (2)$$

[1] See the *Bhāṣya* under *sūtra* 11 of Chapter III in [Umāsvāti 1932, 170]. He is placed in the first century A.D. by [Pingree 1970, 59]. But the date (and even authorship) is controversial.

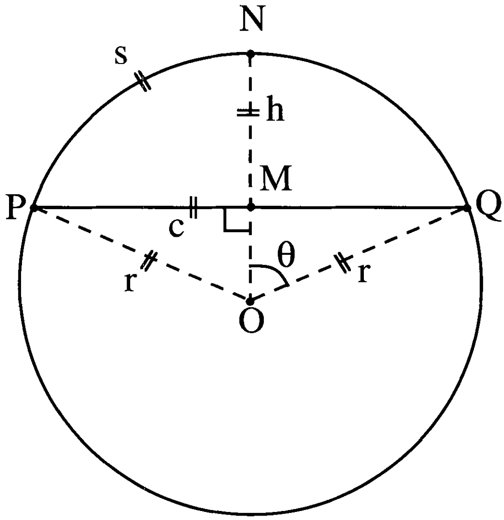


Figure 1: Segment of a circle

$$= \frac{s \cdot r}{2} - \frac{c(r-h)}{2} \quad (3)$$

$$= \frac{r(s-c)}{2} + \frac{c \cdot h}{2}. \quad (4)$$

In terms of the semi-central angle θ subtended by the arc at the center, we have the formulas

$$s = 2r\theta, \quad (5)$$

$$c = 2r \sin\theta, \quad (6)$$

$$h = r(1 - \cos\theta), \quad (7)$$

where

$$\tan\left(\frac{\theta}{2}\right) = \frac{2h}{c}. \quad (8)$$

We see that the use of trigonometric functions and tables makes the computation of the arc-length s and of the area A quite straightforward when any two of the three parameters c, h, and d $(= 2r)$ are known. But when trigonometry was not sufficiently developed, or when its proper use was unknown or avoided, the problem of finding s and A was difficult. In such a situation mathematicians had recourse to devising suitable empirical rules. Practical formulas were found for needful calculations.

In an earlier paper [Gupta 1979], the present author discussed the Indian rules for finding the arc of a circular segment. Most of the formulas used in ancient and medieval India were of the type

$$s = \sqrt{c^2 + kh^2} \tag{9}$$

where k was chosen such that the formula yielded the expected result for the semicircle (which is also a segment with $c = 2r$ and $h = r$). That is,

$$k = \pi^2 - 4. \tag{10}$$

The simplest approximation $\pi = 3$ gives $k = 5$. But the most commonly used value[2] of k was 6, which corresponds to the well-known Jaina approximation $\pi = \sqrt{10}$. For small arcs, Nīlakaṇṭha Somasutvan (ca. 1500 A.D.) [3] found the best formula of the type (9) to correspond to $k = 16/3$. An altogether different formula

$$s = \sqrt{10\left(\frac{c}{4} + \frac{h}{2}\right)^2} \tag{11}$$

is said[4] to be quoted by Bhāskara I in his commentary (A.D. 629) on the *Āryabhaṭīya* of Āryabhaṭa I (born A.D. 476).

2 *Area of a circular segment in other ancient civilizations*

Some rules for computing the area of a segment of a circle are found in Babylonian tablets, but they have not been stated clearly therein nor have they been understood satisfactorily.[5] However, the present author was able [Gupta 2001] to assign some sensible meaning to certain procedures in a Babylonian text to arrive at a few empirical mensurational rules for the arc-length and area of a circular segment. On that basis the Old Babylonian text BM 85194 (ca. 1600 B.C.)[6] can be cited to infer that the Babylonians had used the formula

$$A = ch - kh^2 \tag{12}$$

[2] See [Umāsvāti, 170], and [Gupta 1979, 91–2].
[3] [Gupta 1972], [Gupta 1972–73]; also [Gupta 1979, 93].
[4] [Shukla 1976, LVI, 74] (under II, 10).
[5] For an example see [Katz 1993, 20].
[6] [van der Waerden 1983, 177–9].

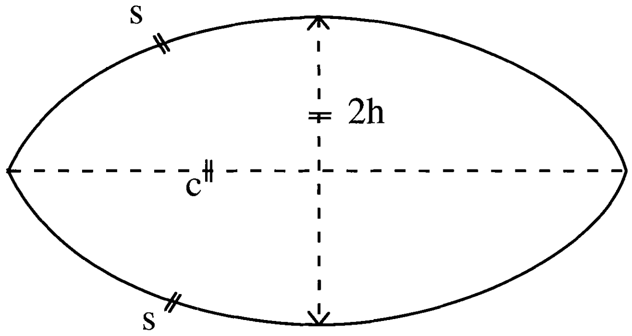


Figure 2: The double-segment

for the area of the segment where k is 1/2 or 1. The value $k = 1/2$ is to be preferred if (12) is expected to give the exact result for a semicircle with the Babylonian value $\pi = 3$. Otherwise $k = 1$ yields better values of A for segments significantly smaller than a semicircle.

Nevertheless there is no doubt that the most popular ancient rule for the area of a circular segment was

$$A = (c + h) \cdot \frac{h}{2}. \tag{13}$$

So far no direct evidence for the use of (13) is found in Babylonian texts. But it easily follows by using the newly discovered formula (for a segmental arc)[7]

$$s = c + h \tag{14}$$

suitably in the case of the figure formed (in the shape of a banana-leaf) by the double-segment (see Fig. 2). For this purpose we use the well-known and universal ancient rule[8] for round figures

$$\text{area} = \frac{(\text{perimeter})(\text{width})}{4} \tag{15}$$

[7] [Gupta 2001] contains details.

[8] For circular areas the rule is found in Umāsvāti's *Bhāṣya*, 170, and in the *Jiu Zhang Suanshu* (I.32) (ca. A.D. 100). See [Lam 1994, 13]; also see [Høyrup 1996, 21–3], and [Hayashi 1990, 5].

When this is applied to the double-segment of area $2A$, we get, by (14),

$$2A = 2\,(c+h)\cdot\frac{2h}{4} \tag{16}$$

thereby getting the expected rule (13).

The formula (13) was used in a Demotic mathematical papyrus of Hellenistic Egypt. It is found in the Papyrus Cairo (JE 89127–30 and 89137–43) written during the third century B.C.[9] In its following equivalent form

$$A = \frac{ch+h^2}{2} \tag{17}$$

the same rule (13) is found in the famous Chinese classic *Jiu Zhang Suanshu* ('Nine Chapters on Mathematical Art') written in the Han Period (206 B.C. to 221 A.D.)[10]

The inaccuracy of the formula (13) was known to the Greek mathematician Heron (A.D. 1st century). He attributed the rule[11] to 'the ancients' and conjectured that it arose by taking $\pi = 3$ in which case it gives the correct area for the semicircle. He further says that those who wanted a better result applied the formula

$$A = (c+h)\cdot\frac{h}{2} + \frac{1}{14}\cdot\left(\frac{c}{2}\right)^2. \tag{18}$$

Since for a semicircle ($c = 2r$, $h = r$) this will give

$$A = \frac{11}{7}\,r^2, \tag{19}$$

it is clear that the correction term in (18) was added by those who accepted the Archimedean value $\pi = 22/7$. However, Heron adds that the use of (18) should be restricted to the range where

$$2 \le \frac{c}{h} \le 3. \tag{20}$$

Where $(c/h) > 3$, Heron recommends the formula

$$A = \frac{2\,ch}{3} \tag{21}$$

---

9 [van der Waerden 1983, 39–40, 172–7].

10 [Lam 1994, 13], and [van der Waerden 1983, 36–40].

11 [Heath 1981, 330].

which is based on assuming the circular arc (of the segment) to be approximated by a parabolic arc in the Archimedean style.[12] Further the *Mensurae* (= *De mensuris*) attributed to Heron contains two more formulas [Heath 1981, 330]

$$A = \frac{h}{2} \cdot (c+h)\left(1+\frac{1}{16}\right) \tag{22}$$

$$A = \frac{h}{2} \cdot (c+h)\left(1+\frac{1}{21}\right) \tag{23}$$

to be used for segments which are smaller and bigger than a semicircle respectively (for semicircle itself the second rule gives exact result with $\pi = 22/7$).

The formula (18) is mentioned by the Roman agrimensor Columella (A.D. 1st century)[13] and is also found in the Hebrew work *Mishnat ha-Middot* which is attributed to Rabbi Nehemiah (ca. A.D. 150).[14]

3 *The classical rule (13) in India*

It has been pointed out above that in Heron's view, the classical rule (13) was based on $\pi = 3$, and that it was modified to the form (18) by those who preferred the value $\pi = 22/7$. A similar thing happened in China. The formula

$$A = \frac{ch + h^2}{2} + (\pi - 3) \cdot \frac{c^2}{8} \tag{24}$$

is found in the *Siyuan yujian* (A.D. 1303) of Zhi Shijie.[15] It is his improvement of the Chinese form (17) with $\pi = 22/7$ and $157/50$.

In India, modifications were done in a different manner which may be compared with those that are represented by (22) and (23). We shall describe them here. But first of all it may be mentioned that the classical rule (13) itself as such is found in the *Gaṇitasārasaṅgraha* (VII, 43) of Mahāvīra (ca. A.D. 850) as well

[12] When h is small, we can neglect h^2 in (1) to get $(c/2)^2 = dh$ which becomes the parabola $y^2 = dx$ with a proper choice of coordinate axes.

[13] [Heath 1981, 303], and [Høyrup 1996, 13, 16].

[14] [Midonick 1968, 197]. For controversy about date and authorship of the Hebrew work, see [Katz 1993, 152], and [Høyrup 1996, 25].

[15] [Martzloff 1997, 327].

as in the *Trilokasāra* (*gātha* 762) of Nemicandra (10th century).[16] Both these authors use $\pi = 3$ as a rough approximation and hence specify the said rule also so which is quite natural. The rule (13) along with (14) is also found in the *Gaṇitakaumudī* (IV, 12) of Nārāyaṇa Paṇḍita (1356) for segments which are smaller than a semicircle.[17]

A rule given by Śrīdhara (ca. A.D. 750) in his *Triśatikā* (*sūtra* 47) is perhaps the earliest modified form of (13) found in India. He says[18]

> *jīvāśaraikyadalahataśarasya vargaṃ daśāhataṃ navabhiḥ*
> *vibhajedavāptamūlaṃ prajāyate kārmukasya phalam*
>
> Take ten times the square of the product of the arrow and half the sum of the chord and arrow, and divide by nine. The square-root of the quotient (so obtained) gives the area of the bow-figure.

That is,

$$A = \sqrt{\left(h \cdot \frac{c+h}{2}\right)^2 \left(\frac{10}{9}\right)}. \tag{25}$$

Clearly this is a modification of (13) based on an adjustment of π from the rough value $\pi = 3$ to the better value $\pi = \sqrt{10}$ which is used in the *Triśatikā* itself (*sūtra* 45) and the accompanying example. The formula (25) is also found in the Prakrit work *Gaṇitasāra* (III, 46) of Ṭhakkura Pherū who, being a Jaina, describes $\pi = \sqrt{10}$ as exact (III, 43).[19]

In the *Mahāsiddhānta* (XV, 89) of Āryabhaṭa II the equivalent of formula (25) is given as a rough rule[20] and the corresponding value $\pi = \sqrt{10}$ is used in the preceding verse (XV, 88). For accurate area, he gives (XV, 93) a verbal rule equivalent to the formula (23) with the corresponding value $\pi = 22/7$ in the preceding verse (XV, 92).[21] Actually Āryabhaṭa II's form for (23) is

$$A = 22 \cdot \frac{c+h}{2} \cdot \frac{h}{21}. \tag{26}$$

[16] [L. C. Jain 1963, 190], and [Viśuddhamati 1975, 597].

[17] [Hayashi 1990].

[18] [Dvivedi 1899, 35].

[19] [Nahata & Nahata 1961, part II, 56].

[20] [Dvivedi 1910, 171] where the reading in the footnote is correct and accepted here.

[21] [Dvivedi 1910, 172]. [Billard 1971, 157–62] shifts the date of the *Mahāsiddhānta* to early 16th century. Also see [Mercier 1993].

It is clear that (25) and (26) imply a modification of (13) to some desired value of π, thereby yielding the general prototype form

$$A = (c+h) \cdot h \cdot \frac{\pi}{6}. \tag{27}$$

Now the Jainas frequently used the approximate formula [Gupta 1975, 43]

$$\sqrt{a^2 + x} = a + \frac{x}{2a}. \tag{28}$$

This will give the approximation $\pi = 19/6$ for $\sqrt{10}$ (with $a = 3$, $x = 1$). The value $\pi = 19/6$ itself is found[22] in the *Gaṇitasāra* (III, 45) of Ṭhakkura Pherū. For such π, the typical rule (27) implies the formula

$$A = (c+h)\left(\frac{h}{2}\right)\left(1 + \frac{1}{18}\right) \tag{29}$$

which in fact is reported to be found in the anonymous Indian work *Pañcaviṃśatikā* (ca. 1400 or earlier) [Hayashi 1991, 399, 411, 436–7]. In addition to (28), the following formula was also used in India [Gupta 1985, 14, 17]:

$$\sqrt{a^2 + x} = a + \frac{x}{2a+1}. \tag{30}$$

This easily enables us to get $\pi = 22/7$ from $\pi = \sqrt{10}$ which was well-known in India.[23] In turn we get (26) as a modification of (13).

It may be that practical geometers or surveyors found that (13) always yielded results in defect of the actual area for segments smaller than a semi-circle or even for semi-circle (because they knew that the actual value of π was greater than 3). So a modifying factor f might have been thought to be a remedy. That is, for better results, a suggested modification could be in the form

$$A = (c+h)\left(\frac{h}{2}\right) \cdot f \tag{31}$$

[22] The use of $\pi = 19/6$ is found earlier in the *Tiloyapaṇṇatī*, I, 118 (see [Viśuddhamati 1984, 26]) and elsewhere (see [Hayashi 1991, 333–5]).

[23] [Gupta 1991]. Alberuni credits Brahmagupta (fl. 628) with a knowledge of $\pi = 22/7$ (see [Sachau 1964, Vol. I, 168]). It was also known to Śrīdhara [Hayashi 1985, 755].

where f is slightly greater than one, or more specifically of the form

$$f = 1 + \frac{1}{N} \tag{32}$$

where N is a suitable positive integer. The choice of $N = 21$ in (23) and of $N = 18$ in (29) was made from a consideration of approximations to π. If the same criterion is applied to $N = 16$ in (22), it would imply the value $\pi = \frac{51}{16}$ which is nowhere mentioned. So there may have been some other considerations for choosing $N = 16$. Of course for a chosen integer N, the corresponding implied value of π will be given by

$$\pi = 3 + \frac{3}{N}. \tag{33}$$

It is possible that a simpler integer was selected for making the calculations more convenient but without affecting the result much. In India, the cases $N = 18$ and 21 are found in (29) and (26). It may be that for convenience of computation, the choice of $N = 20$ was found to be good because it is a decimally simple integer between 18 and 21 (in fact near the better $N = 21$). The new choice yields

$$A = (c + h)\left(\frac{h}{2}\right)\left(1 + \frac{1}{20}\right). \tag{34}$$

And it is interesting to note that this formula was in fact popular in India in the 15th century. It is said to have been used by Viṣṇu Paṇḍita (ca. 1410).[24] Gaṇeśa in his commentary (1545) on *Līlāvatī* (rule 213) states that it was known to his father Keśava (ca. 1500).[25] The Sanskrit text for (34) is

> *cāpe phalaṃ śaro jyeṣuyogārdhaghno nakhāṃśayuk*
>
> In a bow-figure, the area is the product of the arrow and half the sum of the chord and the arrow, increased by its 20th part.

That is,

$$A = h \cdot \frac{c + h}{2} + \frac{1}{20} \cdot \left(h \cdot \frac{c + h}{2}\right). \tag{35}$$

Interestingly, the same Sanskrit hemistich is found in the *Gaṇitapañcaviṃśī* (*sūtra* 25) which is attributed to Śrīdhara but whose authorship of the extant work is said to be doubtful.[26] If

[24] [Datta & Singh 1980, 167].
[25] [Apte 1937, Part II, 218].
[26] [Pingree 1979, 903]; and [Hayashi 1985, 751–6].

Śrīdhara was the original author of (35), it is likely that he got it from his own formula (25) by simplification and using typical ancient Indian techniques of surd-computation to produce[27]

$$\frac{\sqrt{10}}{3} = \frac{\sqrt{4000}}{60} = \frac{63}{60} = \frac{21}{20} = 1 + \frac{1}{20}, \tag{36}$$

the required form.

4 *Special Jaina rule for the area of a segment*

In the Jaina school of Indian mathematics, the computation of the area of the bow-figure was carried out also in another way especially for cosmographical purposes. The method is based on a formula which is explicitly mentioned in the *Tiloyapaṇṇatti* (IV, 2401) of Yativṛṣabha in the following verbal statement:[28]

> *isupāda-guṇida-jīvā guṇidavvā dasapadena jaṃ vaggaṃ* |
> *mūlaṃ cāvāyāre khettetthaṃ hodi suhuma-phalaṃ* ||
>
> The square of the product of a quarter *isu* ($= h$) and chord ($= c$) is multiplied by ten. The square-root of the result is the accurate (*suhuma*) area of the bow-figure.

That is,

$$A = \sqrt{10\left(\frac{c \cdot h}{4}\right)^2} \tag{37}$$

We can say that this formula is based on $\pi = \sqrt{10}$ because in the case of a semicircle ($c = 2r$, $h = r$) it gives the exact area $\pi r^2/2$ for that value of π. The formula (37) is also found in the *Bṛhatkṣetrasamāsa* (I, 122) of Jinabhadra Gaṇi (fl. 609 A.D.).[29] The first half of a *gāthā* quoted by Bhāskara I in his commentary (629 A.D.) on the *Āryabhaṭīya* (under II, 10) reads[30]

> *isupāyaguṇā jīvā dasikaraṇi bhaved vigaṇiya padaṃ*
>
> The product of the chord and a quarter of the arrow, when further multiplied by the square-root of ten, becomes the area of the bow-figure.

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[27] For Śrīdhara's rule $\sqrt{N} = \sqrt{Na^2}/a$, see [Shukla 1959, 175] and *Triśatikā* rule 46.

[28] [Viśuddhamati 1986, 636]. Yativṛṣabha is placed between A.D. 473 and 609.

[29] [Anupam Jain 1990, 163].

[30] [Shukla 1976, 73].

That is,

$$A = \sqrt{10}\, c \cdot \frac{h}{4} \tag{38}$$

which is just a simplified form of (37). This form of the formula is also found in the *Gaṇitasārasaṅgraha* (VII, 70) of Mahāvīra[31] as an accurate rule whereas (13) is given as an approximate rule (VII, 43). This is also the case with another Jaina work, the *Trilokasāra* of Nemicandra (*gāthā* 762).[32]

By making use of (37) the areas of various geographical regions (into which the Jambū Island is divided) were obtained. These areas are found in the *Tiloyapaṇṇatti* (IV, 2402–9) itself and have been shown to be in complete agreement with those which the present writer computed by applying (37) and then simplifying the calculations in the Jaina style.[33] It seems that the formula (37) is older than the *Tiloyapaṇṇatti.*

It may be pointed out that the corresponding formula of the type (37), which is consistent with the rough value $\pi = 3$, would be

$$A = 3\, c \cdot \frac{h}{4}. \tag{39}$$

But it is significant to note that the Jaina authors of *Gaṇitasārasaṅgraha* and *Trilokasāra* (both of which give $\pi = 3$ as a rough value) did not give the formula (39). Instead, both of them preferred to prescribe the popular classical rule (13) for rough calculation. Why was it so? This point is relevant because the formula (39) gives better results than (37) or (38) and even than[34]

$$A = \pi\, c \cdot \frac{h}{4} \tag{40}$$

except for segments which are nearly semicircular.

A more important question is the source or derivation of the peculiar rule (37). It is still likely that (37) was a modification of (39) from $\pi = 3$ to $\pi = \sqrt{10}$. So we consider the rationales of the simple rule (39).

Averaging[35] was a usual and simple method to get empirical results. In Fig. 3, situated on the same base PQ, the area of the

[31] [L. C. Jain 1963, 198].
[32] [Viśuddhamati 1975, 597]; see Section 3.
[33] [Gupta 1987, 52–3].
[34] [Gupta 1989, 21–2].
[35] [Gupta 1981] contains a survey on averaging.

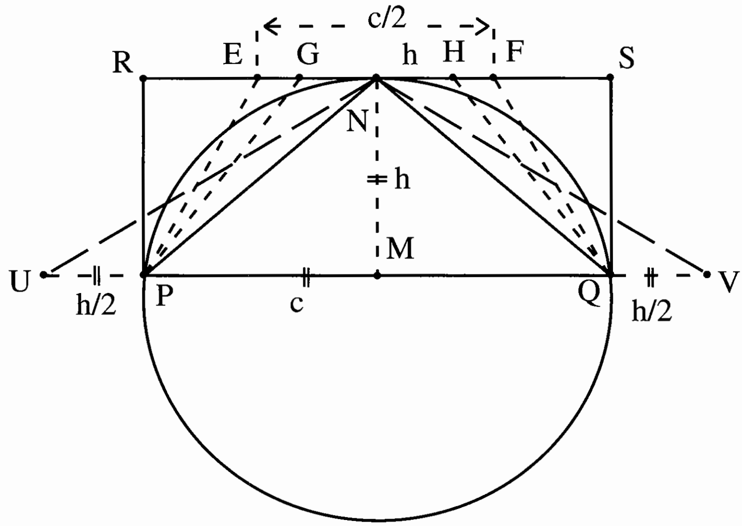


Figure 3: The mean of two areas

inscribed triangle PNQ is $ch/2$, and that of the outer rectangle $PSRQ$ is ch. Taking the average of these two areas,[36] we get (39) by regarding the segmental area as lying midway between those of the triangle and rectangle.

Geometrically the above averaging process amounts to equating the area of the segment with that of the trapezoid $PEFQ$ where $EF = c/2$. Incidentally it may be pointed out that such a technique of replacing a given figure by a simpler figure that is assumed equivalent has often been taken as an explanation or rationale of ancient formulas. Thus the popular classical rule (13) follows by equating the area of the segment to that of the trapezoid $PGHQ$, where $GH = h$. On the other hand, [Eves 1983, 11] obtained (13) by equating the segmental area to that of triangle UNV, where $UP = QV = h/2$.

[36] Ancient mathematicians may have easily noted that the area of a semicircle ($3r^2/2$ with $\pi = 3$) is in fact the mean of the areas of the triangle ($= r^2$) and outer rectangle ($= 2r^2$) on the same base.

5 *Karavinda's segment-area rules*

In closing, some rules found in Karavinda's commentary on the *Āpastamba Śulbasūtra* may be mentioned. One verbal rule reads[37]

> *śarāhatastu kodaṇḍo dalito dhanuṣaḥ phalam*
>
> Half of (the product of) arc as multiplied by arrow is the area of the bow-figure.

That is,

$$A = \frac{s \cdot h}{2} \tag{41}$$

According to Datta,[38] the above Sanskrit line gives the accurate area of the sector of a circle. His interpretation is wrong because *śara* ('arrow') is usually taken as the height of the segment (and not the radius of the circle). However, he is right in pointing out that another rule

$$A = \frac{s}{2} \cdot \frac{h}{2} \tag{42}$$

which Karavinda quotes is incorrect.[39] If we apply the ancient rule (15) to the double-segment in Fig. 2, we get

$$2A = \frac{2s \cdot 2h}{4} \tag{43}$$

which at once gives (41). The analogy of segment with semicircle may be used by writing the latter's area as

$$A = \frac{\pi r^2}{2} \approx \frac{3r^2}{2} = \frac{3r \cdot r}{2} = \frac{3(2r \cdot r)}{4}.$$

In the last two expressions, replacing $3r$, $2r$, and r by s, c, and h (as is true for the semicircle), we get (41) and (39) for the segment analogously.

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[37] [Srinivasachar & Narasimhachar 1931, 124].

[38] [Datta 1991, 17]. Datta's interpretation is good when the bow-string PMQ (Fig. 1) is assumed to take the position POQ in the pulled state.

[39] Ibid., and [Srinivasachar & Narasimhachar 1931, 56]. [Chakravarti 1934, 28] says that the formula $A = ch/2$ is found in the *Śulbasūtras* but gives no details.

Bibliography

D. V. Apte (ed.), *The Līlāvatī of Bhāskara II* (with the commentaries of Gaṇeśa and Mahīdhara), Poona, 1937 (Part II).

Roger Billard, *L'astronomie indienne*, Paris, 1971.

Gurugovinda Chakravarti, 'On the Earliest Hindu Methods of Quadratures', *Journal of the Department of Letters (University of Calcutta)*, 24, 1934, pp. 23–8.

Bibhutibhusan Datta, *The Science of the Śulba*, Calcutta, repr. 1991.

B. Datta and A. N. Singh, 'Hindu Geometry', *Indian Journal of History of Science*, 15, 1980, pp. 121–88.

Sudhākara Dvivedi (ed.), *Triśatikā of Śrīdhara*, Benares, 1899.

——. *Mahāsiddhānta of Āryabhaṭa II*, 3 parts, Benares, 1910.

Howard Eves, *Great Moments in Mathematics Before 1650*, Washington DC, 1983.

R. C. Gupta, 'Nīlakaṇṭha's Rectification Formula', *Mathematics Education*, 6, 1972, section B, pp. 1–2.

——. 'Nīlakaṇṭha's Formula for Finding the Arc or an Angle' (in Hindi), *Racanā*, 1972–73, pp. 33–4.

——. 'Circumference of the Jambūdvīpa in Jaina Cosmography', *Indian Journal of History of Science*, 10, 1975, pp. 38–46.

——. 'Jaina Formulas for the Arc of a Circular Segment', *Jain Journal*, 13 (3), 1979, pp. 89–94.

——. 'The Process of Averaging in Ancient and Medieval Mathematics', *Gaṇita-Bhāratī*, 3, 1981, pp. 32–42.

——. 'On Some Ancient and Medieval Methods of Approximating Quadratic Surds', *Gaṇita-Bhāratī*, 7, 1985, pp. 13–22.

——. 'Chords and Areas of Jambūdvīpa in Jaina Cosmography', *Gaṇita-Bhāratī*, 9, 1987, pp. 51–3.

——. 'On Some Rules from Jaina Mathematics', *Gaṇita-Bhāratī*, 11, 1989, pp. 18–26.

——. 'The Jaina Value of Pi and its Transmission Abroad' (in Hindi), *Proceedings of the International Seminar on Jaina Mathematics and Cosmology*, Hastinapur, 1991, pp. 117–20.

——. 'Mensuration of a Circular Segment in Babylonian Mathematics', *Gaṇita-Bhāratī*, 23, 2001, pp. 12–17.

Takao Hayashi, *The Bakhshali Manuscript*, Ph.D. dissertation, Brown University, 1985.

——. 'Nārāyaṇa's Rule for a Segment of a Circle', *Gaṇita-Bhāratī*, 12, 1990, pp. 1–9.

——. 'Pañcaviṃśatikā in its Two Recensions', *Indian Journal of History of Science*, 26, 1991, pp. 395–448.

T. L. Heath, *A History of Greek Mathematics*, vol. II, repr. New York, 1981.

Jens Høyrup, 'Hero, Ps-Hero, and Near Eastern Practical Geometry', Roskilde Univ. Center, Preprint No. 5, 1996.

Anupam Jain, *Contributions of the Jaina Acharyas to the Development of Mathematics* (in Hindi), Ph.D. dissertation, Meerut University, 1990.

L. C. Jain (ed.), *Gaṇitasārasaṅgraha of Mahāvīra* (with Hindi translation), Sholapur, 1963.

Victor J. Katz, *A History of Mathematics: an Introduction*, New York, 1993.

Lay Yong Lam (tr.), 'Jiu Zhang Suanshu', *Archive for History of Exact Sciences*, 47, 1994, pp. 1–51.

Jean-Claude Martzloff, *A History of Chinese Mathematics*, Berlin, 1997.

Raymond Mercier, 'The Date of Mahāsiddhānta', *Gaṇita-Bhāratī*, 15, 1993, pp. 1–13.

H. Midonick, *The Treasury of Mathematics*, vol. I, Harmondsworth, 1968.

A. Nahata and B. Nahata (eds), *Ratnaparīkṣādi Sapta-grantha-saṅgraha (including Gaṇitasāra) of Ṭhakkura Pherū*, Jodhpur, 1961.

David Pingree, *Census of the Exact Sciences in Sanskrit*, Series A, Vol. I, Philadelphia, 1970.

——. 'The Gaṇitapañcaviṃśī of Śrīdhara', in *Ludwik Sternbach Felicitation Volume*, Lucknow, 1979, pp. 887–909.

Edward C. Sachau (tr.), *Alberuni's India*, repr. Delhi, 1964.

K. S. Shukla (ed.), *Pāṭīgaṇita of Śrīdharācārya*, Lucknow, 1959.

——. *Āryabhaṭīya with the Commentary of Bhāskara I and Someśvara*, New Delhi, 1976.

D. Srinivasachar and S. Narasimhachar (eds), *The Āpastamba-śulbasūtra* (with commentaries of Kapardiswamin, Karavinda, and Sundararāja), Mysore, 1931.

Umāsvāti, *Sabhāṣya-tattvārthādhigamasūtra* (with the Hindi translation of Pandit Khūbacandra), Agāsa, 1932.

B. L. van der Waerden, *Geometry and Algebra in Ancient Civilizations*, Berlin, 1983.

Āryikā Viśuddhamati (tr.), *Trilokasāra of Nemicandra* (with commentary of Mādhavacandra and Hindi translation), Mahāvīrajī, 1975.

——. *Tiloyapaṇṇatī (Triloka-prajñāpti) of Yativṛṣabha* (with Hindi translation), Kota/Lucknow, Vol. I: 1984, Vol. II: 1986, Vol. III: 1988.

A Survey of Rules for Computing the True Daily Motion of the Planets in India

SETSURO IKEYAMA

This is a survey of rules for computing true daily motion of the planets from mean daily motion appearing in Indian astronomical treatises (*siddhāntas* and *karaṇas*). They are arranged in chronological order.

1 *Notation*

$\bar{\alpha}$: manda anomaly
$\tilde{\alpha}$: śīghra anomaly
$\bar{v}$: mean daily motion of the planet
$\tilde{v}$: manda-corrected daily motion
v: true daily motion ($= \tilde{v}$ for the sun and the moon)
v_A: daily motion of manda apogee
v_S: daily motion of śīghra apogee
$\bar{v}_\alpha$: daily motion of manda anomaly ($= \bar{v} - v_A$)
$\tilde{v}_\alpha$: daily motion of śīghra anomaly ($= v_S - \tilde{v}$)
v_α: daily motion of corrected anomaly
ε: correction to mean daily motion
R: radius of the standard circle
C: circumference of the standard circle
r: radius of an epicycle
c: circumference of an epicycle
H: "hypotenuse", true geocentric distance
I: interval of a Rsine table (usually $225'$)
$J[n]$: nth tabulated Rsine
$\Delta J[n]$: nth Rsine-difference ($J[n] - J[n-1]$)
ΔJ_α: Rsine-difference corresponding to an anomaly α

2 *Abbreviation of Sanskrit Texts*

PS: *Pañcasiddhāntikā* of Varāhamihira (composed ca. 550)
BSS: *Brāhmasphuṭasiddhānta* of Brahmagupta (composed 628)
MBh: *Mahābhāskarīya* of Bhāskara I (composed before 629)
LBh: *Laghubhāskarīya* of Bhāskara I (composed after MBh)
KhKh: *Khaṇḍakhādyaka* of Brahmagupta (epoch 665)
SDV: *Śiṣyadhīvṛddhidatantra* of Lalla (composed ca. 750?)
SS: *Sūryasiddhānta* (composed ca. 800)
VS: *Vaṭeśvarasiddhānta* of Vaṭeśvara (composed 904)
LMM: *Laghumānasa* of Mañjula (Muñjāla)[1] (composed ca. 932)
SSE: *Siddhāntaśekhara* of Śrīpati (composed ca. 1050)
MS: *Mahāsiddhānta* of Āryabhaṭa II (composed ca. 950–1000?)
SSI: *Siddhāntaśiromaṇi* of Bhāskara II (composed 1150)
SOS: *Somasiddhānta* (composed before 1400)
KA: *Karaṇāmṛta* of Citrabhānu (composed ca. 1530)

3 *Introduction*

We can find two types of rules in Indian astronomical texts for calculating true daily motion from mean daily motion.
Type 1: convert $\bar{v}$ or $\bar{v}_\alpha$ directly into v or v_α. In the type 1 rules, the true geocentric distance of the planet (*karṇa*, H) is usually used. For example H for p_1 in figure 1 is Op_1 and the true daily motion of p_1 (v), or of anomaly (v_α) is calculated:

$$v = \bar{v} \cdot \frac{R}{H}, \quad v_\alpha = \bar{v}_\alpha \cdot \frac{R}{H}.$$

This rule is sometimes called *karṇabhukti.*
Type 2: calculate the difference between true and mean daily motion ($\varepsilon = |v_\alpha - \bar{v}_\alpha| = |v - \bar{v}|$ in figure 1) and add or subtract it from $\bar{v}$. Most of the type 2 rules use Rsine-differences for calculating ε.
First of all, the Rsine-difference including the mean anomaly of planet $\bar{p}1$ (ΔJ_α) is found from a Rsine table in this way:
when

$$J[n-1] \leq \text{Sin}\,\bar{\alpha} < J[n],$$

[1] About the name of the author, see Shukla 1990, p. 1.

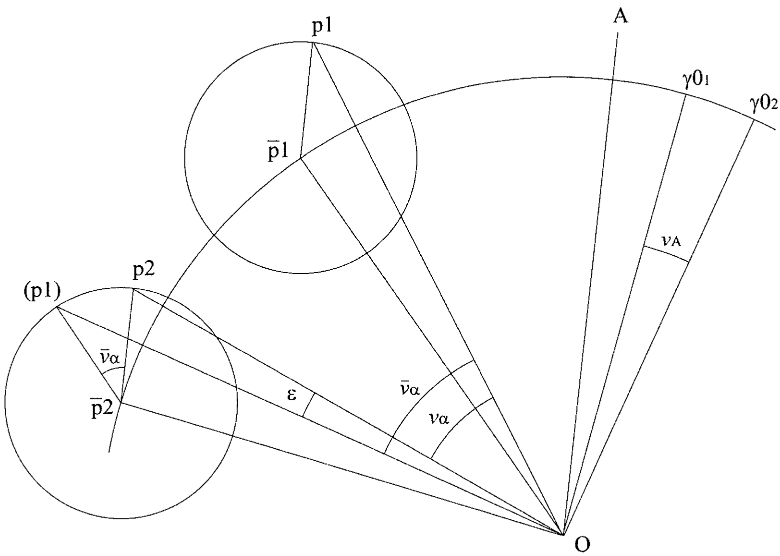


Figure 1: General

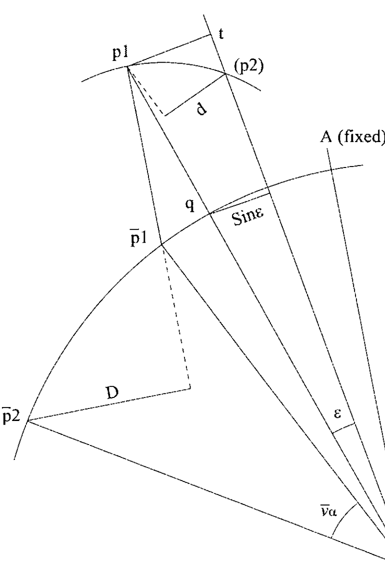


Figure 2: jīvabhukti

$$\varDelta J_\alpha = \varDelta J[n] = J[n] - J[n-1]. \tag{1}$$

Then, in figure 2, an Rsine-difference corresponding to $\bar{v}_\alpha$ (D) is calculated by a proportion from the interval of a Rsine table (I) and $\varDelta J_\alpha$:

$$I : \varDelta J_\alpha = \bar{v}_\alpha : D.$$

Then D is converted into d by another proportion using the circumference of the standard circle (C) and that of an epicycle (c):

$$D : C = d : c.$$

Assuming $d = \mathrm{Sin}\,\varepsilon$, it is converted into arc ε by the third proportion with the first tabulated Rsine ($J[1]$):

$$J[1] : I = \mathrm{Sin}\,\varepsilon : \varepsilon.$$

Combining these three proportions we get:

$$\varepsilon = \bar{v}_\alpha \cdot \frac{\varDelta J_\alpha}{J[1]} \cdot \frac{c}{C}. \tag{2}$$

This rule is called *jīvabhukti* because it uses Rsines (*jīva*). The problem of *jīvabhukti* is that the correction ε does not change continuously; it remains the same while $\mathrm{Sin}\,\alpha_1$, $\mathrm{Sin}\,\alpha_2$, $\mathrm{Sin}\,\alpha_3 \ldots$ are between the same Rsines. Bhāskara I criticizes this in LBh II, 14 and some later astronomers provide inprovements (SS II, 50–51; LMM II, 4cd; MS III, 15–16ab; SSI II, 37).

4 *Pañcasiddhāntikā*

4.1 *PS IX, 12–13: Type 2*

$$\bar{v}_\alpha = \bar{v} - v_A,$$

$$\mathrm{Sin}\,\varepsilon = \bar{v}_\alpha \frac{\varDelta J_\alpha}{225} \cdot \frac{c}{360},$$

$$v = \bar{v} \pm \varepsilon.$$

360 is the circumference of the standard circle expressed in degrees and 225 is the interval of the Rsine table in minutes. Varāhamihira does not mention how to convert $\mathrm{Sin}\,\varepsilon$ into ε here[2].

[2] Formula (2) of Neugebauer and Pingree 1970-71, part II p. 71 does not correctly reflect what the text says.

4.2 *PS IX, 14cd: Type 1*

$$H = \bar{v}\frac{R}{v}.^{3}$$

5 *Brāhmasphuṭasiddhānta*

5.1 *BSS II, 41–42ab: Type 2*

$$\varepsilon = \bar{v}_\alpha \cdot \frac{\Delta J_\alpha}{J[1]} \cdot \frac{c}{360}.$$

5.2 *BSS II, 42cd–44: Type 1*

$$v = v_S - (v_S - \bar{v}) \cdot \frac{\Delta J_\alpha}{J[1]} \cdot \frac{R}{H}.$$

This rule uses crude approximations (See Ikeyama 2003a).

6 *Mahābhāskarīya and Laghubhāskarīya*

6.1 *MBh IV, 14–17; LBh II, 9–13: Type 2*

6.1.1 *MBh IV, 14; LBh II, 9–10*

$$v = \bar{v} \pm \bar{v}\frac{\Delta J_\alpha}{I} \cdot \frac{c'}{80}.$$

Bhāskara clearly mentions that the mean daily motion is divided by 'the arc', I, i.e., 225 minutes. This means that the process for converting $\mathrm{Sin}\,\varepsilon$ into ε is omitted, but this does not lead to a problem in practice because the first Rsine ($J[1]$) is also 225 in MBh whose R is 3438.
2/9 of the circumferences of the epicycles (c') are used here which are given in MBh VII, 13–16 and LBh I, 19–22 as in the *Āryabhaṭīya* I, 8–9.
In the Ārya school $v_A = 0$ (and therefore $\bar{v} = \bar{v}_\alpha$) except in the case of the moon, so that $\bar{v}$ is used instead of $\bar{v}_\alpha$ in this formula.

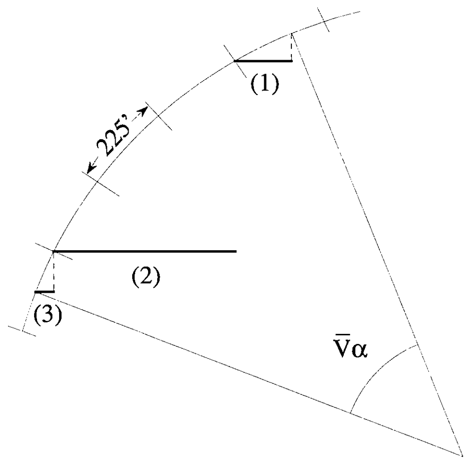


Figure 3: Moon Rule in MBh and LBh

6.1.2 *MBh IV, 15–17; LBh II, 11–13*

For the daily motion of the Moon Bhāskara gives a special rule to calculate D because the motion of the Moon is great and the proportion $D = \bar{v}_\alpha \cdot \Delta J_\alpha / I$, cannot be used (see figure 3). His rule is:

$$D = (1) + (2) + (3),$$

where (2) is the sum of tabulated Rsine-differences and (1) and (3) are computed by means of the proportion.

6.2 *MBh IV, 58–59: Type 2*

The same rule as in MBh IV, 14 and LBh II, 9–10 is used here for the other five star-planets for calculating both manda correction and śīghra correction:

$$\tilde{v} = \bar{v} \pm \bar{v} \frac{\Delta J_\alpha \cdot c'}{18000}; \; v = \tilde{v} \pm (v_S - \tilde{v}) \frac{\Delta J_\alpha \cdot c'}{18000} \cdot \frac{R}{H},$$

where

$$18000 = 225 \times 80.$$

In the case of śīghra correction, Bhāskara converts d into Sin ε by means of a proportion:

$$d : H = \text{Sin}\,\varepsilon : R.$$

6.3 *MBh IV, 13; LBh II, 8: Type 1*

This rule is said to be only for the Sun and Moon.

$$v = \bar{v}\frac{R}{H}.$$

7 *Khaṇḍakhādyaka*

7.1 *KhKh I, 19–20: Type 2*

$$v_{\text{sun}} = \bar{v}_{\text{sun}} \pm \frac{\Delta J_\alpha}{15}; \; v_{\text{moon}} = \bar{v}_{\text{moon}} \pm \frac{\Delta J_\alpha \cdot 7}{8}, \qquad \text{(I, 19)}$$

$$v = \bar{v} \pm \bar{v}_\alpha \frac{\Delta J_\alpha}{900}. \qquad \text{(I, 20)}$$

The first two formulae can be derived from the third when the value of the mean daily motion of anomaly of the sun (about 60′) and of the moon (about 784′) are substituted for $\bar{v}_\alpha$. In the third formula, 900 corresponds to $J[1] \cdot 360/c$. Therefore, if this formula is for all the planets, it follows that the circumferences of the manda epicycles of all the planets would be the same.

8 *Śiṣyadhīvṛddhida*

8.1 *SDV II, 15: Type 2*

$$v_{\text{sun}} = \bar{v}_{\text{sun}} \pm \frac{\Delta J_\alpha}{101}; \; v_{\text{moon}} = \bar{v}_{\text{moon}} \pm \frac{\Delta J_\alpha \cdot 10}{33}.$$

These are the same rules as in SDV III, 11 but applied specifically to the sun and the moon; $J[1]$ of SDV is 225, c' of the sun is 3, and that of the moon is 7.

8.2 *SDV III, 11–13: Type 2*

Exactly the same rule as MBh IV 58–59:

$$\tilde{v} = \bar{v} \pm \bar{v}\frac{\Delta J_\alpha}{225} \cdot \frac{c'}{80}, \qquad \text{(III, 11)}$$

$$v = \tilde{v} \pm (v_S - \tilde{v})\frac{\Delta J_\alpha}{225} \cdot \frac{c'}{80} \cdot \frac{R}{H}. \qquad \text{(III, 12–13)}$$

8.3 *SDV III, 18–19: Type 1*

For śīghra correction Lalla uses the same rule as BSS II, 42cd–44:

$$\tilde{v} = \bar{v} \cdot \frac{R}{H}, \qquad \text{(III, 18ab)}$$

$$v = v_S - (v_S - \tilde{v}) \cdot \frac{\Delta J_\alpha}{J[1]} \cdot \frac{R}{H}. \qquad \text{(III, 18cd–19)}$$

9 *Sūryasiddhānta*

9.1 *SS II, 47–49: Type 2*

$$\tilde{v} = \bar{v} \pm \bar{v}\frac{\Delta J_\alpha}{225} \cdot \frac{c}{360},$$

in the case of the moon,

$$\tilde{v} = \bar{v} \pm (\bar{v}_{moon} - v_A)\frac{\Delta J_\alpha}{225} \cdot \frac{c}{360}.$$

This is the rule only for the manda correction. '225' is the first tabulated Rsine, not I. $\bar{v}$ is used instead of $\bar{v}_\alpha$ because the manda apogees of the planets except the moon are fixed in the *Sūrya-siddhānta* as in the Ārya school.

9.2 *SS II, 50–51: Type 2*

$$v = \tilde{v} \pm (v_S - \tilde{v}) \cdot \frac{|H - R|}{H}.$$

This is a *jīvabhukti* type rule but does not use Rsine-differences. In figure 2, $Oq = R$, $Op1 = H$, $p_1q = |H - R|$, and $\bar{p}_1p_1 = r$.

$$\text{arc}\, \bar{p}_1\bar{p}_2 : R = \text{arc}\, p_1(p_2) : r.$$

Assuming

$$\triangle p_1(p_2)t \propto \triangle p_1\bar{p}_1 q,$$

$$p_1(p_2)(\approx \operatorname{arc} p_1(p_2)) : p_1 t = r : |H - R|,$$

and also

$$H : p_1 t = R : \operatorname{Sin}\varepsilon,$$

combining these three proportions, we get this formula. See Burgess' explanation (Burgess 1860, pp. 91–92) and Sengupta's discussion in the reprint (Burgess 1860, reprint pp. xx–xxvi).

10 *Vaṭeśvarasiddhānta*

10.1 *VS II, i, 97–98: Type 2*

This is a rule for manda correction.

$$\tilde{v} = \bar{v} \pm \bar{v}_\alpha \frac{\Delta J_\alpha}{J[1]} \cdot \frac{c}{360}.$$

10.2 *VS II, i, 99: Type 1*

This is also the rule for manda correction only.

$$v = v_A + \bar{v}_\alpha \cdot \frac{R}{H}.$$

10.3 *VS II, i, 100: Type 2*

The same rule as VS II, i, 97-98 applied specifically to the sun and the moon. In VS, $J[1]$ is 56;15 (225/4), the circumference of the manda epicycle of the sun is 14°, and that of the moon is 31; 30°.

$$v_{\text{sun}} = \bar{v}_{\text{sun}} \pm \frac{\Delta J_\alpha \cdot 7}{172}; \; v_{\text{moon}} = \bar{v}_{\text{moon}} \pm \frac{\Delta J_\alpha \cdot 49}{40}.$$

10.4 *VS II, iii, 18: Type 1*

$$\tilde{v} = v_A + (\bar{v}_\alpha) \cdot \frac{\Delta J_\alpha}{J[1]} \cdot \frac{R}{H}; \; v = v_S - (\tilde{v}_\alpha) \cdot \frac{\Delta J_\alpha}{J[1]} \cdot \frac{R}{H}.$$

This is a BSS type rule but used for both manda and śīghra correction.

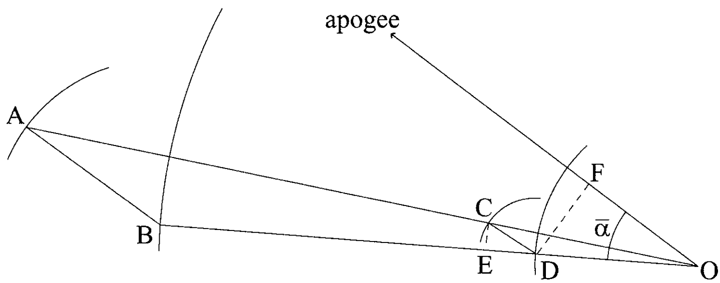


Figure 4: Laghumānasa

11 *Laghumānasa*

11.1 *LMM 14cd (II, 4cd): Type 2*

$$v = \bar{v} \pm \bar{v} \cdot \frac{\text{Cos}\,\bar{\alpha}}{d \pm \frac{\text{Cos}\,\bar{\alpha}}{2}},$$

where $\text{Cos}\,\bar{\alpha}$ is $\overline{OF}$ of figure 4 expressed in degrees and d, which are given in LMM 13 (II, 3), are equal to $R/\bar{r}$; R is $\overline{OD}$ in minutes and $\bar{r}$ is $\overline{AB}$, the radius given in degrees of a manda epicycle corresponding to $\bar{R}$ ($\overline{OB}$). $\bar{v}_\alpha = \bar{v}$ here which means the motions of apogees (v_A) are assumed to be 0.
Using superscripts $\circ$ and m which mean degrees and minutes respectively, we can express this formula like:

$$v^m = \bar{v}^m \pm \bar{v}^m \cdot \frac{\text{Cos}\,\bar{\alpha}^\circ}{\frac{R^m}{\bar{r}^\circ} \pm \frac{\text{Cos}\,\bar{\alpha}^\circ}{2}},$$

Combining a formula which is also found in MBh IV, 58–59 (section 6.2) and SDV III, 12–13 (section 8.2):

$$\varepsilon^m = \bar{v}^m \cdot \frac{\Delta J_\alpha{}^\circ}{J[1]^\circ} \cdot \frac{\bar{r}^m}{\bar{R}^m} \cdot \frac{R^\circ}{H^\circ},$$

and the relationship Bhāskara II mentions (formula (3) in section 14.1 below):

$$\Delta J_\alpha{}^\circ = \text{Cos}\,\bar{\alpha}^\circ \cdot \frac{J[1]^\circ}{R^\circ},$$

we get:

$$\varepsilon^m = \bar{v}^m \cdot \frac{\operatorname{Cos}\bar{\alpha}^\circ \cdot \bar{r}^m}{\bar{R}^m \cdot H^\circ}.$$

Assuming

$$H^\circ(\overline{OC}) \approx \overline{OD} + \overline{DE} = R^\circ + \operatorname{Cos}\bar{\alpha}^\circ \cdot \frac{\bar{r}^m}{\bar{R}^m},$$
$$\bar{R}^m \approx 3600,$$

we finally get:

$$\varepsilon^m = \bar{v}^m \cdot \frac{\operatorname{Cos}\bar{\alpha}^\circ}{R^\circ \cdot \frac{\bar{R}^m}{\bar{r}^m} + \operatorname{Cos}\bar{\alpha}^\circ}$$
$$= \bar{v}^m \cdot \frac{\operatorname{Cos}\bar{\alpha}^\circ}{\frac{\bar{R}^m}{\bar{r}^\circ} + \operatorname{Cos}\bar{\alpha}^\circ}.$$

It seems, as Majumdar points out (Majumdar 1951, p. 53), that $\operatorname{Cos}\bar{\alpha}/2$ of Mañjula's formula should be $\operatorname{Cos}\bar{\alpha}$.
This is a variation of the type 2 rule which does not use any Rsine-difference. Compare Bhāskara's formula (section 14.1) and see the explanations of Majumdar (Majumdar 1951, pp. 45–56) and of Shukla (Shukla 1990, pp. 125–131).

11.2 *LMM 17 (II, 7): Type 1*

$$v = v_S - (v_S - \tilde{v}) \cdot \frac{\text{vyāsa} - \frac{\text{śīghraphala}}{12}}{\text{śīghra divisor}}.$$

The 'śīghra divisors' are calculated in LMM 15–16ab (II, 5–6ab):

$$\text{śīghra divisor} = d_m \cdot \frac{r_m}{r_s} + \frac{\operatorname{Sin}\tilde{\alpha}}{3} \pm \operatorname{Cos}\tilde{\alpha},$$

where r_s is the radius of a śīghra epicycle given in degrees.
Up to now I have not found a satisfactory explanation for this formula. See Majumdar (Majumdar 1951, pp. 50–61) and Shukla (Shukla 1990, pp. 131–136).

12 *Siddhāntaśekhara*

12.1 *SSE III, 40–41ab: Type 2*

$$v = \bar{v} \pm \bar{v}_\alpha \cdot \frac{\Delta J_\alpha}{J[1]} \cdot \frac{c}{360}.$$

12.2 *SSE III, 42–43: Type 1*

$$v = v_S - (v_S - \tilde{v}_\alpha) \cdot \frac{\Delta J_\alpha}{J[1]} \cdot \frac{R}{H}.$$

This is the same rule as BSS II, 42cd–44.

13 *Mahāsiddhānta*

13.1 *MS III, 15cd–16ab: Type 2*

$$v = \bar{v} \pm \bar{v} \cdot \frac{\text{koṭiphala}}{R}.$$

This is the same rule as Bhāskara II (see section 14.1 below) if $\bar{v}_\alpha = \bar{v}$, i.e., v_A of the sun is assumed to be zero.

14 *Siddhāntaśiromaṇi*

14.1 *SSI II, 37: Type 2*

$$\tilde{v} = \bar{v} \pm \bar{v}_\alpha \cdot \frac{\text{koṭiphala}}{R}.$$

Bhāskara says in his auto-commentary that the difference of the true longitudes of the plenet in two successive days is 'crude motion (*sthūlā gatiḥ*)', and motion calculated by this rule is 'accurate and instantaneous (*sūkṣmā tātkārikī*)'.
Bhāskara assumes $\Delta J_\alpha \propto \text{Cos}\,\bar{\alpha}$, so that the equation ε changes continuously. For finding the Rsine-difference corresponding to $\bar{\alpha}$, he does not use the traditional rule (formula (1) of the introduction) but calculates it with a proportion (Chaturvedi 1981, p. 120):

$$\Delta J_\alpha = \text{Cos}\,\bar{\alpha} \cdot \frac{J[1]}{R}. \qquad (3)$$

Then, substituting this 'instantaneous Rsine-difference' for ΔJ_α in the typical *jīvabhukti* (formula (2) of the introduction), we get:

$$\varepsilon = \bar{v}_\alpha \cdot \frac{\operatorname{Cos}\bar{\alpha}}{R} \cdot \frac{c}{C},$$

where $(\operatorname{Cos}\bar{\alpha} \cdot c)/C$ is the *koṭiphala.*

14.2 *SSI II, 39: Type 1*

$$v = v_S - \tilde{v}_\alpha \cdot \frac{\text{Sin}\,(90 - \text{phalāṃśa})}{H}.$$

This is an improvement of Brahmagupta's rule in BSS II 42cd–44 (section 5.2 above). According to Bhāskara's auto-commentary (Chaturvedi 1981, pp. 121–123), when formula (3) is combined with the rule of Brahmagupta:

$$v = v_S - (v_S - \tilde{v}) \cdot \frac{\Delta J_\alpha}{J[1]} \cdot \frac{R}{H}$$

a formula is obtained:

$$v = v_S - (v_S - \tilde{v}) \cdot \frac{\operatorname{Cos}\tilde{\alpha}}{H}$$

and this is the formula mentioned here. Therefore, the word *phalāṃśa*, literally means 'degrees of the result' should be understood as śīghra anomaly $\tilde{\alpha}$.

15 *Somasiddhānta*

15.1 *SOS II, 29: Type 1*

$$v = v_S \pm \tilde{v}_\alpha \cdot \frac{R}{H}.$$

16 *Karaṇāmṛta*

16.1 *KA I, 28cd–29ab: Type 2*

$$\tilde{v} = \bar{v} \pm \bar{v} \cdot \frac{\operatorname{Cos}\bar{\alpha}}{\text{divisor}}.$$

Divisors are given in I, 25–27. KA 28cd is very close to LMM II, 4cd (section 11.1 above). The anonymous commentator interprets the word *koṭi* as *koṭiphala*, $(\operatorname{Cos}\bar{\alpha}\cdot c)/C$ (Namboodiri 1975, p. 14).

16.2 *KA I, 36–37: Type 1*

$$v = v_S - (v_S - \tilde{v}) \cdot \frac{\text{śīghravyāsa} - \frac{\text{doḥphala}}{14}}{\text{śīghra divisor}}.$$

This rule seems to be the same as that of LMM II, 7 (see section 11.2 above) except that the divisor of *doḥphala* is 14 (*śakvarī*), not 12.

Source Text

PS IX 12–13
saptakalā vitryaṃśāś candroccasyendubhuktir anayonā |
kendrasya parijñeyā sphuṭabhuktiś cānayā kāryā || 12
kendrāntarajyāguṇitā tithivargeṇoddhṛtā ca pariṇāmya |
tatkārmukaṃ kṣayacayau bhuktau mṛgakarkaṭādyeṣu || 13
(Neugebauer and Pingree 1970-71, part I pp. 92, 94)

PS IX 14cd
vyāsārdhahatā bhuktiḥ sphuṭabhuktihṛtā sphuṭaḥ karṇaḥ || 14cd
(Neugebauer and Pingree 1970-71, part I p. 94)

BSS II, 41–44
grahamandakendrabhuktir jyāntaraguṇitādyajīvayā bhaktā |
labdhaṃ sphuṭaparidhiguṇaṃ bhagaṇāṃśahṛtaṃ phalakalābhiḥ || 41
mṛgakarkyādāv ūnādhikā svamadhyamagatiḥ sphuṭārkendvoḥ |
śīghragatiṃ mandaphalasphuṭabhuktyūnāṃ kujādīnām || 42
śīghraphalabhogyajīvāsaṃguṇitām ādyajīvayā vibhajet |
phalaguṇitaṃ vyāsārdhaṃ vibhājayec chīghrakarṇena || 43
labdhonā śīghragatiḥ sphuṭabhuktir bhavati labdham adhikaṃ cet |
śīghragateḥ śīghragatiṃ labdhāt saṃśodhya vakragatiḥ || 44
(Ikeyama 2003a)

MBh IV, 13–17
viṣkambhārdhahatā bhuktiḥ sūryācandramasoḥ sadā |
svāviśeṣeṇa karṇena sphuṭabhuktir avāpyate || 13
antyajīvāthavā bhuktyā guṇitā dhanuṣā hṛtā |
svaparidhyāhate 'śītyā labdhe hīnādhike sphuṭā || 14
antyajīvādhanuḥkhaṇḍaṃ kendrabhogād viśodhayet |
tadviśodhya makheḥ śeṣaṃ pātyate 'viṣame tataḥ || 15
uccabhuktivihīnāyā bhukteḥ śītāṃśumālinaḥ |
utkramajyā krame grāhyā kramajyā cotkramasthite || 16
ādyantayoḥ phalaṃ yuktvā guṇayoś cānupātataḥ |
tatphalena vihīnāḍhyā bhuktiḥ sphuṭatarā hi sā || 17
(Shukla 1960, text section p. 23)

LBh II, 8–13
vyāsārdhasaṅguṇā bhuktir madhyā karṇena labhyate |

sphuṭabhuktiḥ sahasrāṃśo śītāṃśor apy ayaṃ vidhiḥ || 8
antyamaurvīhatāṃ bhuktiṃ madhyamāṃ dhanuṣā haret |
labdhaṃ svavṛttasaṃkṣuṇṇaṃ chitvāśītyā viśodhayet || 9
makarādisthite kendre karkaṭādau tu yojayet |
madhyabhuktau sahasrāṃśoḥ sphuṭabhuktir udāhṛtā || 10
utkramakramato grāhyāḥ padayor ojayugmayoḥ |
vartamānaguṇād indoḥ kendrabhukteḥ kalāvaśāt || 11
ādyantadhanuṣor jñeyaṃ phalaṃ trairāśikakramāt |
gatagantavyadhanuṣī kendrabhukter viśodhayet || 12
ittham āptaguṇaṃ hatvā vṛttenāśītisaṃhṛtam |
prāgvat kṣayodayāv indor madhye bhoge sphuṭo mataḥ || 13
(Shukla 1963, text section pp. 5–6)

MBh IV, 58–59
mandāntyajīvāguṇitāṃ svabhuktiṃ bhūyaḥ svavṛttena hatāṃ vibhajya |
rāśeḥ kalābhir daśatāḍitābhir bhuktau dhanarṇaṃ padayuktito 'rdham || 58
śīghroccabhuktes tad apāsya śeṣāt kendrāntyajīvāvidhinā yad āptam |
trijyāhataṃ karṇavibhaktabhedaṃ nyāsena śīghrasya dhanarṇam iṣṭam || 59
(Shukla 1960, text section p. 27)

KhKh I, 19–20
pañcadaśakena vibhajed bhānumato bhogyamānakaṃ piṇḍam |
śaśino 'gaguṇaṃ vasubhiḥ kṣayadhanadhanahānayaḥ svagatau || 19
gatibhogyakhaṇḍakavadhāl labdhaṃ navabhiḥ śatai ravīnduphalam |
prāgvac chukrādīnāṃ kṣayadhanadhanahānayaḥ svagatau || 20
(Sengupta 1941, pp. 22–23)

SDV II, 15
svabhogyakhaṇḍaṃ kṣitikhendubhir hṛtaṃ
raver vidhor digguṇitaṃ suroddhṛtam |
tadūnayukte bhavataḥ sphuṭe gatī
kramāt svakendre mṛgakarkaṭādike || 15
(Chatterjee 1981, part 1 p. 34)

SDV III, 11–13
jyākhaṇḍakena guṇitā mṛdukendrajena
bhuktir grahasya śarayugmayamair vibhaktā |
kṣuṇṇā sphuṭena guṇakena hṛtā khanāgair
liptā gateḥ phalam ṛṇaṃ dhanam uktavac ca || 11
tadvarjitā svacalatuṅgagatiḥ svabhogya-
khaṇḍāhatā śarayamākṣihṛtā hatā ca |
svena sphuṭena guṇakena khanāgabhaktā
trijyāhatā śrutihṛtāśuphalaṃ gateḥ syāt || 12
mandasphuṭā grahagatiḥ sphuṭatām upaiti
yuktonitā virahitā sahitām unā ca |
śīghrābhidhānanijakendrapadakrameṇa
vakrā gatir bhavati ced ṛṇato viśuddhā || 13
(Chatterjee 1981, part 1 p. 46)

SDV III, 18–19
trijyāhatā grahagatir mṛdukarṇabhaktā

mandasphuṭā bhavati tadrahitāśubhuktiḥ |
trijyāhatā svacalakarṇahṛtāśucāpa-
bhogyajjyayā viguṇitā vihṛtādimaurvyā || 18
labdhaṃ tyajet svacalatuṅgagateḥ sadaiva
śeṣaṃ sphuṭā bhavati ca grahabhuktir evam |
labdhaṃ bhaved yad adhikaṃ calatuṅgabhukter
vyastaṃ bhunakti khacaraḥ prativāsaraṃ tat || 19
(Chatterjee 1981, part 1 p. 49)

SS II, 47–51
svamandabhuktisaṃśuddhā madhyabhuktir naśāpateḥ |
dorjyāntarādikaṃ kṛtvā bhuktāv ṛṇadhanaṃ bhavet || 47
grahabhukteḥ phalaṃ kāryaṃ grahavan mandakarmaṇi |
dorjyāntaraguṇā bhuktis tattvanetroddhṛtā punaḥ || 48
svamandaparidhikṣuṇṇā bhagaṇāṃśoddhṛtāḥ kalāḥ |
karkyādau tu dhanaṃ tatra makarādāv ṛṇaṃ smṛtam || 49
mandasphuṭīkṛtāṃ bhuktiṃ projjhya śīghroccabhuktitaḥ |
taccheṣaṃ vivareṇātha hanyāt trijyāntyakarṇayoḥ || 50
calakarṇahṛtaṃ bhuktau karṇe trijyādhike dhanam |
ṛṇam ūne 'dhike projjhya śeṣaṃ vakragatir bhavet || 51
(Chaudhary 1987, pp. 77–80)

VS II, i, 97–100
mandatuṅgagativarjitā gatiḥ kendrabhuktir iha khecarasya sā |
dorguṇāntarahatādyajīvayā bhājitā svamṛduvṛttasaṃguṇā || 97
bhagaṇāṃśahṛtā phalaṃ gatau nijakendre makarādike kṣayaḥ |
dhanam indugṛhādike sphuṭā śravaṇāgre khalu vā gatis tadā || 98
nijakendragatiḥ samāhatā tribhamaurvyā mṛdukarṇabhājitā |
svamṛdūccagatiḥ phalānvitā grahabhuktis tv athavā parisphuṭā || 99
bhujabhojyaguṇāntaraṃ raveḥ svaranighnaṃ dvisvarendubhājitam |
śaśino 'ṅkajalāhataṃ hṛtaṃ khakṛtair bhuktiphalaṃ kalādi vā || 100
(Shukla 1986, part I pp. 102–103)

VS II, iii, 18
nijaphalabhojyajyāghnā kendragatiś cādyajīvayā bhaktā |
trijyāghnā karṇahṛtā labdhonayutā svaśīghramandagatiḥ || 18
(Shukla 1986, prat I, p. 115)

LMM 14cd (II, 4cd)
koṭir gatighnā cchedāptaṃ vyastaṃ gatikalāḥ phalam || 4
(Āpaṭe 1952, p. 10)

LMM 17 (II, 7)
vyāsaṃ śīghraphalārkāṃśabhāgonaṃ grahaśīghrayoḥ |
gatyantaraghnaṃ chedāptaṃ tyaktvā śīghragater gatiḥ || 7
(Āpaṭe 1952, p. 11)

SSE III, 40–43
mandakendragatir arkacandrayor jyāntareṇa guṇitā hṛtādyayā |
jīvayā svapariṇāhatāḍitā kharturāmavihṛtā gateḥ phalam ||40
ādyaturyapadayos tadūnitā madhyayos tadadhikā sphuṭā gatiḥ |
yātayeyadinajagrahāntaraṃ varttamānadivase gatiḥ sphuṭā ||41

cañcalakendragatiḥ phalabhogyajyāguṇitādyaguṇena vibhaktā |
vyāsadalaghnaphalaṃ śrutibhaktaṃ tadrahitāśugatiḥ sphuṭabhuktiḥ || 42
syād avanītanayādikhagānāṃ śīghragateḥ phalam abhyadhikaṃ cet |
tatphalato 'pi viśodhaya śeṣaṃ vakragatir bhavati dyucarāṇām || 43
(Miśra 1932/1947, pp. 172–175)

MS III 15–16ab
mandaphalaṃ kendravaśāt svarṇaṃ sūrye sphuṭo bhavati |
koṭiphalaghnī bhuktir gajyābhaktā kalādiphalam || 15
bhuktau karkimṛgādye kendre svarṇaṃ bhavet spaṣṭā | 16ab
(S. Dvivedī 1910, pp. 58–59)

SSI II, 36cd–39
dināntaraspaṣṭakhagāntaraṃ syād gatiḥ sphuṭā tatsamayāntarāle || 36
koṭīphalaghnī mṛdukendrabhuktis trijyoddhṛtā karkimṛgādikendre |
tayā yutonā grahamadhyabhuktis tātkālikī mandaparisphuṭā syāt || 37
samīpatithyantasamīpacālanaṃ vidhos tu tatkālajayaiva yujyate |
sudūrasaṃcālanam ādyayā yataḥ pratikṣaṇaṃ sā na samā mahatvataḥ || 38
phalāṃśakhāṅkāntaraśiñjinīghnī drākkendrabhuktiḥ śrutihṛdviśodhyā |
svaśīghrabhukte sphuṭakheṭabhuktiḥ śeṣaṃ ca vakrā viparītaśuddhau || 39
(Chaturvedi 1981, pp. 119–121)

SOS II, 29
śīghrakendragatis trijyākṣuṇā karṇoddhṛtā ṛṇam |
śīghroccabhukteḥ syād bhuktir vakrabhuktir viparyaye || 29
(V. P. Dvivedī 1912, p. 10)

KA I, 28cd–29ab
koṭir gatighnī chedāptā vyastaṃ gatikalāphalam || 28cd
tatsaṃskṛtā madhyagatir mandasphuṭagatir bhavet | 29ab
(Namboodiri 1975, p. 14)

KA I, 36–37
śīghravyāso doḥphalataḥ śakvaryaṃśavivarjitaḥ |
sa mandasphuṭaśīghroccagatibhedasamāhataḥ || 36
śīghracchedahṛto labdhaṃ śodhyaṃ śīghragatir gatiḥ |
śodhyāt tyakte tūccabhoge vakrabhuktis tu śiṣyate || 37
(Namboodiri 1975, p. 17)

References

Āpaṭe 1952: Muñjālācārya, *Laghumānasam*, 2nd ed. with Parameśvara's commentary by B. D. Āpaṭe, Ānandāśrama Sanskrit Series 123, Poona, 1952.

Burgess 1860: Ebenezer Burgess, *Translation of the Sūrya-Siddhānta*, the American Oriental Society, New Haven,

1860. Reprinted with P. C. Sengupta's Introduction, Calcutta, 1935.

Chatterjee 1981: Lalla, *Śiṣyadhīvṛddhida Tantra*, edited and translated by Bina Chatterjee. 2 parts: part 1, critical edition and commentary; part 2, translation and mathematical notes, Indian National Science Academy, New Delhi, 1981.

Chatruvedi 1981: *Siddhānta-Śiromaṇi of Bhāskarācārya*, edited with Bhāskara's own commentary Vāsanābhāṣya and Vārttika of Nṛsiṃha by Murali Dhara Chaturvedi, Library Rare Text Publication No. 5, Sampurnanand Sanskrit Universiy, Varanasi, 1981.

Chaudhary 1987: *Sūryasiddhānta*, edited with the 'Tattvāmṛta' Sanskrit commentary, notes, etc. by Kapileśwara Chaudhary, Kāshī Sanskrit Series 144, Chaukhambha Sanskrit Sansthan, Varanasi, 1987 (4th edition).

S. Dvivedī 1910: Āryabhaṭa. *Mahāsiddhānta*, edited with his own commentary by Mahāmahopādhyāya Sudhākara Dvivedī, Benares Sanskrit Series 148–150 (Pingree No. 36), 3 fascs., Braj Bhushan Das and Co., 1910.

V. P. Dvivedī 1912: *Somasiddhānta*, edited by Vindhyeśvarī Prasād Dvivedi with the *Brahmasiddhānta*, Benares Sanskrit Series No, 152 (Pingree No. 39), Braj Bhushan Das and Co., 1912.

Ikeyama 2003a: Setsuro Ikeyama, *Calculation of True Daily Motion: Two Rules in the* Brāhmasphuṭasiddānta, SCIAMVS, 2003 (forthcoming).

Majumdar 1951: Muñjalācārya, *Laghumānasam*, edited with English translation and critical notes by N. K. Majumdar, Indian Positive Sciences Series No. 2, Indian Research Institute, Calcutta, 1951.

Miśra 1932/47: Śrīpati, *Siddhāntaśekhara*, edited by Babuāji Miśra. Part I: edited with the commentary of Makkibhaṭṭa (Chaps. I–IV) and an original commentary (chaps. IV–XII), University of Calcutta, 1932; part II: edited with an

original commentary (chaps. XIII–XX), University of Calcutta, 1947.

Namboodiri 1975: Citrabhānu, *Karaṇāmṛtam*, edited by V. Narayanan Namboodiri, Trivandrum Sanskrit Series No. 240, The Oriental Research Institute and Manuscript Library University of Kerala, Trivandrum, 1975.

Neugebauer and Pingree 1970-71: Varāhamihira, *The Pañcasiddhāntikā*, edited and translated by O. Neugebauer and D. Pingree, 2 vols., København, 1970–1971.

Sengupta 1941: Brahmagupta, *Khaṇḍakhādyaka with the Commentary of Caturveda Pṛthūdaka Svāmin*, edited by Prabodhchandra Sengupta, University of Calcutta, 1941.

Shukla 1960: Bhāskara, *Mahābhāskarīya*, edited and translated into English with explanatory and critical notes, and comments, etc., by K. S. Shukla, Bhāskara I and his works part II, Department of Mathematics and Astronomy, Lucknow University, 1960.

——. 1963: Bhāskara, *Laghubhāskarīya*, edited and translated into English with explanatory and critical notes, and comments, etc., by K. S. Shukla, Bhāskara I and his works part III, Department of Mathematics and Astronomy, Lucknow University, 1963.

——. 1986: *Vaṭeśvara-Siddhānta and Gola of Vaṭeśvara*, critically edited with English translation and commentary by K. S. Shukla, 2 vols., Indian National Science Academy, New Delhi, 1986.

——. 1990: K. S. Shukla, *A Critical Study of the Laghumānasa of Mañjula*, Indian National Science Academy, New Delhi, 1990.

The Problem of the Sun's Corner Altitude and Convergence of Fixed-Point Iterations in Medieval Indian Astronomy

KIM PLOFKER

1 *Introduction*

A remarkable rule for calculating the altitude of the sun above the horizon first appeared in a Sanskrit treatise of the seventh century. As noted in [Pingree 1978, 571–2], it is an elaborately complicated formula which is applicable only under very restricted circumstances: when the sun is exactly at one of the four intercardinal directions, that is, when the angle between its vertical circle and the prime vertical circle passing through the east and west points is 45°. This condition gave to the problem the name *koṇaśaṅku*, 'corner gnomon' or 'corner altitude'.

As it happened, the demonstrably exact solution to this problem in the abovementioned seventh-century text was the prelude to the development of various approximate ones over the course of the next several centuries. These took the form of *asakṛt* ('not just once') or iterative rules, a very common type of computational technique in Sanskrit texts [Plofker 2002]. In particular, these approximations were of the sort now known as 'fixed-point' iterations, whose convergence can be quite unreliable—a fact that has helped shape entire subfields of modern mathematics, such as chaos theory. Using some of the mathematical tools now employed in these disciplines, we can see that Indian fixed-point iterations for the corner altitude were in fact susceptible to certain convergence problems, which were to some extent solved by later modifications. In this paper, we examine the mathematical behavior of these iterations, and reconstruct how some of their users apparently recognized and attempted to deal with the problems inherent in them.

2 *The original rule for the sun's corner altitude*

The problem of finding the sun's altitude a above the horizon, given its declination δ, its corner direction d (measured southward from the east point, so that the southeastern d is 45°, the southwestern 135°, etc.) and the terrestrial latitude ϕ, is almost entirely peculiar to Indian astronomy.[1] Except in its level of difficulty, the corner altitude is typical of the problems in the Sanskrit astronomical topic known as 'Three Questions' (*tripraśna*), which deals with the various relationships between quantities determining direction, terrestrial location, and time: e.g., computing the local latitude from the noon equinoctial shadow of a gnomon, or the current time from the latitude and the sun's declination and altitude, etc. These are generally straightforward calculations based on the similarity of right triangles inside a hemisphere bounded by the celestial sphere and the observer's horizon. It is not certain what practical motivation (if any) inspired astronomers to tackle the esoteric problem of the corner altitude,[2] nor who first solved it successfully. The first known solution is attested in the chapter on 'Three Questions' of the *Brāhmasphuṭasiddhānta* (628 CE) of Brahmagupta, who states:

> Half the square of the radius, decreased by the square of the Sine of the sun's rising amplitude, multiplied by the square of twelve, is the 'First'. The 'Second' is the product of the Sine of the rising amplitude and twelve and the [noon] equinoctial shadow [s_0]. When the two are divided by the square of the equinoctial shadow added to 72, the square root of the First increased by the square of the Second, increased or decreased [when the sun is] in the northern or southern hemisphere [respectively] by the Second, is the Sine of altitude [for $d = 45°$].[3]

Brahmagupta says nothing about how this rule was derived or why it works. We shall explain its meaning in modern terms as follows. Figure 1 illustrates the corner altitude for a typical case

[1] See, however, [Kennedy and Debarnot 1979].

[2] Some *śilpaśāstra* or architectural texts mention the auspicious qualities of the various directions, but apparently without reference to the position of the sun.

[3] *Brāhmasphuṭasiddhānta* 3, 54–5 [Dvivedi, 66]. Here and in all subsequent quotations, square brackets are used to indicate clarifying interpolations that do not appear in the original Sanskrit.

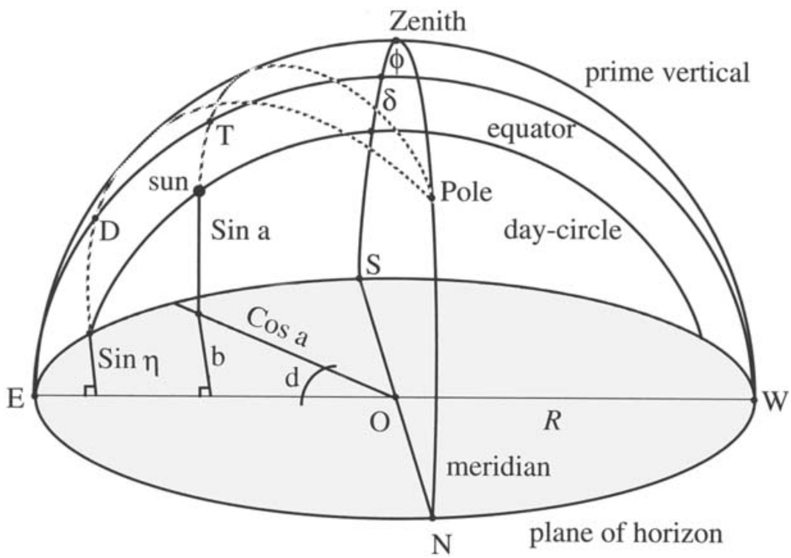


Figure 1: The celestial hemisphere above the horizon

in the northern hemisphere (only northern or positive latitudes are generally considered in Indian astronomy). An analytic solution can be derived geometrically from consideration of the quantities in the same three-dimensional picture and in its analemma (two-dimensional orthographic projection) in Figure 2, as follows: The sun's rising amplitude η, i.e., the arc between its rising-point and the east point E, is defined by

$$\frac{\mathrm{Sin}\,\eta}{\mathrm{Sin}\,\delta} = \frac{R}{\mathrm{Cos}\,\phi}, \tag{1}$$

where the capitalized 'Sin', for instance, indicates the usual sine function scaled by some non-unity value of the trigonometric radius R. A southern declination is considered negative and a northern δ positive; we shall take the sign of η to correspond to that of δ, though Sanskrit texts treat its Sine as always positive and modify their addition and subtraction procedures depending on its direction. (In fact, they treat all Sines and Cosines as positive, but we shall follow modern practice in this regard.) The distance b between the east-west line EW and the point on the

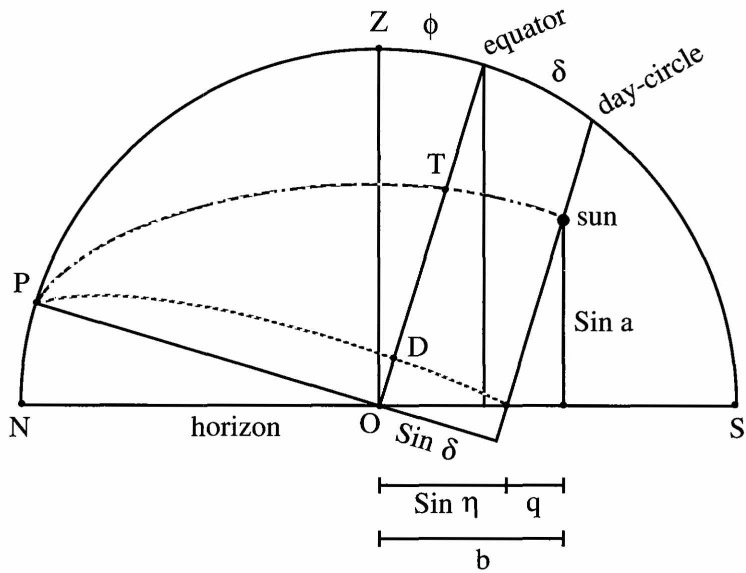


Figure 2: Analemma projection of celestial hemisphere

plane of the horizon directly beneath the sun is given by

$$\frac{b}{\operatorname{Cos} a} = \frac{\operatorname{Sin} d}{R}, \tag{2}$$

and we shall assume d and therefore b to be positive when (as is most usual in northern latitudes) the sun is south of the prime vertical. The solar altitude a can then be derived from these two quantities and the latitude ϕ as follows:

$$\frac{\operatorname{Sin} a}{b + \operatorname{Sin} \eta} = \frac{\operatorname{Cos} \phi}{\operatorname{Sin} \phi}. \tag{3}$$

Setting $R = 1$ so that we may use the familiar modern (lower-cased) trigonometric functions, we can reduce these rules to the expression

$$\frac{\sin a}{\cos a \sin d + \dfrac{\sin \delta}{\cos \phi}} = \frac{\cos \phi}{\sin \phi}. \tag{4}$$

This can be recast as a quadratic equation in terms of $\sin a$ as follows:

$$\sin^2 a(\tan^2 \phi + \sin^2 d) - \sin a \left(2\frac{\sin\delta}{\cos\phi}\tan\phi\right) + \left(\frac{\sin^2\delta}{\cos^2\phi} - \sin^2 d\right) = 0 \tag{5}$$

(and for simplicity's sake, we shall leave this and related expressions in terms of $\sin a$, rather than complicating the expressions further with inverse sines). The values of the root are found from the quadratic formula:

$$\sin a = \frac{\dfrac{\sin\delta}{\cos\phi}\tan\phi \pm \sqrt{\sin^2 d \tan^2\phi - \sin^2 d \cdot \dfrac{\sin^2\delta}{\cos^2\phi} + \sin^4 d}}{\tan^2\phi + \sin^2 d}. \tag{6}$$

The astronomical context constrains the type and sign of the roots by limiting the possible values of the parameters ϕ and δ. In the general quadratic equation $Ax^2 + Bx + C = 0$, neither of the roots $x = (-B \pm \sqrt{B^2 - 4AC})/2A$ can be real if $4AC > B^2$. Given A positive as in equation (5), if $C < 0$, both roots are real and of opposite signs; if $0 < 4AC < B^2$, both are real and have the same sign as $-B$. Substituting for A, B, and C in the above conditions the actual coefficients from equation (5)—and noting that ϕ is non-negative and δ must fall within the limits of the sun's yearly north-south motion in declination, which are considered in the Indian tradition to be $\pm 24°$—we can deduce the constraints on its roots as follows:

$$|\delta| \begin{cases} < |d| : \quad 1 - \dfrac{\sin^2\delta}{\sin^2 d} \begin{cases} > \sin^2\phi \Rightarrow & \text{2 real roots of opposite signs} \\ < \sin^2\phi \Rightarrow & \text{2 real roots of same sign as } \delta \end{cases} \\ \\ > |d| : \quad \dfrac{\sin^2\delta - \sin^2 d}{1 - \sin^2 d} \begin{cases} > \sin^2\phi \Rightarrow & \text{no real root} \\ < \sin^2\phi \Rightarrow & \text{2 real roots of same sign as } \delta \end{cases} \end{cases} \tag{7}$$

Physically, the absence of any real root $\sin a$ means that at the given latitude, the sun at the given declination never attains the direction either of d or of $-d$; two real roots of the same sign imply

that it reaches both d and $-d$ either above the horizon (when the roots are positive) or below the horizon (when they are negative); and if there are two real roots of opposite signs, the sun moves to d during the daytime and to $-d$ in the night, or vice versa. Since the sun's declination is considered as constant throughout a given day, its day-circle is symmetric about the prime meridian, so its altitude is the same at d and $(180 - d)$.

Because, as we stated above, d is an odd multiple of 45°, $|\sin d| = \sin 45° = 1/\sqrt{2}$; so equation (5) reduces to

$$\sin^2 a\,(\tan^2\phi + \frac{1}{2}) - \sin a\left(2\frac{\sin\delta}{\cos\phi}\tan\phi\right) + \left(\frac{\sin^2\delta}{\cos^2\phi} - \frac{1}{2}\right) = 0 \tag{8}$$

and equation (6) to

$$\sin a = \frac{\dfrac{\sin\delta}{\cos\phi}\tan\phi \pm \sqrt{\dfrac{1}{2}\tan^2\phi - \dfrac{1}{2}\dfrac{\sin^2\delta}{\cos^2\phi} + \dfrac{1}{4}}}{\tan^2\phi + \dfrac{1}{2}}. \tag{9}$$

With d thus restricted, $|\delta|$ is always less than $|d|$. Equation (7) then implies that there will be one real positive root $\sin a$ if and only if $\sin^2\phi < 1 - 2\sin^2\delta$; this is always true for any $\phi < 55°$ approximately, which includes all possible latitudes on the Indian subcontinent. So there should always be a unique positive real value for the corner altitude in our calculations.

Now we can rewrite the procedure from Brahmagupta's verses in algebraic notation, employing our sign convention for η defined above:

$$\operatorname{Sin} a = \sqrt{\frac{\left(\dfrac{R^2}{2} - \operatorname{Sin}^2\eta\right)\cdot 12^2}{72 + s_0{}^2} + \left(\frac{12s_0\cdot\operatorname{Sin}\eta}{72+s_0{}^2}\right)^2} + \frac{12s_0\cdot\operatorname{Sin}\eta}{72+s_0{}^2}. \tag{10}$$

Recall from equation (1) that $\operatorname{Sin}\eta = R\cdot\operatorname{Sin}\delta/\operatorname{Cos}\phi$; moreover, since the noon equinoctial shadow s_0 of a standard twelve-digit gnomon is also measured in digits, $s_0/12 = \tan\phi$ (see Figure 3). Given these identities, it is straightforward (though somewhat laborious) to show that Brahmagupta's rule—which became standard in later treatises—indeed reproduces the positive root from equation (9).

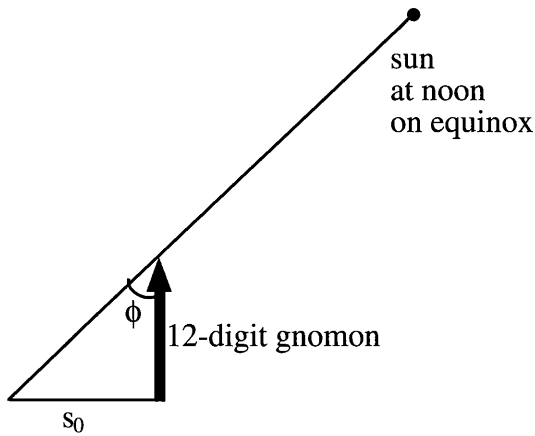


Figure 3: Noon equinoctial shadow

3 *A new solution by means of fixed-point iteration*

The first known iterative alternative to Brahmagupta's closed-form solution for $\sin a$ appeared a century or two after Brahmagupta's work, in the 'Three Questions' chapter of the *Śiṣyadhīvṛddhidatantra* of Lalla (which does not mention the closed-form version):

> When the square of the Sine of the sun's rising amplitude added to [some arbitrary] desired [amount, q_0] is multiplied by two and subtracted from the square of the radius, the square root [of that] is the Sin altitude in the corner direction. Then when [it] is multiplied by the noon equinoctial shadow and divided by twelve, the quotient is the [new] desired [quantity, q_1]. The Sine of the rising amplitude is increased by that, as before. Thus the iterative rule for the altitude [when] the sun is in the south. When [it is] in the northern hemisphere, this rule [is modified by using] the difference of the Sine of the rising amplitude and the desired [quantity, rather than their sum].[4]

In more general terms, and again employing our own sign convention for η,

$$\mathrm{Sin}\, a_0 \quad = \quad \sqrt{R^2 - 2(q_0 - \mathrm{Sin}\,\eta)^2},$$

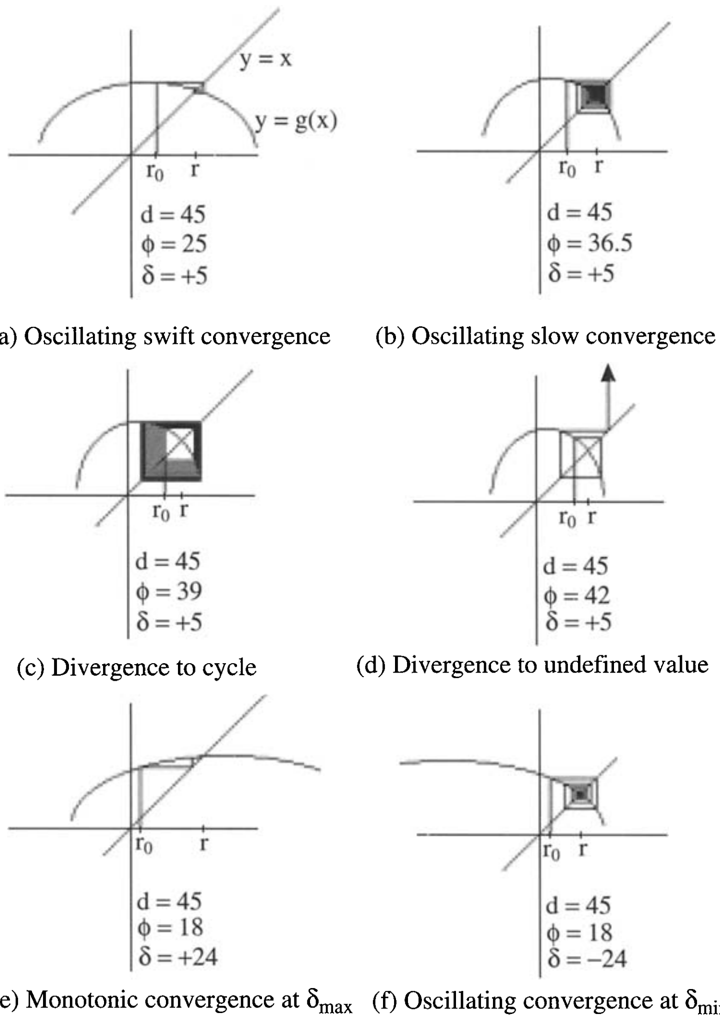
[4] *Śiṣyadhīvṛddhidatantra* 4, 34–5 [Chatterjee, I, 72].

$$q_1 = \operatorname{Sin} a_0 \cdot \frac{s_0}{12} \qquad \left[= \operatorname{Sin} a_0 \cdot \frac{\operatorname{Sin}\phi}{\operatorname{Cos}\phi}\right],$$
$$\operatorname{Sin} a_1 = \sqrt{R^2 - 2(q_1 - \operatorname{Sin}\eta)^2}, \qquad (11)$$

and so on until the true value of $\operatorname{Sin} a$ is reached. Here Lalla introduces a new quantity that we shall call q, which the successive 'desired amounts' q_i are supposed to approximate. Evidently, q is the sum of b and $\operatorname{Sin}\eta$: for it is clear from Figure 1that when $d = 45°$, $\operatorname{Cos}^2 a = 2b^2$, and we see from similar right triangles in Figure 2that the sum of (negative) $\operatorname{Sin}\eta$ and (positive) b is indeed equal to $\operatorname{Sin} a \cdot \operatorname{Sin}\phi / \operatorname{Cos}\phi$. In physical terms, the user is being asked to guess the distance b between the east-west line and the point directly beneath the sun; then, assuming that that point lies in the corner direction (which is usually astronomically impossible for the estimated b at the given ϕ and δ), to calculate its height; and, supposing that altitude to be on the given day-circle, to calculate a new distance b, and so on. But will the mere repetition of these computations starting with some arbitrarily guessed q_0 in place of the required sum $b + \operatorname{Sin}\eta$ actually come up with the correct answer, and if so, why?

To see the mathematical implications of this procedure, let us consider the modern definition of a fixed-point iteration.[5] Briefly, when a root r of a function $f(x)$ is sought, a fixed-point iteration finds it by employing some auxiliary function $g(x)$ such that the desired root of f is also a fixed point $r = g(r)$ of g. This fixed point is found by choosing some initial value or 'seed' r_0 and then computing the successive values $r_1 = g(r_0)$, $r_2 = g(r_1)$, and so forth. If the iteration is convergent, these successive values (the so-called 'orbit' of the seed r_0) will approach closer and closer to the 'attracting fixed point' r, which falls at the intersection of the graph of the auxiliary function $y = g(x)$ and the straight line $y = x$, as illustrated in Figure 4. On the other hand, if the iteration does not converge, the successive r_i may move farther and farther away from r, which in this case is called a 'repelling'

[5] A more detailed treatment of this topic can be found in [Devaney 1992]. I thank Jared Herzberg for providing the reference, and Davide P. Cervone and Homer White for many helpful comments on this section. The calculations for the iterative functions were carried out in Waterloo Maple V, and their orbit diagrams were plotted using John Hubbard's and Beverly West's Analyzer 9.0.



(a) Oscillating swift convergence (b) Oscillating slow convergence

(c) Divergence to cycle (d) Divergence to undefined value

(e) Monotonic convergence at δ_{max} (f) Oscillating convergence at δ_{min}

Figure 4: Orbits of $g(\sin a)$ for various ϕ and δ

fixed point; or the r_i may approach a 'cycle' or oscillation between other values at a finite distance from r, which is then said to be 'neutral', neither attracting nor repelling.

Finding an auxiliary function $g(x)$ that has a fixed point where $f(x)$ has a root is generally just a matter of algebraic reshuffling of the terms of f; however, finding such a $g(x)$ that will converge reliably can be more difficult. The behavior of convergent and non-convergent iterations is demonstrated by the sample orbit diagrams of Figure 4, showing various forms of an iterative function $g(x)$ (defined in the following section) whose graph in each case is a semi-ellipse. In these diagrams, the initial value of the function at the seed r_0 falls on the vertical line extending from the x-axis at r_0 up to the graph of $g(x)$. Then a horizontal line is drawn from that point $(r_0, g(r_0))$ over to the point $(g(r_0), g(r_0)) = (r_1, r_1)$ on the line $y = x$. A vertical line is then extended from that point to the point $(r_1, g(r_1))$ on the graph of $g(x)$, and the process continues as the orbit 'walks' toward or away from the fixed point r. As shown in the figure, the 'steps' of the orbit may approach or recede from the fixed point from one side only (monotonic convergence or divergence, when the slope of the curve at the fixed point is positive) or may jump between too-small and too-large values (oscillating convergence or divergence at a negative slope).

The pictured orbit diagrams also suggest that in a convergent iteration, the differences between the successive horizontal steps get smaller and smaller as the size of the steps themselves approaches zero; if, on the other hand, the successive differences increase, or the step size approaches some finite non-zero amount, or both, then the iteration will not ultimately converge. For smooth and well-behaved functions such as the ones we shall deal with here, this is a reasonably accurate statement of the convergence condition.

Somewhat more formally, we say that convergence for all possible seeds in some interval around r requires that for all the r_i in the orbit of a seed r_0 in that interval,

$$|r_i - r_{i+1}| > |g(r_i) - g(r_{i+1})| \qquad \text{or} \qquad \left| \frac{g(r_i) - g(r_{i+1})}{r_i - r_{i+1}} \right| < 1.$$

This is roughly equivalent to requiring that the absolute value of the first derivative g' of the function at r must be strictly less

than 1. (For a more intuitive justification, recollect that the first derivative is just the slope of the tangent line to the function at that point. Since each successive function value $g(r_i)$ becomes the next input r_{i+1} to the function, a continual decrease in the size of the step-differences means that each vertical step is smaller than the preceding horizontal step: i.e., the y-values change more slowly than their corresponding x-values, so the absolute value of the slope is less than 1.) When $|g'(r)| > 1$, the iteration will diverge; for a neutral fixed point, $|g'(r)| = 1$. Values of $|g'(r)|$ close to 1 mean that the iteration will converge or diverge slowly; the speed of convergence increases as the derivative approaches zero. (When the first derivative at r actually is zero, i.e., the tangent line to the curve at that point is horizontal, the order of convergence is quadratic instead of linear, and the fixed point is called 'superattracting'.)

4 *Evaluating Lalla's fixed-point technique*

We can now return to Lalla's iterative rule for the corner altitude, as represented by equation (11), and assess how effective it is according to the analysis of fixed-point algorithms presented above. We wish to know, first: is it in fact a valid auxiliary function with respect to the earlier closed-form solution, that is, does it have a fixed point where the quadratic in equation (8) has a root? And second: if so, does the iteration actually converge to that fixed point, and how quickly? We start by rewriting Lalla's procedure as a single iterative equation in terms of $\sin a$ with modern trigonometric functions:

$$\sin a_{n+1} = g(\sin a_n) = \sqrt{1 - 2\left(\sin a_n \tan\phi - \frac{\sin\delta}{\cos\phi}\right)^2}. \qquad (12)$$

We solve for the fixed point $\sin a = g(\sin a)$ by setting $\sin a_{n+1} = \sin a_n = \sin a$, and can show after some algebraic manipulation that it occurs at

$$\sin a = \frac{2\dfrac{\sin\delta}{\cos\phi}\tan\phi \pm \sqrt{2\tan^2\phi - 2\dfrac{\sin^2\delta}{\cos^2\phi} + 1}}{2\tan^2\phi + 1}, \qquad (13)$$

which is evidently (after canceling 2's) exactly the same as the expression for the roots of the quadratic in equation (9). Further-

more, taking a (positive) square root at every iteration means that all the successive $\sin a_n$ will be positive. So the fixed point of Lalla's iteration is indeed identical to the positive real root of Brahmagupta's quadratic.

The question of whether the iteration will actually converge to this fixed point is somewhat more complicated. Recall that convergence is dependent on the size of the first derivative of the function at the fixed point, which is dependent on the values of the terrestrial latitude ϕ and the solar declination δ. Strictly speaking, both the quadratic equation (8) and the iterative equation (12) represent not individual functions but families of functions, parametrized by the constants ϕ and δ. We have now shown that the fixed points of the iterated function will always be the same as roots of the corresponding quadratic; but the orbits produced by the iteration will change with the values of these two parameters. We must therefore modify our question and ask which values of ϕ and δ, if any, will produce an iteration with an attracting fixed point.

Differentiating the iterative function in equation (12) with respect to $\sin a$ gives

$$g'(\sin a) = \frac{-2\tan\phi\left(\sin a\tan\phi - \dfrac{\sin\delta}{\cos\phi}\right)}{\sqrt{1 - 2\left(\sin a\tan\phi - \dfrac{\sin\delta}{\cos\phi}\right)^2}}. \tag{14}$$

To satisfy our convergence condition, the absolute value of this first derivative at the fixed point must be less than 1. So setting $|g'(\sin a)| < 1$, and substituting into the above expression for g' the value of the positive fixed point $\sin a$ from equation (13), we find that the absolute value of the quantity

$$\frac{-2\tan\phi\left(\dfrac{2\dfrac{\sin\delta}{\cos\phi}\tan\phi + \sqrt{2\tan^2\phi - 2\dfrac{\sin^2\delta}{\cos^2\phi} + 1}}{2\tan^2\phi + 1}\cdot\tan\phi - \dfrac{\sin\delta}{\cos\phi}\right)}{\sqrt{1 - 2\left(\dfrac{2\dfrac{\sin\delta}{\cos\phi}\tan\phi + \sqrt{2\tan^2\phi - 2\dfrac{\sin^2\delta}{\cos^2\phi} + 1}}{2\tan^2\phi + 1}\cdot\tan\phi - \dfrac{\sin\delta}{\cos\phi}\right)^2}}$$

must be strictly less than 1.

It is not intuitively obvious from this untidy expression which particular values of ϕ and δ will satisfy the required condition. In broad terms, though, using $\delta < 0$ lowers the maximum ϕ for which the inequality will hold, and using $\delta > 0$ raises it. (The reason is illustrated by the orbit diagrams shown for different values of ϕ and δ in Figure 4: in general, increasing ϕ increases the second derivative or rate of change of the slope of the function, making a 'steeper' semi-elliptical arc, while decreasing δ shifts its midpoint (at which the slope is zero) towards the left, meaning that the curve then intersects the $y = x$ line at a point where its slope is steeper. Increasing δ shifts the graph toward the right, with the opposite effect.) More precisely, we can set δ equal to some chosen value and then solve the inequality for the corresponding maximum ϕ. We find[6] that when $\delta = 24°$ at the summer solstice, the upper bound on ϕ is about 49°, for $\delta = 0$ at equinoxes it is about 35°, and for $\delta = -24°$ it is about 21.5°.

Therefore, an astronomer using Lalla's iterative technique to compute the southeast *koṇaśaṅku* at, say, Ujjain ($\phi \approx 23°$) on the day of the summer solstice, using a standard Sine table with $R = 3438$ and initially guessing q_0 to be 1000, would find the approximate results (3359, 3435, 3437, 3437)—a short sequence converging quickly to the same value of Sin a that Brahmagupta's closed-form solution produces. But the same astronomer performing the same calculation with, for example, $q_0 = 500$ at the winter solstice would come up with the successive values 1915, 971, 2088, 501, 2414, and the square root of -1125271, a bewildering and completely useless result. Moreover, since the speed of convergence decreases as the decreasing δ lowers the upper bound on ϕ, even iterations that ultimately do converge to a fixed value can require long and wearisome toil in order to get there. Our hypothetical Ujjain astronomer at an equinox, for example, starting with $q_0 = 1000$, would have to plod through the computation of 3134, 2878, 2972, 2939, 2951 and 2946 before eventually coming

[6] The expression used in Maple to produce the upper bound on ϕ for, e.g., $\delta = 24°$ is as follows:

```
solve (1 = abs( subs( d = (24 *3.14159 / 180),
subs( x=( (2 * tan(f) * (sin(d) / cos(f)) + sqrt(2 * tan(f)^ 2
- 2 * (sin(d)^ 2 / cos(f)^ 2) + 1))/ (2 * tan(f)^ 2 + 1) ),  (
(-2 * tan(f)*( tan(f) * x - (sin(d) / cos(f)) ))/  sqrt(1 - 2 *
(tan(f) * x - (sin(d) / cos(f)))^ 2) )) ) ), f).
```

to rest at $\mathrm{Sin}\, a = 2948$. At $\delta = -10°$ he would need to perform some fifteen iterations to get a result precise to the nearest integer, and at $\delta = -20°$ he would require more than ninety before settling into a cycle between two repeating values and never reaching the true $\mathrm{Sin}\, a$ at all.

It is thus clear that the ingenious method of Lalla (or his unidentified source) for the corner altitude, although procedurally somewhat simpler than Brahmagupta's rule, is liable to crippling malfunctions. Yet Lalla's text gives no clue that he was aware of their existence, although since his own local latitude was probably 21° or thereabouts,[7] he could hardly have avoided encountering some of them if he used the method frequently. Various later authors also discussed the same rule without offering any caveats: e.g., Govindasvāmin (9th c., $\phi \approx 10°$?), Vaṭeśvara (c. 904, $\phi = 23;45°$), and Lalla's own commentator Bhāskara II (b. 1114, $\phi \approx 24°$).[8] Apparently these astronomers (at least the ones at higher latitudes) either never made much use of this iterative rule, or else discovered some of its shortcomings but deliberately refrained from commenting on them. The syncretic nature of Sanskrit astronomy lends credence to both explanations: respect for earlier authorities and appreciation of computational ingenuity for its own sake, combined with the rather free organization of the texts, permitted the survival of many techniques that had little practical value, as well as some that were known to be inaccurate. When we consider that Lalla's rule requires computing one Sine and one square root at every iteration, whereas Brahmagupta's requires only one of each to produce the exact result, it is tempting to conclude that the iterative formula was probably preserved more as an interesting mathematical curiosity than as part of the working toolkit of practicing astronomers.

5 *Evidence for recognition of the method's failure*

Hints in some later texts, however, strongly suggest that the convergence problems with the *koṇaśaṅku* rule were in fact noticed,

[7] For the little that is known about Lalla's place of origin and his life in general, see [Pingree 1970, V, 545] and [Chatterjee, II, xv].

[8] These references occur in the commentary on *Mahābhāskarīya* 3, 41 [Kuppanna Sastri, 155–8]; *Vaṭeśvarasiddhānta* 3, 12, 3–4 [Shukla, I, 204–9]; and *Siddhāntaśiromaṇi* Gaṇita 3, 30 [Śāstrī, 75–7].

and that partially successful methods were developed to cope with them. (Of course, these developments were unrelated to the modern tools and concepts we used in the previous sections to investigate convergence issues: *asakṛt* rules for the Indian astronomer were a matter not of graphs, functions, and derivatives, but merely of successive answers produced by repeated calculations.) What may be the first such hint is seen in the *Siddhānta-śiromaṇi* of Bhāskara II, where in presenting the same iterative rule, he remarks:

> When one has subtracted the square of the Sine of the rising amplitude, multiplied by two, from the square of the radius, that square-root is now the 'corner gnomon'...
>
> [Commentary:] Here, because of [our] ignorance of the 'corner gnomon', there is [also] ignorance of [the distance of] the base of the 'gnomon' [from the east-west line, i.e., b]. Only the Sine of the rising amplitude is known. That is initially considered [as] the segment [b]. Hence the 'corner gnomon' resulting from [the rule] beginning 'the square of the Sine of the rising amplitude, multiplied by two, from the square of the radius...' is approximate. Then, by means of the rule repeatedly [applied], it becomes correct.[9]

The seed q_0 is here specified as zero, meaning that b when $d = 45°$ is initially guessed to be the same as $\mathrm{Sin}\,\eta$. Obviously, since the sun's daily motion always—except for observers at the equator—takes it south of its rising amplitude, a small nonzero q_0 would be a better estimate. Though this use of a zero seed seems counterproductive, it may possibly have originated as a diagnostic technique. For it is evident from equation (11) that for negative δ, when q_0 is as small as possible (i.e., zero), $\mathrm{Sin}\,a_0$ and therefore q_1 will be as large as possible. If this q_1 does not cause the iteration to fail by producing a negative number under the square-root sign in the computation of $\mathrm{Sin}\,a_1$, the user can be confident that no subsequent (smaller) q_n will do so, and thus the iteration will at least not diverge to an undefined value.

This notion of the zero seed as a diagnostic test for divergence remains speculative, since Bhāskara does not describe it as such; he seems rather to be suggesting that we are forced to approximate our first b by $\mathrm{Sin}\,\eta$ since 'only $\mathrm{Sin}\,\eta$ is known'. Later in the

[9] *Siddhāntaśiromaṇi* Gaṇita 3, 30 [Śāstrī, 75–7].

same comment he does enumerate the various directions in which the *koṇaśaṅku* may appear at latitudes of 55° or more, but does not illustrate the use of the iterative rule in such cases or mention the possibility of its failure.

A different and more constructive modification of Lalla's original rule appears in a commentary on Lalla's work by Mallikārjuna Sūri, who worked c. 1178 at a latitude of approximately 18° [Chatterjee, II, xxiii–xxvi]. Mallikārjuna's remarks on the relevant verses of the *Śiṣyadhīvṛddhidatantra* run thus:

> The Sine of the sun's declination, multiplied by the equinoctial hypotenuse, divided by twelve, is the Sine of the sun's rising amplitude. The square of that, multiplied by two, is subtracted from the square of the radius. The square-root of that, multiplied by the noon equinoctial shadow and divided by twelve, is the first desired [quantity, q_1]. That quantity, when the declination is south, is added to the Sine of the sun's rising amplitude, or subtracted from the Sine of the sun's rising amplitude when the declination is north. If the desired [quantity] is greater than the Sine amplitude, [use] their difference. Then, when one has multiplied the square of that by two, it is subtracted from the square of the radius. The square-root of that is the corner altitude.
>
> Having multiplied that by the equinoctial shadow, one should divide it by twelve. The quotient is the second desired [quantity, q_2]. When one has considered half the sum of these second and first quantities as the desired quantity, as before it is to be added to the Sine of the sun's rising amplitude when [the sun is] in the south, [or] subtracted in the north. Then when one has multiplied the square of that by two and subtracted it from the square of the radius, the square-root is the corner altitude. After multiplying that separately by the noon equinoctial shadow, one should divide it by twelve. The quotient is a [new] desired [quantity, q_3]. As before, when one has corrected that quantity by the Sine of the rising amplitude, multiplied its square by two, [and] subtracted it from the square of the radius, the square-root [of the result] is the corner altitude. [Computing] by iteration in this way again and again, the corner altitude is determined. When it is equal to [the value from the previous [iteration], the corner altitude is accurate.[10]

Here, Mallikārjuna has worked into his explication of Lalla's rule (again, without articulating any criticism of it) a significant alter-

[10] Comm. on *Śiṣyadhīvṛddhidatantra* IV, 34–5 [Chatterjee, I, 73].

ation.[11] His verbal procedure can be recast symbolically in our usual way as follows:

$$\begin{aligned}
\operatorname{Sin} a_0 &= \sqrt{R^2 - 2(0 - \operatorname{Sin}\eta)^2} \qquad [q_0 = 0], \\
q_1 &= \operatorname{Sin} a_0 \cdot \frac{s_0}{12} \qquad \left[= \operatorname{Sin} a_0 \cdot \frac{\operatorname{Sin}\phi}{\operatorname{Cos}\phi}\right], \\
\operatorname{Sin} a_1 &= \sqrt{R^2 - 2(q_1 - \operatorname{Sin}\eta)^2}, \\
q_2 &= \operatorname{Sin} a_1 \cdot \frac{s_0}{12}, \\
\operatorname{Sin} a_2 &= \sqrt{R^2 - 2\left(\frac{q_1 + q_2}{2} - \operatorname{Sin}\eta\right)^2}, \\
q_3 &= \operatorname{Sin} a_2 \cdot \frac{s_0}{12}, \\
\operatorname{Sin} a_3 &= \sqrt{R^2 - 2\left(q_3 - \operatorname{Sin}\eta\right)^2}.
\end{aligned} \tag{15}$$

The first four steps of the algorithm obviously mirror those of Lalla in equation (11), except that like Bhāskara, Mallikārjuna uses a zero q_0. But the next step deviates from the original procedure, being equivalent to the expression

$$\sin a_2 = \sqrt{1 - 2\left(\frac{\sin a_0 + \sin a_1}{2} \tan\phi - \frac{\sin\delta}{\cos\phi}\right)^2}.$$

In other words, Mallikārjuna recommends finding the third approximation to $\sin a$ by plugging into the iterative equation not the second approximation, as Lalla directs, but the mean of the first and second approximations. Iterating this step is equivalent to defining a new iterative function which we shall call $\gamma(\sin a)$:

$$\sin a_{n+1} = \gamma(\sin a_n) = \frac{\sin a_n + g(\sin a_n)}{2} \tag{16}$$

It is clear from the form of the above expressions that $\gamma(\sin a)$ has the same fixed point, and is defined over the same domain,

[11] It is quite possible, of course, that Mallikārjuna's modifications are not originally due to Mallikārjuna, nor Lalla's rule to Lalla, for that matter. The dearth of specific attributions in texts like these, and the vast number of such texts that remain unpublished, mean that the ultimate origins of these and similar innovations remain very doubtful. The fact that Mallikārjuna's commentary on this rule immediately goes on to describe Brahmagupta's closed-form solution as an alternative *may* imply that he was personally aware of the problems of the iterative approach, but then again it may not.

as $g(\sin a)$ in equation (12). But its first derivative is different:

$$\gamma'(\sin a) = \frac{1}{2} + \frac{g'(\sin a)}{2}. \tag{17}$$

Evidently, γ will pass the first-derivative test for an attracting fixed point in more cases than g will—specifically, $|\gamma'| < 1$ when $-3 < g' < 1$.[12] This implies that at $\delta = -24°$, the fixed point of the function γ is attracting for ϕ up to about 33°.

Also, this new method often converges more quickly than the original one. This makes intuitive sense when we consider an orbit oscillating between too-small and too-large approximations to the fixed point: splitting the difference between two successive values generally cuts down on the size of the oscillation. More precisely, a comparison of the absolute values of the first derivative tells us that

$$|\gamma'| \begin{cases} < |g'| & \text{for} \quad -1 < g' < -1/3 \\ > |g'| & \text{for} \quad -1/3 < g' < 1. \end{cases}$$

Recall that even the largest positive δ does not shift the graph of g very far to the right of the y-axis, so the greatest positive slope ever actually attained at the fixed point will be little larger than zero. So in most of the possible cases where the iteration of g will converge, that of γ will converge faster, as in Figure 5; and it will also converge in some cases where g does not.

Unfortunately, the latter advantage is largely nullified by Mallikārjuna's requiring a seed of zero, which, as discussed above, will cause an immediate failure for any iteration of Lalla's algorithm that would ultimately diverge. Thus a user at higher latitudes, working with values of ϕ and δ for which g would diverge but γ converge, would see his iteration crash in the computation of $\mathrm{Sin}\, a_1$ before he could apply γ. At Mallikārjuna's own latitude of about 18°, however, Lalla's original iteration g is everywhere convergent, so this would not be a problem for him.

6 *The generalization of Lalla's rule by Parameśvara*

A new refinement of the iterative rule for the corner altitude emerged in the work of Parameśvara (c. 1400, $\phi = 10;51°$), the

[12] I'm indebted to John Feroe for pointing out this feature, and for the following explanation of it.

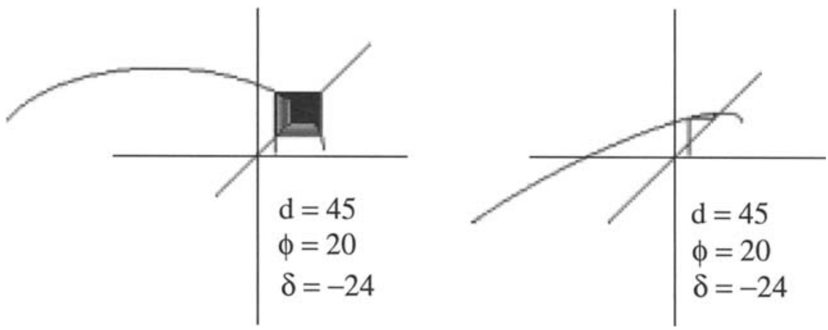


Figure 5: Orbits of $g(\sin a)$ and $\gamma(\sin a)$ for identical ϕ and δ

student of the famous leader of the Kerala school in Indian astronomy and mathematics, Mādhava of Saṅgamagrāma [Pingree 1970, IV, 187–92]. Parameśvara was familiar with Lalla's iteration, having written a supercommentary on the work of Govindasvāmin that included a version of it. In this supercommentary, he also presented a generalized form of this rule for arbitrary d, that is, an iterative solution to the quadratic in equation (5):[13]

> The determination of the '[great] shadow' [Cos a] and so forth should be [made] not only when the sun is standing in a corner direction, but it is computed also when [the sun is] in [any] desired direction. How? It is said:
>
> When one has estimated a desired 'gnomon-distance' [q], the segment [equal to the distance between the base of the altitude and the east-west line, b] is [found] from the sum or difference of that and the Sine of the rising amplitude. The segment [b], multiplied by the radius and divided by the Sine of the desired [azimuth] dependent on the radius, is the 'Sine of visibility' [Cos a]. The square root of the difference of the squares of the radius and that [Cos a] is the 'gnomon' [Sin a]. The 'gnomon' is multiplied by the Sine of the latitude and divided by the Cosine of the latitude. And that is [a new] 'gnomon-distance' [q]. ¿From the sum or difference [of that] and the Sine of the rising amplitude the

[13] Interestingly, neither Parameśvara nor anyone else seems to have sought a non-iterative solution by generalizing Brahmagupta's closed-form rule itself.

> [new] segment [b], and then in the same way the 'Sine of visibility' and so forth, [are computed]. And having made them again, one should [continue to] iterate.[14]

Rewriting this symbolically,

$$\begin{aligned} \operatorname{Cos} a_0 &= \frac{(q_0 - \operatorname{Sin}\eta)\cdot R}{\operatorname{Sin} d}, \\ \operatorname{Sin} a_0 &= \sqrt{R^2 - \operatorname{Cos}^2 a_0}, \\ q_1 &= \operatorname{Sin} a_0 \cdot \frac{\operatorname{Sin}\phi}{\operatorname{Cos}\phi}, \\ \operatorname{Cos} a_1 &= \frac{(q_1 - \operatorname{Sin}\eta)\cdot R}{\operatorname{Sin} d}. \end{aligned} \tag{18}$$

This expanded rule (which may be of Parameśvara's own devising—he does not discuss its origin) can easily be recast as a generalized version of equation (12):

$$\sin a_{n+1} = g(\sin a_n) = \sqrt{1 - \frac{1}{\sin^2 d}\left(\sin a_n \tan\phi - \frac{\sin\delta}{\cos\phi}\right)^2}. \tag{19}$$

The value of the fixed point $\sin a$ and the form of the first derivative $g'(\sin a)$ are identical to the expressions in equations (13) and (14) respectively, except that all the factors of 2 are replaced by the more general $1/\sin^2 d$. Therefore the fixed point of the new iteration will again be the same as a root of the corresponding quadratic—in this case, equation (5)—but as indicated by equation (7), the existence of a unique real positive root is no longer guaranteed.

Even when a real positive $\sin a$ exists, the iteration of this generalized $g(\sin a)$ will not always converge to it. The factor of $1/\sin^2 d$ in our new version of g' means that $|g'|$ will tend to exceed 1 as d gets smaller, so the upper bound on ϕ for which the iteration for a given δ will fail decreases with d. For example, when $d = 22.5°$ and $\delta = -20°$, the constraints in equation (7) imply that there will be one positive value of $\sin a$ for any $\phi < 26°$. But as illustrated in Figure 6, the generalized iterative equation fails to converge to that value for ϕ as small as $10°$. In fact, at

[14] *Siddhāntadīpikā* on *Mahābhāskarīya* III, 41 [Kuppanna Sastri, 158–9].

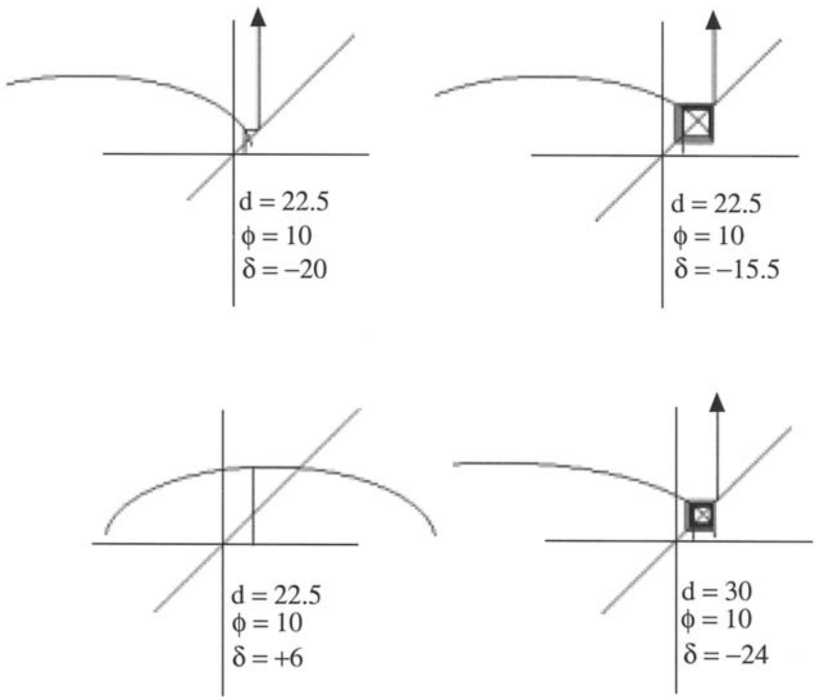


Figure 6: Orbits of generalized $g(\sin a)$ with $d < 45°$

that latitude, no iteration with $d = 22.5°$ will converge unless δ is greater than about $-15.5°$, and convergence remains sluggish as long as δ is less than $-12°$ or so. So even Parameśvara at his low latitude on the southwestern coast must have run into trouble if he tested the rule for sufficiently small d and negative δ—though, as usual, we see no allusion in the text to any such difficulty.

7 *Parameśvara's new iterative method for the* koṇaśaṅku

Nonetheless, there is reason to suspect that Parameśvara eventually abandoned this iterative approach precisely on account of its convergence problems. For in another (probably subsequent) work, the *Goladīpikā* [Pingree 1970, V, 188–9], he again addresses

the problem of the sun's corner altitude, but does not refer to the now-standard technique of Lalla. Instead, he prescribes the following more complicated rule (with his own commentary on the verses):

> An iterative method should be performed with intelligence for the sake of finding the corner altitude. [When the sun is] in the south, if the 'leg' is small[er than the 'arm'], something should be subtracted from the *ghaṭikās* [of time] past [since sunrise] or to come [till sunset]. If the 'arm' in that case is small[er], something should be added to the past or future *ghaṭikās*. When the sun stands in the north and moves only on that [northern] side [of the prime vertical circle], [the same corrections should be made] in that way. If the motion is on both [sides of the prime vertical circle], [first make the correction] as before to obtain a southern Sine altitude, [and then make the corrections] in reverse to obtain the northern Sine altitude. Here, the amount to be subtracted etc. is to be guessed by means of [one's] intelligence. By others, the difference between the 'arm' and the 'leg' is subtracted from or added to [the time] in *asus*. [Repeat the procedure] in just this way.
>
> [Commentary:] Whenever the 'arm' [b] and 'leg' [the east-west distance between the end of the segment b and the north-south line] of the 'shadow'-hypotenuse [$\operatorname{Cos} a$] are equal, then the sun has arrived at a corner direction. Therefore the 'gnomon' [or height, $\operatorname{Sin} a$] observed at that time is the 'corner gnomon'. When one has produced equality [between] the 'arm' and 'leg' by an iterative rule, the corner altitude is determined. He [i.e., Parameśvara himself] states the iterative method [in the lines beginning] '[When the sun is] in the south...'. When the sun is in a direction south of the east-west line and a southern corner altitude is to be found, if the 'leg' is smaller than the 'arm' of the great 'shadow'-hypotenuse attained at a given [amount of] past or future *ghaṭikās*, some amount of time should be subtracted from the past or future *ghaṭikās*. In that case, the sun is south of the corner: that is the meaning. Likewise, if the 'arm' is smaller than the 'leg', some amount of time is added to the past or future *ghaṭikās*. Thus the rule [when the sun is] in the southern hemisphere. If the sun, standing in a northern direction, moves only in the northern part [of the sky], then for the sake of determining the Sine altitude standing in a northern [corner] direction, the rule is [applied] just in that [same] way. And if the sun in the northern hemisphere moves to a southern [corner] direction too, then in order to determine the Sin altitude in the southern

> [corner] direction, the subtraction and addition are [prescribed] as before. Then in order to determine the northern Sine altitude, the subtraction and addition are reversed. Here the amount of the subtracted or added [quantity] is to be guessed with intelligence. Otherwise, *asus* measured by the difference of the 'arm' and 'leg' are subtracted from or added to the past or future *asus*. One should do [this] in the same way again until there is equality [between] the 'arm' and 'leg'. Then the iterated altitude becomes the corner altitude.[15]

The *ghaṭikā* is a time-unit equal to one-sixtieth of a day, and an *asu* is 1/360 of that, or four seconds; if a day is taken to equal 360° of revolution of the celestial equator, then an *asu* is the time it takes for one minute of arc to pass a given point. (Since $R = 3438 \approx 360 \cdot 60/2\pi$, *asus* are like our modern radians in allowing sines and angles to be expressed in the same units.) Sanskrit mathematical texts generally distinguish between the two legs of a right triangle with terms translated here as 'arm' (referring in this case to our b) and 'leg' (the other leg of the right triangle containing b and the hypotenuse or 'great shadow' $\operatorname{Cos} a$).

Here, Parameśvara is recommending that the user compare the sizes of b and the 'leg' $\sqrt{\operatorname{Cos}^2 a - b^2}$ at a particular moment in time: when the sun is in a corner direction and the hypotenuse $\operatorname{Cos} a$ makes a 45° angle with both, they will be equal. If they are not equal at the chosen moment, one must adjust the time till they become so. That is, if the 'leg' is longer than b, the sun is north of its corner location, and the time t since sunrise (or till sunset, in the case of a western *koṇaśaṅku*) must be increased; if b is longer, the sun is south of the corner position and t must be decreased. So the new time estimate should be the sum of t and the positive or negative quantity $\sqrt{\operatorname{Cos}^2 a - b^2} - |b|$. (As Parameśvara notes, however, the sign of the correction must be reversed for the first of two corner positions occurring in the same half-day. As noted in the discussion of equations (7)–(9) above, this is not possible at latitudes lower than about 55°; so for the present, we shall neglect this possible application of Parameśvara's rule and concentrate on explaining how it works for more typical corner altitudes.)

[15] *Goladīpikā* 4, 15–18ab [Sarma, 47].

Since Parameśvara does not explain in detail how Sin a and b are to be found from t, we shall use in our analysis his worked example from a subsequent verse and its commentary:

> [When] the true sun stands in the middle of Aries, ... the Sine altitude in the direction of Agni [i.e., south-east] is to be stated, and [the Sine altitude] standing in the middle [between] the directions of Agni and Indra [east]. The latitude-shadow [Sin ϕ] is 647.
>
> [Commentary:] He states an example [beginning] '[When] the true sun...' When the true sun is at the middle of Aries, ... the Sine altitude of the sun at the corner [direction] at that time is to be stated, and the Sine altitude of the sun at the middle [between the directions] of Agni and Indra is to be stated [as well]... These Sine altitudes are to be stated [for] the place where Sine latitude is equal to 647; this is the statement of the example. Here the Sine latitude is equal to 647; therefore the Cosine latitude is determined [as] equal to 3377. Because of that [longitude of] the sun, the Sine of the rising amplitude is determined [to be] equal to 368...
>
> For the sake of [computing] the corner altitude, the elapsed *ghaṭikās* of the day are considered [to be] 8. [On computing] with those, the Sine altitude is determined [to be] equal to 2516. The 'gnomon-distance' [q] is equal to 482. Its 'shadow' [Cos a] is equal to 2343; that is the hypotenuse. Here the Sine of the rising amplitude is equal to 368. The difference of the gnomon-distance and the Sine of the rising amplitude in different directions is the 'arm' [b] of the 'shadow'-hypotenuse, and that is equal to 114. The square-root of the difference of the squares of that 'arm' and the 'shadow' is the 'leg', and that is equal to 2340. Then, because of the inequality of the 'arm' and the 'leg', their difference is to be added (because of the 'leg's' [being] greater) to the previously determined elapsed *ghaṭikās* of the day. Then the [new] elapsed *asus* of the day are determined [to be] equal to 5106. [Computing] with those, the Sine altitude is determined [to be] equal to 3407; the 'shadow' is equal to 461.[16] The 'arm' of the 'shadow' is equal to 285. Its 'leg' is equal to 362. Here too, because of the 'leg's' [being] greater, the difference between 'arm' and 'leg' is to be added to the elapsed *asus* of the day. The [new] elapsed *asus* of the day, thus arrived at, are equal to 5183. And when one has found with those the Sine altitude etc., [the procedure] is

[16] The calculations do not support the edition's reading of 468 for 461, so we follow the variant reading in two manuscripts of *śaśi* (one) for *vasu* (eight).

> done again [until] there is no difference from the previously found [value]. In [this] iterative rule, when the 'leg' is smaller than the 'arm', then the Sine altitude etc. is to be calculated after one has subtracted the difference of 'arm' and 'leg' from the elapsed *asus* of the day. Here, the corner altitude thus iterated becomes equal to 3414...[17]

(The omitted portions treat the computation of $\mathrm{Sin}\, a$ when the sun is on the prime vertical circle, an easy problem in similar right triangles which does not concern us here.) In this example, we are given the solar longitude Aries 15°, corresponding to a solar declination of about $+6°$, and the noon equinoctial shadow s_0 normalized to $R = 3438$, which is $\mathrm{Sin}\,\phi = 647$, implying a latitude of about $10;51 = 10.85°$. We then find $\mathrm{Sin}\,\eta = R \cdot \mathrm{Sin}\,\delta / \mathrm{Cos}\,\phi = 368$. Parameśvara also provides an initial guess at the time t since sunrise when the south-east *koṇaśaṅku* will occur: 8 *ghaṭikās* or 2880 *asus* or $360 \cdot 8/60 = 48$ time-degrees of the equator, represented by the equatorial arc TD in Figure 1.

To compute $\mathrm{Sin}\, a$ from this information alone, we must find (though Parameśvara does not tell us so) the hypotenuse of the right triangle in Figure 2 containing $\mathrm{Sin}\, a$ and q. This is done by exploiting the similarity of the Sine of any arc θ' measured on the sun's day-circle for a non-zero δ to that of the corresponding arc of time θ on the equator: to wit, $\mathrm{Sin}\,\theta : \mathrm{Sin}\,\theta' :: R : \mathrm{Cos}\,\delta$. We determine for the given δ and ϕ the arc of the half-equation of daylight, whose Sine OD (from similar right triangles in Figure 2) is $R{\cdot}\tan\delta\tan\phi = 69$, and subtract it from the time t since sunrise: $2880 - \mathrm{arcSin}\, 69 = 2811$ *asus*. (Bear in mind that δ and the quantities that depend on it in sign are positive in Parameśvara's example but negative in the figures.) The Sine of this result is the line segment $OT = 2508$, and the Sine of the corresponding arc on the day-circle is $OT \cdot \mathrm{Cos}\,\delta/R = 2494$. Our desired hypotenuse H is the sum of this amount and the Sine of the day-circle arc corresponding to OD on the equator: $2484 + \mathrm{Sin}\,\delta \cdot \mathrm{Sin}\,\phi / \mathrm{Cos}\,\phi = 2494 + 69 = 2563$.

It is then a simple matter to find $\mathrm{Sin}\, a = H \cdot \mathrm{Cos}\,\phi/R = 2517$ (slightly different, due to rounding and interpolation inaccuracies, from Parameśvara's 2516) and the 'gnomon-distance' $q = H \cdot \mathrm{Sin}\,\phi/R = 482$. Trivially, $\mathrm{Cos}\, a = \sqrt{R^2 - \mathrm{Sin}^2 a} = 2343$, and

[17] *Goladīpikā* 4, 23 [Sarma, 49–50].

$b = q - \mathrm{Sin}\,\eta = 482 - 368 = 114$. Then the other leg of the triangle containing b and $\mathrm{Cos}\,a$ is $\sqrt{\mathrm{Cos}^2 a - b^2} = 2340$, which when diminished by b is 2226, which is added to the original time estimate in *asus*: $2880 + 2226 = 5106$. New values of all the above quantities are then calculated from the new values of t in successive iterations; the resulting sequence of approximations to $\mathrm{Sin}\,a$ is $(2516, 3407, 3414, 3414)$. (Parameśvara's rule also permits one to adjust t by an arbitrarily chosen amount rather than by the difference $\sqrt{\mathrm{Cos}^2 a - b^2} - b$, but since the former technique is too ill-defined to evaluate analytically, we shall concentrate on the latter alternative.)

We can recapitulate the steps of the complete iterative procedure, given only the known ϕ and δ and an arbitrarily selected t_0, as follows:

$$\begin{aligned}
H_0 &= \frac{\mathrm{Cos}\,\delta}{R} \cdot \mathrm{Sin}\left(t_0 - \mathrm{arcSin}\left(R \cdot \frac{\mathrm{Sin}\,\delta}{\mathrm{Cos}\,\delta}\frac{\mathrm{Sin}\,\phi}{\mathrm{Cos}\,\phi}\right)\right) \\
&\quad + \frac{\mathrm{Sin}\,\delta\,\mathrm{Sin}\,\phi}{\mathrm{Cos}\,\phi}, \\
\mathrm{Sin}\,a_0 &= \frac{H_0 \cdot \mathrm{Cos}\,\phi}{R}, \\
q_0 &= \mathrm{Sin}\,a_0 \cdot \frac{\mathrm{Sin}\,\phi}{\mathrm{Cos}\,\phi} = \frac{H_0 \cdot \mathrm{Sin}\,\phi}{R}, \\
b_0 &= q_0 - R \cdot \frac{\mathrm{Sin}\,\delta}{\mathrm{Cos}\,\phi}, \\
t_1 &= t_0^{(\mathrm{asus})} + \sqrt{(R^2 - \mathrm{Sin}^2 a_0) - {b_0}^2} - |b_0|. \qquad (20)
\end{aligned}$$

In modern notation,

$$\begin{aligned}
t_{n+1} = t_n^{(\mathrm{radians})} &+ \sqrt{1 - \sin^2 a_n - \left(\sin a_n \tan\phi - \frac{\sin\delta}{\cos\phi}\right)^2} \\
&- \left|\sin a_n \tan\phi - \frac{\sin\delta}{\cos\phi}\right|. \qquad (21)
\end{aligned}$$

Upon setting $t_{n+1} = t_n = t$, equation (21) reduces to the quadratic of equation (8), confirming that the value of $\sin a$ at its fixed point t is indeed the desired corner altitude.

We then confront the perennial question of whether and when the prescribed iteration will actually converge to that fixed point. Rewriting it solely in terms of ϕ and δ as a new iterative function

$h(t)$, we get

$$t_{n+1} = h(t_n) = t_n + \sqrt{1 - \cos^2\delta \sin^2(t_n - \sin^{-1}(\tan\delta\tan\phi)) + \frac{\sin^2\delta}{\cos^2\phi}(\sin^2\phi - 1)} - \left|\sin\phi\cos\delta\sin(t_n - \sin^{-1}(\tan\delta\tan\phi)) + \sin\phi\sin\delta\tan\phi - \frac{\sin\delta}{\cos\phi}\right|. \tag{22}$$

This function is very cumbersome to evaluate analytically, but we can get an idea of its behavior by inspecting its sample orbit diagrams in Figure 7. As shown therein, the iterative function's graph is no longer a semi-ellipse but a curve oscillating about the $y = x$ line with multiple fixed points (in fact, it has an infinite number of fixed points, since the terms depending on $\sin(t_n)$ are identical for t_n, $t_n + 360°$, $t_n + 2 \cdot 360°$, etc.). However, for values of ϕ and δ for which there is a unique positive real corner altitude, there is a unique fixed point of $h(t)$ occurring in the interval from $t = 0$ to the cusp at $t = 90° + \sin^{-1}(\tan\delta\tan\phi)$—that is, in the half-day (after sunrise or before sunset) equal to 90 time-degrees plus the current half-equation of daylight. And the convergence of $h(t)$ to that fixed point is in each case sure and swift, even for latitudes as unrealistically high as 54°. Furthermore, at still higher latitudes where (for $\delta \gg 0$) there are two such fixed points, representing the occurrence of a northern corner altitude followed by a southern one in the same day, $h(t)$ will converge to the latter of these (as Parameśvara noted, addition and subtraction would have to be reversed in order to compute the former).

The extra computation required by Parameśvara's rule for the corner altitude, compared to that of Lalla, is thus rewarded by its significantly better success. In essence, while Lalla's method undertakes eventually to reconcile incompatible positions of the sun on its day-circle and on its altitude-circle, Parameśvara's computes in a more self-consistent way, keeping the sun on its own day-circle and calculating altitudes and distances that actually occur at the given day and place. Although Parameśvara nowhere discusses these issues in his explanation, it seems likely that he was inspired to experiment with *koṇaśaṅku* iterations at least in part by the desire to obtain more reliable convergence.

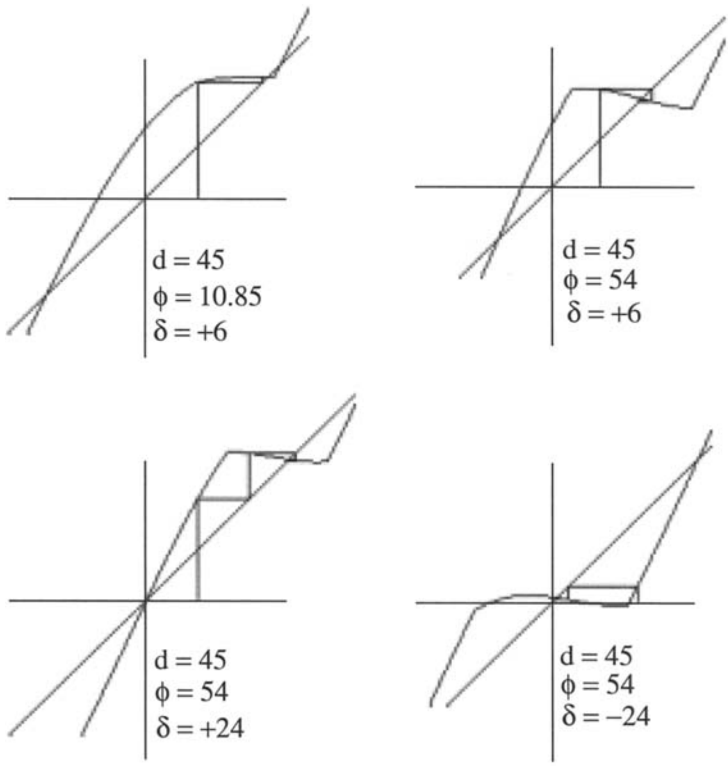


Figure 7: Orbits of $h(t)$

8 *Parameśvara's generalization of the new iterative rule*

Further evidence in support of this suggestion is provided by Parameśvara's subsequent verses, in which he combines this new iterative approach with his attempts to generalize the solution for the corner direction to any desired *d*:

> When one has thought of [some] desired direction [*d*] in a circle of [standard] radius, the Sine produced from that [angle] is determined, multiplied by the 'shadow', and divided by the radius. That is the 'arm' [or north-south distance, *b*] in the [given] direction. If the [actual] 'arm' of the [current] 'shadow' is equal to that, then the sun is standing in the desired direction. If it is not equal, their equality is to be established as before, from iteration.

The difference of the 'arms' is to be subtracted or added as before to the [past or future time in] *asus*, according to the [previous] explanation. And the result—plus one-half [of itself] or doubled in the case of slowness of approach to the desired amount, or minus one-third or halved if [the approach] is too fast—is always to be subtracted or added in this way in the iterative rule. [The procedure] is to be performed just like that, [and] the Sin altitude standing in the desired direction is fixed in this way by the stated rule.

[Commentary:] When one has determined the corner altitude in this way, again he states an iteration in the case of the determination of the altitude standing in [any] desired direction, [in the lines beginning] 'When one has thought of [some] desired direction...'. When the sun stands in a corner direction in a circle of [standard] radius, the Sine of one and a half [zodiacal] signs [i.e., 45°] should be the 'arm'. When [it is] in the middle [between] the directions of Agni [i.e., south-east] and Indra [east], the 'arm' is the Sine of a sign less one-quarter. When [it is] in the middle [between the directions] of Agni and Yama [south], the 'arm' is the Sine of two signs plus one-quarter. Having considered [any] desired direction in this way, and multiplied that 'arm'-Sine by the great 'shadow' [$\operatorname{Cos} a$] at that time, one should divide it by the radius. Then the quotient is the 'arm' for the [desired] direction in the given circle of [radius equal to] the given 'shadow'. Again, one should compute as before [the 'arm'] of the 'shadow'-hypotenuse at that time. Then if the 'shadow-arm' is equal to the 'arm' for the desired direction, the sun is standing in the desired direction. If [it is] not equal, [their] equality should be calculated by iteration. In that case, subtraction and addition are to be done according to the [previous] explanation. One who knows the explanation is an authority on this: this is the meaning. Here, *asus* equal to the difference between the 'shadow-arm' and the 'arm' for the direction are to be added or subtracted. The approach to the quantity determined by the iterative method is not always quick. So in the case of slowness of approach, when one has increased the amount to be [e.g.,] subtracted by one-half or doubled it, and subtracted or added [that quantity], the procedure is to be done [as specified]. When, because of excessive quickness of approach, the obtained quantity exceeds the desired quantity, then [that] diminished by one-third or halved is to be taken as the amount to be added or subtracted. And in this way the altitude in [any] desired direction is determined by an iterative rule.[18]

[18] *Goladīpikā* 4, 18cd–22 [Sarma, 48].

Now Parameśvara compares b not with its 'leg' $\sqrt{\mathrm{Cos}^2 a - b^2}$ but with the Sine of the desired direction-angle d scaled to the current Cos a, or Sin $d \cdot$ Cos a/R. Labeling this 'arm in the desired direction' b_d, we can express the steps of the iterative procedure as follows:

$$\begin{aligned}
H_0 &= \frac{\mathrm{Cos}\,\delta}{R} \cdot \mathrm{Sin}\left(t_0 - \mathrm{arcSin}\left(R \cdot \frac{\mathrm{Sin}\,\delta}{\mathrm{Cos}\,\delta}\frac{\mathrm{Sin}\,\phi}{\mathrm{Cos}\,\phi}\right)\right) \\
&+ \frac{\mathrm{Sin}\,\delta\,\mathrm{Sin}\,\phi}{\mathrm{Cos}\,\phi}, \\
\mathrm{Sin}\,a_0 &= \frac{H_0 \cdot \mathrm{Cos}\,\phi}{R}, \\
\mathrm{Cos}\,a_0 &= \sqrt{R^2 - \mathrm{Sin}^2\,a_0}, \\
q_0 &= \mathrm{Sin}\,a_0 \cdot \frac{\mathrm{Sin}\,\phi}{\mathrm{Cos}\,\phi} = \frac{H_0 \cdot \mathrm{Sin}\,\phi}{R}, \\
b_0 &= q_0 - R \cdot \frac{\mathrm{Sin}\,\delta}{\mathrm{Cos}\,\phi}, \\
b_{d0} &= \frac{\mathrm{Sin}\,d \cdot \mathrm{Cos}\,a_0}{R}, \\
t_1 &= t_0^{(\mathrm{asus})} + (b_{d0} - |b_0|).
\end{aligned} \tag{23}$$

Rewritten as a new iterative function $k(t)$, this becomes

$$\begin{aligned}
&t_{n+1} = k(t_n) = t_n + \\
&\sin d\sqrt{1 - (\cos\phi\cos\delta\sin(t_n - \sin^{-1}(\tan\delta\tan\phi)) + \sin\delta\sin\phi)^2} - \\
&\left|\sin\phi\cos\delta\sin(t_n - \sin^{-1}(\tan\delta\tan\phi)) + \sin\phi\sin\delta\tan\phi - \frac{\sin\delta}{\cos\phi}\right|.
\end{aligned} \tag{24}$$

In the formulation and discussion of this rule we see at last an explicit recognition of convergence problems.[19] Because obtaining the desired equality of b and b_d 'is not always quick', Parameśvara recommends multiplying their difference by a scale factor before

[19] Parameśvara also addressed the problem of convergence speed directly in a different work, when discussing an iterative approximation for the Sine of a given angle (see [Plofker 1996]). There too, he makes no reference to the fact that the slowly-converging iteration in question often fails to converge at all—although the new iteration he substitutes for it happens to solve both these problems.

adding it to the elapsed *asus*. The scale factor, which we shall call p, is 1.5 or 2 'in the case of slowness of approach', i.e., slow monotonic convergence; for 'excessive quickness', i.e., continually overshooting the fixed point via slow oscillating convergence, p is 2/3 or 1/2. We can rewrite this modified version of $k(t)$ as a closely related iterative function $\kappa(t)$:

$$t_{n+1} = \kappa(t_n) = t_n + p \cdot (k(t_n) - t_n). \qquad (25)$$

Its behavior is illustrated by the final sample problem in verse 4, 23, quoted above: namely, calculating Sine altitude in the direction east-southeast ($d = 22.5°$). The remainder of Parameśvara's commentary on that verse explains the solution:

> [Commentary:] ... Now, for the sake of determining the Sine altitude in the middle [between the directions] of Indra and Agni, the elapsed *ghaṭikās* of the day are considered [to be] 8. [On computing] with these, the Sine altitude is determined [to be] equal to 2516. Its 'shadow' [Cos a] is equal to 2343. The 'shadow-arm' [b] should be equal to 114. Here, because the Sine altitude [is] in the middle [between the directions] of Indra and Agni, the assumed 'arm', in a circle of [standard] radius, should be [the Sine of] a fourth part of a quadrant of a circle. Its 'arm' multiplied by the 'shadow', divided by the radius, is the 'arm in the given direction'. When the arm is so much in the circle of [standard] radius, then how much [is it] in the circle [with radius] Cosine altitude? this is the proportion. The 'arm in the given direction' thus determined should be equal to 896. Here, because of the inequality of the 'arms' and the greater [size] of the 'arm in the given direction', when one has added the difference of those 'arms' to the elapsed *asus* of the day, and calculated as before the Sine altitude etc. corresponding to that elapsed [amount] of the day, the two 'arms' are to be determined [afresh]. Therefore the elapsed *asus* of the day, added to the difference of the 'arms', should be equal to 3662. The Sine altitude calculated from that is equal to 2971. Its Cosine altitude is equal to 1730. The 'shadow-arm' is equal to 201; the 'arm in the given direction' is equal to 662.
>
> Here, slowness of approach [between] the 'shadow-arm' and the 'arm in the given direction' is apparent. So when one has added their difference, doubled, to the elapsed *asus* of the day, the [quantities] corresponding to that, beginning with Sine altitude, are to be determined. The elapsed *asus* of the day, so computed, should be equal to 4584; the Sine altitude computed from that is equal

> to 3314, the Cosine altitude equal to 915. The 'shadow-arm' is equal to 266, the 'arm in the given direction' equal to 350. Their difference also, doubled, is to be added to the elapsed *asus* of the day. Then the elapsed *asus* of the day are equal to 4752, the Sine altitude equal to 3352, the Cosine altitude equal to 764. The 'shadow-arm' is equal to 274, the 'arm in the given direction' equal to 292. When one has added the difference of the 'arms', doubled in this case too, to the elapsed *asus* of the day, the resulting *asus* are equal to 4788. Their Sine altitude is equal to 3360, the Cosine altitude equal to 728. The 'shadow-arm' is equal to 276, the 'arm in the given direction' equal to 278. Now the difference of the 'arms' is only 2. The increase in the Sine altitude from the addition of that, doubled, to the elapsed *asus* of the day should be [a length corresponding to] only one arcminute. So the Sine altitude in the middle [between the directions] of Indra and Agni is equal to 3361. The Sine altitude in [any] desired direction is to be determined in the same way.[20]

Is $\kappa(t)$, the generalization of $h(t)$ to arbitrary d, an improvement upon Parameśvara's previous generalization of $g(\sin a)$? Comparing the sample results in Figure 8 with the corresponding ones in Figure 6, we observe that once again the new approach of approximating t instead of $\sin a$ is far more reliable (although in the case of Parameśvara's example, the new function actually converges a little more slowly than the old one would). So all the serious convergence problems with the original *koṇaśaṅku* iteration and its variants are at this point successfully resolved, some seven centuries after its initial appearance in Lalla's text.

9 *Conclusion*

Although fixed-point iterations play an important supporting role in much of ancient and medieval mathematical astronomy, it is very difficult to get a clear idea of how their inventors thought about them and developed them. The *koṇaśaṅku* iterations in Sanskrit texts shed some light on aspects of this question in the medieval Indian tradition. As these texts' profusion of iterative rules on similar topics suggests, there were few or no methodological qualms about the use of such approximations in place of available exact solutions. Apparently the former were not even

[20] *Goladīpikā* 4, 23 [Sarma, 51–2].

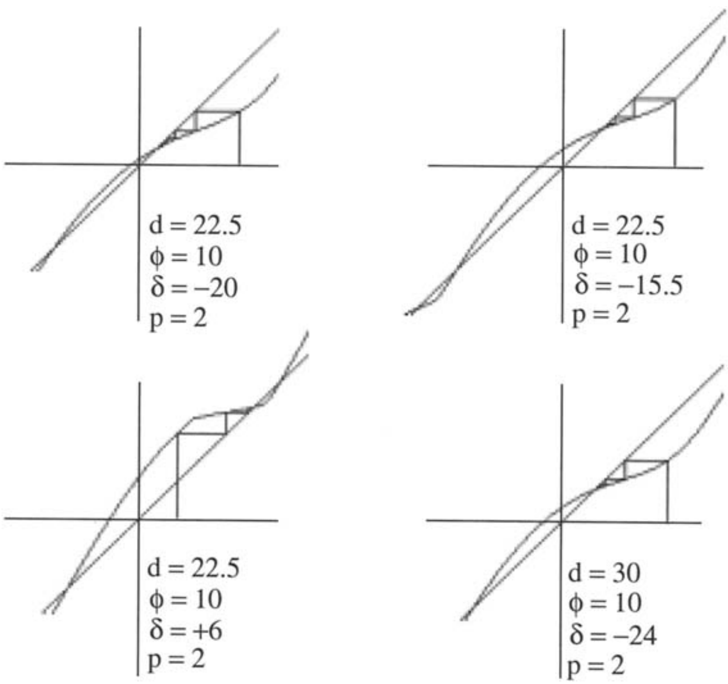


Figure 8: Orbits of $\kappa(t)$ with $d < 45°$

distinguished from the latter by terms such as 'rough' or 'approximate', as were many practical procedures like estimating π as $\sqrt{10}$ or interpolating in abbreviated Sine tables with large intervals. Evidently, it was understood that the ultimate fixed result of a (convergent) *asakṛt* or iterative rule was as accurate as the equivalent from a *sakṛt* or closed-form one.
Almost nothing is recorded about the opinions of mathematicians in this tradition concerning iterations that were not convergent. But our scrutiny of *koṇaśaṅku* rules and their results indicates that mathematicians knew a good deal more about them than met the reader's eye. The innovations that appear in the work of Mallikārjuna and Parameśvara are very satisfactorily explained as attempts to deal with the cases where Lalla's original iteration converged slowly or not at all. In Parameśvara's case, the impetus

to improve the original rule very likely came from his efforts to generalize it for arbitrary d, where its convergence failures become even more noticeable.

In fact, it is Parameśvara's work that contains the first known explicit reference to convergence problems in these rules; he also mentions in passing the existence of different approaches to the new *koṇaśaṅku* iteration, where some users 'guess' the successive approximations 'by means of one's intelligence' while 'others' rely on a deterministic algorithm. Such parenthetical remarks, in addition to the analyses discussed above, reveal glimpses of active mathematical experiment and debate among Indian mathematicians concerning the behavior of iterative rules, much richer and more complex than we might infer from their terse formulaic statements of the rules themselves.

Bibliography

B. Chatterjee, *Śiṣyadhīvṛddhidatantra of Lalla*, 2 vols, New Delhi, 1981.

R. L. Devaney, *A First Course in Chaotic Dynamical Systems: Theory and Experiment*, Reading MA, 1992.

S. Dvivedi, *Brāhmasphuṭasiddhānta of Brahmagupta*, Benares, 1901/1902.

E. S. Kennedy & M.-Th. Debarnot, 'Al-Kāshī's Impractical Method of Determining the Solar Altitude', *Journal for the History of Arabic Science*, 3, 2, 1979, pp. 219–27.

T. S. Kuppanna Sastri, *Mahābhāskarīya of Bhāskarācārya*, Madras, 1957.

D. Pingree, *Census of the Exact Sciences in Sanskrit*, Series A, vols 1–5, Philadelphia, 1970–94.

——. 'History of Mathematical Astronomy in India', in *Dictionary of Scientific Biography*, vol. 15, New York, 1978, pp. 533–633.

K. Plofker, 'Use and transmission of iterative approximations in India and the Islamic world', in *From China to Paris: 2000*

Years Transmission of Mathematical Ideas, ed. Y. Dold-Samplonius et al., Stuttgart, 2002, pp. 167–86.

——. 'An example of the secant method of iterative approximation in a fifteenth-century Sanskrit text', *Historia Mathematica*, 23, 1996, pp. 246–56.

K. V. Sarma, *The Goladīpikā by Parameśvara*, Adyar Library Pamphlet Series, 32, Madras, 1956–7.

B. Śāstrī, *The Sūryasiddhānta*, rev. G. D. Śāstrī, Varanasi, 1989.

K. S. Shukla, *Vaṭeśvarasiddhānta and Gola of Vaṭeśvara*, 2 vols, New Delhi, 1986.

Sanskrit Scientific Texts in Indo-Persian Sources, with special emphasis on *siddhāntas* and *karaṇas*

S. M. Razaullah Ansari

1 *Introduction*

The pioneering work of Prof. David Pingree on the transmission of ancient Indian astronomy into Islamic countries in the early period is quite well known.[1] We may mention here briefly that at least two Indian *siddhāntas* are considered to have been transmitted during or shortly after Caliph al-Manṣūr's reign (754–775): namely, the *Āryabhaṭasiddhānta* of Āryabhaṭa (born in 476 A.D.) and a work (possibly entitled *Mahāsiddhānta*) of the school of the *Brāhmasphuṭasiddhānta* (written in 628 A.D. by Brahmagupta, born in 598). The latter was translated from Sanskrit into Arabic by Muḥammad ibn Ibrāhīm al-Fazārī and colleagues in about 775 as the *Zīj al-Sindhind*, or *Zīj al-Sindhind al-Kabīr*.[2] The *Sindhind* tradition was employed by al-Fazārī's contemporary Yaʿqūb ibn Ṭāriq (d. 796) and particularly by Muḥammad ibn Mūsā al-Khwārizmī (d. 850) in his *Zīj*, through which it spread even to the scholars of Muslim medieval Spain (*al-Andalus*).[3]

So far as the genre of Indian astronomical literature called *karaṇas* is concerned, we may cite here the *karaṇa* of Brahmagupta, the *Khaṇḍakhādyaka* (epoch 665), which was translated or adapted into Arabic in 735 in Sind (India) as *Zīj al-Arkand*,[4] two excerpts from which are known as *Zīj al-Jāmiʿ* and *Zīj al-Hazūr*.[5] The latter was compiled in Qandahar (formerly in India, now in Afghanistan).

[1] [Pingree 1981], wherein many of his other works on the subject are cited.

[2] [Pingree 1981, 18–19, 21 n. 51]; [Sezgin 1978, 16–19], and for Indian sources *ibid.*, pp. 116–20.

[3] [Pingree 1996a].

[4] [Pingree 1981, 33]; see also [Sezgin 1978, 120].

[5] [Sezgin 1978, 120] quotes (in n. 3) the Arabic text of the relevant passage from *Kitāb ʿIlal az-Zījāt* by al-Hāshimī (fl. 9th/10th c.), who reported this information.

For the record, we may also mention here that the Arabic translation of Vijayananda's[6] *Karaṇatilaka*[7] (compiled in 966 A.D.), carried out by Abū al-Rayḥān al-Bīrūnī (973–1048), is extant even today in the private collection of Dargāh Pīr Muḥammad Shāh in Ahmedabad (India).[8] Its Arabic title is *Ghurrāt al-Zījāt.* The Arabic text with a facsimile of the manuscript has been published by N. A. Baloch,[9] and an English translation by F. M. Quraishi.[10]

2 *Astronomy in Muslim India*

In order to appreciate the astronomical context of this period, we summarise in this section the development of astronomy during the pre-Mughal (1191–1526 AD) and Mughal (1526–1857) eras of Indian history. Here we are confining ourselves only to Arabic and Persian sources. It is clear that during this time, the above-mentioned trend of transmission reversed its direction: sources from the Islamic scientific traditions in the West Asia found their way to India.

Astronomical sciences developed in the Islamic countries, particularly during the ʿAbbāsid Caliphate and its successors, passed from the early assimilation stage to the creative stage[11] which in turn culminated in the establishment of Naṣīruddīn al-Ṭūsī's Marāgha school of Islamic theoretical astronomy.[12] Is-

[6] In the Arabic text the name is Bijyānand son of Jayānand.

[7] The translator, al-Bīrūnī, uses the title *Zīj-i Bijyānand.*

[8] It is a unique manuscript; no other copy has been found to date. Furthermore, the Sanskrit original seems to have been lost. The manuscript was first discovered by M. Nizam [Nizam 1929–30].

[9] [Baloch 1973], with an excellent introduction (of 74 pages), and appended excerpts from other works of al-Bīrūnī wherein he refers to the *Karaṇatilaka*; namely, *Kitāb Taḥqīq mā li'l-Hind* (*Indica*), *Al-Qānūn Masʿūdī*, *Risāla Tamhīd al-Mustaqarr*, and *Ifrād al-Maqāl.*

[10] [Quraishi 1978] presents an English translation of the Arabic text with calculations, notes, and references (the Arabic text itself is also appended), but without mentioning the *first* study of the *Karaṇatilaka* of [Rizvi 1963–65]. However, he refers to the discovery of this manuscript of [Nizam 1929–30].

[11] [Sezgin 1978] deals expertly in his Introduction with the assimilation of scientific astronomy (Sec. D, pp. 16–19) and with the beginning of the 'creative' period (Sec. E, pp. 19–36).

[12] Research on this school was initiated by E. S. Kennedy and his colleagues. It is based particularly on the Arabic sources pertaining to the critique of

lamic practical astronomy is embodied in the *Zījes* and writings on astronomical-mathematical instruments.[13] It was transferred to India during the aforementioned periods, when scholars from West and Central Asia (including Iran) flocked to the courts of Indian Sultans and Mughal emperors. That exodus was a boon to the development of Islamic astronomy in India. The standard astronomical and mathematical sources in Arabic and Persian accompanied those scholars, and multitudes of those primary sources are still extant in various Indian libraries. For instance, we possess in manuscript form treatises on both theoretical and practical astronomy by Naṣīruddīn al-Ṭūsī, Quṭbuddīn Shīrāzī, Maḥmūd al-Chaghmīnī, Jamshīd Ghayāthuddīn al-Kāshī, Ulugh Beg, ʿAlā'uddīn ʿAlī Qushchī, and Bahā'uddīn al-ʿĀmilī, to name just a few.[14] All this material and the patronage of the Islamic astronomers by Indian rulers led to the development of Indo-Islamic astronomical literature, particularly in the Indo-Persian language.[15] For instance, the following *Zījes* may be noted:[16]

1. *Zīj-i Nāṣirī*, dedicated to Naṣīruddīn Maḥmūd bin Sulṭān Shamsuddīn Iltutmish (ruled in Delhi 1246–65), the author being Maḥmūd bin ʿUmar (MS in Tabriz, Iran).

2. *Zīj-i Jāmiʿ Maḥmūd Shāhī Khiljī*, compiled sometime during 1438–60 by an anonymous Indian scholar (MS in Oxford), dedicated to Sulṭān Maḥmūd Shāh Khiljī (reigned 1435–69).

Ptolemaic astronomy; cf. [Saliba 1994], in which relevant articles on this topic are referenced.

13 Cf. [King 1987], the standard work particularly on Islamic astrolabes. See also [Ansari & Ghori 1985–87], especially the Introduction (pp. 215–16). We may add, for instance, that a large number of copies of the famous treatise of al-Ṭūsī, *Twenty Chapters on the Astrolabe*, are extant in India and Pakistan; this work was even translated into Sanskrit. Moreover, a couple of manuscript copies of the standard work on the astrolabe by al-Bīrūnī, *Istiʿāb al-Wujūh al-Mumkina fī Ṣanʿat al-Asṭurlāb*, exist in the libraries at Rampur and Aligarh Muslim University.

14 Cf. [Ansari 1995], in the appendix of which we have listed almost all the astronomical treatises and their commentaries available in manuscript collections of Indian libraries.

15 Cf. [Ansari 1997/2001], in which we have presented the first study on this topic.

16 Cf. [Ghori 1985], [Ansari 1995, 281–4]. See also '*Note added in proof*'.

3. *Tashīl Zīj-i Ulugh Beg*, a commentary on Ulugh Beg's Tables (ZUB) by Shaykh Chānd ibn Bahāuddīn, the court astronomer of the emperors Humāyūn (reigned 1530–56) and his son Akbar (reigned 1556–1605). It may be noted that Akbar ordered during his reign the translation of ZUB into Sanskrit, which was carried out by a team of Muslim and Hindu scholars. One of its copies is extant in the City Palace Museum of Jaipur (India).

4. *Zīj-i Shāhjahānī*, dedicated to the emperor Shāh Jahān (reigned 1628–58), compiled by the court astronomer Farīduddīn bin Masʿūd Dehlawī (d. 1630). The emperor's Hindu court astronomer Nityānanda translated it into Sanskrit. One copy of this translation is in the City Palace Museum (Jaipur); three manuscript copies are in the Khaṣ Mōhar Collection (Jaipur).[17]

5. *Zij-i Raḥimi*, compiled also by Farīduddin Dehlawī in about 1628. Its unique manuscript is extant in the Holy Shrine Library in Mashhad (Iran).[18]

6. *Zīj-i Muḥammad Shāhī* (ZMS), compiled for Maharaja Sawai Jai Singh (1686–1743) and dedicated to the emperor Muḥammad Shāh, has already been treated elsewhere.[19] This is the most important Zīj of Mughal India. In fact it replaced throughout much of the Islamic world (e.g., Iran and Central Asia) even the standard *Zīj-i Ulugh Beg* prepared in the fifteenth century at Samarqand. ZMS was compiled by Mirzā Khayrullāh Muhandis (d. 1747) who belonged to a distinguished family of mathematicians. (The critical edition of the Persian text of ZMS will appear shortly.)[20]

7. *Zīj-i Ashkī*, by Kundan Lāl Ashkī, son of Mannū Lāl Falsafī, written in 1816. An autograph manuscript of 62 pages is in

[17] For details about this Sanskrit translation, see [Ansari 1995, 277] and [Pingree 1999, 77].

[18] For the first short description, see [Ghasemlou & Naderi 2002].

[19] Cf. [Ghori 1985, 36–41]; [Ansari 1995, 283]. For its relation to de La Hire's tables, see [Pingree 1999] and [van Dalen 2000].

[20] See the forthcoming series of articles by us on 'ZMS and its Significance in the Zīj-Literature', to be published in the *Indian Journal of History of Science*.

Hyderabad. This *Zīj* is written in the traditional style.

8. *Zīj-i Bahādurkhānī*, by Ghulām Ḥusayn Jaunpūrī, written in 1838 and printed in 1855 in Benares. It is largely based on the *Zīj-i Muḥammad Shāhī*. Evidently it follows the style of Central Asian *Zīj*es.[21]

Besides the astronomical and mathematical tables, a number of treatises particulary on astrolabe were also written.[22] In fact, a whole school of astrolabe makers in India sprang up, which is known as the Lahore astrolabists, the manufactured specimens of which are to be found through out the world.[23]

A by-product of the promotion of the science of astronomy during the period in question was the interaction between the scholars of traditional Indian astronomy (the Sanskritists) and those of Islamic astronomy (scholars of Arabic-Persian). That interaction gave birth to the translation of several Arabic-Persian sources into Sanskrit: for instance, the translation of *Zīj-i Ulugh Beg* by a team of Muslim and Hindu scholars during Emperor Akbar's reign, or that of the *Taḥrīr al-Majisṭī*, the recension of Ptolemy's *Almagest* by Naṣiruddīn al-Ṭūsī.[24] Without going into the details of this interaction,[25] we attempt in the following an account of the reverse trend: that is, the translation of Sanskrit texts into the Indo-Persian language.

3 *Persian Translation of Scientific Texts in Sanskrit*

3.1 *Mathematical Texts*

[21] [Ghori 1985, 42–4]; [Ansari 1995/96], 'Ghulām Ḥussain Jaunpūrī and his Zīj-i Bahādurkhānī.

[22] We may mention here particularly a Sanskrit treatise on the astrolabe, translated or adapted from some Arabic or Persian work, entitled *Yantrarāja* ('King of Instruments'), composed by Mahendra Sūrī in about 1370. Mahendra was a court astrologer of Sulṭān Fīrūz Shāh Tughlaq (reigned 1351–88). A number of commentaries were written on this text, of which there are extant about 100 manuscript copies [Sarma 1999, 147]; see [Ohashi 1977, 211] for details.

[23] Cf. Sarma, S.R. (1994 a,b)

[24] The title of this Sanskrit translation is *Samrāṭ Siddhānta*; it was carried out by Jagannātha (b. 1652), and was commissioned and/or sponsored by Maharaja Sawai Jai Singh [Sen 1966, 90].

[25] See [Ansari 1995], especially pp. 276–9; see also [Sarma 1998].

For the sake of completeness, we may mention two non-astronomical mathematical works of Bhāskara II (b. 1114) which, as is well known, were translated into Persian in medieval India:

1. *Līlāvatī*, translated by the famous scholar Abu'l Fayḍ Fayḍī (1547–96) in 1587 at the instance of Emperor Akbar. A large number of manuscript copies are extant in the libraries of India and Pakistan. The text was published lithographically from Calcutta in 1827, 1832 and 1854. It was also translated into English by J. Taylor (published from Bombay, 1816) and partly by H. T. Colebrooke (London, 1817).[26]

2. *Bījagaṇita*, translated by ᶜAtā'ullāh Rushdī or Rāshidī (son of Aḥmad Maᶜmār, the architect of the Taj Mahal), dedicated to the emperor Shāh Jahān (reigned 1628–58) and composed in 1634–5. The English translation of the Persian text was made by E. Strachey, London 1813.[27]

3.2. *Astronomical Texts*

3.2.1 *Astrological-astronomical Works.* We know of two such texts which were translated from Sanskrit into Indo-Persian in the Sultanate (pre-Mughal) period:[28]

1. *Dalā'il-i Fīrūzī*, translated by ᶜIzzuddīn Khālid Khānī (or Khafī) by the order of Sulṭān Fīrūz Shāh Tughlaq (1351–88). The author 'was [then] one of the poets and *munshīs*. It was an astrological tract in verse, and dealt with the rising and setting of the seven planets, and their good and evil import, and of auguries and omen.' The translation was seen in Lahore in 1591 by ᶜAbdul Qādir Badaoni (Badāyūnī), who is the author of this report.[29]

2. *Tarjumah-i Bārāhī* or *Kitāb Bārāhī Sanghtā*, a translation of Varāhamihira's *Bṛhatsaṃhitā*, by ᶜAbdul ᶜAzīz Shams

[26] [Sen et al. 1966, 25–26]; [Storey 1972, 4–5].
[27] [Sen et al. 1966, 20–22]; [Storey 1972, 5].
[28] [Jalali & Ansari 1985].
[29] [Badaoni 1973, 332]. Badaoni (1540–1615) was a famous translator of Sanskrit classics (for instance, the *Mahābhārata*, the *Rāmāyaṇa*, and a *History of Kashmir*) into Persian at the instance of Emperor Akbar.

Thānesarī, by the order of Sulṭān Fīrūz Shāh Tughlaq. Six manuscript copies of the Persian text have been found by us so far: namely, 2 in Aligarh Muslim University, 1 in the India Office (London), 1 in A. P. State Central Manuscript Institute/Library (Hyderabad), and 2 in the Shīrānī collection at the Punjab University Library (Lahore).[30]

Another translation of the same Sanskrit text with the title *Nujūm Mālā* was composed by Pandit Rāj Bhīm at the instance of Nawāb Ḥaydar Beg Khān Nuṣrat Jang in 1789–90. Monzavi lists four manuscripts (3 in Lahore, 1 in Karachi).[31] One manuscript of the same is in the Subhanullah Collection, MS 200/6, in M.A. Library (Aligarh Muslim University).

Recently we have found another manuscript of the Persian translation of the *Bṛhatsaṃhitā* by Kirpa Nath Khatri ibn Rai Lahorimal of Sialkot district in Punjab, entitled *Zā'ichā Nāmah*, in the Library of Rajasthan Institute of Persian and Arabic (Tonk). There are two manuscripts, No. 3205 (copied in 1824) and No. 3267 (copied in 1822).

3.2.2. *Karaṇas.*

1. *Zīj-i Muẓaffarshāhī*, written during the reign of Sulṭan Muẓaffar Shāh II (1511–26), son of Maḥmūd Shāh Begarah Gujrātī. The unique anonymous manuscript copy of this *Zīj* is in the Shīrānī Collection, Punjab University Library, Lahore (Pakistan), as MS 6261/1, with 34ff. It was composed in 931 AH/1525 AD. The text of the *Zīj* has been published by Aftāb Asghar [Asghar 1980], who conjectured that the scribe was one Gul Muḥammad. In the colophon, a sort of title is noted by the scribe, viz., *'al-Shams w'al-Qamar'*, which in turn is presumably taken from the Quranic verse: *'Ash-Shamsu w'al-Qamaru bi Ḥusbān'* (55:5). The present title, *Zīj-i Muẓaffarshāhī*, is given by Ḥāfiẓ Maḥmūd Shīrānī, who argued that according to the *Zīj* tradition it may be named after the patron in whose reign the work was compiled. See 'Note added in proof'.

[30] See details in [Jalali & Ansari 1985].
[31] [Monzavi 1983, 305–306].

The author of the *Zīj* mentions in an introductory note that this *Zīj* is based on a compilation by Indian scholars (*Ḥukamā'-i Hind)*. Although the author quotes no source, on the basis of the Sanskrit technical terms used in the concise prescriptions for calculating the planetary parameters, we have identified it as a *karaṇa.* The text is divided into ten short chapters, each of which consists of six or seven sections *(faṣl)*. There are chapters corresponding to each planetary body, the moon, and its nodes; the last chapter is on miscellaneous topics. (An English translation of this work is currently in preparation.) The importance of this text lies in the fact that it is the second Persian exemplar of the pre-Mughal period which was translated from Sanskrit and which is extant today—the first being the translation of Varāhamihira's *Bṛhatsaṃhitā*, as mentioned above.

2. *Sharḥ Frankūhal* (or *Frankōhal*) is a commentary on the *Karaṇakutūhala*, written in 1183 AD by Bhāskara II.[32] The commentary is anonymous; it was composed in 1809 Bikrāmī/1752 AD and its unique manuscript of 159ff, No. sh 520 bhā, is extant in the Punjab Public Library (Lahore).[33]

3. *Karaṇkatū(ō)hal* is another anonymous manuscript extant in the collection of Punjab University Library (Lahore). It was copied by Gul Muḥammad, who also copied the *Zīj-i Muẓaffarshāhī* (see No. 1 above). Therefore this manuscript copy (No. sh /3/102/6261, of 68ff) may be dated to the sixteenth century. It has not been identified by the cataloguer Monzavi.[34] We conjecture that it may be a Persian translation of Bhāskara's *Karaṇakutūhala.*

3.2.3. *Siddhāntas.* The practical Islamic astronomy developed during the medieval Indian period was based solely on the Central Asian *Zīj* tradition, as briefly delineated in section 2 above, and we have not found any reference to date for the direct utilisation of *siddhāntic* (theoretical) astronomy during the 12th–

[32] [Sen et al. 1966, 31–32].
[33] [Monzavi 1983, 289–90]. See also [ʿAbbāsī 1963, 270].
[34] [Monzavi 1983, 363] (entry no. 690). Monzavi does not identify the author of the Sanskrit original. See also 'Note added in proof'.

17th centuries by Arabic/Persian-knowing Indian astronomers.[35] However, this situation changed at the close of the 18th century and during the first half of the 19th century. In the following we present our findings on this shift of interest and the resulting transmission.

1. One of the very few commentaries on *Zīj-i Muḥammad Shāhī* (compiled in the 18th century and sponsored by Maharaja Sawai Jai Singh) is by an Indian scholar ʿAbdullāh bin ʿAẓm̃uddīn bin Muḥammad Khān, called Mahārat Khān (fl. 18th c.). A number of its manuscript copies are available in various libraries.[36] The title of the commentary is *Tashīl Zīj-i Muḥammad Shāhī*; it is a sort of astronomical ready-reckoner with a large number of auxiliary tables.

   Here we are particularly interested in MS No. 3641 of the Library of the Arabic and Persian Research Institute of Rajasthan (Tonk, India). In this manuscript we find an appendix to the subsection 'Crescent Visibility according to the method of Indian scholars (*Ḥukamā'-i Hind)*'.[37] It comprises a number of chapters, each of which consists of a number of sub-sections (*faṣl)*.[38] Their headings are: About crescent visibility; true daily motion of a planet (*karaṇabhukti)*; appearance and disappearance of planets; knowing about the lunar and solar eclipses; about solar ingress into a zodiacal sign (*saṅkranti)*, and about the ascendant (*spaṣṭalagna*). At several places in the text, the scribe of this appendix (not Mahārat Khān) named his sources. In Persian orthography they are given as: *Grahalāghū(va)*, *Siddhāntmañjarī*, *Siddhāntdinchandrī*, *Bhāstī* and *Siddhāntacharat(t)ar*; the last one has not been identified. At one place he names also '*ṣāḥib-i* [author of] *Līlāvatī*', i.e., Bhāskara II. Besides these

[35] However, the converse was not true. For instance, the Hindu court astronomer of Emperor Shāh Jahān, Nityānanda, in his *Sarvasiddhāntarāja* (written in 1639) employed Islamic planetary models and even adapted Islamic mathematical astronomy for the framework of Indian *yuga* astronomy. See details in the very interesting paper [Pingree 1996b].

[36] See [Storey 1972, 94] for details.

[37] We are at present unable to compare other manuscript copies of Mahārat Khān's commentary to find out whether or not such an appendix is included in any other copy.

[38] Tonk MS 3641, ff. 150–56.

Sanskrit sources he also used the *Zīj-i Sulaymānjāhī*.[39] We have identified most of his Sanskrit sources as follows:

1.1. *Grahalāghava*, also known as *Siddhāntalāghava* (written in 1520), is the very famous work of Gaṇeśa Daivajña (b. 1507), son of Keśava Daivajña of Nandīgrāma (near Bombay). It is actually a *karaṇa*, since without using trigonometrical calculations, it gives simple arithmetical methods for carrying out astronomical calculations. It is in use even today in many Indian states by calendar (*pañcāṅg*) makers.[40]

1.2. *Siddhāntamañjarī* is an elementary treatise on astronomy, authored by Mathurānātha Vidyālaṅkāra (ca. 1609). Note that two other titles of this treatise are *Suryasiddhāntamañjarī* and *Ravisiddhāntamañjarī*.[41] The author of the appendix mentions also that it is based on the '*Sūraj Siddhānt*'.

1.3. *Siddhāntdinchandrī* is known to us simply as *Dinācandrikā*. The author was Rāghavānanda Cakravartin (ca. 1599). It is actually a set of astronomical tables with brief instructions for the construction of a calendar.[42]

1.4. *Bhāstī*, i.e., *Bhāsvatī*, is a well known *karaṇa* written in ca. 1099. It is based on Varāhamihira's work and the *Suryasiddhānta*. The author is Śatānanda who lived in the sacred town of Pūrī. With the rules given in this work, calculations for the occurrence of eclipses can be carried out accurately. In fact, our author of the appendix mentions this source in the section 'Lunar and Solar Eclipses', especially for calculating the position of the first lunar node (*Rāhu*).[43]

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[39] See section 2.7 below. In two marginal notes he mentions Imāmuddīn (Riyāḍī) and (Mirzā) Khayrullāh Muhandis also. For the former, cf. section 2.1 below and for the latter n. 52.

[40] Cf. [Sen et al. 1966, 64]; also [Bose et al. 1971, 100], and for greater detail [Rao 2000, 158–69].

[41] [Sen et al. 1966, 143].

[42] [Sen et al. 1966, 175].

[43] [Sen et al. 1966, 193–4].

The author of the appendix illustrates his method of calculations by examples in which the years 1257, 1258 and 1259 AH appear (that is, the years 1841–43 AD), which thus could be taken as the time of composition of this appendix, and for that matter, this copy of the commentary.[44] Therefore we assume that the author of this appendix is the scribe of this copy of the commentary. The city of Lucknow is mentioned in several places. The copious use of Sanskrit terminology and methods indicates the author's in-depth knowledge of ancient Indian astronomy and its Sanskrit sources.

2. Apart from the above-mentioned appendix, our survey has brought to light a number of *Zīj*es in Persian which are either direct translations or adaptations of well-known *Siddhāntas.* We list them in the following.[45]

   2.1. *Zīj-i Āṣaf Jāhīl* (18th c.). It purports to be based on the *Sūryasiddhānta*; excerpts from it are found in a *Biyāḍ* (a Notebook) of the famous mathematician Imāmuddīn Riyāḍī (d. 1732), son of Luṭfullāh Muhandis. The manuscript of this Notebook is in the Salar Jung Museum Library (Hyderabad). This *Zīj* is dedicated to the first ruler (*Niẓām*) of Hyderabad, Niẓām al-Mulk Āṣaf Jāh (reigned 1720–48).[46]

   2.2. *Zīj-i Niẓāmī*, (1780), compiled by Khwājah Bahādur Ḥusayn Khān alias Sayyid Abu'l Fatḥ.[47] It is dedicated to the fifth ruler of Hyderabad, Nawwāb Niẓām ʿAlī Khān (reigned 1762–1802).[48] There are two

[44] Note that the earliest manuscript of this commentary by Mahārat Khān is in Leiden and is dated 1770, which is chronologically followed by another manuscript copy in the Mullāh Fīrūz collection (C. R. Cama Research Library) in Bombay, dated 1791; see [Storey 1972, 94].

[45] The following set of Indian *Zīj*es has been described in some detail in [Ansari 1995, 283–4]. For lack of space we can give here only a little more information; we intend to give a detailed account of all these *Zīj*es elsewhere.

[46] For the first mention of this *Zīj*, see [Ansari 1996–97, 15].

[47] In MS 296, f. 3a–b, the author gives his family tree: his forefathers had migrated from Bukhara to India in 1657 and had been in the service of Emperor Aurangzeb initially, and later shifted to the Deccan (Hyderabad) to serve the first Niẓām.

[48] The author's title is actually *Zīj-i Niẓām ʿAlī Khānī*, as given on f. 11b

manuscripts of this work, Riyāḍī 112 and Riyāḍī 296. Both seem to be autographs. The second manuscript is more detailed. Both are in A. P. Government Oriental Manuscripts Library (Hyderabad).[49] This *Zīj* is mainly based on the *Sūryasiddhānta*, but the author mentions treatises of 'other predecessors' which he studied: namely, *Grahalāghava*, *Tithicintāmṇī*, *Laghucintāmṇī*, *Brahamatul(ya)*,[50] *Narasiṃha* and *Rāmvinod*.[51] It is interesting that the author sometimes quotes Sanskrit verses (*ślokas*) in Arabic *naskh* script with marks for vowels.[52]

2.3. *Zīj-i Sarūmanī* (1797), translated by Ṣafdar ʿAlī Khān from the *Siddhāntaśiromaṇī* of Bhāskara II, written by him in 1150 AD. Information about the translation is given in *Zīj-i Ṣafdarī* (see 2.6 below). This *Zīj* was dedicated to Arasṭū Jāh.

2.4. *Zīj-i Hindī* (1804/5), compiled by Gul Beg Munajjim ('the astronomer'), whose grandfather was the son of Mirzā Khayrullāh Khān.[53] Only two manuscripts of this *Zīj* are extant, one in Raza Library (Rampur, India), MS No. 1221, ff. 106–30; the other in the National Museum (Karachi, Pakistan), MS No. 1959-

of MS 296, with the date of writing 1194 AH/1780 AD. *Sharḥ Zīj-i Niẓāmī* is the title under which it is indexed in the library. [Storey 1972, 100] could not identify it, since it was not available to him.

[49] This is the successor of the famous Āṣafyah Library or State Central Library, Hyderabad.

[50] MS 296, f. 5b. The first three works are by Gaṇeśa Daivajña (see section 1.1 above). *Tithicintāmaṇi* and *Laghutithicintāmaṇi* are identical; see [Sen et al. 1966, 66]. For a recent work refer to [Ikeyama & Plofker 2000]. *Brahamatul(ya)* or *Brahmasiddhāntatulya* is in fact the *Karaṇakutūhala* of Bhāskara II; see [Sen et al. 1966, entry 5, 31].

[51] These two names are mentioned on f. 2a, MS 112. Narasiṃha (ca. 1687–1747) wrote a treatise on the determination of *tithis* etc. [Sen et al. 1966, 149]. The *Rāmavinoda* of Rāma (or Rāmacandra) is based on the *Sūryasiddhānta*. It is interesting to note that 'this work was written at the instance of Mahārāja Rāmdāsa, a minister at the court of Akbar' [Sen et al. 1966, 179].

[52] For instance, in MS 296, f. 8a, a quotation from the *Sūryasiddhānta* is given.

[53] Khayrullāh son of Luṭfallāh was actually the author of *Zīj-i Muḥammad Shāhī*. He was the director at Jai Singh's observatory at Delhi.

409/2, pp. 148–76.[54] We have estimated the date of the composition by converting the 1726 'Śākha' (Śaka) mentioned (on f. 107a of the Rampur MS) in connection with the determination of the *tithi*. On f. 106b, the author declares; 'I wish to translate Makrandī, i.e., *Zīj-i Hindī*, into Persian, so that men of this field can be benefited by it and can be able to compile an Indian horoscope (*Patrah-i Hindī*).' This Makrandī may be identified with the Makaranda whose work *Makarandasāraṇī* or *Tithipatra* (written in 1478 AD) is known to us. It is an astronomical work for the compilation of calendars, based on the *Sūryasiddhānta*.[55] It contains many tables; Gul Beg's treatise comprises also 34 tables (ff. 113–30).

2.5. *Zīj-i Mīr ʿĀlamī* (1807/8) by Ṣafdar ʿAlī Khān bin Muḥammad Ḥusayn Khān bin Muḥammad Ismāʿīl Shīrāzī, extant only as a unique manuscript, MS No. Riyāḍī 301, with 162ff., in A. P. Govt. Oriental Manuscripts Library (Hyderabad). It appears to be dedicated to Mīr ʿAlam (d. 1808).[56] On f. 1b, the author states that this is a translation of *Kitāb Grahchandrikā*. Actually this is the first draft of the following *Zīj*.

2.6. *Zīj-i Ṣafdarī* (1819) by Ṣafdar ʿAlī Khān, MS Hayat 15 in Salar Jung Museum Library (Hyderabad), with 183ff. It seems to be an autograph copy. It is also a translation of *Grahchandrikā*, which may be identified with the *Grahacandrika Gaṇita* ('Calculations for Planets and Moon') by Appaya, son of Marla Perubhaṭṭa (ca. 1491).[57] Besides him he quotes also *Grahalāghva*. In connection with the fractional part of the solar year, he lists its values according to Ptolemy, Battānī, Muḥīuddīn Maghrabī, again the (author of)

[54] [Monzavi 1983, 279].

[55] Cf. [Sen et al. 1966, 135].

[56] [Storey 1972, 97] cites this information from his vol. I, part 1, p. 751. We have taken the date of composition 1807/08 from f. 2b and f. 8b, wherein the argument of the tables begins with the year 1729 'Śākha'/1807.

[57] See [Sen et al. 1966, 7]. The author declares on f. 1b that it is the most reliable of the Indian *Zījes*.

Grahacandrika and at one point ‹Abdurāḥmān al-Ṣūfī.

2.7. *Zīj-i Sulaymān Jāhī* (1839) by Sayyid Rustam ‹Alī Raḍwī, dedicated to the ruler of Avadh, Naṣiruddīn Ḥaydar (reigned 1827–37). A unique manuscript is extant in Raza (Raḍā) Library (Rampur), old MS No. 1224 (new No. 1229),[58] with 77ff. It was copied by Muḥammad Akbar Dehlavī for Muftī Sharfuddīn Rāmpūrī. According to the author (f. 1a), he belonged to the city of Shāhjahānābād (Delhi), had been a pupil of Śrī Dhar of Benares and of Muftī Blīgh al-‹Ālam Khān of the city of Murshidābād. He studied ancient and modern astronomical *Zīj*es and particularly *Siddhānt-charat(t)ar* (?), *Siddhānt-dinchandrī* and *Siddhāntbhāstī* etc.,[59] on which this *Zīj* is based. We may refer to sections 1.3 and 1.4 above for details about the last two sources.

4 *Concluding Remarks*

The foregoing account of the development of mathematical astronomy in Muslim India shows clearly that from the very beginning there had been substantial interaction between Arabic/Persian- and Sanskrit-knowing scholars, which in turn gave rise to a wealth of literature. We have delineated here briefly the most significant works, and further conjecture that a class of Indo-Persian sources with the general title *Risālah-i Nujum* may belong to this category of medieval Indian astronomical literature. In any case the aforementioned interaction culminated during the 18th century, when Maharaja Sawai Jai Singh established the school of translation to which we have already referred.[60] We have remarked elsewhere [Ansari 1995, 286] that

> the scientific renaissance which was gradually building up during the Mughal period and which gathered momentum through the efforts of Maharaja Sawai Jai Singh (1686–1743) could not be even sustained, due to the tremendous political turmoil and instability at the close of the 18th and beginning of the 19th centuries. The result was that the scholars and also their schools

[58] Cf. the new [*Catalogue Rampur* 1994, 346].
[59] We use here the Persian orthography.
[60] See [Sarma 1998] for details.

could neither be patronised by the central Mughal authority in Delhi, nor by rajas, maharajas or nobles of Mughal India, hence the decline of traditional sciences in the first half of the 19th century.

However, that indigenous nascent scientific renaissance of the late medieval period was revived again by those Indian scholars who interacted with the European scholar-administrators. They responded positively to the 'New Astronomy' (*Hay'at-i Jadīd*) and acquired it quite eagerly. A number of Indo-Persian sources have been found by us in which modern European astronomy has been explained quite well.[61] This genre of writings culminated in the compilation of a treatise in Persian by a Hindu scholar, Raja Ratan Singh (d. 1851),[62] entitled *Hada'iq al-Nujūm* ('The Gardens of Astronomy'), which was lithographed at Lucknow in 1841. In this excellent and systematic treatise of 1158 pages, he mentions the astronomical work and discoveries of Copernicus, Tycho Brahe, Kepler, Galileo and Newton, and the then-recent work of Hevelius, Flamsteed, John Herschel, Cassini, Laland, to name just a few.[63] Evidently he was well aware of the works of his European contemporaries. That new development was then brought full circle when during the British period of Indian history modern astronomical observatories were established not only by the Government of India, but also by Indian monarchs,[64] which were the precursor of Indian efforts to contribute to world knowledge of astronomy and astrophysics in the last fifty years of independent India.

[Note added in proof]
To item 1, p.xxx: See [Storey, 52, n. 90]. Another manuscript of this *Zīj* is extant in Āyatullāh Marʿashī's Library, Qum (Iran); see the article by van Dalen's in this volume.

To §3.2.2.1: Recently I have found another manuscript of *Zīj-i Muẓaffarshāhī* in the Raza Library (Rampur). It is in the Persian collec-

[61] [Ansari 2002]. In this paper we have dealt with a number of Indo-Persian sources.
[62] Ratan Singh, with *nom de plume* 'Zakhmī' ('the Wounded'), belonged to the Indo-Persian *literati* of Mughal Indian society, see [Ansari 2002, 139–141].
[63] Cf. [Ansari 2003], section 6, wherein an appreciative account of Ratan Singh's treatise is given.
[64] [Ansari 1977], revised and expanded as [Ansari 1985].

tion: *Rasā'il Hay'at*, Ms. No. 1185 b, ff. 144b–159b. The text coincides *Verbatim* with that of the manuscript of Shīrānī collection. The scribe is also anonymous as the author.

To Note 34: I have found a complete anonymous manuscript copy of the Persian translation of *Karaṇakutūhala*, in Raza Library (Rampur), Ms. No. 1185 b, ff. 118a-143b. The scribe did not date it. But from the three years mentioned in the text, viz., 801 Yazdagird/1431 AD, 803 Yazdagird /1434 AD, and 810 Yazdagird /1441 AD — the last two for the lunar and solar eclipses observed by the author in Delhi —, this translation could be dated as of 15th century, i.e., of pre-Mughal India. I intend to publish its detailed study shortly.

Bibliography

Manẓūr Aḥsan ‹Abbāsī, *Comprehensive Catalogue of Persian Manuscripts in Punjab Public Library* (in Urdu), Lahore, 1963.

S. M. R. Ansari, 'On the Early Development of Western Astronomy in India and the Role of Royal Greenwich Observatory', *Archives Internationales d'Histoire des Sciences*, 27, 101, 1977, pp. 237–52.

——. *Introduction of Modern Astronomy in India during 18th-19th Centuries*, New Delhi, 1985.

——. 'On the Transmission of Arabic-Islamic Astronomy to medieval India', *Archives Internationales d'Histoire des Sciences*, 45, 135, 1995, pp. 273–97.

——. 'Ghulām Ḥussain Jaunpūrī and his Zīj-i Bahādurkhānī', *Studies in History of Medicine & Science*, XVI, 1–2, New Series, 1995/96, pp. 181–8.

——. 'Rare Arabic-Persian Manuscripts in Salar Jung Museum', *Salar Jung Museum Biannual Journal* 33–34, 1996–97, pp. 13–20.

——. 'Promotion of Astronomy by Indian Monarchs During the 15th–18th Centuries', in *Astronomy in the Time of King*

Sejong, eds Kyung J. Sim and Changbom Park, Daejeon (Korea), 2001, pp. 58–75.

——. 'Modern Astronomy in Indo-Persian Writings', in *History of Oriental Astronomy, Proceedings of the IAU Joint Discussion-17, Kyoto,*, ed. S. M. Razaullah Ansari, Dordrecht, 2002, pp. 133-44.

S. M. R. Ansari & S. A. Khan Ghori, 'Two Treatises on Astronomical Instruments by ʿAbd al-Munʿim al-ʿĀmilī and Qāsim ʿAlī al-Qāinī', in *History of Oriental Astronomy,* ed. G. Swarup et al., Cambridge, 1987, pp. 215–25.

Aftab Asghar, 'Zīj-i Muẓaffar Shāhī', *Majallah-i Taḥqīq* (Lahore, Pakistan), 3, 1, 1980, pp. 113–152.

ʿAbdul Qādir Badaoni (Badāyūnī), *Muntakhab al-Tawārīkh*, vol. I, Patna, 1973.

N. A. Baloch, *Ghurrāt Al-Zījāt or Karaṇatilaka*, Hyderabad Sind (Pakistan), 1973.

D. M. Bose, S. N. Sen, & B. V. Subbarayappa, *A Concise History of Science in India*, New Delhi, 1971.

Catalogue of Persian Manuscripts of Rampur Raza Library, vol. 1, Rampur, 1994.

Benno van Dalen, 'Origin of the Mean Motion Tables of Jai Singh', *Indian Journal of History of Science*, 35, No.1, 2000, pp. 41–66.

Farid Ghasemlou & Negar Naderi, 'The Persian Astronomical Tables Composed in India', in *500 Years of Tantrasamgraha, A Landmark in the History of Astronomy*, eds. M.S. Sriram, K. Ramasubrahmanian & M.D. Srinivas, Indian Institute of Advanced Study, Simla (India), 2002, pp. 137-44.

S.A. Khan Ghori, 'Development of Zīj-Literature in India', *Indian Journal of History of Science*, Vol. 20 (1985), pp. 21–48.

S. Ikeyama & K. Plofker, 'The *Tithicintāmaṇī* of Gaṇeśa: A Medieval Indian Treatise on Astronomical Tables', *SCIAMVS*, 2, 2001, pp. 251–89.

S. F. A. Jalali & S. M. R. Ansari, 'Persian Translation of Varāhamihira's Bṛhatsaṃhitā', *Studies in History of Medicine and Science*, 9, 1985, pp. 161–9.

David A. King, *Islamic Astronomical Instruments*, London, 1987.

Ahmad Monzavi, *A Comprehensive Catalogue of Persian Manuscripts in Pakistan*, Vol. I, Islamabad (Pakistan), 1983.

M. Nizam, 'A Unique Manuscript of Astronomy', *Archaeological Survey of India, Annual Report*, 1929–30, pp. 232–3 (reprinted in [Quraishi 1978] as an appendix).

Yukio Ohashi, 'Early History of the Astrolabe in India', *Indian Journal of History of Science*, 32, 3, 1997, pp. 199–295.

David Pingree, *Jyòtiḥśāstra*, Wiesbaden, 1981.

——. (1996a), 'Indian Astronomy in Medieval Spain', in *From Baghdad to Barcelona, Studies in the Islamic Exact Sciences in Honour of Prof. Juan Vernet*, 2 vols, eds J. Casulleras & J. Samsó, Barcelona, 1996, Vol. I, pp. 39–48.

——. (1996b), 'Indian Reception of Muslim Versions of Ptolemaic Astronomy', in *Tradition, Transmission and Transformation,* eds F. Jamil Ragep and Sally P. Ragep, Leiden, 1996, pp. 471–85.

——. (1999), 'An Astronomer's Progress', *Proceedings of the American Philosophical Society*, 143, 1, 1999, pp. 73–85.

M. F. Quraishi,*Ghurrat al-Zījāt or Karana Tilaka by Abū al-Raiḥan Muḥammad bin Aḥmad al-Bīrūnī*, Lahore, 1978.

S. Balachandra Rao, *Indian Mathematics and Astronomy, Some Landmarks*, Bangalore, 2000.

S. S. Hussain Rizvi, (1963-65): 'A Unique and Unknown Book of Al-Beruni', *Islamic Culture*, 37, 1963, pp. 112–30, 167–187, 223–245; 38, 1964, pp. 47–74, 195–212; 39, 1965, pp. 1–26, 137–80.

G. Saliba, *A History of Arabic Astronomy*, New York, 1994.

S. R. Sarma (1994a), 'From Al-Kura to Bhagola: On the Dissemination of the Celestial Globe in India', *Studies in History of Medicine & Science*, XIII, 1, New Series, 1994, pp. 69–85.

——. (1994b), 'The Lahore Family of Astrolabists and their Ouvrage', *Studies in History of Medicine & Science*, XIII, 2, New Series, 1994, pp. 205–24.

——. 'Translation of Scientific Texts into Sanskrit under Sawai Jai Singh', *Sri Venkateswara University Oriental Journal*, XLI, Part 1 & 2, 1998, pp. 68–87.

——. 'Yantrarāja: The Astrolabe in Sanskrit', *Indian Journal of History of Science*, 34, 2, 1999, pp. 145–58.

S. N. Sen, A. K. Bag, & S. R. Sarma, *A Bibliography of Sanskrit Works of Astronomy and Mathematics*, Part I, New Delhi, 1966.

C. A. Storey, *Persian Literature, A Bio-bibliographical Survey*, Vol. II, Part 1, London, 1972.

F. Sezgin, *Geschichte des Arabischen Schrifttums*, Vol. VI, Leiden, 1978.

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The Fragments of Abū Sahl al-Kūhī's Lost Geometrical Works in the Writings of al-Sijzī

J. LEN BERGGREN, JAN P. HOGENDIJK

1 *Introduction*

Abū Sahl Wījan ibn Rustam al-Kūhī was a mathematician from Ṭabaristan[1] who flourished in the second half of the tenth century under the patronage of at least three kings of the Buyid Dynasty: ʿAḍud al-Daula, Ṣamṣām al-Daula and Sharaf al-Daula, whose combined reigns over much of Iraq and Western Iran extended over the twenty-seven years from 962 to 989. His extant writings have been the particular study of the first-named author for the last twenty years, and a survey of his work – based on this study – has appeared in [Berggren 2003]. The tenth century was a time of intense geometrical research in medieval Islam and included most of the working lives of such geometers as Ibrāhīm ibn Sinān (909 - 946), Aḥmad al-Ṣaghānī (fl. ca. 970), Abū Ṣaʿd al-ʿAlāʾ ibn Sahl, Aḥmad ibn Muḥammad ibn ʿAbd al-Jalīl al-Sijzī (fl. 970), Abū Naṣr ibn ʿIrāq (died between 1018 and 1036), and Abū al-Wafāʾ al-Būzjānī (940 – 997/8). Both Ibrāhīm and al-Sijzī are directly connected with the work of al-Kūhī, that we survey in the present paper, and, in addition, al-Ṣaghānī, and al-Būzjānī worked with al-Kūhī on solar observations during the reign of Sharaf al-Daula in 988.[2] In short, al-Kūhī was an active member (and arguably the best geometer) in a community of tenth-century mathematicians who knew not only each other's work but, in many cases, each other as well.

Aḥmad ibn Muḥammad ibn ʿAbd al-Jalīl al-Sijzī [Sezgin 1974, 329-334; Sezgin 1978, 224 - 26; Sezgin 1979, 177–182, 333–334]

[1] See the *Fihrist* of Ibn al-Nadīm, who died in 995 [Dodge 1970, 669].

[2] The information included on dates here is from Sezgin 1974, which still represents the state of our knowledge about the dates of these individuals. It is striking how many major figures left so little biographical trace.

was a mathematician of the 10th century whose active career over, at least, the thirty-five year period from 963 - 998 led him into contact with a number of mathematicians of his time. Among them were al-ʿAlā' ibn Sahl and Abū Sahl al-Kūhī. [3] And it was the people he knew and the writings he read that create al-Sijzī's importance for us. For, rather like those of Pappus for Greek geometry, the writings of al-Sijzī are important for modern historians because of the information they provide about the writings of some of the great geometers up to and including his time. The second-named author of this paper has read through all known works of al-Sijzī and, in [Hogendijk 1986], has mined this lode for references to the works of the Greek geometer, Apollonius. He has, at the same time, collected references it contains to other major mathematicians, such as al-Kūhī , and it is the dual purpose of this paper to publish all the information available about al-Kūhī in these writings and to relate this information to what is already known from al-Kūhī's extant writings.

Since most of the thirty-three extant works of al-Kūhī have been published one would be tempted to say that we have a fairly complete picture of his work. Unfortunately, of the many propositions in several pages of material from al-Kūhī's writings that al-Sijzī provides, only two are found in al-Kūhī's extant writings. Thus this study furnishes one more reminder, if one were necessary, of the incomplete state of our knowledge of mathematics in the medieval Islamic world.

The primary source of information from al-Sijzī on al-Kūhī's writings is al-Sijzī's *Book on the Selected Problems that Were Discussed by Him and the Geometers of Shīrāz and Khorāsān, and his (own) Annotations* [Sezgin 1974, 333 no. 23], a work we shall follow al-Sijzī in refering to simply as *Geometrical Annotations.* The *Geometrical Annotations* is a rather disorganized and inhomogeneous collection of about 45 problems and their solutions, compiled by al-Sijzī. We are confident that he wrote such a treatise, not only because the incipit refers to him as the author but because in another work of his, the *Treatise on Parabolic and Hyperbolic Cupolas* [Sezgin 1974, 331, no. 3 = no. 5], he refers to the *Geometrical Annotations* as a work of his which is on the point

[3] For details on this and the rest of the scanty additional biographical information available on al-Sijzī see [Hogendijk 1986, 192-193].

of completion. However, the fact that existing versions of the work show slight variations in the material they contain shows that the work may have existed in different versions, and it is certainly possible that material was added or subtracted during the active career of the treatise.

The references to al-Kūhī in the *Geometrical Annotations* belong to two categories:

1. Problems by al-Kūhī or others, with answers by al-Kūhī;

2. Problems by al-Kūhī, with solutions by al-Sijzī and others.

We shall give an edited Arabic text and translation of those of the first type in their entirety, but for reasons of space we must give text and translation of only the statement of the problem for those of the second type. In the present paper, we have assigned the numbers 1 through 10 to the ten continuous fragments of the *Geometrical Annotations* which we have edited. We call "Problem 8" the geometrical problem of al-Kūhī that al-Sijzī discusses in fragment 8. If a fragment consists of different problems, we have used a notation such as Problem 10,2 to indicate the second problem in fragment 10. The six problems in fragment 10 are, from the third onward, introduced in the manuscripts by the words "third," "fourth," etc.

In fragments 11,1 and 11,2 below we have edited two references to al-Kūhī in another work by al-Sijzī entitled *An Answer to Geometrical Questions Asked to Him by People from Khorāsān* [Sezgin 1974, 333 no. 22]. Finally, fragment 12 is a "lemma by Abū Sahl al-Kūhī" in al-Sijzī's *Treatise on the Division of the Rectilineal Angle into Three Equal Parts* [Sezgin 1974, 331 no. 7]. The fragments in this paper have never been edited or translated before; only fragment 12 has been summarized in [Woepcke 1855b, 118].

The main interest of the references and fragments in this paper is that they preserve a small part of the lost work of al-Kūhī, who was probably one of the most knowledgeable geometers of the entire Islamic tradition. Although al-Sijzī was also famous in his time and had a large number of students, the numerous mathematical errors[4] in his extant work show that he was much

[4] Below we shall see examples where al-Kūhī's work was misunderstood by al-Sijzī, see for example Problems 2, 5, 6,1 and 10,4 below.

less talented than al-Kūhī.

The fragments in this paper also provide new information about the interaction between the two mathematicians. As we shall see, some easy questions by al-Kūhī with answers by al-Sijzī suggest that al-Kūhī and al-Sijzī had to some extent a teacher-student relationship. And, in this regard, it is interesting that the teaching was evidently at the intermediate/advanced level, since it is clear that al-Kūhī felt that al-Sijzī knew his way around in the *Elements* and was at the stage where he could begin to work on some problems that either themselves involved conics or had extensions that did.

2 *Overview of the problems*

It is our purpose in the mathematical discussions in this commentary on the problems to give not a detailed account of the proofs, but enough of an outline to illustrate both their general approach and a reasonable notion of the geometrical methods used. The details, of course, are in the text and its translation.

Of the problems, six of the first seven are closely related to the works of Apollonius and bear directly on matters discussed in his *Conics, Cutting-off of a Ratio, Plane Loci,* and *Determinate Section.* Additionally, Problem 8, although not specifically related to the Apollonian corpus, is very much in the spirit of the problems in the now-lost works of that geometer. If one accepts our view, stated above, that the problems were to some extent set by al-Kūhī to develop al-Sijzī's skill as a geometer, one has to conclude that a mastery of Apollonian problems and techniques was very much a part of what al-Kūhī thought every skilled geometer should possess. The two remaining problems from the first group, found in fragments 3 and 9, relate to problems of inscribing one figure in another. Note that neither problem is of the 'how to inscribe figure X in figure Y' type but both compare the sizes of figures of a certain type inscribed in figures of another type.

Problems 10,1 through 10,6 seem to represent an independent composition of al-Kūhī's, which he wrote in response to what was evidently a series of geometrical questions that had been posed to him. Given that both Ibrāhīm ibn Sinan and al-Sijzī authored

works of this genre[5] it would not be at all surprising to learn that al-Kūhī, too, had written such a work. Al-Kūhī's *Two Geometrical Questions* [Berggren and Van Brummelen 2000a] contains solutions to two unrelated geometrical questions and may therefore be viewed as a work in the same category as well. We add that both Problems 10,1 and 10,2 are closely related in subject matter to Problem 7.

The first two fragments from al-Kūhī are theorems in the *Conics*, specialized to circles. The first is the specialization of *Conics* III, 53, where it is stated for any central conic [Ver Eecke 1959, 273; Taliaferro 1998, 259-60]. Other than the interchange of the labels B and G, al-Kūhī's lettering for Problem 1 is that of Apollonius,[6] and, apart from this interchange, so is his proof, which needs no further comment. The inclusion of problems of this sort, special cases of theorems whose more general forms were the common knowledge of competent geometers in al-Kūhī's time, is further confirmation of our earlier suggestion that at least some of these problems were meant as instructional material for al-Sijzī, and that the relationship between al-Kūhī and Sijzī was to some extent that of teacher and student.

In Problem 2, al-Kūhī considers two tangents at points E and Z of a circle that intersect at point A, which is outside the circle (Figure 1). He proves that if any line through A intersects the circle at points T and H and ZE at I, then[7] $HA : AT = HI : IT$. In *Conics* III, 37 the proposition is stated for any section of a cone, including the two branches of a hyperbola [Ver Eecke 1959, 249-250, Taliaferro 1998, 235-237]. The lettering in the argument which, according to al-Sijzī, "Abū Sahl (al-Kūhī) mentioned on the authority of Apollonius," differs from that in the *Conics,* and al-Kūhī's proof is more complicated than the proof found there (although it is more elementary in the sense that it does not

[5] Ibrāhīm wrote the *Selected Problems* [Sezgin 1974, 294 no. 6], see [Saidan 1983] and the recent edition by Bellosta in [Rashed & Bellosta 2000]. The two works by al-Sijzī of which we have published fragments in this paper also belong to this genre.

[6] In medieval Arabic translations of Greek geometrical works, Greek labels of points and lines in geometrical figures were usually transcribed by Arabic letters with the same numerical value, see [Toomer 1976, 32-33].

[7] In modern terms, line ZE is the polar of point A with respect to the circle. The theorem is to the effect that for any secant through A, the four points A, I, T, H are a harmonic set.

depend on earlier propositions in *Conics* III.)

In Figure 1, K is the center of the circle, line AK intersects the circle at points B and G and line ZE at D, and N is the midpoint of HT. Al-Kūhī's proof of Problem 2 consists of the following three parts.

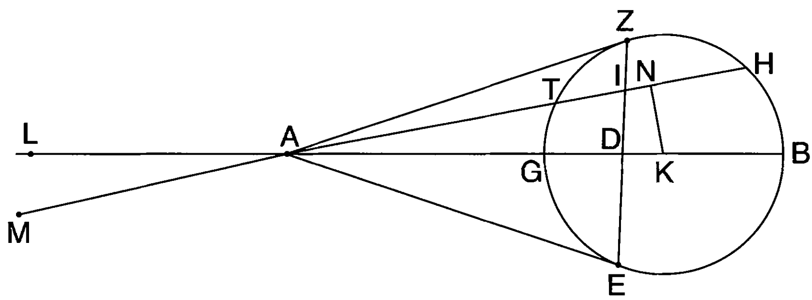


Figure 1

(1) $BA : AG = BD : DG$ and therefore, after some work, $BA \cdot AG = AD \cdot AK$;
(2) the two identities $BA \cdot AG = HA \cdot AT$ and $AD \cdot AK = AN \cdot AI$;
Now, as a consequence of (1) and (2),
(3) $HA \cdot AT = AN \cdot AI$, which implies $HA : AT = HI : IT$.

The first part of (1) is a special case of the theorem to be proved when the secant, ATH, is the diameter AGB. Al-Kūhī says it is "clear." The first identity in (2) follows from *Elements* III, 36, and the second from the similarity of triangles ADI and ANK. Al-Kūhī's proof of (3) is similar to his proof of (1) in reverse order.

Al-Kūhī's proof can be simplified, and his use of the auxiliary line LA in Figure 1 avoided, by using properties of right triangles and tangents established in the *Elements* to show that AZ^2 is equal both to $AD \cdot AK$ and $AH \cdot AT$. However, in the present, complicated, form, the proof can also be used to prove the specialization of *Conics* III:38 for the circle [Ver Eecke 1959, 251-252].[8]

Fragment 3 is a theorem that may have come out of practical work with the design of mosaics. Al-Kūhī considers a triangle ABG with $AB > BG$. He proves that the inscribed square

[8] In modern terms, this is the analogous theorem for the case where point A is inside the circle and its polar ZE does not meet the circle.

with its base on side AB is less than the inscribed square with its base on side BG (see Figure 2). Elsewhere in the *Geometrical Annotations*,[9] al-Sijzī constructs a square in a given triangle, and al-Kūhī's contemporary Abū al-Wafā' discusses two such constructions in Chapter 7 of his *Geometrical Constructions Necessary for the Craftsman* [Qurbani 1992, 60-61].[10] In any case, al-Kūhī was also interested in the problem of inscribing a polygon inside another one, and [Hogendijk 1985] has published his work on inscribing an equilateral pentagon in a given square.

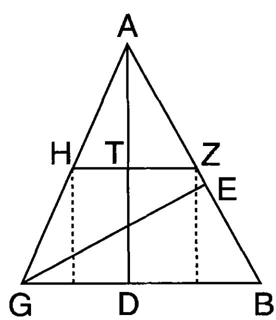


Figure 2

The idea of al-Kūhī's proof is as follows (Figure 2). Suppose that AD and GE are altitudes of the triangle and that HZ is one of the sides of the inscribed square constructed on side BG. If HZ intersects AD at T, then $DT = HZ$.
(1) Al-Kūhī shows that two times the area of the triangle ABG is equal to the rectangle contained by $BG + AD$ and DT.
(2) By a similar argument, two times the area of triangle ABG is also equal to the rectangle contained by $AB + GE$ and the side of the inscribed square with its base on side AB.
(3) Al-Kūhī applies *Elements* V, 25, to conclude from the proportion $AB : BG = AD : GE$ that $AB + GE > BG + AD$. The theorem now follows.

This proof provides a nice application of *Elements* V 25. In order to apply it however, al-Kūhī must establish the proportion

[9] See the manuscripts C 50b:5-11, I 59a:3-9 = IF 57:3-9.

[10] See also the summary of a Persian version of the work in [Woepcke 1855a, 336]; according to Woepcke the triangle should be equilateral, but in the Arabic original, Abū al-Wafā' presents the two constructions for an arbitrary triangle.

$AB : BG = AD : GE$, which he does on the basis of the similarity of triangles ABD and GBE, and then establish that AB and GE are the largest and smallest of its four terms, which he states but does not establish. Perhaps he thought the verification of this fact would provide a nice exercise for al-Sijzī.[11]

Fragment 4 is an analysis of the general problem of Apollonius's *Cutting-Off a Ratio*, namely: "Given two straight lines ... and a fixed point on each line, to draw through a given point a straight line which shall cut off segments from each line (measured from the fixed points) bearing a given ratio to one another." In Figure 11 below, the two given straight lines are BG, KD, he fixed points are B, D, and the given point is A. For reasons unknown to us, al-Kūhī's statement of the problem assumes as given two points on each of the lines (points B, G, D, K), although he only uses one on each line (points B, D). Al-Kūhī's analysis differs from that of Apollonius [Heath 1921 II, 76], and is much easier than that of the same problem by Ibrāhīm ibn Sinan [Saidan 1983, 210-213; Bellosta 2000, 668-673].

Problem 5 concerns the construction of an inscribed triangle of given area in a given semicircle ADB (Figure 12 below). This problem, whose solution it is very difficult to believe that al-Kūhī did not know, suggests to us that he and al-Sijzī had a teacher-student relation. Al-Sijzī's solution to this problem, which we have edited and translated below, suggests that he was unfamiliar with conic sections. The problem is an ancient one, and [Heath 1921 I, 299-301] discusses the diorismos of this problem in connection with Plato. Note that the fact that ADB is a semicircle is not used in the problem, and the same solution can be used for any given curve ADB, something al-Kūhī would have been aware of. See, for example, his solution of the last problem in his *On the Ratio of the Segments of a Single Line that Falls on Three Lines* in [Berggren and Van Brummelen 2000b, 38-43.]

Both problems in fragment 6 are related to the first locus in Book 1 of Apollonius's lost *Plane Loci*. According to Pappus of Alexandria, Apollonius showed in his first locus, among other things, that, "[i]f two straight lines are drawn from a given point, and containing a given angle, and either holding a [given] ratio

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[11] $AB > GE$ follows from the hypothesis $AB > BG$ combined with $BG > BE$.

to one another or containing a given area, and the end of one touches a plane locus [i.e., a straight line or a circle] given in position, the end of the other will touch a [second] plane locus given in position" [Jones 1986 I, 106]. Fragment 6 is the first evidence of knowledge of Book 1 of the *Plane Loci* in the Arabic tradition; for traces of Book II see [Hogendijk 1986]. In the first problem 6,1 solved by al-Kūhī (Figure 3), the given point is A, the given angle is H, the given area is EZ, and the given plane locus is a given line ℓ_1 (DG in Figure 3). Al-Kūhī considers another given line ℓ_2 (BG in Figure 3) that intersects ℓ_1 at the given point G. His problem is to find points B on ℓ_2 and L on ℓ_1 such that $BA \cdot AL = EZ$ and $\angle BAL = H$. Al-Kūhī considers an arbitrary point D on ℓ_1 and finds point T such that $DA \cdot AT = EZ$ and $\angle DAT = H$, and he constructs a segment of a circle through A and T which contains an angle equal to angle ADG. He then finds B as a point of intersection of the segment of the circle and ℓ_2. It is easy to show that the circle is independent of point D and T, so it must be the "[second] plane locus given in position" mentioned by Pappus. The problem, of course, has a solution if and only if the "second locus" and the line GB have a common point. It would be interesting to know to what extent al-Kūhī's solution of problem 6,1 was modelled after Apollonius's construction of the locus in question.

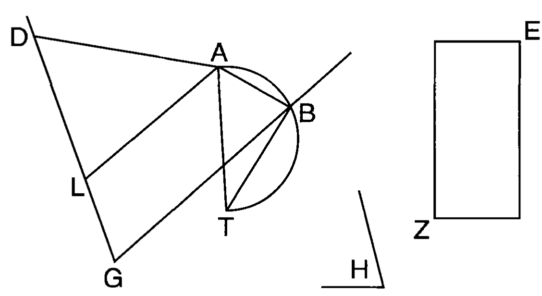


Figure 3

In problem 6,1, al-Kūhī uses an idea that he also exploits successfully in his solution to a problem that was put to him by Abū Isḥāq al-Ṣābī (see [Berggren 1983]). In the case of this problem, the idea is to construct in Figure 3 triangle ADL and then to construct a scale model of it (triangle ABT) on AB but so that

AB corresponds to AD and BT to DL. (Hence TA corresponds to LA.) He only knows angle ADL and AD and $AL \cdot AB$, but then by similarity and the fact that AB corresponds to AD he will know $AT \cdot AD$. Hence he will know AT, since he knows AD and since $\angle TAD = \angle BAL$. So all he need do now is draw a circular arc on AT admitting angle ABT (= angle ADL.) In the *Geometrical Annotations*, al-Sijzī presents two other solutions of al-Kūhī's problem 6,1 (without reference to al-Kūhī).[12] These solutions boil down to the construction of the above mentioned "[second] locus" and they are therefore variations on al-Kūhī's solution.

One finds the same problem as No. 37 in Ibrāhīm ibn Sinan's *Selected Problems.* Given the relative dates of the two authors, it is quite possible that Ibrāhīm's problem was the motivation for al-Kūhī's treatise.

In problem 6,2, al-Kūhī wants to construct point L so that $\angle BAL = H$ and $BA : AL$ is a given ratio.[13] Al-Kūhī's solution of the second problem is the same as that of Ibrāhīm ibn Sinan in his *Selected Problems* [Saidan 1983, problem 37], [Bellosta 560, 732-735]. Al-Kūhī and Ibrāhīm utilize a circle in their solution, but one can, using Apollonius's lost *Plane Loci*, construct a solution omitting the circle, and the synthesis could have been simplified. In the *Geometrical Annotations*, al-Sijzī also presents a simple solution to the second problem.[14]

Problems 6,1 and 6,2 correspond to the second and first problems (respectively) in al-Kūhī's treatise *On Drawing Two Lines from a Point at a Known Angle, by the Method of Analysis* [Berggren and Van Brummelen, 2001]. In that treatise the two given straight lines of Problem 6 are replaced by a single line (or circle or arbitrary curve). In the treatise one finds only analyses of the problem, whereas one finds the syntheses in Problem 6 as well, ostensibly due to al-Kūhī.

Fragment 7 is a question concerning the division of a triangle. The clear, but very long-winded proof[15] is apparently al-Sijzī's

[12] See the manuscripts, C 51b:25-34, 34-37, I 61a: 7-16, 16-20 = IF 61:7-16, 16-20.

[13] We use the notation of the first problem here, see Figure 3.

[14] See the manuscripts, C 52a:2-10, I 61b:1-9, IF 62:1-9.

[15] The proof runs for the better part of a page in the most important manuscript, until C 40a:28, I 41a:10 = IF 21:10.

answer to this question by al-Kūhī. Following this proof, the manuscripts have a "lemma for this proposition from the Book of Apollonius on *Cutting the Lines in Ratios*" (i.e. the *Cutting-off of a Ratio*), and then another solution of the problem, probably by al-Sijzī.

Al-Kūhī uses the method of analysis to reduce Problem 8 to one he solved in his lost treatise *Producing Points on Lines in the Ratio of Rectangles.*[16] That problem is the following: Given a line segment $ATKB$, where T and K are given in position on AB and in the order indicated, find a point H on the segment TK so that the ratio of the two rectangles, $AH \cdot HT : BH \cdot HK$, is equal to a given ratio (Figure 19 below). Al-Kūhī refers to his *Producing Points on Lines in the Ratio of Rectangles* in at least two of his treatises, namely *On the Construction of the Astrolabe* and *On the Complete Compass.* Indeed, the first of the two problems from *Producing Points ...* that al-Kūhī refers to in his *On the Astrolabe* is a slight variant of the above-mentioned problem, namely one that requires finding H on KT so that $AH \cdot HK : BH \cdot HT$ is equal to a given ratio.[17]

Fragment 9 is closely related to Fragment 5, the construction of a right-angled triangle GDB of given area in a given semicircle AGB (Figure 20 below). In fragment 9, al-Kūhī asks al-Sijzī to prove that for a given semicircle AGB, the maximal triangle that can be inscribed this way is obtained when arc AG is one-third of the semicircle. This proposition provides the diorism for Problem 5 above. Of course, *finding* the condition [AG one third of the semicircle] is the real work. By means of the hyperbola construction in Problem 5, and the property of the internal and external division of a diameter of a circle by a chord perpendicular to that diameter with which al-Kūhī begins the proof of Problem 1, the diorismos can be found easily (Figure 4).

For, suppose that the circle is tangent at G to a hyperbola with asymptotes BA and BZ, the line though B perpendicular to BA. Let the common tangent to the circle and hyperbola at G intersect BA extended at E and BZ at Z. Then we have $EG = GZ$, because EZ is tangent to the hyperbola, and so $ED = DB$.

[16] Its Arabic title is *iḥdāth al-nuqaṭ ʿalā al-khuṭūṭ fī nisab al-suṭūḥ.*

[17] For the problems in *Producing Points ...* that al-Kūhī refers to in his *On the Astrolabe* see [Berggren 1994, 176 - 178] and [Rashed 1993, 225-228]

By the property of the circle referred to above, $EA : EB = DA : DB$, hence $DA : DB = (DA+EA) : (EB+DB) = DE : 3DE = 1 : 3$.

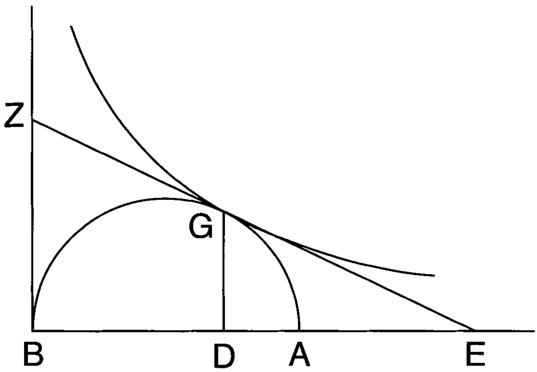


Figure 4

Fragment 10 is a series of problems which seem to be taken from a single source, probably independent of the sources for the previous problems since it appears that al-Kūhī composed it as a unit. This is suggested (i) by the heading "Answer by Wayjan, known as Abū Sahl al-Kūhī, to the geometrical questions, and they are six propositions," (ii) the reference to the "answer" (singular) to six questions, and (iii) the fact that here alone the text refers to al-Kūhī simply as "Wayjan."

Problems 10,1 and 10,2 are two cases of one problem which may be viewed, on the one hand, as examples of verging constructions, for one is given a triangle ABG and a point D (on a side or its extension) and is required to draw a straight line verging towards the given point D and cutting the other two sides of triangle ABG in such a way that the (areas of the) triangles $\Delta(ADF), \Delta(MFG)$ formed at two of the vertices have to each other a given ratio $E : Z$ (Figures 5, 22). A similar construction, with the given triangle replaced by a square and the given ratio being that of identity, underlies the construction of the heptagon ascribed to Archimedes in Arabic sources.[18] On the other

[18] On Archimedes's construction and the controversy that ensued about the legitimacy of the so-called 'moving' geometry that it and some other verging constructions necessitated see [Hogendijk 1984].

hand, the problem reminds one of problems in Euclid's *Division of Figures*, a work known in medieval Islam [Sezgin 1974, 118; Hogendijk 1993]. In *Divisions of Figures*, the typical problem is to draw a line through a given point dividing a given figure, usually a triangle or some kind of quadrilateral, into two parts that stand in a given relation to each other. In *Divisions of Figures* one finds the application of areas with square deficit solved and applied to the problem of dividing a triangle into two equal parts by a line passing through a point inside the triangle, and in problems 10,1-2 al-Kūhī uses application with square excess.[19]

Problems of type 10,1-2 seem to have been very popular in tenth-century Iran, and al-Sijzī solves similar problems in the *Geometrical Annotations.*[20] The proof of problem 10,1-2 is long but, for the most part, straightforward, and the non-trivial mathematical machinery appears at the beginning, in the definition of the point M. In order to construct the figure, al-Kūhī defines BK and DN as perpendiculars from B and D onto the side AG, and GO as a parallel to AB. He then uses the given magnitudes, E and Z, and the perpendiculars BK and DN to define segments T and H as fourth proportionals in two proportions, namely $E : T = (BK/2) : BA$ and $H : Z = BG : (BK/2)$. He then uses T and H to define an area α according to the proportion $AD^2 : \alpha = T : H$. Neither the Hellenistic geometers, nor their Islamic successors, were at all shy about defining a magnitude in this way, as a fourth proportional when the three other terms are known. In general, such a fourth proportional is not constructible by Euclidean tools, but in this case it is possible to construct by Euclidean tools a rectangle of area α. Al-Kūhī now applies this rectangle as a rectangle on the given segment BD, which may be done by *Elements* I, 44. A side of this rectangle perpendicular to AD is then cut off of BG extended, as GL; thus $BD \cdot GL = \alpha$. He now uses *Elements* VI, 29[21] to apply to GL a rectangle equal in

[19] The application of areas - with or without excess and deficit - was an important tool of the Greek geometers which, among other uses, provided the basis for Apollonius's definition of the conic sections. For a brief account of the method see [Heath 1921 I, 150 - 154].

[20] See the manuscripts: C 41a:7-22, 43b:19-30, 45a:2-23; I 42b:5-22 = IF 44:5-22, I 47a:16-47b:4 = IF 43:16-44:4, I 49b:4-50a:1 = IF 38:4-39:1.

[21] [Heath 1956 II, 267] points out in his commentary on this proposition that for rectangles and squares, which are the figures al-Kūhī uses, the construction can be done with *Elements* II, 5.

area to rectangle $BG \cdot GL$ so that the rectangle so formed exceeds the part of it lying along GL by a square figure, Q (not labeled in the text). If D is on AB al-Kūhī defines M on BL extended so that LM is equal to a side of Q, but if D is on AB extended he defines M on BG so that GM is equal to a side of Q. Thus, in both cases $GM \cdot LM = BG \cdot GL$.

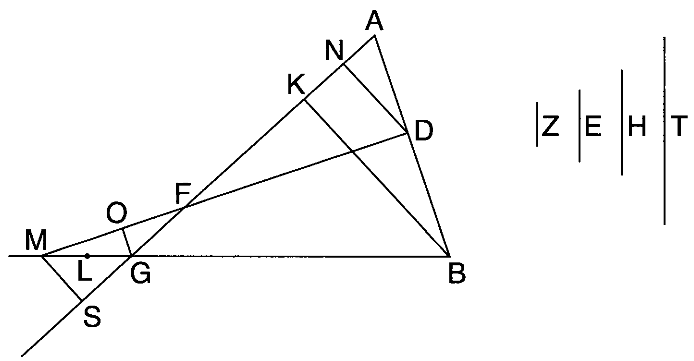


Figure 5

If F is the point where DM (extended, in the second case) meets AG then triangle ADF has to triangle FMG the given ratio of E to Z.

The main steps of the proof are as follows:
(1) It follows from the definition of T that $E : T = \Delta(AFD) : DA \cdot AF$, which holds for any segment AF.
(2) From the definitions of L, M and O it follows that $DB \cdot GL = MG \cdot GO$, and hence that $T : H (= AD^2/DB.GL) = AD^2/MG \cdot GO$.
(3) $AD^2/MG.GO = DA.AF : MG.GF$, which follows from the similarity of triangles ADF and GOF.
(4) From (1) - (3), and the fact that $E : H$ is composed of $E : T$ and $T : H$, it follows that $E : H = \Delta(ADF) : MG \cdot GF$.
(5) Al-Kūhī now tacitly introduces MS perpendicular to AG (extended if necessary) and uses similar triangles to conclude from the proportion defining H that $H : Z = GM : (MS/2)$, and hence - taking GF as a common height - that $H : Z = GM.GF : \Delta(MGF)$.
(6) From (4) and (5) it follows that $E : Z = \Delta(ADF) : \Delta(MGF)$.

Problems 10,1 and 10,2 can be reduced to Problem 10,6: Construct a line through a given point that cuts the sides of an angle given in magnitude and position so that the triangle formed at the vertex of the angle is given in magnitude, i.e. has a given area. In this case, the given angle is angle OGM, which is equal to angle B, the given point is D, and the line al-Kūhī produces is DOM, which cuts the given angle OGM in such a way that $AD^2/MG \cdot GO = H/T$. This implies that the rectangle $MG \cdot GO$ is given in magnitude, since both AD^2 and the ratio H/T are known. But, since the angle MGO is known, Prop. 66 of Euclid's *Data* implies that $MG \cdot GO : \Delta MGO$ is known, where ΔMGO the area of triangle MGO. And, hence, ΔMGO is known. The text, however, contains no hint that al-Kūhī was aware of this reduction.

The text of Problem 10,3 is defective, but the meaning of the problem is clear (Figure 6). Given triangle ABG, the point D on BG extended, and the ratio $E : Z$, construct points L and M on AG and AB respectively such that $LM//AD$, and $(BM \cdot LG) : (BG \cdot LM) = E : Z$. The problem reminds one of a verging problem, although the line to be constructed must verge not to a given point but in a given direction. The construction is reminiscent of that in Euclid's *Divisions of Figures,* 19-20 [Archibald 1915, 52-55; Hogendijk 1993, 150-152], in that one first applies an area to a given line segment (to obtain another line segment as width) and then applies another area to that (with square excess in the case of Problem 10,3 and square deficit in the case of *Divisions of Figures,* 19-20.) No diorismos is required, because one can make the ratio of the areas as small as one likes by letting the line LM approach BG and as large as one likes by letting the line LM approach A. Here again, as in the two previous problems, the key to the solution lies in two applications of areas, one exactly and the other with square excess. The first of these is the application of an area α (defined by $BA \cdot AG : \alpha = E : Z$) to the line segment AD to produce a rectangle whose other side is then cut off on GB as GT. The second of these is the application of the area $BD \cdot GT$ to TB with square excess equal to BK^2, thus defining K on GB extended. It follows from the the definition of T that $BA \cdot AG : AD \cdot TG = E : Z$. Then, the application of areas with square excess, routine manipulation of ratios, and the similarity of triangles ADK and

NBK show that $KG \cdot BN = DA \cdot TG$. From this it requires only routine manipulation of ratios and the similarity of two pairs of triangles (ABN and BML; AKG and LBG) to establish that $BM \cdot LG : BG \cdot LM = E : Z$. One notes that the line the desired line is required to be parallel to is not just any line but a line through the vertex of the triangle. Mathematically, of course, this latter condition is irrelevant, but precisely for that reason it may provide a hint as to the origin of the problem. Another hint may be contained in the fact that the direction is defined by a line through A to a point on BG extended rather than by a line through G to a point on the segment AB. But we confess that the context of this problem still eludes us.

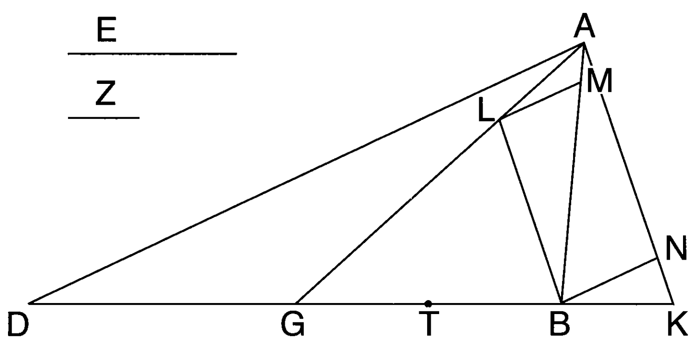


Figure 6

Problem 10,4 is as follows in modern terms (Figure 7). Construct a right-angle triangle so that if the altitude, h, to the hypotenuse, c, divides it into two parts j and k (with $j < k$), then $(c/2 - h)j = A$, a given area. (Note that in any right triangle, other than an isosceles right triangle, $c/2 - h > 0$.) This is an odd problem, because for each shape of a right-angled triangle, there is one size which will produce a solution. Al-Kūhī constructs a solution such that also $c/2 - h = j$. Again, the context for this problem eludes us. The methods in its solution do, however, remind one of those in the solution of the previous three problems, for this problem relies on *Elements* II, 14, to transform a given rectangle to a square, and then on *Elements* II, 5 (which can also be used for application of areas with square excess) to transform the combination of a rectangle and a square into a single square.

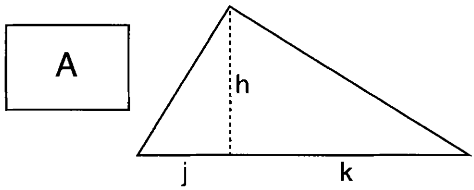


Figure 7

In Problem 10,5, al-Kūhī posits two given circles and he constructs a straight line of which the two circles cut off two chords of given lengths. Clearly some diorismos is needed to solve this problem, since, for example, if the two given circles are concentric a given segment in one circle uniquely determines the size of the segment in the other. In general, however, al-Kūhī was more interested in solving geometrical problems under the assumption that everything would work out than he was in hacking through all possible cases to see where possible problems with the general procedure might arise. His attitude seems to be well summed up by the following quote from his correspondence with al-Ṣābī [Berggren 1983, 70] : "If I had ... used ... division [into cases] and diorismos, as Apollonius did in some of his theorems, our composition would be [very] long." In the *Geometrical Annotations*, al-Sijzī presents a somewhat different solution of the same problem.[22]

Problem 10,6 is again a verging construction whose concern with areas reminds one of Archimedes's requirement in his construction of the regular heptagon. In this case we are given an angle, an area, and a point, and we are required to produce a line passing through the point cutting off from the given angle a triangle whose area is equal to the given area. It is, however, much easier than Archimedes' problem, because in this case only one area, not two related areas, depend on the line drawn through the given point. Again, too, we have the same two cases of application of areas as in Problems 10,1-3 playing the major role in the solution to the problem. The difference, that in this case one does not first apply a rectangle to a given line segment but constructs a triangle with a given point as vertex having its base on a given line, is only apparent. For, in terms of actual

[22] See the manuscripts C 43b:30-40, I 47b:5-15 = IF 34:5-15.

construction, this latter problem is most easily effected by the procedure in *Elements* I, 42, which involves applying a rectangle whose base is half that of the given triangle to the perpendicular from the given point A to the given side, ZB, of the angle ZBG (Figure 26 below). Ibrāhīm ibn Sinan also solves this problem in his *Selected Problems*, see [Saidan 1983, 237-238], [Bellosta 2000, 714-171], but al-Kūhī's solution is easier.

In Fragment 11 we have edited two references by al-Sijzī to al-Kūhī in another work, the *Answer to Geometrical Questions Proposed to Him (=al-Sijzī) by People from Khorasan.*

Fragment 11,1 is a reference to Proposition 6 of *On the Centers of Tangent Circles on Lines, by Way of Analysis*, although al-Sijzī's labeling of the various parts of the figure differs from that of al-Kūhī. Two works of al-Kūhī had this title, but only the one referred to here is known. It has been published in [Abgrall 1995].

In Fragment 11,2, al-Sijzī refers to a special case of a problem solved in al-Kūhī's lost work *Producing Points on Lines in the Ratio of Rectangles.* The problem in question is the second one from that work that al-Kūhī refers to in his treatise *On the Astrolabe*, and it requires that, given a line segment AGB, one find on segment GB a point Z so that $AG \cdot GZ : AZ \cdot ZB$ is equal to a given ratio (Figure 28 below). Al-Sijzī solves the special case where the ratio is one of identity. The problem is equivalent to a quadratic equation, and al-Kūhī solved it by application of areas of Book VI of Euclid's Elements, but al-Sijzī uses the intersection of a circle and a parabola in his solution.

Our last fragment 12 is a "lemma by Abū Sahl al-Kūhī" from al-Sijzī's treatise on the trisection of an angle. This lemma is a reduction of the problem of trisecting a given angle to the problem of constructing for an arbitrary angle ABG (taken as the supplement of the angle to be trisected) a triangle ABG so that if GD is the bisector of angle G then BG is the mean proportional between AB and BD (Figure 29 below). Al-Kūhī shows in a brief treatise [Saidan 1984] how to construct such a triangle by means of an equilateral hyperbola.

3 *Manuscripts and Editorial Procedures*

Al-Sijzī presents ten references to and fragments of al-Kūhī's works in the *Book on the Selected Problems Which Were Discussed by Him and the Geometers of Shīrāz and Khorāsān, and His (i.e., al-Sijzī's) Annotations* [Sezgin 1974, 333 no. 23].

This work survives in the manuscripts C = Dublin, Chester Beatty 3652, ff. 35a-52b, see [Arberry 1955-1964 III, 59], and I = Istanbul, Resit 1191, 31a-62b, corresponding to pp. 2-63 in the facsimile edition [al-Sijzī 2000]. C is dated 612 AH / AD 1215, and [Arberry 1955-1964 III, 60] states that C was copied from an autograph by al-Sijzī. I is an undated manuscript. Since most readers will have access to I in the facsimile edition [al-Sijzī 2000], we use the notation IF 3 for page 3 in the facsimile, corresponding to f. 31b in I. A version[23] of the beginning of the work has come down in a third manuscript T = Dublin, Chester Beatty, 3045, 74a-89b [Arberry I, 19]. T is dated 699 AH / 1299 AD The title of this version is *Geometrical Annotations*, see [Sezgin 1974, 333 no. 27]. This text contains the first two fragments of al-Kūhī only.

The comparison between these three manuscripts shows that the text in T was slightly edited with respect to style.[24] The following two curious errors in I show that I is in all likelihood dependent on C. In I 53a:15 = IF 45:15, the text reads *wa-naṣilu khaṭṭ* alif-ṭā' *wa-nukhriju ʿalā* jīm *istiqāmatihi ilā* lām, meaning "we join line AT and extend it in G a straight line to L". In manuscript C, however, the text reads "we join line AT and extend it in a straight line to L." In manuscript C, the word *ʿalā* (translated by "in") is at the end of the line f. 47b:19, and next to it appears the letter *jīm* = G, which is a label of a point in the geometrical figure. Thus a scribe who copied C must have

[23] Although this version is shorter, it contains some material not found in the longer version. One possibility is that al-Sijzī wrote the version we have as the longer one, later added some additional material relevant to the treatise, thus creating an expanded version from which the shorter version was made.

[24] For example, in fragment 2, T has the reading TI which is mathematically correct, and C has the reading NT, We have assumed that al-Sijzī's text has IT which is easily misread as NT, and that the editor of T changed IT in TI, which is more easily distinguishable in Arabic. Probably for the same reason, the editor of T also changed LA in the original, which can easily be misinterpreted as the word *lā*, meaning "not," to AL. See further our critical apparatus.

mistaken this letter as part of the text, and I must be dependent on C. A similar error can be detected by comparing I 37b:22 = IF 14:22 with C 38b:5. In I, the letter *hā'*=*E* appears in the text in a nonsensical way, but in C the letter appears immediately after the end of the line, as a label in the geometrical figure. However, there are also a few words and passages which are unclear in C and clear in I. Perhaps the scribe of I (or of an archetype of I) used C as his main source but checked other manuscripts occasionally. The dependency of I on C was first stated in [Rashed 1991, 34], who compared the manuscript texts in C 68b-78b and I 1b-30b of the treatise *On Analysis and Synthesis* by Ibn al-Haytham.

In editing the text we have not noted the few places in which we have changed the orthography to standard classical orthography, nor the several places where we have corrected trivial grammatical errors.[25] In the case of scribal errors in copying letters referring to points in geometrical diagrams, our policy is that if the form of the letter in the text could reasonably be construed as being the correct letter then we have taken it so. Otherwise, we have made a note in our critical apparatus. We note that the labels *kāf* (*K*) and *lām* (*L*) are not always clearly distinghuishable in the manuscripts, and the same is true for or *ḥā'* (*H*) and *jīm* (*G*).

Our guiding editorial principle has been to restore al-Sijzī's (not al-Kūhī's) text. We have assumed (1) that the manuscript C differs from al-Sijzī's original only by mechanical scribal errors, and (2) that al-Sijzī's original text contained few if any mathematical mistakes. Thus we have attempted to emend some passages in the text for mathematical sense. There are certain passages in the text which are mathematically unsatisfactory and which cannot be emended in a paleographically plausible way. For example, it would be paleographically implausible to rewrite entire sentences. In such cases, we feel forced to assume that al-Sijzī made a mathematical error in his interpretation of al-Kūhī's work. In the present article we have not always tried to identify modifications in al-Kūhī's proofs due to al-Sijzī.

In our translations, we use parentheses as punctuation, although (like other punctuation - periods, commas, etc.) they are

[25] For example, *yakūn nisba* in C 37b:15 has been silently corrected to *takūn nisba*, in the word *muʿṭā* the final alif has been changed to final yā', etc.

foreign to medieval Arabic. Square brackets enclose explanatory remarks that we have inserted into the text. Pointed brackets enclose translations of material we have inserted (also in pointed brackets) in the Arabic text to restore what we conjecture are missing portions. Our system for transliterating letters referring to points in geometrical diagrams is that of [Hermelink and Kennedy 1962]. Within the Arabic text the initial consonant in the name Kūhī is represented in two ways, sometimes by a qāf and sometimes by a kāf. We have not altered the text or the transliteration of the name, but have faithfully represented the name Qūhī or Kūhī as it occurs in the manuscripts in the various places.

Al-Sijzī makes two references to al-Kūhī in the *Answer to Geometrical Questions Proposed to Him by People from Khorasan* [Sezgin 1974, 333 no. 22]. This treatise has survived in the manuscripts C 53a-60a (see [Arberry 1955-1964 III, 59]) and I 110b-123b (= IF 156-182). The references are presented in fragments 11 below.

Finally, al-Sijzī presents a "lemma by Abū Sahl al-Kūhī" in the *Treatise on the Division of the Rectilineal Angle into Three Equal Parts* [Sezgin 1974, 331 no. 7]. This treatise has come down to us in the manuscript L = Leiden, University Library, Or. 168, 23a-35b [Voorhoeve 1957, 306].

4 *Translations*

Fragment 1. C 35a:6-8, I 31b:2-4 = IF 2:2-4, T 74a:3-6.

[Figure 8] Question by Abū Sahl al-Qūhī. Answer by Aḥmad ibn Muḥammad ibn ʿAbd al-Jalīl. We assume semicircle AGB, the diameter is AB, and we draw two perpendiculars BE, AD onto AB (and extend them) indefinitely. We draw AGE arbitrarily and we join BG and extend [it] towards AD. I say that EB times AD is equal to the square of AB. Proof ...

C 35a:15-18, I 31b:15-17 = IF 2:15-17, T 74b:6-11.

...What Abū Sahl mentioned on the authority of Apollonius[26] is this: since the ratio of AD times BE to the square of AB is

[26] Compare *Conics* III:53.

compounded of the ratio of AD to AB and the ratio of BE to AB, that is to say, of the ratio of GZ[27] to ZB and the ratio of GZ to ZA, the ratio compounded of [28] the ratio of the square of GZ to AZ times ZB is this same ratio. But the square of GZ is equal to AZ times ZB, so AD times EB is equal to the square of AB, and that is what we wanted to prove.

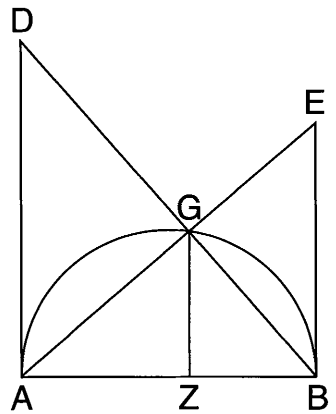


Figure 8

Fragment 2. C 35a:37-35b:7, I 32a:14-25 = IF 3:14-25, T 75a:12-75b:14.

[Figure 9] Solution by Abū Sahl al-Kūhī. Line AH cuts the circle at T, and AZ, AE are two tangents to the circle. We join ZIE. I say that the ratio of HI to IT is as the ratio of HA to AT.

Proof: We draw AB which passes though the center. Then it is clear that the ratio of BD to DG is as the ratio of BA to AG. We make AL equal to AG.

Since the ratio of BD to DG is as the ratio of BA to AG, the ratio of BG to DG is as the ratio of BL to LA, and the ratio of half of it, that is GK, to GD is as the ratio of half of it, that is KA, to AG.

Therefore the ratio of KG and KD taken together to GD is as the ratio of KA and KG taken together to AG, that is to say,

[27] GZ is perpendicular to AB.

[28] The three words "ratio compounded of" makes no sense here, they may have been added by al-Sijzī.

the ratio of BG to GD is as the ratio of BL to AG. So the ratio of KA to AG is as the ratio of KG to GD as we have mentioned.[29]

So the ratio of the first to the second is as the ratio of the first plus the third to the second plus the fourth.[30] So the ratio of AK together with KG, that is AB, to AD [$=AG+GD$], is as the ratio of KA to AG. So AB times AG is equal to AK times AD.

But the product AK times AD is equal to the product NA times AI because of the similarity of the two triangles,[31] and AB times AG is equal to AH times AT.[32] So AH times AT is equal to AN times AI.

So the ratio of HA to AI is as the ratio of AN to AT, and as the ratio of the remainder HN to the remainder IT. But HN is equal to NT, so the ratio of NA to AT is as the ratio of NT to TI. So the ratio of twice NA, that is HM,[33] to MA, which is equal to AT, is as the ratio of twice NT, that is HT, to IT. Separando, the ratio of HA to AT is as the ratio of HI to IT. That is what we wanted to prove.

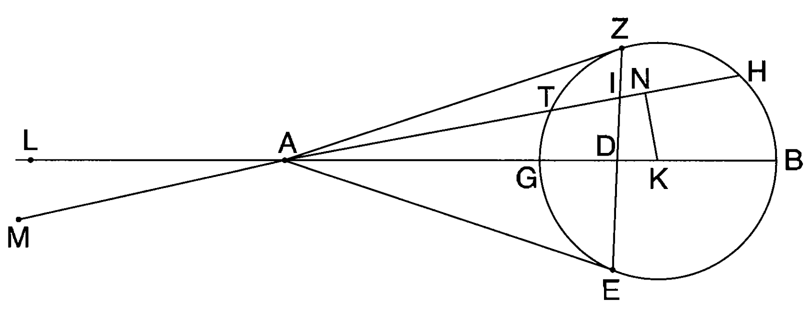


Figure 9

[29] The paragraph "Therefore the ratio of KG to ... we have mentioned" is mathematically superfluous and may be due to al-SijzīThe text here concludes from $GK : GD = KA : AG$, and the rule $a : b = c : d \rightarrow [a + (a - b)] : b = [c + (c - d)] : d$, that $GK + KD : GD = KA + KG : AG$. Then, using the fact that $KG = KB$, the author concludes that $BD : GD = BA : AG$, as had been stated in the beginning. The text then repeats $GK : GD = KA : AG$ in the form $KA : AG = KG : GD$.

[30] See *Elements* V, 12.

[31] The similar triangles are ADI and ANK. Point N is the midpoint of HT.

[32] Euclid's *Elements* III, 36.

[33] Point M has to be defined on HA extended such that $AM = AT$.

Fragment 3. C 37a:38-37b:9, I 35b:22-36a:10, IF 10:22-11:10.

[Figure 10][34] Solution by Abū Sahl al-Qūhī. Triangle ABG is assumed, and BG is less than AB. We want to prove that the square inscribed in the triangle and constructed on side AB is smaller than the square circumscribed by the triangle and constructed on BG.

Thus let us drop perpendiculars AD, GE onto BG, BA. We draw ZH equal to the side of the square constructed on BG [i.e., the square circumscribed by the triangle]. Then, since triangle ABD is similar to triangle BEG, the ratio of AB to BG is as the ratio of AD to GE, but AB is greater and GE is smaller, so AB and GE taken together are greater than BG and AD taken together.[35]

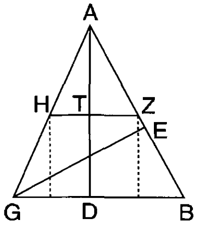


Figure 10

Again, since the ratio of BG to ZH is as the ratio of BA to AZ, and [this is] as the ratio of DA to AT,[36] the ratio of BG to ZH is as the ratio of DA to AT. So the product of BG times AT is equal to the product AD times ZH, that is, times DT. We add the product DT times BG. Then the product BG times AT and times DT, that is BG times AD, is equal to the product DT times AD and times BG taken together. So the side of the square constructed on BG times lines AD and BG together is equal to the product AD times BG, which is equal to the product GE times BA.

By this reasoning also, the product of the < side of the > square constructed on line AB times AB and GE together is equal to the product GE times AB.

[34] We have altered the figure in the manuscript, which shows an isosceles triangle with $AB = AG$.

[35] *Elements* V,25.

[36] Point T is the intersection of AD and HZ.

So the product of the side of the square constructed on BG times lines AD and BG together is equal to the product of the side of the square constructed on AB times lines GE and AB together. But we have proved that AB and GE together are greater than BG and AD together. So the side of the square constructed on AB is smaller than the side of the square constructed on BG, so its square is smaller than its square. That is what we wanted to prove.

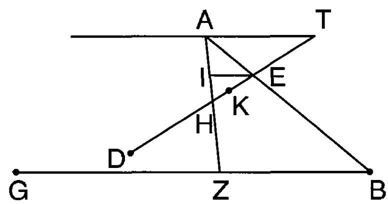


Figure 11

Fragment 4. C 37b:10-17, I 36a:11-19 = IF 11:11-19.

[Figure 11] The analysis of this proposition of Apollonius [is] by Abū Sahl. Point A is known and lines BG, DL[37] are known in position and magnitude.[38] We want to draw [a line] such as AZ such that the ratio of HD to BZ is equal to an assigned ratio. Analysis: The ratio of HD to BZ is known. We join AB and we draw EI parallel to BG, and we extend DE to T.[39] Then the ratio of BZ to EI is known since it is as the ratio of AB, which is known, to AE, which is known. So the ratio of DH to EI is known. But the ratio of AT to EI is made as the ratio of LD to DH.[40] Then the ratio of TH to HE is equal to the ratio of LD

[37] Point L in the text corresponds to point K in the figure. This point K in the figure may be a scribal error.

[38] The proof assumes that points B and D are known and that lines BG and DT are "known in position," but it is not necessary to assume that points G and L are known, so segments BG and DT do not need to be "known in magnitude." In the proof, Al-Kūhī actually defines the position of point L on line DL.

[39] Point T is chosen in such a way that AT is parallel to BG.

[40] If point L is defined this way on line DL, $LD : AT = DH : EI$, which is a known ratio. Since AT is known in magnitude, LD is also known in magnitude, so point L is known in position. The text is somewhat odd, and it is possible that al-Kūhī said instead of *lākin ... tujʿalu* the word *l-yakun*

to DH. Convertendo,[41] the ratio of DL to LH is equal to the ratio of HT to TE, so the product DL times ET, both of which [segments] are known, is equal to the product TH times HL, so TH times HL is known. But TL is known, so LH is known,[42] so point H is known, and that is what we wanted to prove.

Fragment 5. C 38a:9-18, I 37a:6-16 = IF 13:6-13.

[Figure 12] Abū Sahl al-Qūhī asked me [this question] so I solved it by means of the hyperbola. A semicircle AB is given and the diameter is AB. We want to draw from the diameter to the circumference a straight line perpendicular to the diameter, and such that the product of one of the parts of the diameter and the perpendicular drawn onto it is equal to the given square EZ.

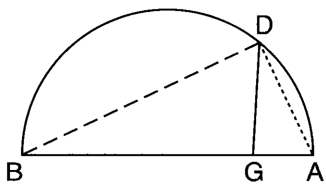


Figure 12

[The following solution is by al-Sijzī] [Figure 13] Thus let us extend EM, EN indefinitely.[43] We apply to point Z the [conic] section KZI with asymptotes ET, EH. We make EH equal to AB. We describe on diameter EH semicircle ELH and we drop from the point of its intersection and the intersection of the [conic] section, that is L,[44] a perpendicular LM onto EH. We make AG equal to EM and we draw GD perpendicularly to AB. I say that this is what was required, that is to say, that AG times GD is

..., in which case his text meant "let the ratio of AT to EI be as the ratio of LD to DH."

[41] This operation is defined in *Elements* V, def. 17.

[42] See Euclid, *Data*, 58-59 [Thaer 1962, 40-41].

[43] Al-Sijzīexpects his reader to deduce from the figure that the "given square" is $EMZN$, point H is on EM extended and point T is on EN extended.

[44] The terminology "its intersection and the intersection of the [conic] section" is strange and shows al-Sijzī's unfamiliarity with conic sections. Al-Sijzī does not bother to ask whether a point of intersection L exists. See fragment 9 below.

equal to the square of EZ. Its proof: since ML times ME is equal to the square EZ, therefore AG times GD is equal to the square EZ. So we have constructed what we wanted, and that is what we wanted to prove.

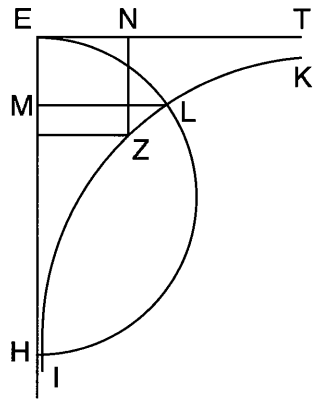


Figure 13

Fragment 6. C 38b:4-38, I 37b:21-38b:7 = IF 14:21-16:7.

[Problem 6,1] By Abū Sahl. We want to draw from the [given] point A to the two lines BG, GD [given in position] two straight lines such that their product is equal to the [given] rectangle EZ and they contain an angle equal to the [given] angle H.

[Synthesis: Figure 14] Thus we mark on GD an arbitrary point, and let it be D, and we join AD. We draw from point A a straight line AT in such a way that the product AD times AT is equal to rectangle EZ, and they $[AD, AT]$ contain an angle equal to angle H. We construct on AT a segment of a circle, containing an angle equal to angle ADG, namely segment ABT. We join AB, BT. We set up on AB an angle equal to angle TAD, namely angle BAL.

Then angle BAL is equal to angle TAD, so we subtract the common angle TAL. Then angle BAT is equal to angle DAL, but angle GDA is [also] equal to angle TBA, so triangle ATB is similar to triangle DAL. So the ratio of AL to AT is as the ratio of AD to AB, and so the product AL times AB is equal to the product AD times AT. But the product AD times AT is equal to rectangle EZ. So the product AL times AB is equal to

rectangle EZ, and they $[AL, AB]$ contain an angle equal to angle H. And that is what we wanted to prove.

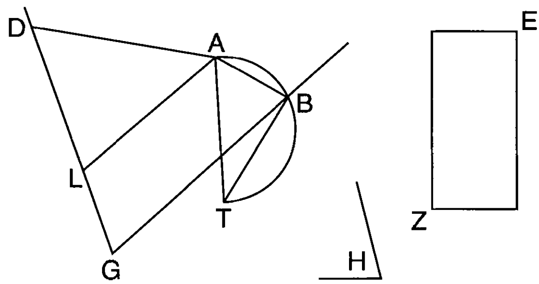


Figure 14

His [i.e., al-Kūhī's] analysis: [Figure 15] We also draw from the known point A to the two lines BG, GD, which are both known in position, two lines AB, AD, and they contain the known angle BAD. I say that the product AB times AD is known.[45]

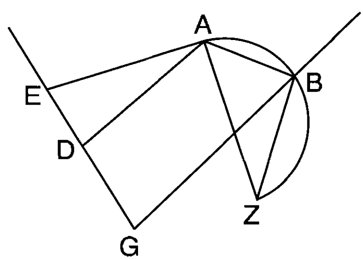


Figure 15

By way of analysis, we say: The product AB times AD is known. We draw from point A to < line GD > line AE which contains together with line EG a [i.e., an arbitrary] known angle. We[46] make the product AE times AZ equal to the product AB times

[45] Here al-Sijzīshould have said: ...two lines AB, AD, which contain the known angle DAB and whose product AB times AD is known. I say that points B, D are known." Al-Sijzī was evidently misled by the beginning of al-Kūhī's analysis: "By way of analysis, we say: The product AB times AD is known."

[46] Here one would expect: "we draw the line AZ so that angle AEZ is equal to the given angle DAB."

AD, and we join BZ. Then the product AE times AZ is [also equal to the] known [area], so[47] the ratio of AD to AE is as the ratio of AZ to AB. So triangle ABZ is similar to triangle ADE, and so angle ABZ is equal to the known angle AED, so angle ABZ is known. So a known circular segment passes through point B, namely ABZ, and so point B is known. So line AB is known in position, so angle BAD is known, so line AD is known in position, and so point D is known. That is what we wanted to prove.

[Problem 6,2] Also by him [i.e., al-Kūhī] [Figure 16] We want to draw from the known point A to the two lines BG, GD, which are both known in position,[48] two lines AB, AD that contain a known angle BAD, such that the ratio of AB to AD is known.

By analysis, we say that the ratio of AB to AD is known. Through points A, B, D some circle passes, namely ABD. We draw from point A two lines AE, AZ and we join BD. Then, since the ratio of AB to AD is known, and angle BAD is known, triangle ABD is known in form, so angle ABD is known. But it is equal to angle AZD, so angle AZD is known, so line AZ is known in position,[49] and so point Z is known. Again, angle ADB is known, but it is equal to angle AEB, so angle AEB is known, so line AE is known in position, so point E is known. So the two points Z, < E are known, and point A is known, so circle AEZ is known, so the points B, > D are known, and this is what we wanted to prove.

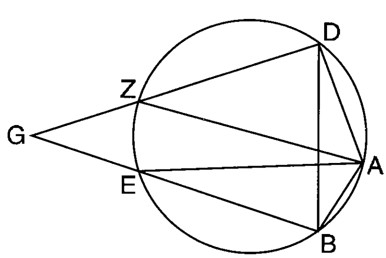


Figure 16

[47] "and" would be correct here.

[48] The notion "known in position" does not imply that the positions of points B, D on the two lines BG, GD are known.

[49] See Euclid, *Data* 30 [Thaer 1962, 23].

[Figure 17] The synthesis: We want to draw from point A to the two lines BG, GD, two straight lines which contain angle T and such that the ratio between them is as the ratio of HT to TK. So we draw from point A to BG line AB which contains with line $< BE >$ [I f. 16] angle ABE equal to angle K. We also draw from point A to line GD line AD which contains together with line DZ angle ADZ equal to angle H. We let pass through points A, B, D a certain circle, namely AB. We join AE, AZ, ZE. Then angle ADZ is equal to angle AEZ, and equal to angle H, so angle AEZ is equal to $<$ angle $>$ H. Then, angle ABE is also equal to angle AZE and equal to angle K, so by subtraction, angle EAZ is equal to angle T. Thus triangle AEZ is similar to triangle HTK, so the ratio of AE to AZ is as the ratio of HT to TK, and that is what we wanted to prove.[50]

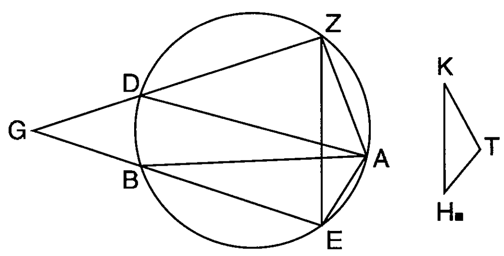


Figure 17

Fragment 7. C 39b:41-40a:1, I 40b:2-3 = IF 20:2-3.

[Figure 18] My answer to Abū Sahl. If there are triangle ABG and line BD, and we want to draw from point A a straight line such as AEZ, in such a way that the ratio of triangle ADE to triangle ABZ is as the ratio of H to T[51] ...

[50] Ibrāhīm ibn Sinān also solved Problem 6,2 in his *Selected Problems*. [Bellosta 2000, 560, 732-735] failed to discuss the diorismos of the problem: if $\angle ZGE + \angle T \neq 180^\circ$, the problem has one solution. If $\angle ZGE + \angle T = 180^\circ$ the problem has infinitely many solutions if $D = B = G$ and no solutions if $B \neq G$ (in which case points A, D, B are on a straight line). Al-Sijzīpresents a simple solution to the problem in C 52a:2-10, I 61b:1-9, IF 62:1-9. In the notation of Figure 17, he constructs a figure $HTKG'$ similar to $EAZG$. Using the facts that the angles ZGE and ZGA are known, he finds G' as the intersection of two circular segments.

[51] Point D is on side AG of the triangle, point E is on BD and Z on BG.

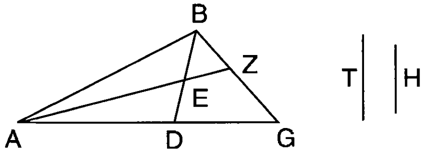


Figure 18

Fragment 8. C 41b:18-28, I 43b:13-25 = IF 26:13-25.

[Figure 19] By Abū Sahl. Let us assume that two lines AB, GD are bounded and that point E is assumed. We want to draw [a line] such as line EZ, so that we divide the two lines [segments AB, GD] at H, Z in such a way that the ratio of the product GZ times AH to the product HB times ZD is as an assumed ratio.

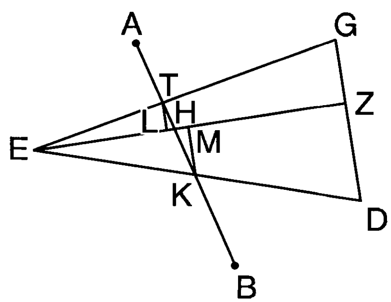


Figure 19

Thus let us draw EZ by way of analysis, such that $<$ the ratio of $>$ GZ times AH $<$ to $>$ HB times ZD is as an assumed ratio. We join EG, ED, then they meet line[52] AB at points T, K. We draw TL, KM parallel to line GD. Then, since the ratio of AH times GZ to HB times ZD is known[53] and the ratio of AH times GZ to AH times TL is known because it is as the ratio of GE to ET, which is [a] known [line segment], the ratio of AH times TL to HB times ZD is known. But the ratio of HB times ZD to HB times KM is known because it is as the ratio of DE to EK, so the ratio of AH times TL to HB times KM is known. But

[52] The manuscripts have: "It meets the two lines."

[53] There are many grammatical errors in the Arabic text of the following passage.

it is compounded of the ratio of AH to HB and the ratio of TL to KM, < which > is as the ratio of TH to HK because of the similarity of the two triangles.[54] So the ratio compounded of the ratio of AH to HB and the ratio of TH to HK is known, and it is as the ratio of the rectangle AH times HT to the rectangle BH times HK. Thus the ratio of the rectangle AH times HT to the rectangle BH times HK is known, so point H is known from *Producing Points on Lines in the Ratios of Rectangles.*[55] That is what we wanted to prove.

Fragment 9. C 42b:15-18, I 45a:22-45b:1 = IF 29:22-30:1.

[Figure 20] Question by Abū Sahl on the diorismos of a proposition involving a semicircle. AGB is given and we have divided the arc into two parts in such way that AG is one-third of the semicircle. BG has been drawn and we drop perpendicular GD onto AB. I say that triangle GDB is greater than any triangle that results from drawing a line from B to the semicircle BGA and the perpendicular drawn from the endpoint of that line to line AB. Answer by Aḥmad ibn Muḥammad ibn ʿAbd al-Jalīl: ...

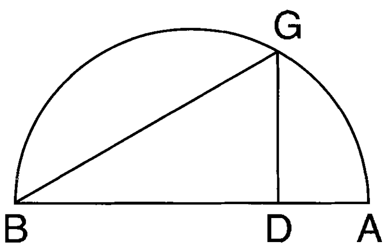


Figure 20

Fragment 10. C 46a:20-47a:25, I 51b:4-53a:22 = IF 42:4-45:22.

Answer by Wayjan, known as Abū Sahl al-Kūhī, to the geometrical questions, and they are six propositions.

We want to draw from point D, which is on side AB in the first example [Problem 10,1, Figure 21] [56] and on its rectilinear

[54] The two similar triangles are TLH, KMH.
[55] See our commentary.
[56] The word "example" really means "case" here.

extension in the second example [Problem 10,2, Figure 22], a straight line in such a way that the ratio of the triangle produced below angle A to the triangle produced below angle G is as the ratio of E to Z.

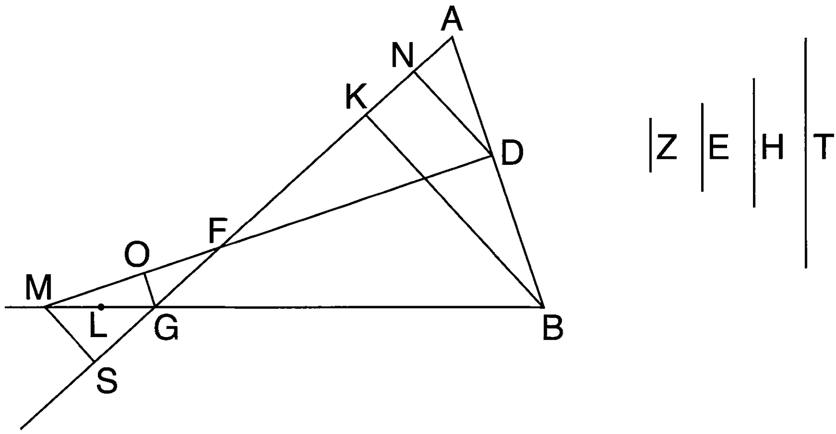


Figure 21

Thus we make the ratio of E to another line, namely T, as the ratio of half the line BK perpendicular to line AG, to BA, and we also make the ratio of another line, namely H, to Z as the ratio of BG to half the line BK perpendicular to line AG. We apply to line DB a rectangle such that the ratio of the square of AD to it is as the ratio of T to H, and let the width which is produced be equal to line GL. We apply to line GL a rectangle equal to the rectangle BG times GL, and which exceeds its completion by a square, and let the side of the exceeding square in the first example be LM and in the second example GM.[57] We draw line $DMFO$. I say that the ratio of triangle ADF to triangle GFM is as the ratio of E to Z.

Proof: we make each of the lines ND, MS perpendicular to line AG, and line GO parallel to line AB. Then, since the ratio of E to T is as the ratio of half of line BK to line BA, and the ratio of half of line BK to line BA is as the ratio of half of ND to DA because of the similarity of the two triangles, the ratio of E to T is as the ratio of half of ND to DA. If we make AF an

[57] In both cases $MG \cdot ML = BG \cdot GL$.

altitude common to the two of them, the ratio of E to T is as the ratio of half of ND times AF, that is triangle ADF, to the rectangle DA times AF.

Again, since the ratio of T to H is as the ratio of the square of AD to the rectangle DB times GL, and the rectangle DB times GL is equal to the rectangle MG times GO, because the rectangle BG times GL is equal to the rectangle MG times ML, the ratio of BG to GM is as the ratio of ML to LG. So *componendo* in the first example and *separando* in the second example, the ratio of BM to MG, that is the ratio of DB to GO because of the similarity of the two triangles, is as the ratio of MG to GL. So the rectangle MG times GO is equal to the rectangle DB times GL.

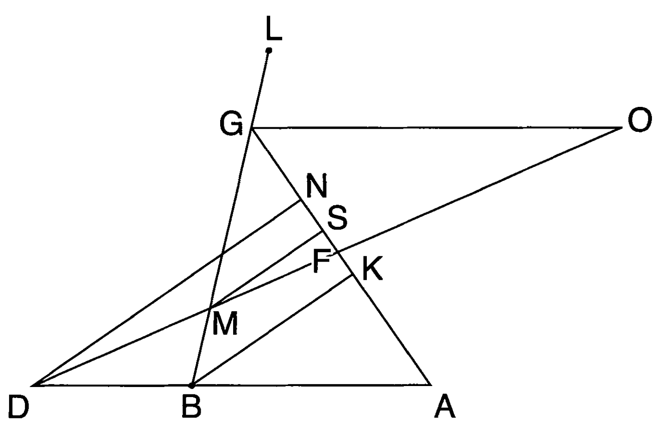


Figure 22

But the ratio of the square of AD to the rectangle DB times GL was [shown to be] as the ratio of T to H, so the ratio of T to H is as the ratio of the square of AD to the rectangle GM times GO. But the ratio of the square of AD to the rectangle GM times GO is as the ratio compounded of the ratio of AD to GM and of the ratio of AD to GO; so, the ratio of T to H is as the ratio compounded of the ratio of DA to MG and of the ratio of DA to GO, that is to say, as[58] the ratio of AF to FG, because of the similarity of the two triangles. So the ratio of T to H is as the

[58] It would have been correct to say "the ratio" (*nisba*), not "as the ratio" (*ka-nisba*).

ratio compounded of the ratio of DA to MG and of the ratio of AF to FG. But the ratio compounded of the ratio of DA to MG and of the ratio of AF to FG is as the ratio of the rectangle DA times AF to the rectangle MG times GF. So the ratio of T to H is as the ratio of the rectangle DA times AF to the rectangle MG times GF.

But the ratio of triangle ADF to the rectangle AD times AF was [shown to be] as the ratio of E to T. So, ex aequali, the ratio of triangle ADF to the rectangle MG times GF is as the ratio of E to H.

Again, since the ratio of H to Z is as the ratio of GB to half of BK, and the ratio of BG to half of BK is as the ratio of GM to half of MS because of the similarity of the two triangles, so if we make FG an altitude common to them, the ratio of H to Z is equal to the ratio of the rectangle MG times GF to the rectangle half of MS times GF. But half of MS times GF is equal to triangle MGF. So the ratio of H to Z is as the ratio of rectangle MG times GF to triangle MGF. But the ratio of E to H was [shown to be] equal to the ratio of triangle ADF to rectangle $<$ MG times $>$ GF, so ex aequali, the ratio of E to Z is as the ratio of triangle ADF to triangle MGF in both examples. That is what we wanted to prove.

The third [Problem 10,3, Figure 23.[59] We want to draw from side AB of triangle ABG toward side AG a straight line parallel to line AD which is known in position, in such a way that the ratio of the product of one of the remaining two sides times the other $< \ldots >$ [60] is as the ratio of E to Z.

If AD is parallel to BG, its construction is easy.[61] If it is not parallel to it, then they meet, and let this be at point D. We apply to line AD a rectangle such that the ratio of BA times AG to it is as the ratio of E to Z, and let the width which is produced

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[59] The figure in the manuscript shows lines E, Z of the same length, but we have changed the figure to represent the general situation.

[60] The text is defective here. The problem is as follows. Given: triangle ABG, the point D on BG extended, and the ratio $E : Z$. Required: points L and M on AG and AB respectively such that $LM//AD$ and $(BM \cdot LG) : (BG \cdot LM) = E : Z$.

[61] Al-Sijzīsolves the easy case $AD//BG$, $E = Z$, in C 43a:15-25, I 46a:18-46b:3 = IF 31:18-32:3.

be equal to line GT.[62] We apply to line TB a rectangle equal to the product BD times GT and which exceeds its completion by a square, and let the side of the exceeding square be BK.[63] We join AK and we draw from point B a line parallel to line AK, namely BL. We draw from point L a line parallel to line AD, namely LM. I say that the ratio of the product BM times LG to the rectangle BG times ML is as the ratio of E to Z.

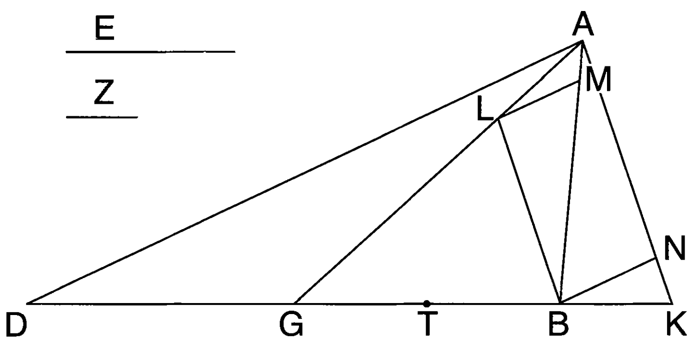


Figure 23

Proof of this: We make BN parallel to AD. Then, since the product TK times KB is equal to the product TG times BD, the ratio of KT to TG is as the ratio of DB to BK. *Componendo*, the ratio of KG to GT is as the ratio of DK to KB. But the ratio of DK to KB is as the ratio of AD to NB because of the similarity of the two triangles. So the ratio of KG to GT is as the ratio of AD to NB. So the product KG times BN is equal to the product AD times TG.

But the ratio of the product BA times AG to the product AD times TG was [assumed to be] as the ratio of E to Z, so the ratio of E to Z is as the ratio of [the product] BA times AG to the product GK times BN. But the ratio of the product BA times AG to the product KG times BN is compounded of two ratios, one of which is the ratio of AB to BN and the other the ratio of AG to GK. So the ratio of E to Z is compounded of two ratios, one of which is AB to BN and the other AG to GK. But the ratio of AB to BN is as the ratio of BM to ML because of the

[62] In modern notation $(BA \cdot AG)/(AD \cdot G) = E/Z$.
[63] In modern notation $BD \cdot GT = TK \cdot BK$.

similarity of the two triangles, and the ratio of AG to GK is as the ratio of LG to GB because AK, LB are parallel lines. So the ratio of E to Z is compounded of the ratio of BM to ML and of the ratio of LG to GB, which is as the ratio of the product BM times LG to the product BG times ML. So the ratio of E to Z is as the ratio of the product BM times LG to the product BG times ML. That is what we wanted to prove.

The fourth [Problem 10,4, Figure 24]. We want to find a right-angled triangle such that [1] the excess of half the hypotenuse over the perpendicular drawn from the right angle times [2] one of the two parts which the perpendicular drawn to it cuts off [from the hypotenuse] is equal to the known rectangle A.

Thus we make the square of BG equal to the rectangle A. We draw BE perpendicular to BG. We make the square of GD twice the square of DB.[64] We make each of the lines DE, DZ equal to line DG, and we join ZE, GE. I say that triangle GEZ is the required [triangle], that is to say that angle ZEG is [a] right [angle] and the excess of half of ZG, which is the hypotenuse, over EB the perpendicular, times BG which is one of the parts [of the hypotenuse] is equal to rectangle A.

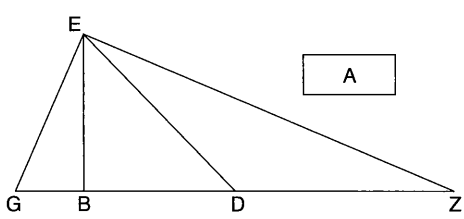


Figure 24

Proof of this: Since lines DE, DZ, DG are equal, point D is the center of a circle whose circumference passes through the points Z, E, G, so angle ZEG is [a] right [angle]. Again, since line ZG is bisected at point D and divided into two unequal parts at point

[64] Curiously, al-Sijzīdoes not explain the construction of point D. This construction could be carried out as follows: Choose point F on GB extended such that $BF = BG$, and then construct point D on BF extended such that $BD \cdot FD = GB^2$ by Euclid's *Elements* VI, 29 [Heath 1956 II, 265]. An easier construction is possible because $\angle EGB = 67.5^\circ$.

B, the rectangle ZB times BG plus the square of DB is equal to the square of DG.[65] But the square of DG is equal to twice the square of BD. So the rectangle ZB times BG plus the square of DB is equal to twice the square of DB. If we remove the common square DB, the rectangle ZB times BG is equal to the square of DB. But the rectangle ZB times BG is equal to the square of BE, so the square of BE is equal to the square of DB. Thus the excess of GD, which is half of the hypotenuse, over EB, is BG, since EB is equal to DB. So line BG, which is the excess of half ZG over EB, is BG,[66] and BG times itself is equal to the rectangle A since we have made its square equal to the rectangle A. That is what we wanted to prove.

The fifth [Problem 10,5, Figure 25]. We want to draw in two circles $ABGD, EZHT$ one straight line such that there falls in circle $ABGD$ a line equal to line GD and in circle $EZTH$ a line equal to line HT.

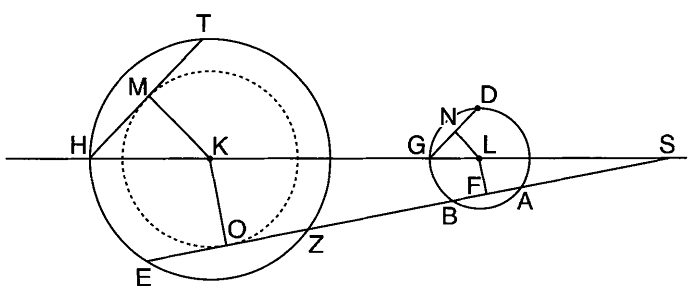


Figure 25

Thus we drop from the centers K, L two perpendiculars KM, LN onto lines GD, HT. If KM is equal to LN, its construction is easier than if it is not equal. If they are not equal, one of them in longer than the other, so let the longer [line] be KM. Then we make the ratio of KS to SL as the ratio of KM to LN. We draw from point S line SEZ tangent to the circle with radius KM and center the point K.[67] I say that line AB, i.e., the part of this line that falls in circle $ABGD$, is equal to line GD.

[65] Euclid, *Elements* II, 5.

[66] The passage "So line BG, which is the excess of half ZG over EB, is BG" is superfluous, and probably an addition made by al-Sijzī.

[67] This circle is not drawn in the figure in the manuscripts.

Proof of this: We make lines KO, LF perpendiculars to line $ABEZ$. Then, since the ratio of KM, which is equal to KO, to line LN is as the ratio of KS to SL, and the ratio of KS to SL is as the ratio of KO to LF, the ratio of KO to each of the lines LN, LF is the same, so LN is equal to LF. Hence line AB is equal to line GD. Line EZ is equal to line HT since their distances [KO, KM] from the center [K] are the same. So we have drawn in the circles $ABGD, EZHT$ one straight line in such a way that the magnitude of the part of it that falls inside circle $ABGD$ is equal to a known line, and the part of it that falls inside circle $EZHT$ is equal to another known line. That is what we wanted to prove.

The sixth [Problem 10,6, Figure 26]. We want to draw from [the given] point A to [the given] lines BG, $< BZ >$ a straight line in such a way that below angle B a triangle is produced which is equal to the known rectangle E.[68]

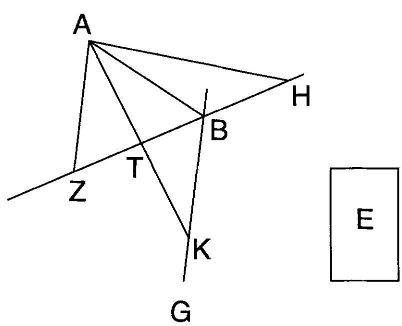


Figure 26

Thus we draw from point A a line parallel to line BG, namely AZ. We join line AB, and we make triangle ABH equal to rectangle E. We apply to line HB a rectangle equal to rectangle ZB times BH such that it exceeds its completion by a square, and let the side of the exceeding square be line BT. We join line AT and we extend it rectilinearly towards K. I say that triangle BKT is equal to rectangle E.

[68] The problem is stated incompletely. Line BD is also a given line. Required: to construct a straight line ATK such that T is on BD, K is on BG, and triangle BTK is equal in area to E.

Proof of this: Since rectangle ZB times BH is equal to rectangle HT times TB, the ratio of ZB to BT is as the ratio of TH to HB. *Separando*, the ratio of ZT to TB $<$ is as the ratio of TB to BH. But the ratio of ZT to TB $>$ is as the ratio of AT to TK, so the ratio of AT to TK is as the ratio of TB to BH.

But the ratio of AT to TK is as the ratio of triangle ABT to triangle TBK, and, similarly, the ratio of TB to BH is as the ratio of triangle ATB to triangle ABH. So the ratio of triangle ATB to each of the triangles ABH, KBT is the same. So triangle KBT is equal to triangle ABH. But triangle ABH was [supposed to be] equal to rectangle E. So triangle KBT is equal to rectangle E. That is what we wanted to prove.

Fragment 11. C 55b:22-25, I 115a:21-23 = IF 165:21-23.

[Fragment 11,1, Figure 27] We construct a circle which is tangent to circle EZ, and meets point A, and the center of which is on line GD, as Wayjan ibn Rustam proved in the sixth proposition of his book *On the Centers of Tangent Circles on Lines, by Way of Analysis.*

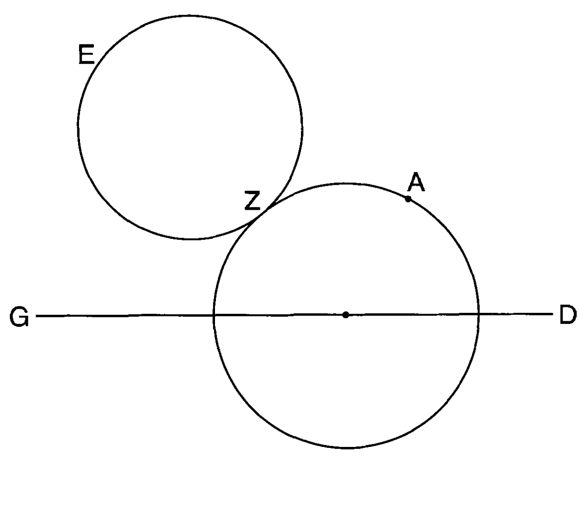


Figure 27

[Fragment 11,2, Figure 28] Synthesis of a proposition of Wayjan ibn Rustam, and another method for the proof of the question before this proposition. Namely: How do we divide the straight line AB, [already] divided at G, into two parts, for example at Z,

in such a way that the ratio of ZB to AG is as the ratio of ZG to AZ.[69]

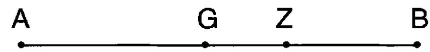


Figure 28

Fragment 12. L 24b:1-14.

[Figure 29] Lemma by Abū Sahl al-Kūhī. Lines AB, BG are assumed and contain angle B, and BG is [extended] indefinitely. We want to draw AG and DG in such a way that DG is equal to DA and the ratio of AB to BG is as the ratio of BG to BD. If this is achieved, the division of angle EBG into three equal parts is also achieved if AB is extended towards E.

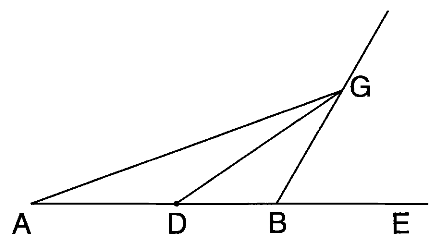


Figure 29

This is because triangle ABG is similar to triangle DBG because angle B is common, so angle BGD in triangle BGD is equal to angle GAB in triangle GAB, and angle GDB is equal to angle AGB, so angles D, G in triangle BDG are equal to angles G, A in triangle BAG. But angle BDG is twice angle A so it is twice angle DGB, and angle EBG is equal to angles BGD [plus] BDG. So angle BDG is two-thirds of angle EBG and angle BGD is one-third of it. Thus the division of the angle into three equal parts is achieved by means of this lemma.

[69] Al-Sijzīpresents a construction of Z by intersecting a semicircle on diameter AB with a parabola with vertex G, axis GB and latus rectum AG. Algebraically, the problem is equivalent to a quadratic equation. Al-Kūhī would have solved the problem somewhat as follows: ¿From $ZB : AG = ZG : AZ$ we obtain $ZB \cdot AZ = AG \cdot ZG$, and, by addition of $ZB \cdot AG$, we obtain $ZB \cdot (AZ + AG) = AG \cdot BG$. Define H on BA extended such that $AH = AG$. Then $ZB \cdot ZH = AG \cdot BG$, and point Z can now be found by Euclid's *Elements* VI, 29.

5 *Arabic Texts*

Fragment 1. C 35a:6-8, 15-18, I 31b:2-4, 15-17 = IF 2:2-4, 15-17, T 74a:3-6, 74b:6-11.

السؤال عن[1] أبي سهل القوهي الجواب لأحمد[2] بن محمد بن عبد الجليل.
نفرض نصف[3] دائرة اجب والقطر اب ونخرج به اد عمودين[4] على اب إلى غير نهاية ونخرج اجه كيفما وقع ونصل بجـ ونخرج إلى اد[5]. أقول إن هب في اد يعدل مربع[6] اب. برهانه ....

وأما ما ذكر أبو سهل عن أبلونيوس فهو هذا[7] فلأن نسبة اد في به إلى مربع اب مؤلّفة من نسبة اد إلى اب ومن نسبة به إلى اب أعني من نسبة جز إلى زب ومن نسبة جز إلى زا تكون النسبة المؤلّفة من نسبة مربع جز إلى از في زب هذه النسبة بعينها. لكن مربع جز يعدل[8] از في زب فـ اد في هب يعدل[9] مربع اب وذلك ما أردنا أن نبين.[10]

Fragment 2. C 35a:37-35b:7, I 32a:14-25 = IF 3:14-25, T 75a:12-75b:14.

استخراج أبي[11] سهل الكوهي. خط اح قطع الدائرة على ط و از اه مماسا[12] الدائرة ونصل زيه. أقول[13] إن نسبة حي إلى يط كنسبة حا إلى اط. برهانه أن نخرج اب يجوز على المركز فيبين أن نسبة بد إلى دجـ كنسبة با إلى اجـ. ونجعل ال[14] مثل اجـ. فمن أجل أن نسبة بد إلى دجـ كنسبة با إلى اجـ تكون نسبة[15] بجـ[16] إلى دجـ كنسبة بل إلى لا[17] ونسبة نصفه وهو جك[18] إلى جد كنسبة نصفه وهو كا إلى اجـ.

---

1 السؤال عن CI: سؤال T. 2 القوهي الجواب لأحمد CI: الكوهي جواب احمد T.
3 نصف CI: - T. 4 عمودين T: عمودان CI. 5 ونخرج إلى اد CI: ونخرجه إلى د T.
6 يعدل مربع CI: مساو لمربع T. 7 وأما ···· فهو هذا CI: طريق أبلونيوس في هذا T.
8 يعدل CI: مثل T. 9 يعدل CI: مثل T. 10 أن نبين CI: بيانه T.
11 استخراج أبي CI: لأبي T. 12 مماسا CI: عاسان T. 13 أقول CI: فأقول T.
14 ال CT: ابل I. 15 بد إلى دجـ ···· تكون نسبة T, in marg. D, - I. 16 بجـ TI: ابجـ C, the ا indicates the position of the marginal passage in the text, see the previous footnote. 17 لا C: ال T, كر I. 18 جك: جل CIT.

فإذًا نسبة كج[1] و كد بالتركيب إلى جد[2] كنسبة كا مع كج بالتركيب إلى اج أعني نسبة بج إلى جد كنسبة بل[3] إلى اج · فنسبة كا إلى اج كنسبة كج إلى جد كما ذكرنا فنسبة الأول إلى الثاني كنسبة الأول والثالث إلى الثاني والرابع· فنسبة اك[4] مع كج أعني اب إلى اد كنسبة كا إلى اج ف اب في اج مساو ﻟ اك[5] في اد · لكن ضرب اك[6] في اد مساو لضرب نا في اي لتشابه المثلثين و اب في اج مساو ﻟ اح في اط ف اح في اط[7] مساو ﻟ ان في اي[8] · فنسبة حا إلى اي[9] كنسبة ان إلى اط وكنسبة حن[10] الباقي إلى يط الباقي· و حن[11] مساو ﻟ نط · فنسبة نا إلى اط كنسبة نط إلى طي[12] · فنسبة ضعف نا أعني حم إلى ما المساوي ﻟ اط كنسبة ضعف نط أعني[13] حط إلى يط[14] · وإذا فصلنا نسبة حا إلى اط كنسبة حي إلى يط وذلك ما أردنا أن نبين·

Fragment 3. C 37a:38-37b:9, I 35b:22-36a:10 = IF 10:22-11:10.

استخراج أبي سهل القوهي· مثلث ابج مفروض و بج أصغر من اب · نريد أن نبين أن المربع المضاف إلى ضلع اب الكائن في المثلث أصغر من المربع المضاف إلى بج المحيط به المثلث·

فلنخرج اد جه عمودين على بج با و نخرج زح مساويًا لضلع المربع المعمول على بج · فلأن مثلث ابد يشبه مثلث بهج تكون نسبة اب إلى بج كنسبة اد إلى جه لكن اب أعظم و جه أصغر ف اب جه مجموعين أعظم من بج اد مجموعين· وأيضًا لأن نسبة بج إلى زح كنسبة با إلى از وكنسبة دا إلى اط فنسبة بج إلى زح كنسبة دا إلى اط فضرب بج في اط مساو لضرب اد في زح أعني في دط · نجعل ضرب دط في بج مشتركًا يكون ضرب بج في اط وفي دط أعني بج في اد مساويًا[15]

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[1] كج CI: كع T. [2] إلى جد CI : - T. [3] بل CI: بك T. [4] اك CI: ال T.
[5] اك CI: ال T. [6] اك CI : ال T. [7] ف اح في اط CT: - I. [8] اي CT : الى I.
[9] اي CT : الى I. [10] حن T : حر CI. [11] حن CI: حي T. [12] طي T : طن CI. [13] حم إلى ما ... أعني CI: in marg. T. [14] يط : نط CI, طي T.
[15] مساويًا : مساو CI.

لضرب دط في اد وفي بج جميعًا· فضلع المربع المعمول على بج في خطي اد بج جميعًا مساو لضرب اد في بج المساوي لضرب جه في با ·

وبهذا التدبير أيضًا يكون ضرب < ضلع > المربع المعمول على خط اب في اب و جه جميعًا مساو لضرب جه في اب · فضرب ضلع المربع المعمول على بج في خطي اد بج مساو لضرب ضلع المربع المعمول على اب في خطي[1] جه اب مجموعًا· وقد بيّنّا أن اب جه مجموعين أعظم من بج اد· فضلع المربع المعمول على اب أصغر من ضلع المربع المعمول على بج فمربعه أصغر من مربعه· وذلك ما أردنا أن نبيّن·

Fragment 4. C 37b:10-17, I 36a:11-19 = IF 11:11-19.

لأبي سهل تحليل هذا الشكل لأبللونيوس· نقطة ا معلومة وخطا بج دل معلوما الوضع والمقدار· نريد أن نخرج مثل از يكون نسبة حد[2] إلى زب كنسبة مفروضة·

فعلى التحليل نسبة حد[3] إلى بز معلومة· ونصل اب و نخرج هي يوازي بج و نخرج ده[4] إلى ط · فنسبة بز إلى هي معلومة لأنّها كنسبة اب المعلوم[5] إلى اه المعلوم[6] فنسبة دح إلى هي معلومة· لكن نسبة اط إلى هي تجعل كنسبة لد إلى دح فنسبة طح إلى حه كنسبة لد إلى دح · وإذا قلبنا تكون نسبة دل إلى لح كنسبة حط إلى طه · فضرب دل في هط المعلومين مساو لضرب طح في حل ف طح في حل معلوم و طل معلوم فلح معلوم فنقطة ح معلومة[7] وذلك ما أردنا أن نبين·

Fragment 5. C 38a:9-18, I 37a:6-16 = IF 13:6-13.

سألني أبو سهل القوهي فاستخرجته بالقطع الزائد·

نصف دائرة اب معطى والقطر اب ونريد أن نخرج من القطر إلى المحيط خطًا مستقيمًا يكون عمودًا على القطر ويكون ضرب أحد

---

[1] خطي C: خط I. [2] حد : حه CI. [3] حد : حه CI. [4] ده C: ج I.
[5] المعلوم : المعلومة CI. [6] المعلوم : المعلومة CI. [7] معلومة : معلوم CI.

قسمي[1] القطر في العمود الواقع عليه يعدل مربع هز المعطى.
فلنخرج هم هن إلى غير نهاية ونضيف إلى نقطة ز قطع كزي لا يلتقيانه هط هح. ونجعل هح مثل اب وندير على قطر هح نصف دائرة هلح ونخرج من نقطة تقاطعها وتقاطع القطع أعني ل عمود لم على هح ونجعل اج مثل هم ونخرج جد عمودًا[2] على اب. أقول إنه هو المطلوب أعني أن اج في جد مثل مربع هز. برهانه لأن مل في مه يعدل مربع هز ف اج في جد إذًا مثل مربع هز فقد عملنا ما أردنا وذلك ما أردنا أن نبين.

Fragment 6. C 38b:4-38, I 37b:21-38b:7 = IF 14:21-16:7.

لأبي سهل. نريد أن نخرج من نقطة ا إلى خطي بج جد خطين مستقيمين حتى يكون ضرب أحدهما في[3] الآخر مثل سطح هز ويحيطان بزاوية مثل زاوية ح[4].
فنتعلم على جد نقطة كيف ما وقعت ولتكن د ونصل اد. ونخرج من نقطة ا خطًا مستقيمًا[5] وهو اط حتى يكون ضرب اد في اط مثل سطح هز ويحيطان بزاوية مثل زاوية ح. ونعمل على اط قطعة دائرة تقبل زاوية مثل زاوية ادج وهي قطعة ابط. ونصل اب بط ونقيم على اب زاوية مثل زاوية طاد وهي زاوية بال.
فزاوية بال مثل زاوية[6] طاد فنلقي زاوية طال المشتركة فتبقى زاوية باط مثل زاوية دال وزاوية جدا مثل زاوية طبا فمثلث اطب شبيه بمثلث دال فنسبة ال إلى اط كنسبة اد إلى اب فضرب ال في اب مثل ضرب اد في اط. وضرب اد في اط مثل سطح هز فضرب ال في اب مثل سطح هز ويحيطان بزاوية مثل زاوية ح وذلك ما أردنا أن نبين.

[1] قسمي : قسم CI. [2] عمودًا : عمود CI. [3] في C: في ه I. This error proves that I is dependent on C, because the في in C is at the end of a line and followed by the label ه which belongs to the figure. [4] ح C: ج I. [5] خطًا مستقيمًا : خط مستقيم CI. [6] زاوية C : - I.

تحليله ونخرج أيضًا من نقطة ا[1] المعلومة إلى خطي بج جد المعلومي الوضع خطي اب اد[2] ويحيطان بزاوية باد المعلومة · أقول إن ضرب اب في اد معلوم·

فعلى التحليل نقول إن ضرب اب في اد معلوم ونخرج من نقطة ا إلى < خط جد > خط اه يحيط مع خط هج بزاوية معلومة ونجعل ضرب اه في از مثل ضرب اب في اد[3] ونصل بز · فضرب اه في از معلوم فنسبة اد إلى اه كنسبة از إلى اب فمثلث ابز[4] شبيه بمثلث اده فزوية ابز مثل[5] زاوية اهد المعلومة فزاوية ابز[6] معلومة· فتجوز بنقطة ب[7] قطعة دائرة معلومة وهي ابز[8] · فنقطة ب معلومة فخط اب معلوم الوضع فزاوية باد معلومة فخط اد معلوم الوضع فنقطة د معلومة وذلك ما أردنا أن[9] نبين·

أيضًا له نريد أن نخرج من نقطة ا المعلومة إلى خطي بج جد المعلومي الوضع خطي اب اد ويحيطان بزاوية باد المعلومة حتى تكون نسبة اب إلى اد معلومة·

فعلى التحليل نقول أن نسبة اب إلى اد معلومة· وتمرّ بنقط[10] ا ب د دائرة ما وهي ابد ونخرج من نقطة ا خطي اه از ونصل بد ·

فلأن نسبة اب إلى اد معلومة وزاوية باد معلومة فمثلث ابد معلوم الصورة فزاوية ابد معلومة وهي مثل زاوية ازد فزاوية ازد معلومة فخط از معلوم الوضع فنقطة ز معلومة· وأيضًا فإن زاوية ادب معلومة وهي مثل زاوية اهب فزاوية اهب معلومة فخط اه معلوم الوضع فنقطة ه معلومة· فنقطتا ز < ه معلومتان ونقطة ا معلومة فدائرة اهز معلومة فنقطتا ب> د معلومتان وذلك ما أردنا أن نبين·

التركيب نريد أن نخرج من نقطة ا[11] إلى خطي بج جد خطين مستقيمين يحيطان بزاوية ط ونسبة أحدهما إلى الآخر كنسبة حط[12] الى طك[13] ·

[1] ا : - I. [2] اد : اه CI. [3] مثل ضرب اب في اد C: - I. [4] ابز C: از I.
[5] ابز مثل C: از مثل مثل I. [6] ابز C: .. ا I. [7] ب C: اب I. [8] ابز C: .. ا I.
[9] أن I: أن أن C. [10] بنقط C: بنقطة I. [11] ا C: اه I. [12] حط C: ج ط I.
[13] طك : طل CI.

فنخرج من نقطة ا إلى بجـ[1] خط اب يحيط مع خط < به > بزاوية ابه مساوية[2] لزاوية ك . ونخرج أيضًا من نقطة ا إلى خط جد خط اد[3] يحيط مع خط دز[4] بزاوية ادز مساوية لزاوية ح . ونجوز على نقط[5] ا ب د دائرة ما وهي اب ونصل اه از زه[6] فزاوية ادز مساوية لزاوية اهز ومساوية لزاوية ح فزاوية اهز مساوية < لزاوية > ح . فزاوية ابه[7] أيضًا مساوية لزاوية ازه ومساوية لزاوية ك فتبقى زاوية هاز مساوية لزاوية ط فمثلث اهز يشبه مثلث حطك[8] فنسبة اه إلى از كنسبة حط الى طك[9] وذلك ما أردنا أن نبين.

Fragment 7. C 39b:41-40a:1, I 40b:2-3 = IF 20:2-3.

جوابي لأبي سهل إذا كان مثلث ابجـ وخط بد ونريد أن نخرج من نقطة ا خطًا مستقيمًا[10] مثلًا اهز حتى تكون نسبة مثلث اده إلى مثلث ابز كنسبة ح إلى ط .

Fragment 8. C 41b:18-28, I 43b:13-25 = IF 26:13-25.

لأبي سهل لنفرض خطي اب جد محدودان ونقطة ه مفروضة . نريد أن نخرج مثلًا خط هز فيقسم الخطين على ح ز تكون نسبة ضرب جز في اح إلى حب[11] في زد كنسبة مفروضة.

فلنخرج هز على سبيل التحليل تكون < نسبة > جز في اح < إلى > حب في زد كنسبة مفروضة. ونصل هجـ هد فيلقيان خط[12] اب على نقطتي ط ك . ونخرج طل كم موازيًا لخط جد .

فلأن نسبة اح[13] في جز إلى حب في زد معلومة[14] ونسبة اح في جز إلى اح في طل معلومة[15] لأنها كنسبة جه إلى هط المعلوم فنسبة اح في طل

1 بجـ : جـ CI. 2 مساوية : متساوية CI. 3 اد : اب CI. 4 دز : دب CI.
5 نقط : نقطة CI. 6 از زه : ازه CI. 7 ابه : اجه CI. 8 حطك : حطل CI. 9 طك : طل CI. 10 خطًا مستقيمًا : مستقيم I, خط مستقيم C.
11 إلى حب I: in margin of C. 12 فيلقيان خط : فيلقى خطي CI.
13 اح C: - I. 14 معلومة : معلومًا CI. 15 معلومة : معلومًا CI.

إلى حب في زد[1] معلومة[2] · ونسبة حب في زد إلى حب في كم معلومة[3] لأنّها كنسبة ده[4] إلى هك فنسبة اح في طل إلى حب في كم معلومة وهي مؤلّفة من نسبة اح إلى حب ومن نسبة طل إلى كم < التي هي > كنسبة طح إلى حك من جهة تشابه المثلثين· فالنسبة المؤلّفة من نسبة اح إلى حب ومن نسبة طح إلى حك معلومة وهي كنسبة سطح اح في حط إلى سطح بح في حك فنسبة سطح اح في حط إلى سطح بح في حك معلومة فنقطة ح معلومة من إحداث النقط على الخطوط في نسب السطوح وذلك ما أردنا أن نبين

Fragment 9. C 42b:15-18, I 45a:22-45b:1 = IF 29:22-30:1.

سؤال أبي سهل الكوهي في تحديد شكل بنصف دائرة· اجب معطى وقد قسمنا القوس بقسمين يكون اج ثلث نصف الدائرة وقد[5] خرج جب ونخرج جد عمودًا على اب · أقول إن مثلث جدب أعظم من كل مثلث يحدث من إخراج خط من نقطة ب إلى نصف دائرة بجا ومن عمود مخرج من طرف ذلك الخط على خط اب · جواب أحمد بن محمد بن عبد الجليل ...

Fragment 10. C 46a:20-47a:25, I 51b:4-53a:22 = IF 42:4-45:22.

جواب ويجن المعروف بأبي سهل الكوهي للمسائل الهندسية وهي ستة أشكال·

نريد أن نخرج من نقطة د التي على ضلع اب في المثال[6] الأول أو على استقامته في المثال[7] الثاني خطًا مستقيمًا[8] حتى تكون نسبة المثلث الحادث تحت زاوية ا إلى المثلث الحادث تحت زاوية ج كنسبة ه إلى ز ·

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[1] زد C: ز I. [2] معلومة : معلومًا CI. [3] معلومة : معلومًا CI. [4] ده : جـ I.
[5] وقد C: و I. [6] المثال : مثال CI. [7] المثال : مثال CI.
[8] خطًا مستقيمًا : خط مستقيم CI.

فنجعل نسبة ه إلى خط آخر وهو ط كنسبة نصف خط بك[1] العمود على خط اجـ إلى با ونجعل أيضًا نسبة خط آخر وهو ح إلى ز كنسبة بجـ إلى نصف خط بك العمود على اجـ ·

ونضيف إلى خط دب سطح نسبة مربع اد إليه كنسبة ط إلى ح والعرض الذي يحدث فليكن مساويًا لخط جل · ونضيف إلى خط جل سطحًا مساويًا لسطح بجـ في جل ويزيد على تمامه مربعًا[2] وليكن ضلع المربع الزائد في المثال الأول لم وفي الثاني جم · ونخرج خط دمفع ·

فأقول إن نسبة مثلث ادف إلى مثلث جفع كنسبة ه إلى ز ·

برهان ذلك إنا نجعل كل واحد من خطي ند مس عمودًا على خط اجـ وخط جع موازيًا[3] لخط اب · فلأن نسبة ه إلى ط كنسبة نصف خط بك إلى خط با ونسبة نصف خط بك إلى خط با كنسبة نصف[4] ند إلى دا من جهة تشابه المثلثين فنسبة ه إلى ط كنسبة نصف ند إلى دا · وإن جعلنا اف ارتفاعًا مشتركًا بينهما تكون نسبة ه إلى ط كنسبة نصف ند في اف أعني مثلث ادف إلى سطح دا في اف ·

وأيضًا لأن نسبة ط إلى ح كنسبة مربع اد إلى سطح دب في جل وسطح دب في جل مساويًا لسطح مجـ في جع من أجل أن سطح بجـ في جل مساويًا لسطح مجـ في مل فنسبة بجـ إلى جم كنسبة مل إلى لجـ[5] وإذا ركبنا في المثال الأول وفصلنا في المثال الثاني[6] كانت نسبة بم إلى مجـ أعني نسبة دب إلى جع من أجل تشابه المثلثين كنسبة مجـ إلى جل فسطح مجـ في جع مساويًا لسطح دب في جل ·

وقد كانت نسبة مربع اد إلى سطح دب في جل كنسبة ط إلى ح فنسبة ط إلى ح كنسبة مربع اد إلى سطح جم في جع · لكن نسبة[7] مربع اد إلى سطح جم في جع هي كالنسبة[8] المؤلّفة من نسبة اد إلى جم ومن نسبة اد إلى جع فنسبة ط إلى ح كالنسبة[9] المؤلّفة من نسبة دا إلى مجـ

[1] بك C: - I. [2] مربعًا : مربع CI. [3] موازيًا I: مواز C. [4] نصف C: - I.
[5] لجـ: اجـ CI. [6] الثاني C: الثا I. [7] نسبة C: - I. [8] كالنسبة : كنسبة CI.
[9] كالنسبة : كنسبة CI.

ومن نسبة دا إلى جع[1] أعني نسبة اف[2] إلى فج من أجل تشابه المثلثين فنسبة ط إلى ح كالنسبة[3] المؤلّفة من نسبة دا[4] إلى مج ومن نسبة اف إلى فج لكن النسبة[5] المؤلّفة من نسبة دا إلى مج ومن نسبة اف إلى فج هي كنسبة سطح دا في اف[6] إلى سطح مج في جف فنسبة ط إلى ح كنسبة سطح دا في اف إلى سطح مج في جف · وقد كانت نسبة مثلث ادف إلى سطح اد في اف كنسبة ه إلى ط · فبالمساواة نسبة مثلث ادف إلى سطح مج في جف كنسبة ه إلى ح ·

وأيضًا لأن نسبة ح إلى ز كنسبة جب إلى نصف بك ونسبة بج إلى نصف بك هي كنسبة جم إلى نصف مس من جهة تشابه المثلثين وإن جعلنا فج ارتفاعًا مشتركًا[7] الا بينهما تكون نسبة ح إلى ز كنسبة سطح مج في جف إلى سطح نصف مس في جف · ونصف مس في جف مساو لمثلث[8] مجف فنسبة ح إلى ز كنسبة سطح مج في جف إلى مثلث مجف · وقد كانت نسبة ه إلى ح كنسبة مثلث ادف إلى سطح < مج في > جففبالمساواة كانت نسبة ه إلى ز كنسبة مثلث ادف إلى مثلث مجف في جميع المثالين وذلك ما أردنا أن نبين·

الثالث نريد أن نخرج من ضلع اب من مثلث ابج إلى ضلع اج خطًا مستقيمًا موازيًا لخط اد المعلوم الوضع حتى تكون نسبة ضرب أحد الضلعين الباقيين في الآخر < ... > كنسبة ه إلى ز ·

فإن كان اد موازيًا لـ بج فإن عمله سهل·

وإن كان غير موازٍ له فيلتقيان وليكن على نقطة د · ونضيف إلى خط اد سطح نسبة با في اج إليه كنسبة ه إلى ز وليكن العرض الذي يحدث مساويًا لخط جط · ونضيف إلى خط طب سطحًا مساويًا لضرب بد في جط ويزيد على تمامه مربعًا[9] وليكن ضلع المربع الزائد بك · ونصل اك ونخرج من نقطة ب خطًا موازيًا لخط اك وهو بل ونخرج من نقطة ل

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[1] ومن نسبة اد... إلى جع C: - I. [2] اف C: اد I. [3] كالنسبة : كنسبة CI.
[4] دا C : اد I. [5] النسبة : نسبة CI. [6] في اف I: in marg. C.
[7] ارتفاعًا مشتركًا : ارتفاع المشترك CI. [8] مثلث : This word looks almost like مثل in C, and yet the scribe of I wrote it correctly. [9] مربعًا : مربع CI.

خطًا موازيًا لخط اد[1] وهو لم · فأقول إن نسبة ضرب بم في لجـ إلى سطح بجـ في مل كنسبة ه إلى ز ·

برهان ذلك إنا نجعل بن موازيًا لـ اد فلأن ضرب طك في كب مساو لضرب طجـ في بد فنسبة كط إلى طجـ كنسبة دب إلى بك · وإذا ركبنا كانت نسبة كجـ إلى جط كنسبة دك إلى كب · ولكن نسبة دك إلى كب كنسبة اد إلى نب[2] من أجل تشابه المثلثين فنسبة كجـ إلى جط كنسبة اد إلى نب[3] فضرب كجـ في بن مساو لضرب اد في طجـ ·

وقد كانت نسبة ضرب با في اجـ إلى ضرب اد في طجـ كنسبة ه إلى ز · فنسبة ه إلى ز كنسبة با[4] في اجـ إلى ضرب جك في بن · ولكن نسبة ضرب با[5] في اجـ إلى ضرب كجـ في بن مؤلفة من نسبتين إحداهما نسبة اب إلى بن والأخرى نسبة[6] اجـ إلى جك · فنسبة ه إلى ز مؤلفة من نسبتين إحداهما اب إلى بن والأخرى اجـ إلى جك · ونسبة اب إلى بن كنسبة بم إلى مل من جهة تشابه المثلثين ونسبة اجـ إلى جك كنسبة لجـ إلى جب من أجل أن اك لب متوازيان[7] فنسبة ه إلى ز مؤلفة من نسبة بم إلى مل ومن نسبة لجـ إلى جب < و > هي كنسبة ضرب بم في لجـ إلى ضرب بجـ في مل فنسبة ه إلى ز كنسبة ضرب بم في لجـ إلى[8] ضرب بجـ في مل[9] وذلك ما أردنا أن نبين·

الرابع نريد أن نجد مثلثًا قائم الزاوية حتى يكون فضل نصف وتر الزاوية القائمة على العمود المخرج من الزاوية القائمة في أحد قسمي القاعدة الذي فصله[10] العمود المخرج إليها مساويًا[11] لسطح ا المعلوم.[12] فنجعل مربع بجـ مساويًا لسطح ا ونخرج به عمودًا على بجـ ونجعل مربع جد ضعف مربع دب · ونجعل كل واحد من خطي ده دز مساويًا لخط دجـ ونصل خطي زه جه· فأقول إن مثلث جهز هو المطلوب أعني زاوية

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[1] اد : اجـ CI. [2] نب : ثب C, بن I. [3] نب : ثب C, نب I. [4] با : باا CI.
[5] با : بك C, بد I. [6] نسبة : I نسبة نسبة C. [7] متوازيان : متوازيين .
[8] إلى : اي C. [9] فنسبة ه··· مل: - I. [10] فصله : فضله I.
[11] مساويًا : مساوٍ CI. [12] المعلوم C: المعلومة I.

زهج قائمة وفضل نصف زج الذي هو وتر الزاوية القائمة على هب العمود في بج الذي هو أحد القسمين مساو لسطح آ .

برهان ذلك لأن خطوط ده دز دج متساوية فنقطة د مركز دائرة يجوز محيطها على نقط[1] ز ه ج فزاوية زهج قائمة. وأيضًا لأن خط زج مقسوم بنصفين على نقطة د وبقسمين مختلفين على نقطة ب فسطح زب في بج مع مربع دب مساو لمربع دج . ومربع دج مساو لضعف مربع دب فسطح زب في بج مع مربع دب مساو لضعف مربع دب . فإذا ألقينا مربع دب المشترك فيبقى[2] سطح زب في بج مساو[3] لمربع دب لكن سطح زب في بج مساو[4] لمربع به فمربع به مساو لمربع دب . ففضل جد الذي هو نصف وتر الزاوية القائمة على هب هو بج لأن هب مساو[5] ل دب . فخط بج الذي هو فضل نصف زج على هب هو بج و بج في نفسه مساو لسطح آ لأنا جعلنا مربعه مساو لسطح آ وذلك ما أردنا أن نبين.

الخامس نريد أن نخرج في دائرتي ابجد هزحط خطًا واحدًا مستقيمًا حتى يقع في دائرة ابجد خط مساو[6] لخط جد وفي دائرة هزحط خط مساو[7] لخط حط .

فنخرج من مركزي ك ل عمودي كم لن على خطي جد حط . فإن كان كم مساويًا[8] ل لن فعمله أسهل من الذي ليس بمساوي. وإن لم يكونا متساويين فأحدهما أطول من الآخر وليكن الأطول كم . فنجعل نسبة كس إلى سل كنسبة كم إلى لن ونخرج من نقطة س خط سهز مماس الدائرة التي نصف قطرها كم ومركزها نقطة ك . فأقول إن خط اب الذي يقع في دائرة ابجد من هذا الخط مساو لخط جد .

برهان ذلك إنا نجعل خطي كع لف عمودين على خط ابهز . فلأن نسبة كم الذي هو مساو ل كع[9] إلى خط لن كنسبة كس إلى سل ونسبة كس

[1] نقط C: نقطة I. [2] فيبقى C: يبقى I. [3] مساو : مساويًا CI. [4] مساو : مساويًا CI. [5] مساو : مساويًا CI. [6] خط مساو : خطًا مساويًا CI. [7] خط مساو : خطًا مساويًا CI. [8] مساويًا : مساوٍن CI. [9] ل كع C: ول كع I. [10] لف : كف CI.

إلى سل كنسبة كع إلى لف[10] فنسبة كع إلى كل واحد من خطي لن لف واحدة فـ لن مساو ا لف[11] لخط اب مساو لخط جد وخط هز مساو لخط حط لأن بعديهما من المركز سواء. فأخرجنا في دائرتي ابجد هزحط خطًا واحدًا مستقيمًا مقدار الذي يقع منه في دائرة ابجد مساو لخط معلوم والذي يقع في دائرة هزحط[1] مساو لخط آخر معلوم وذلك ما أردنا أن نبين.

السادس نريد أن نخرج من[2] نقطة ا إلى خطي بج بز[3] خطًا مستقيمًا حتى يقع تحت زاوية ب مثلث مساو[4] لسطح ه المعلوم.
فنخرج من نقطة ا خطًا موازيًا لخط بج وهو از ونصل خط اب ونعمل مثلث ابج[5] مساويًا لسطح ه ونضيف إلى خط حب سطحًا مساويًا لسطح زب في بج[6] حتى يزيد على تمامه مربعًا وليكن ضلع المربع الزائد خط بط. ونصل خط اط ونخرج على[7] استقامته إلى ك.
فأقول إن مثلث بكط مساو لسطح ه.
برهان ذلك لأن سطح زب في بج مساو لسطح حط في طب فنسبة زب إلى بط كنسبة طح إلى حب. وإذا فصلنا كانت نسبة زط إلى طب < كنسبة طب إلى بج ونسبة زط إلى طب > كنسبة اط إلى طك فنسبة اط إلى طك كنسبة طب إلى بج. ولكن نسبة اط إلى طك كنسبة مثلث ابط إلى مثلث طبك[8] وكذلك نسبة طب[9] إلى بج كنسبة مثلث اطب إلى مثلث ابج. فنسبة مثلث اطب إلى كل واحد من مثلثي ابج كبط واحدة فمثلث كبط مساو لمثلث ابج. وقد كان مثلث ابج مساو لسطح ه فمثلث كبط مساو لسطح ه و ذلك ما أردنا أن نبين.

Fragment 11. C 55b:22-25, I 115a:21-23 = IF 165:21-23.

ونعمل دائرة تماس دائرة هز وتلقى نقطة ا ويكون مركزها على خط جد

[11] ا لف C : ولا لف I. [1] هزحط : حرحط CI. [2] من : من من I.
[3] خطي بج بز : خط بج CI. [4] مثلث مساو : مثلثا مساويًا CI. [5] ابج : ابج CI. [6] بج : بج CI. [7] Here I adds ج. The letter ج appears in C after the word على, as a label of a point in the geometrical figure. This proves that I is dependent on C. [8] طبك C: طل I.
[9] وكذلك نسبة طب : ولذلك نسبة طف CI. [10] رستم : وستم CI.

على ما بيّنه ويجن بن رستم[10] في الشكل السادس من كتابه في مراكز الدوائر المتماسة على خطوط بطريق التحليل

C 55b:28-30, I 115b:2-4 = IF 166:2-4.

تركيب شكل ويجن بن رستم[1] وطريق آخر لبرهان المسألة التي قبل هذا الشكل وهو كيف نقسم خط اب المستقيم المقسوم على ج بقسمين مثلًا على ز تكون نسبة زب إلى اج كنسبة زج إلى از ·

Fragment 12. L 24b:1-14.

مقدّمة أبي[2] سهل الكوهي

خطا اب بج مفروضان يحيطان بزاوية ب و بج غير متناهي · نريد أن نخرج اجد يكون دج مثل دا وتكون نسبة اب إلى بج كنسبة بج إلى بد· وإذا تهيّأ ذلك فيتهيّأ انقسام زاوية هبج بثلاثة أقسام متساوية إذا أخرج اب إلى ه ·

وذلك لأن مثلث ابج يكون شبيهًا بمثلث دبج لاشتراك زاوية ب فتكون[3] زاوية بجد من مثلث بجد مساوية لزاوية جاب من مثلث جاب وزاوية جدب مساوية لزاوية اجب فتكون زاويتا د ج من مثلث بدج متساويتين[4] لزاويتي ج ا من مثلث باج · لكن زاوية بدج مثلي زاوية ا فهي مثلي زاوية[5] دجب وزاوية هبج مثل زاويتي بجد بدج[6] فزاوية بدج ثلثي زاوية هبج وزاوية بجد ثلثها· فقد تهيّأ بهذه المقدمة قسمة الزاوية بثلاثة أقسام متساوية ·

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1 رستم : وستم CI. 2 أبي : ابو L. 3 فتكون : وتكون L.
4 متساويتين : متساويتان L. 5 زاوية : زاويتي L. 6 بدج : بدب L.

References

P. Abgrall, 'Les cercles tangents d'al-Qūhī', *Arabic Sciences and Philosophy*, 5, 1995, pp. 263 - 295.

A. Arberry, *The Chester Beatty Library: A Handlist of Arabic Manuscripts*, Dublin, 1955-1964, 7 vols.

R.C. Archibald, *Euclid's Book of Divisions of Figures, with a Restoration Based on Woepcke's Text and on the Practica Geometriae of Leonardo Pisano*, Cambridge, 1915.

H. Bellosta, see R. Rashed & H. Bellosta.

J.L. Berggren, 'The correspondence of Abū Sahl al-Kūhī and Abū Isḥāq al-Ṣābī', *Journal for History of Arabic Science*, 7, 1983, pp. 39-124.

——. 'Abū Sahl al-Kūhī's Treatise on the Construction of the Astrolabe with Proof: Text, Translation and Commentary', *Physis Nuova Serie*, 31, 1994, pp. 141-252.

——. 'Abū Sahl al-Kūhī's "On Drawing Two Lines from a Point at a Known Angle, by the Method of Analysis," *Suhayl*, 2, 2001, pp. 161-198.

——. 'Tenth-Century Mathematics Through the Eyes of Abū Sahl al-Kūhī', in *The Enterprise of Science in Islam: New Perspectives*, eds J.P. Hogendijk, A.I. Sabra, Cambridge, 2003 pp. 177-196.

Berggren and Van Brummelen 2000a: J.L. Berggren & G. Van Brummelen, 'Abū Sahl al-Kūhī on "Two Geometrical Questions"', *Zeitschrift für Geschichte der arabisch-islamischen Wissenschaften*, 13, 2000, pp. 165-187.

Berggren and Van Brummelen 2000b: 'Abū Sahl al-Kūhī's "On the Ratio of the Segments of a Single Line that Falls on Three Lines" ', *Suhayl*, 1, 2000, pp. 11-56.

B. Dodge, *Ibn al-Nadīm, Kitāb al-Fihrist*, 2 vols, New York, 1970.

T.L. Heath, *A History of Greek Mathematics,* 2 vols, Oxford 1921.

——. *The Thirteen Books of Euclid's Elements*, reprint of the 2nd edn, New York, 1956.

H. Hermelink & E.S. Kennedy. 'Transcriptions of Arabic Letters in Geometrical Figures', *Journal of the American Oriental Society*, 82, 1962, p. 204, also in *Sudhoffs Archiv*, 45, 1969, p. 85, reprinted in: E.S. Kennedy et al, *Studies in the Islamic Exact Sciences*, Beirut 1983, p. 745.

J.P. Hogendijk, 'Greek and Arabic constructions of the regular heptagon', *Archive for History of Exact Sciences*, 30, 1984, pp. 197-330.

——. 'Al-Kūhī's construction of an equilateral pentagon in a given square', *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 1, 1985, pp. 100-144.

——. 'Arabic Traces of Lost Works of Apollonius', *Archive for History of Exact sciences*, 35, 1986, pp. 187-253.

——. 'The Arabic Version of Euclid's *On Divisions*', In: *Vestigia Mathematica. Studies in Medieval and Early Modern Mathematics in Honour of H.L.L. Busard*, eds M. Folkerts, J.P. Hogendijk, Amsterdam 1993, pp. 143-162.

A. Jones, *Pappus of Alexandria: Book 7 of the Collection*, 2 vols, New York, 1986.

A. Qurbani, *Būzjānī-Nāmeh*, Tehran, 1371 AH (solar) / AD 1992. [in Persian]

R. Rashed, La philosophie mathématique d'Ibn al-Haytham I: L'Analyse et la synthèse, *Mélanges de l'Institut Dominicain d'Études Orientales du Caire*, 20, 1991, pp. 31-231.

——. *Géométrie et Dioptrique au X^e^ siècle. Ibn Sahl, al-Qūhī et Ibn al-Haytham,* Paris, 1993.

R. Rashed & H. Bellosta, *Ibrāhīm ibn Sinān: logique et géométrie au X^e^ siècle*, Leiden, 2000.

A.S. Saidan, *Rasā'il ibn Sinān* (The Works of Ibn Sinān), Kuwait, 1983. [in Arabic]

——. 'Tathlīth al-zāwiya fi'l-ʿuṣūr al-Islāmiyya', RIMA 28, 1984, pp. 99-137.

F. Sezgin, *Geschichte des arabischen Schrifttums*, vol. 5: Mathematik bis ca. 430 H., Leiden, 1974; vol. 6: Astronomie bis ca. 430 H., Leiden, 1978; vol. 7: Astrologie, Meteorologie und Verwandtes bis ca. 430 H., Leiden, 1979.

al-Sijzī, *Collection of Geometrical Works, Majmūʿa min rasa'il handasiyya, by Al-Sijzī, Abū Saʿīd Aḥmad b. Muḥammad b. ʿAbdaljalīl*, ed. F. Sezgin, Frankfurt, 2000.

R.C. Taliaferro, *Apollonius of Perga*, Santa Fe, 1998.

C. Thaer, *Die Data von Euklid*, Göttingen - Heidelberg, 1962.

G.J. Toomer, *Diocles on Burning Mirrors*, New York, 1976.

P. Ver Eecke, *Les Coniques d'Apollonius de Perge*, reprint ed, Paris, 1959.

P. Voorhoeve, *Handlist of Arabic Manuscripts in the Library of the University of Leiden and Other Collections in The Netherlands,* Leiden, 1957.

Woepcke 1855a: F. Woepcke, 'Analyse et extrait d'un récueil de constructions géométriques par Aboûl Wafâ', *Journal Asiatique*, 5th ser. 5, 1855, pp. 218-256, 309-359, reprinted in [Woepcke 1986 I, 483-572] and also in: *Abu l-Wafā' al-Būzajānī, Texts and Studies, Collected and Reprinted, II,* ed. F. Sezgin, Islamic Mathematics and Astronomy no. 61, Frankfurt 1998.

Woepcke 1855b: *l'Algèbre d'Omar Alkhayyami*, Paris, 1855, reprinted in [Woepcke 1986 I, 49-256].

——. *Études sur les mathématiques arabo-islamiques,* ed. F. Sezgin, 2 vols, Frankfurt 1986.

A Hellenistic Astrological Table Deemed Worthy of Being Penned in Gold Ink: the Arabic Tradition of Vettius Valens' Auxiliary Function for Finding the Length of Life

DAVID A. KING

To Abū Kayd from Abū Max,
alas not in gold

1 *Vettius Valens' table for determining the length of life*

A basic concern of classical astrology was the prediction of the circumstances of an individual's life from the celestial configuration at the time of birth, and the prediction of his appearance and characteristics from the celestial configuration at the time of conception. The time of birth can be measured and the instantaneous celestial configurations relative to the local horizon determined by mathematical procedures; the time of conception is in general somewhat more tricky for an outsider to pinpoint, but rules were formulated to determine it from the time of birth and the period of gestation.

One aspect of the individual's circumstances which was of particular interest is the length of his life,[1] and already in classical Antiquity it was proposed that the maximum length of life in years for a specific location is the maximum rising time of a quadrant of the ecliptic over the local horizon, that is, one-half of the maximum length of daylight, in equatorial degrees.[2] The late-second-century astrologer Vettius Valens, of Antioch and, later, Alexandria, contributed to this doctrine a scheme relating the length of an individual's life, subject to this condition for the

[1] A brief summary of this paper was published as 'Some Arabic Copies of Vettius Valens' Table for Calculating the Duration of Life', in *Symposium Graeco-Arabicum II*, Gerhard Endress, ed., Amsterdam, 1989, pp. 25-8.

[2] Neugebauer, *HAMA*, II, p. 721.

maximum, to the longitude of the horoscopus or ascendant, that is, the point of the ecliptic instantaneously rising over the eastern horizon, at the time of birth.

The *Anthology* of Vettius Valens was very popular in late Antiquity, not least for the obscurity of its formulations.[3] The available Greek text in nine books is in bad shape; it was published by Wilhelm Kroll in 1908 on the basis of three Byzantine manuscripts (four others are derivative from these). Over 100 horoscopes in the *Anthology* were investigated by Otto Neugebauer and H. B. van Hoesen in 1959.[4] A more authoritative text was published by David Pingree in 1986 using a few more manuscripts than were available to Kroll.[5] Only one of these copies, used already by Kroll, contains the table whose Arabic descendant we shall be presenting in this paper, namely, MS Oxford, Bodleian Library, Seld. 22 and 20, originally a single codex of 286 folios. This is a late, corrupt Byzantine copy which was already in the possession of the French lawyer Christophe Longueil, who died in Padua in 1522, and then passed in 1556 to the library of John Dee and thence to John Selden (1585–1654).[6]

The fate of the *Anthology* in the East was first investigated by Carlo Alfonso Nallino in the 1890s and more recently has been pursued by Fuat Sezgin.[7] A Middle Persian commentary on the *Anthology* was prepared in the sixth century by the Sasanid minister Buzurjmihr. This commentary, now lost, was apparently arranged in ten books and was called *Vizīdhak* (Arabic, *Bizīdhaj*) which means simply '*Anthology*'.[8] It was in this form, rather than the original Greek, that Vettius Valens' work became available to the astrologers of late-eighth- and early-ninth-century Baghdad. This commentary was translated into Arabic as *Kitāb al-Bizīdhaj*,

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[3] On Vettius Valens in Antiquity see the outdated article by Emilie Boer in Pauly-Wissowa, *Realencyclopädie*, 2-VIII,2, cols 1871-3, and more especially the reference in the next note.

[4] See Neugebauer and van Hoesen, *Greek Horoscopes*, pp. 176-85.

[5] Kroll, *Vettius Valens*, pp. 321-8; and Pingree, *Vettius Valens*, pp. 308-11.

[6] Pingree, *Vettius Valens*, p. x.

[7] Nallino, *'Ilm al-falak*, pp. 192-6 (Arabic) and idem, *Scritti*, VI, especially pp. 291-6; and Sezgin, *GAS*, VII, pp. 38-41 and 80. See also Ullmann, NGWI, pp. 281-2 and 297. (MS Istanbul Nurosmaniye 2920,3 of part of the *Kitāb al-Asrār* attributed to Wālīs does not contain material on our subject.) See also T. Fahd in the article 'Aḥkām al-Nudjūm' in EI_2, especially p. 106b.

[8] Sezgin, *GAS*, VII, p. 80; and Ullmann, *NGWI*, p. 297.

but it is no longer extant in this form either. Vettius Valens was known to the Arabs as Wālīs, sometimes as *Wālinṭiyus*, and sometimes with the appellation al-Rūmī, 'the Byzantine', which would correspond to his accepted provenance of Antioch, but also sometimes as al-Miṣrī, 'the Egyptian', or, more specifically, al-Iskandarānī, 'the Alexandrian' (see below). Two fragments directly attributed to Wālīs in the manuscript sources have been identified by Fuat Sezgin and remain to be properly studied.[9] There are numerous references to Wālīs and Buzurjmihr in several major Arabic astrological works, notably the *Kitāb al-Mughnī* of the tenth-century Christian Arab astrologer Ibn Hibintā[10] and the *Safīnat al-aḥkām*, an astrological compendium by the thirteenth-century polymath Naṣīr al-Dīn al-Ṭūsī.[11] But none of these, as far as I am aware, deals with the table which is the subject of this paper. As I shall show (see Section 3, *sub* MS A), there is some possibility that the table was known in medieval Europe, at least in Palermo in the thirteenth century.

The material of Vettius Valens on the length of life is found at the end of Book VIII of the *Anthology* in the form of two tables, the only ones in the entire work. As noted above, this material survives in only one late Byzantine manuscript. The first table, which has already been discussed by Otto Neugebauer,[12] bears the title *Kanónion a(lpha) kaì plinthíon* and variations thereon on subsequent pages of subtables, essentially meaning 'small table no. 1 also (known as) *plinthíon*'.[13] *Plinthíon* literally means 'small brick' but by extension 'a lattice of bricks' and we shall return to the use of this term when we consider the internal structure of the table. The latter displays values of an auxiliary function

[9] Sezgin, *GAS*, VII, p. 41, and Ullmann, *NGWI*, pp. 281-2, esp. n. 4 on p. 282. See also Pingree, 'Astronomy and Astrology in Iran', pp. 241-2, on MS London BL Add. 23400 (see text to n. 66 below).

[10] See Sezgin, *GAS*, VII, pp. 162-4. The two manuscripts of the main fragments of this work are published in facsimile in Ibn Hibintā, *al-Mughnī*.

[11] Ibid., pp. 22-4.

[12] Neugebauer and van Hoesen, *Greek Horoscopes*, pp. 174-5.

[13] *Plinthíon* is the diminutive of *plinthos* and means literally 'little brick'. By extension it was used to refer to rectangular boxes, checker-boards, squares of tartar, etc. (Liddell and Scott, *Lexicon*, p. 1421b) and as such corresponds to the Middle Persian *zīk*, meaning thread or cord and by extension the warp of a fabric, which came to be used in Arabic as *zīj* for 'astronomical handbook with tables' (Kennedy, '*Zīj* Survey', pp. 123-4, and King and Samsó, 'Islamic Astronomical Tables', p. 12, n. 2).

ζ, labelled simply *arithmos*, 'number', and the length of life, L, in years and months, with occasional entries of an additional 15 days. Both quantities are tabulated as a function of the longitude of the horoscopus λ_H at the time of birth, and the second is dependent on terrestrial latitude ϕ, in this case that of Alexandria, although this is not stated. Values are given for each degree of each sign beginning with Libra (see Figs. 1a-b). Each group of 5 entries is associated with the sun, moon or one of the five planets, but these were not used in known Greek horoscopes. Neugebauer has shown that the tabulated function is defined by:

$$L(\lambda_H) = \zeta(\lambda_H)/60 \times 2D(\lambda_H)..............(1)$$

where $2D$ is the length of daylight corresponding to a solar longitude equal to λ_H. The equinox was taken at Aries 8° as in Babylonian System B solar theory,[14] and the length of daylight in the surviving Greek table was computed using a linear zigzag function having the traditional extremal values 210° and 150° (ratio 7 : 5), a standard scheme for Alexandria.[15] The latitude of Alexandria corresponds to the first climate in the scheme of climates proposed elsewhere in the *Anthology* by Vettius Valens.[16] Only the table of years for Alexandria is mentioned in the published text, and Vettius Valens writes that this table is 'for the sake of example', and that intelligent people will be able to calculate such tables for each *klima*.[17] In only two of the many Greek horoscopes investigated by Neugebauer and van Hoesen is the table of life used. For two nativities in AD 419, both in the climate of Spain, the *arithmoí* are taken from Vettius Valens' first table and the lengths of daylight are computed because the climate is different from that of Alexandria.[18] Again the arithmetical schemes typical of the Babylonian tradition are used rather

[14] Neugebauer, *HAMA*, I, pp. 369-72, and II, pp. 594-8. See also Cumont, *Astrology and Religion*, p. 36.

[15] See, for example, Neugebauer and van Hoesen, *Greek Horoscopes*, p. 3.

[16] Neugebauer, *HAMA*, II, p. 728, and also n. 10 on p. 718.

[17] Pingree, ed., *Vettius Valens*, p. 283 (end of 8.1).

[18] Neugebauer and van Hoesen, *Greek Horoscopes*, pp. 136-8 (L 419) and 140 (L 431).

Unfortunately (at least for me), most of the 65-odd horoscopes out of the 200-odd astronomical and astrological fragments from Roman Egypt (more precisely, from Oxyrhyncus, near Bahnasa, some 150 km south of modern Cairo) so masterfully studied by Alexander Jones (*Astronomical Papyri from*

than values for the length of daylight computed trigonometrically, such as could easily have been derived from Ptolemy's tables of rising times for the climates in the *Almagest.*

VETTIVS VALENS

| *Κανόνιον α′· πλινθίον*

	Ζυγοῦ					Σκορπίου					Τοξότου				
μοῖραι		ἀριθμοί	ἔτη	μῆνες	ἡμέραι		ἀριθμοί	ἔτη	μῆνες	ἡμέραι		ἀριθμοί	ἔτη	μῆνες	ἡμέραι
α	☄	β	ς	α	ιε	♂	ιδ	λθ	ο		♃	κς	ξζ	ς	
β		δ	ιβ	γ			ις	μδ	ε			κη	οβ	ς	
γ		ς	ιη	γ			ιη	μθ	ϙ			λ	οζ	⟨ς⟩	
δ		η	κδ	δ			κ	νε	β			β	ε	β	
ε		ι	λ	γ	ιε		κβ	ξ	ς			δ	ι[ε]	γ	ιε
ς	♂	ιβ	λς	ζ		⟨♃⟩	κδ	ξε	ϙ		♄	ς	ιε	ε	
ζ		κς	οη	ϙ			η	κα	ι[α]			κ	να	β	
η		κη	πδ	ο			ι	κζ	β			κβ	νς	β	
θ		λ	πθ	ια			ιβ	λβ	ζ			κδ	ξα	γ	ιε
ι		β	ς	ο			ιδ	λη	ο			κς	ξς	γ	
ια	♃	δ	ια	θ		⟨♄⟩	ις	μγ	γ		⟨☿⟩	κη	οα	δ	
ιβ		ς	ιζ	ϙ			ιη	μη	ζ			λ	⟨ο⟩ς	ς	
ιγ		κ	νθ	[ι]α			β	ε	δ			ιδ	λε	ζ	ιε
ιδ		κβ	ξδ	ε	ιε		δ	ι	θ			ις	μ	ζ	ιε
ιε		κδ	ο	ς			ς	ις	α			ιη	με	η	
ις	♄	κς	ος	η		⟨☿⟩	η	κα	ε		♀	κ	ν	θ	
ιζ		κη	πα	γ			ι	κς	θ			κβ	νε	θ	
ιη		λ	πζ	δ			ιβ	λβ	ο			κδ	ξ	ι	
ιθ		ιδ	μ	ια			κς	ξθ	β			η	κ	ι	
κ		ις	μς	θ			κη	οδ	δ			ι	κε	δ	
κα	☿	ιη	να	ε		⟨♀⟩	λ	οθ	ς		☄	ιβ	λ	ε	
κβ		κ	νζ	ια			β	ε	γ			ιδ	λε	ε	
κγ		κβ	ξβ	ε	ιε		δ	ι	ς			ις	μ	ε	ι
κδ		κδ	ξη	η			ς	ιε	θ			ιη	με	ε	ιε
κε		η	κβ	θ			κ	νβ	ς			β	ε	γ	
κς	⟨♀⟩	ι	κη	ε	ιε	☄	κβ	νζ	ϙ		♂	δ	ι	α	
κζ		ιβ	λγ	ι⟨α⟩			κδ	ξβ	ϙ			ς	ιε	α	
κη		ιδ	λθ	ε			κς	ξζ	ια			η	κ	γ	
κθ		ις	μδ	ι	ιε		κη	οβ	ια			ι	κε	β	
λ		ιη	ν	δ			λ	οζ	ζ			ιβ	λ	β	

Figure 1a: An extract from the first table in the edition of the *Anthology* of Vettius Valens in the new edition by David Pingree. [From Pingree, *Vettius Valens*, p. 308.]

Oxyrhyncus) predate the compilation of the *Anthology*, but even those from after ca. 300 (about 20 in number) show no traces of calculations of the length of life, in spite of the relative proximity of Alexandria.

Sign Argument	VII	VIII	IX
1	2	14	26
2	4	16	28
3	6	18	30
4	8	20	2
5	10	22	4
6	12	24	6
7	26	8	20
8	28	10	22
9	30	12	24
10	2	14	26
11	4	16	28
12	6	18	30
13	20	2	14
14	22	4	16
15	24	6	18
16	26	8	20
17	28	10	22
18	30	12	24
19	14	26	8
20	16	28	10
21	18	30	12
22	20	2	14
23	22	4	16
24	24	6	18
25	8	20	2
26	10	22	4
27	12	24	6
28	14	26	8
29	16	28	10
30	18	30	12

Figure 1b: The values of the auxiliary function given in Figure 1a, which are sufficient to give an idea of the underlying structure ofthe entire table.

Neugebauer also showed how the function $\zeta(\lambda_H)$ could be generated from a scheme of the even numbers from 2 to 30, which, he wrote, 'has no astronomical significance whatever'. Indeed, he added that the theory well illustrates 'the combination of serious

astronomical concepts with arbitrary manipulation of numbers in order to obtain new complex astrological rules'.[19] Two of the numerous Greek horoscopes investigated by Neugebauer and H. B. van Hoesen used Vettius Valens' first table and actually mentioned the associated length of life as displayed in the surviving table; however, since they were not computed for Alexandria they used different values for 2D to compute the proper life-span.[20]

The following description of the function ζ differs but slightly from that of Neugebauer but takes into consideration Vettius Valens' second table which Neugebauer did not discuss (Figure 2),[21] and perhaps casts just a little light on the inherent mystery. All values are *modulo* 30 with the convention that $30 \equiv 30$ and not 0, and the value of ζ at Libra 0° (= Virgo 30°) is 30. The increase for each single degree is 2 (that is, 6 for each 3° or 12 for each 6°) with an additional 12 after each 6° and an extra 12 after each 30° (except at the end of Virgo). Vettius Valens' second table has different values for the first and second increases. This having been said, I confess that I was until recently at a loss to explain the motivation for these schemes. Certainly the tables are easier to use than the procedures for finding the life-span advocated by, say, Dorotheos of Sidon.[22]

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[19] Neugebauer and van Hoesen, *Greek Horoscopes*, pp. 174-6. See also Neugebauer and Saliba, 'Greek Numerology', on Hellenistic schemes for predicting the chances for the survival or death of a sick person.

[20] See n. 18 above.

[21] The second table is of the same kind and format (but without the planetary associations) and immediately follows the first table: see Kroll, *Vettius Valens*, pp. 325-8, Pingree, *Vettius Valens*, pp. 312-15, and Neugebauer and van Hoesen, *Greek Horoscopes*, p. 174, n. 93. It is labelled simply *kanónion deuteron kai plinthíon* and variations thereon. Different values of the auxiliary function are given, now to two sexagesimal digits, although the second digit is always 0, 20 or 40. The underlying scheme is similar to that in the first table with the following changes: the value for Libra 0° = Virgo 30° is again 30 but the value for Libra 1° has been fiddled to be 0;20; the increase for each single degree of argument is now 2;20 (that is, 7 for each 3° or 14 for each 6°) with an additional increase of 10 after each 6° and an extra 12 after each 30° (except at the end of Virgo). By virtue of the choice of increments, the resulting values are of the same order as those in the first table. The length of life in years and months in this second table is based on these values of the auxiliary function and on the same scheme for the length of daylight as in the first table.

[22] See Pingree, *Dorotheos*, pp. 237-41. See also Ibid., pp. 246 and 264, on Vettius Valens in Dorotheos. On Dorotheos in the Arabic tradition see

A possible solution was suggested to me by José Chabas. The solution lies in the use of the term '*plinthíon*' meaning a 'lattice of bricks', or, more precisely, a 'square matrix of bricks'. Let brick A be the sequence of the first six even numbers:

$$A = 2, 4, 6, 8, 10, 12.$$

Now, let brick B = A + 6, brick C = B + 6, and so on. Thus,

$$B = 8, 10, 12, 14, 16, 18$$
$$C = 14, 16, 18, 20, 22, 24$$
$$D = 20, 22, 24, 26, 28, 30$$
$$E = 26, 28, 30, 2, 4, 6.$$

Then the 'Table of Life', as some of the Arabic sources call it, reduces to the matrix:

C	*E*	*B*	*D*	*A*
B	*D*	*A*	*C*	*E*
A	*C*	*E*	*B*	*D*
E	*B*	*D*	*A*	*C*
D	*A*	*C*	*E*	*B*

As José Chabas further pointed out, this matrix has different interesting symmetries, and changing the order of the columns does not modify the argument. Each brick appears only once in each row and each column. The sum of the entries in row 1 is 70; in row 2, 80; and in row 3, 90; and the basic pattern (70, 80, 90) repeats over and over for the following rows. Similarly, each column has exactly 2 'twos', 2 'fours', 2 'sixes', etc. Thus, the sum of all entries in any row is 2 × (2 + 4 + ... + 30) = 480. These and other features indicate that the matrix is 'well balanced' and 'fair' to all individuals!

During the course of my survey of the medieval scientific manuscripts in the Egyptian National Library in Cairo in the 1970s, I came across a number of Arabic copies of Vettius Valens' table of the auxiliary function $\zeta(\lambda_H)$ not previously known in the Islamic sources.[23] The copies were all of late medieval Egyptian provenance but for one copied from an Egyptian manuscript in Istanbul. Related research in other manuscript libraries in Europe,

Sezgin, *GAS*, VII, pp. 32-8, esp. p. 37.

[23] The various Cairo manuscripts are listed in *Cairo ENL Survey*, no. A21.

the Near East, and India, yielded only two other copies of this table, namely, one in another Egyptian manuscript in the Escorial and the second in a Syrian recension of a thirteenth-century Tunisian *zīj* now preserved in Hyderabad. I do not doubt that other copies will come to light now that attention has been drawn to the table.

VETTIVS VALENS

| *Κανόνιον δεύτερον καὶ πλινθίον*

	Ζυγοῦ				Σκορπίου				Τοξότου			
μοῖραι	ἀριθμοί	συγ	ἔτη	μῆνες	ἀριθμοί	συγ	ἔτη	μῆνες	ἀριθμοί	συγ	ἔτη	μῆνες
α	ο	κ	α	ο	ιβ	κ	λδ	δ	κδ	κ	ξγ	α
β	β	μ	η	β	ιδ	μ	μ	γ	κς	μ	ξθ	ο
γ	ε	ο	ιε	γ	ιζ	ο	μη	θ	κθ	ο	οδ	ια
δ	ζ	κ	κβ	δ	ιθ	κ	νγ	δ	α	κ	γ	α
ε	θ	μ	κθ	δ	κα	μ	νθ	δ	γ	μ	θ	ο
ς	ιβ	ο	λς	γ	κδ	ο	ξε	ια	ς	ο	ιε	δ
ζ	κδ	κ	ογ	γ	ς	κ	ιζ	δ	ιη	κ	μς	ια
η	κς	μ	π	ο	η	μ	κγ	η	κ	μ	να	ο
θ	κθ	ο	πζ	θ	ια	ο	κθ	⟨ια⟩	κγ	ο	νη	η
ι	α	κ	δ	ια	ιγ	κ	λς	ο	κε	κ	ξδ	ζ
ια	γ	μ	ια	θ	ιε	μ	μβ	⟨ε⟩	κζ	μ	ο	ε
ιβ	ς	ο	ιη	ι[β]	ιη	ο	μη	ζ	λ	ο	ος	α
ιγ	ιη	κ	νε	ι	ο	κ	ο	ια	ιβ	κ	λα	ο
ιδ	κ	μ	ξβ	ς	β	μ	ζ	β	ιδ	μ	λζ	β
ιε	κγ	ο	ξθ	θ	ε	ο	ιγ	γ	ιζ	ο	μγ	α
ις	κε	κ	ος	η	ζ	κ	ιθ	ζ	ιθ	κ	μθ	ζ
ιζ	κζ	μ	πγ	η	θ	μ	κε	ι	κα	μ	νε	ο
ιη	λ	ο	ς	θ	ιβ	ο	λβ	ο	κδ	ο	ξ	⟨ι⟩α
ιθ	ιβ	κ	λζ	δ	κδ	κ	ξδ	θ	ς	κ	ις	α
κ	ιδ	μ	μδ	ια	κς	μ	ο[β]	ι	η	μ	κα	ια
κα	ιζ	ο	να	ς	κθ	ο	ος	ι	ια	ο	κζ	ι
κβ	ιθ	κ	νη	α	α	κ	γ	ς	ιγ	κ	λγ	η
κγ	κα	μ	ξε	ς	γ	μ	θ	η	ιε	μ	λθ	γ
κδ	κδ	ο	οβ	γ	ς	ο	ιε	ι	ιη	ο	μς	δ
κε	ς	κ	ιθ	ια	ιη	κ	μη	α	ο	κ	ο	ι
κς	η	μ	κς	ο	κ	μ	νδ	α	β	μ	ς	η
κζ	ια	ο	λγ	ο	κγ	ο	ξ	α	ε	ο	ιβ	ζ
κη	ιγ	κ	μ	ς	κε	κ	ξε	ια	ζ	κ	ιη	⟨ς⟩
κθ	ιε	μ	μζ	ια	κζ	μ	οα	ια	θ	μ	κδ	δ
λ	ιη	ο	νδ	δ	λ	ο	οη	ο	ιβ	ο	λ[β]	β

Figure 2: An extract from the second table in the *Anthology*. [From Pingree, *Vettius Valens*, p. 312.]

2 *The Arabic tradition*

The table was clearly first copied in Arabic in the Abbasid period, more specifically in early-ninth-century Baghdad. Our sources maintain that it was copied in gold ink, which must have been a rare occurrence, but not unique. Indeed, in this case, as in the case of the surviving fragments of the 'Blue Qur'ān' from tenth-century (Fatimid) Qayrawan also written in gold ink, the inspiration may have come from Byzantine practice.[24] The legitimacy of writing or decorating the text of the *Qur'ān* in gold was discussed by some early Muslim authorities, most of whom did not approve of this.[25] However, in Fatimid Egypt there were dozens of *Qur'ān* copies in gold ink.[26]

In view of the purported utility of the table I find it surprising that it was not incorporated into the major Arabic astrological treatises and widely used all over the Muslim world thereafter. The fact that the underlying theory is absurd is of course no reason why the table should not have been popular.

The appointed term of a man's life or the date of his death (Arabic, *ajal*) was a topic discussed in the early Muslim literature on scholastic theology (*kalām*), inevitably without reference to astrological sources. The *Qur'ān* often emphasises the *ajal* as the irrevocable period of life assigned by God, and, according to Muslim tradition, *ajal* is determined for a man whilst still in the womb.[27] It would be interesting to know how the 'Table of Life' was received when it was first made available in Arabic.[28]

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[24] See Bloom, 'The Blue Koran', especially p. 98b. On ink in Islamic society see the article 'Midād' in *EI*2. I owe these references to the kindness of Dr. Jan-Just Witkam.

[25] Jeffery, *Materials*, pp. 150-2, presents opinions recorded by Ibn Abī Dā'ūd (d. 928) in his *Kitāb al-Maṣāḥif.*

[26] Bloom, 'The Blue Koran'. On the preparation of gold ink as described in the treatise by the Zīrid ruler of Ifrīqiya, al-Muʿizz ibn Bādīs (ca. 1025), see Levey, 'Mediaeval Arabic Bookmaking', pp. 9, 25, and 32-3.

[27] See the article 'Adjal' by Ignaz Goldziher and W. Montgomery Watt in *EI*2. On some medieval Jewish notions on this subject see Weil, *Maimonides über die Lebensdauer*, and Kaufmann, 'R. Haja über die Lebensdauer'.

[28] It would not surprise me if a discussion was found in one or other of two works of al-Bīrūnī entitled *Kitāb al-Tanbīh ʿalā ṣināʿat al-tamwīh*, 'An Exposé of the Art of Deception', in which he criticises astrologers who use the planets to determine the length of life, and *Kitāb al-Shumūs al-shāfiya li-'l-nufūs*, 'The Suns which Heal the Soul', in which he presents 'the best method'.

As far as the later history of the table is concerned, the manuscripts I have located prove only that it was known in Tunis in the thirteenth century and then in Cairo from the fifteenth century onwards, as well as in Istanbul in the nineteenth century. I have a hunch that it was known in Seville or Cordova in the twelfth century; if this is so, then it was probably also known in Palermo in the thirteenth century (see MS A in Section 3).

There appear to be at least three different Arabic traditions of the table. In the first, represented by the Escorial manuscript (MS B) and one Cairo copy made from it (MS C), the table itself is accompanied by a text—see Appendix B for a translation—in which the table is attributed to Wālīs, further identified as Wālinṭiyus (*W-'-l-x-ṭ-y-s*), where 'x' represents an undotted carrier for a medial *nūn*, etc.), and is labelled *al-hashtaq*, a word new to the modern literature. Now the Middle Persian for 'brick' is *ishti-*, and the New Persian, *khisht*; a small brick in New Persian is *khishtak*.[29] I suspect that *plinthion* was translated into Middle Persian as *ishtak* and that this was rendered into Arabic as *hashtaq*. The author of this text, which is clearly part of a larger work on astrology, had seen the table of the auxiliary function in its original form, since he mentions that Vettius Valens had begun with entries for Libra. He had also seen the accompanying text in which Vettius Valens generously offered the table to his readers.[30] The table is also labelled *jadwal al-ḥayā wa-kamiyyat ʿumr al-mawlūd*, 'table of life and the length of life of the newborn'. In the Arabic text there are two examples for the latitude of Baghdad, which, as we shall see, are based on parameters from the ninth century.

Alas neither of these works is extant. See further al-Bīrūnī, *Chronology*, p. 79 (text) and p. 92 (trans.), and Sezgin, *GAS*, VII, p. 191, no. 9, and also n. 37 below.

[29] Buck, *Dictionary*, pp. 603-4; Steingass, *Dictionary*, p. 461b.

[30] This appears to be a reference to a remarkable passage in the *Anthology* (8.5, ed. Pingree, *Vettius Valens*, p. 288). The following is an extract from a translation kindly provided by Alexander Jones:

'Hence, for those who most desire to adhere to every methodical system, since each of those who have written treatises has, after working out the system in a distinctive and complicated manner, secretively and grudgingly omitted the solutions without presenting them, I, who have sought (them) through much labour and much trial, have set them out. ... ... if I very often remind you of my generosity and straightforwardness, what I say should be forgiven. ... .'

In the second tradition, represented only by the Hyderabad manuscript (MS A), the table is labelled *jadwal al-t-n-w-r*, which I read as *al-tanawwur*, 'table of illumination',[31] a curious development from 'little brick', and it is attributed to Wālīs. There is no accompanying text (at least not in MS A). The same title for the table and a corrupt version occur in two of the Egyptian manuscripts (MSS D and J).

In the third tradition, represented by the remaining Cairo manuscripts, the table, now labelled *jadwal al-ḥayā*, 'table of life', is accompanied by a short text with a numerical example for an unspecified latitude (actually about 36°). There is no attribution to Wālīs. In one copy (MS G) there is an additional example for an unspecified latitude, this time for Cairo (see Appendix C).

In the Egyptian manuscript in the Escorial, as in six of the seven Cairo manuscripts, it is stated that the table 'was found in a work entitled *Kitāb al-Kāmil fī 'l-nujūm* (=*The Complete / Perfect Book on Astronomy / Astrology*), in the treasury of ʿAbdallāh ibn Ṭāhir ibn al-Ḥusayn (sent to him) by (*ʿan*) al-Ma'mūn, (and) written in gold'. ʿAbdallāh ibn Ṭāhir (798-844) was a well-known general, governor, and confidant of the Caliphs of Baghdad,[32] including al-Ma'mūn (reigned from 813 to 833), who was particulary interested in astronomy.[33] ʿAbdallāh ibn Ṭāhir was governor of Khurasan, with its capital in Nishapur, and had a palace in Baghdad. It seems more likely that his treasury would have been in Baghdad. However, we should keep in mind that in the first and third traditions identified above we have worked examples for the use of the table respectively for the latitude of Baghdad and for an unspecified location with latitude 36°, which could serve Nishapur.[34] The *Kitāb al-Kāmil fī 'l-nujūm* remains unidentified; it is not one of the various titles attributed to Wālīs or his com-

[31] The expression *al-jadwal al-mutanawwar* would be more acceptable. I have preferred the reading *al-tanawwur* to *al-tannūr*, 'baking oven', which makes little sense. Note that in MS J *al-tanawwur* becomes *al-tanwīr*. In my first study of these materials (see n. 1) I chose *al-tannūr*.

[32] See the article ''Abd Allāh b. Ṭāhir' by E. Marin in *EI*$_2$.

[33] See Sayılı, *Observatory in Islam*, pp. 50-87, on al-Ma'mūn's astronomical activity, and also pp. 8-49 on astrology in Islam. A new account of the geodetic activities sponsored by him is in King, 'Earliest Muslim Geodetic Measurements'.

[34] See Kennedy and Kennedy, *Islamic Geographical Coordinates*, p. 245.

mentator by the tenth-century bibliographer Ibn al-Nadīm,[35] and it is not attested as a title of any known astrological work from the early Abbasid period.[36]

In the text accompanying the table in the Escorial manuscript it is stated that Wālīs wrote another tract of the variety known as *namūdār*: a procedure for finding the longitude of the horoscopus at birth when the time of birth is not known exactly.[37] I have come across two copies of this text and am confident that others exist in the vast sources available for the further study of Islamic astrology. This method is outlined in Section 4 and the text is translated in Appendix A.

3 *The Arabic sources*

Before investigating the table and the accompanying instructions I list the ten copies of the table (A-K), two copies of the *namūdār* (L-M), and various other relevant sources (N-Q):

A: MS Hyderabad Andra Pradesh State Central Library 298, ca. 200 folios, copied ca. AH 800 [≈ AD 1400] in Homs. This is a unique copy of a recension of the *zīj* by the thriteenth-century Tunisian astronomer Ibn Isḥāq, which is perhaps the most important source for the history of post-classical Islamic astronomy

[35] Cited in Sezgin, *GAS*, VII, p. 41.

[36] A certain *al-Kitāb al-Kāmil* is attributed to Abū Maʿshar (787-886, active in Baghdad) by Ibn al-Nadīm: see Sezgin, *GAS*, VII, p. 151, and David Pingree in the *DSB* article 'Abū Maʿshar' (I, p. 38, nos. 22 and 24). Another work with the same title was penned by Ibn Nawbakht in 994 (Sezgin, *GAS*, VII, p. 172); it is an astrological history, published as Labarta, *Horóscopos históricos*. The other titles of the same kind associated with al-Farghānī (Sezgin, *GAS*, VI, pp. 150-1) and Abu 'l-Wafā' (Ibid., p. 223) are not astrological works.

[37] On the concept of the *namūdār* see al-Bīrūnī, *Astrology*, pp. 328-9; Sédillot-*fils*, *Ouloug Beg*, II, pp. 201-5 (Ulugh Beg on Ptolemy, Hermes and Zoroaster); North, *Horoscopes and History*, pp. 51-2 (Ptolemy, from *Tetrabiblos*, III.2); and Elwell-Sutton, *Horoscope*, p. 83 (on a late Persian work). For a short discussion of other methods for finding the length of life see al-Bīrūnī, *Chronology*, pp. 78-80 (text) and pp. 90-2 (trans.). See also Tokan, 'al-Bīrūnī on the Length of Life' (abridged Arabic text with facsimile of the unique Konya manuscript), on Indian methods (Sezgin, GAS, VII, p. 92, and p. 190, no. 2), and n. 28 above.

to be discovered in the past few years.[38] Of the 360 tables in this work, no. 171, squeezed in between other astrological tables, is entitled *Jadwal al-tanawwur li-Wālīs li-maʿrifat ʿumr al-mawlūd*, 'Table of illumination by Wālīs for finding the length of life of the newborn (i.e. the age to which a newborn will live)'. A preliminary investigation of the 188 pages of the introduction to the *Zīj* did not reveal any associated text.

There are, however, various tables for determining the period of gestation in the *Zīj* of Ibn Isḥāq,[39] and in the text the twelfth-century astronomer Ibn al-Kammād, who hailed from Seville but was later active in Cordova,[40] is extensively cited. The writings of Ibn al-Kammād on this subject survive in an incomplete version of his *Kitāb Mafātīḥ al-asrār*—see MS Q—and have been studied in an article by Juan Vernet.[41] In this treatise Ibn al-Kammād cites Wālīs on the subject of gestation but does not mention the 'Table of Life'.[42] I conjecture that Ibn Isḥāq found the table in the missing part of the *Mafātīḥ al-asrār*.

As noted by Vernet, this treatise of Ibn al-Kammād was excerpted in Latin by Johannes de Dumpno in Palermo in 1260.[43] A copy survives in MS Madrid Biblioteca Nacional 10023, but this does not contain the 'Table of Life'. Ibn al-Kammād's tables for gestation, on the other hand, had a life of their own in the corpus of tables associated with Abraham Zacut.[44] As far as I am aware, and the same holds for various colleagues consulted, there is no mention of the table in other European sources.

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[38] On the author see Suter, *MAA*, no. 356; and, since the rediscovery of the Hyderabad manuscript, King, 'Astronomy in the Maghrib', pp. 31-5, Samsó, 'Maghribi *Zījes*', p. 93, and now Mestres, 'Hyderabad Manuscript', and idem, Zīj *of Ibn Isḥāq*, esp. p. 294, where the table is mentioned in passing.

[39] Mestres, Zīj *of Ibn Isḥāq*, p. 292. See also n. 85.

[40] Suter, *MAA*, no. 487, and 'Nachträge', no. 487; Brockelmann, *GAL*, S I, p. 864; Millás, *Estudios sobre Azarquiel*, pp. 345-7; Sarton, *IHS*, III.2, pp. 1514-15; and Vernet, 'Tractat', pp. 72-3.

[41] Ibid. See also Chabás and Goldstein, '*Zīj* of Ibn al-Kammād', p. 37, and n. 44 below.

[42] Vernet, 'Tractat', p. 86. Ibn al-Kammād states that Wālīs (al-Rūmī !) had compiled a work on the subject of gestation but that this had become corrupted over the centuries through continuous copying.

[43] See also Carmody, *Bibliography*, p. 165. Johannes de Dumpno is apparently not mentioned in Sarton, *IHS*, and I have not encountered him elsewhere.

[44] Chabás and Goldstein, *Zacut*, pp. 152-3.

B: MS Escorial ar. 961, copied in Cairo in AH 863 [= AD 1459] by ʿAlī ibn Muḥammad al-Dalāmī, known as a copyist of several significant astronomical manuscripts,[45] is an important compendium of Abbasid and Mamluk treatises on astronomical instruments.[46] Here the table, with title *jadwal al-ḥayā wa-kamiyyat al-ʿumr li-'l-mawlūd*, occurs on fol. 44r, amidst an introduction on fols 43r-44v (see Figure 3a). The preceding and subsequent texts are quite unrelated to this section, which was inadvertently overlooked by H.-P.-J. Renaud when he catalogued the scientific manuscripts in the Escorial. (This explains how it remained unknown until the Cairo manuscript survey.)

C: MS Cairo Ṭal'at *mīqāt* 102, 89 fols, copied ca. AH 1200 [≈ AD 1800] apparently in Istanbul from the very manuscript now in the Escorial (B), contains (fols 83v-85v) the same table and treatise with numerous copyist's errors. Here the treatise bears the spurious title *Risāla fī Ṭāliʿ al-mawlūd mina 'l-asṭurlāb*, 'Treatise on (finding) the ascendant of the newborn with the astrolabe'.[47] See also MS K.

D: MS Cairo Muṣṭafā Fāḍil *mīqāt* 58, 15 fols, copied in Cairo ca. AH 850 [≈ AD 1450], contains some anonymous tables for sundial construction (fols 1v-2v) and timekeeping (fols 2v-12v), amidst the latter of which (fol. 6r) occurs the table, here entitled *jadwal al-tanawwur alladhī yustakhraj minhu ʿumr al-mawlūd.*[48] It is stated that the table was found in the treasury of ʿAbdallāh ibn Ṭāhir: see Figure 3b. There are some notes in a hand which is recognizable as that of the Cairo astronomer Ibn Abi 'l-Fatḥ al-Ṣūfī (ca. AD 1500),[49] and it may be that MS G was copied from this.

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[45] *Cairo ENL Catalogue*, I, p. 704.
[46] *Escorial Catalogue*, pp. 97-100.
[47] *Cairo ENL Survey*, no. A21.
[48] Ibid., no. C141.
[49] Suter, *MAA*, nos. 447 and 460 (confused); and *Cairo ENL Survey*, no. C98.


نوّسبو الهشتق لو البس الذي وضعه بعنو كلّ ولا نقاسه
لمعرفه مبلغ العمر وان والبس وهو والطيس عمل سوذا ب
يعرف سنة صحة درجه الطالع وساعة المولود حتى لا يلوب فيها
شك وقد ذكرناه وشرحناه في باب المودارات من كتابنا
هذا ونحن نستغني عن اعادته هاهنا وحد ولا سماه الهشتق
بدا فيه بالميزان وجعل بازا اول درجه منه ب وبازا الثانيه
د وبازا الثالثه و وبازا الرابعة ح وبازا الخامسه ى وبازا
السادسة يب ثم زاد على من الاثني عشر اربعه عشر
فصار بازا الدرجة السابعة كو وعاد فزاد عليها ب في الثامنه
فصار بازايها كح وبازا التاسعة ل وبازا العاشرة لب
وبازا الحادية عشر لد وبازا الثانيه عشر لو وزاد هاهنا
عم فصار بازا الثالثة عشر كو ورجع الى ان يزيد الدرجتين
فصار بازا الرابعه عشر درجه كح وبازا خامسه عشر ل وبازا
السادسه عشر لو وبازا السابعة عشر ع وبازا الثامنه عشر
ل وزاد هاهنا عم فصار بازا التاسعه عشر درجه لد
وبازا العشرين درجه لو وبازا الحادي والعشرين ع وبازا
الثاني والعشرين ك وبازا الثالث والعشرين كب وبازا

Figure 3a-1: The text in MS **B**, f. 43r [Courtesy of the Biblioteca de El Escorial.]



الرابع والعشرين كد وزادها هنا ع١٢ دضار بازا الخامس
والعشرين ح وعاد الى الدرجتين وصاد بازا السته والعشرين
يب وبازا السبعة والعشرين سه وبازا الثامنة والعشرين
لد وبازا التسعه والعشرين نو وبازا الثلاثين كح وجعل
الاسد والحوت مثله وبازا اول درجه من العقرب لد والحق
به الزيادة على هذا الترتيب وجعل الحمل والسنبله مثله
وبازا اول درجه من القوس كو والحق به الزيادة لذلك وجعل
الثور مثله وبازا اول درجه من الجدى ح والحق به الزيادة
على الرسم وجعل الجوزا مثله وبازا اول درجه من الدلو ك
والسرطان مثله وكان يزيد زيادته ان يزيد اسين اسين فاذا
مضت خمس درج زاد ع١٢ ثم عاد فزاد اسين اسين ايضا كذلك
حتى يستوفي جميع البروج ودرج الزيادة في اوايل البروج سته
لانه ليس للدرجه الاولى اشتراك والباقيه مشتركه فهي خمس
درج تماس كل ع١٢ مرتين وبين اختها وقال والبس ايضا
اذا زدت على ما بازا اول درجه من كل برج ١٢ حرج ما يجب
ان يكون بازا اول درجه من البرج الذي يتلوه على نسقه ما كما
ذلك فكان صحيحا ولم يذكر العلة في ممارسته من هذ ٥

Figure 3a-2: The text in MS **B**, f. 43v [Courtesy of the Biblioteca de El Escorial.]



Figure 3a-3: The text in MS **B**, f. 44r [Courtesy of the Biblioteca de El Escorial.]


عمره ان شا الله تعالى فمثاله ان الطالع سبع درج من الاسد
واجزا ساعات الدرجه السابعه من الاسد بمدينة السلام برج
فلما ضربناها في ١٢ خرج الضرب ره لو كان باز اسبع درجات
من الاسد بالمشرق لكون نسبتها من س ثلث وعشر فاذا
اخذنا بقسطها من الماسين وخمس درجات ودقائقها الذي بيناها
كان ذلك ٨٩ درجة وخمس دقائق بنوبها تسع وثمانون سنة وشهر
واحد وايضا فنقول الطالع ست درجات من السرطان واجزا
ساعاته برمه فاذا ضربناها في ١٢ صار [illegible] ونابها في المشرق
لنسبتها من س النصف فلما اخذنا نصف ما خرجه الضرب
كان مائه وستنه وستة اشهر وعلى هذا يجري العمل في سائر
الباب وبحسب في كل بلد باجزا ساعاته وقد رسمنا جدول
المشرق لدرجه درجه من درج البروج كما عرفت من غير
عمل السنين والشهور اذا كان ذلك يعنون في البلدان
بحسب اختلاف اجزا ساعات نهارها وبحسبما نرى يجب ان
يكون هذا الباب معينا على معرفة مقدار العمر وان نقاس به
ما يدل على الكدخداه ومن الدوه وما توجبه التسييرات
الى اجساد المحوس من القطوع فيستظهر بعض ذلك على بعض
لنعلونه الصحيح من هذا العلم الشريف والله سبحانه وتعالى اعلم

Figure 3a-4: The text in MS **B**, f. 44v [Courtesy of the Biblioteca de El Escorial.]

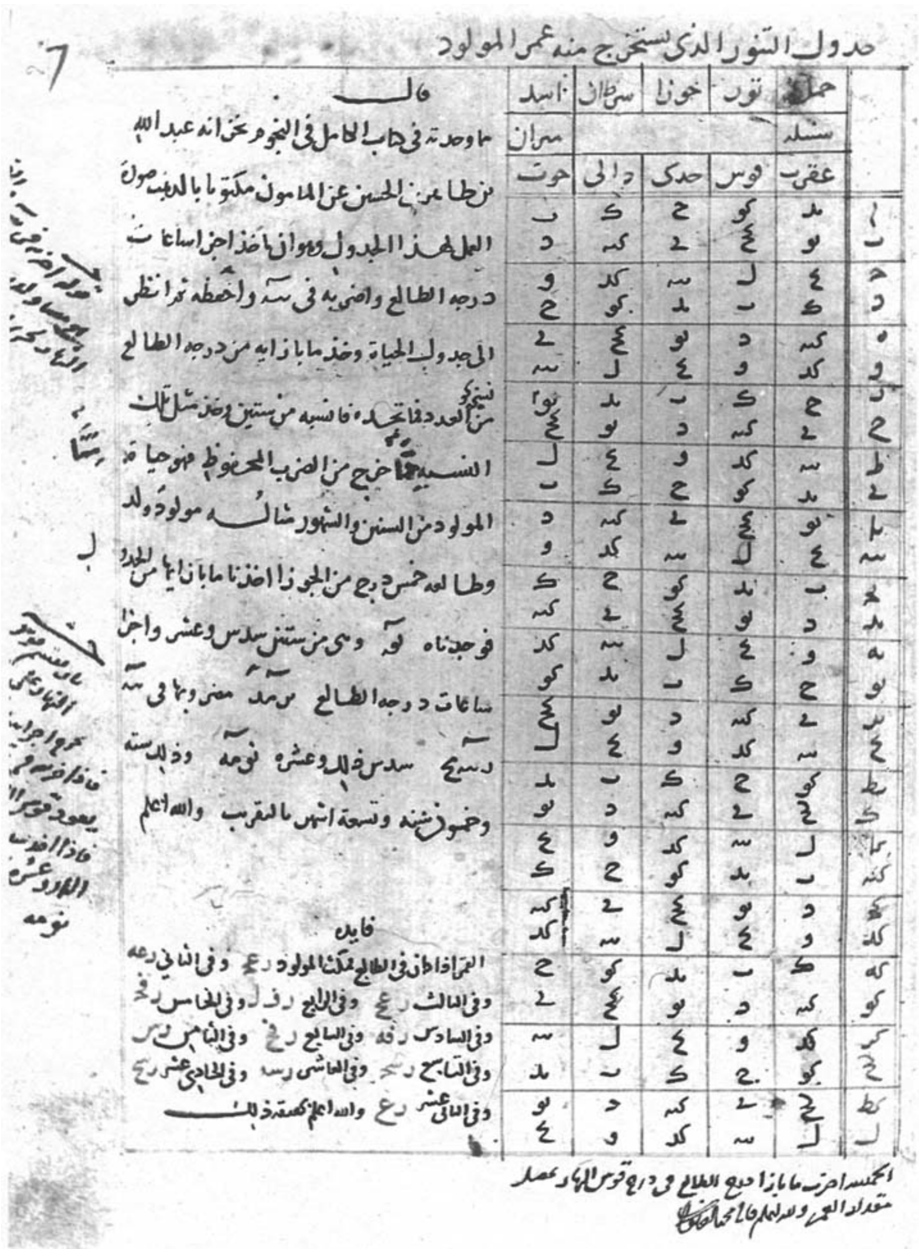


Figure 3b: The table in MS **D**. [Courtesy of the Egyptian National Library.]

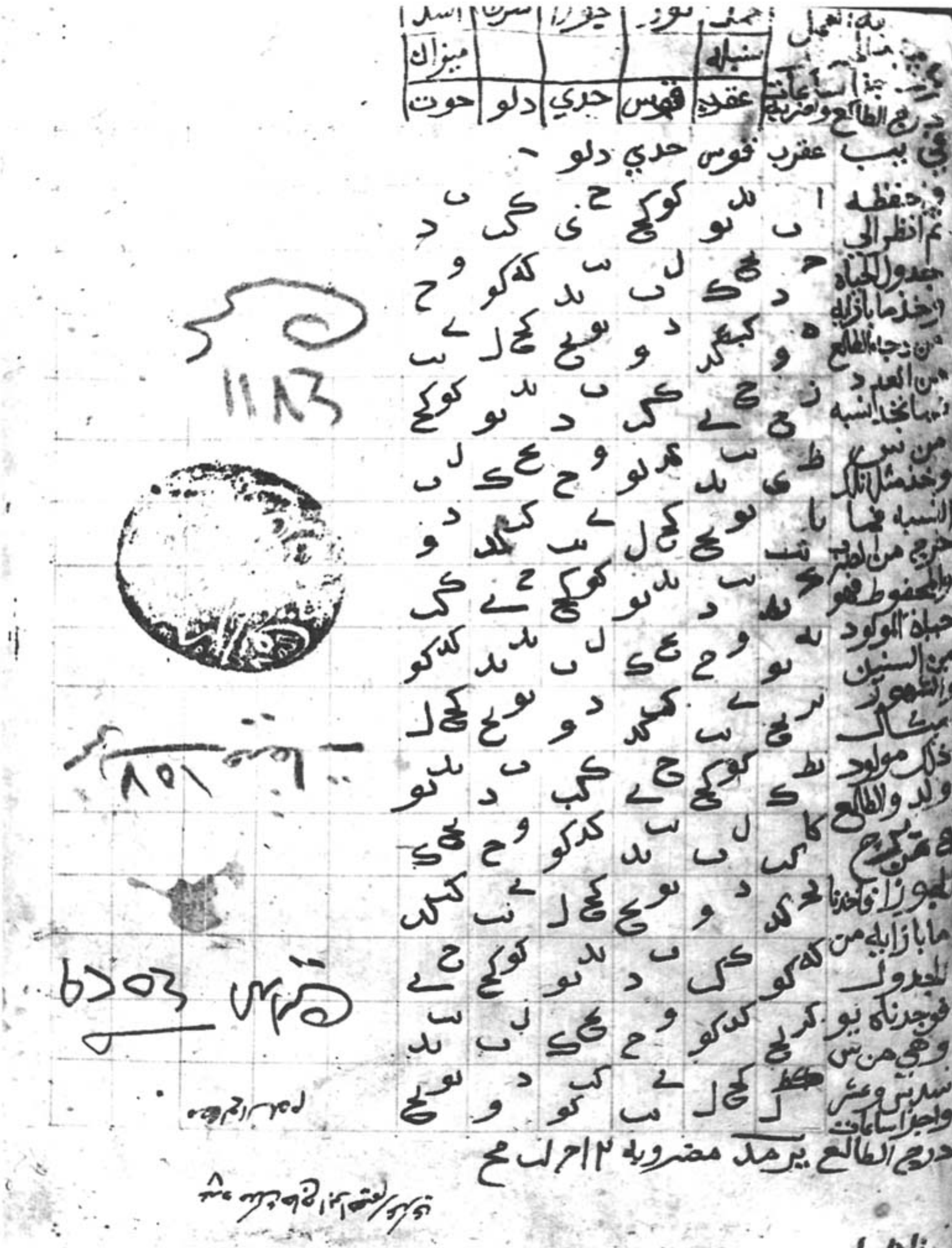


Figure 3c: The table in MS **E**. [Courtesy of the Egyptian National Library.]



Figure 3d: The table in MS **F**. [Courtesy of the Egyptian National Library.]



Figure 3e: The table in MS **G**. [Courtesy of the Egyptian National Library.]

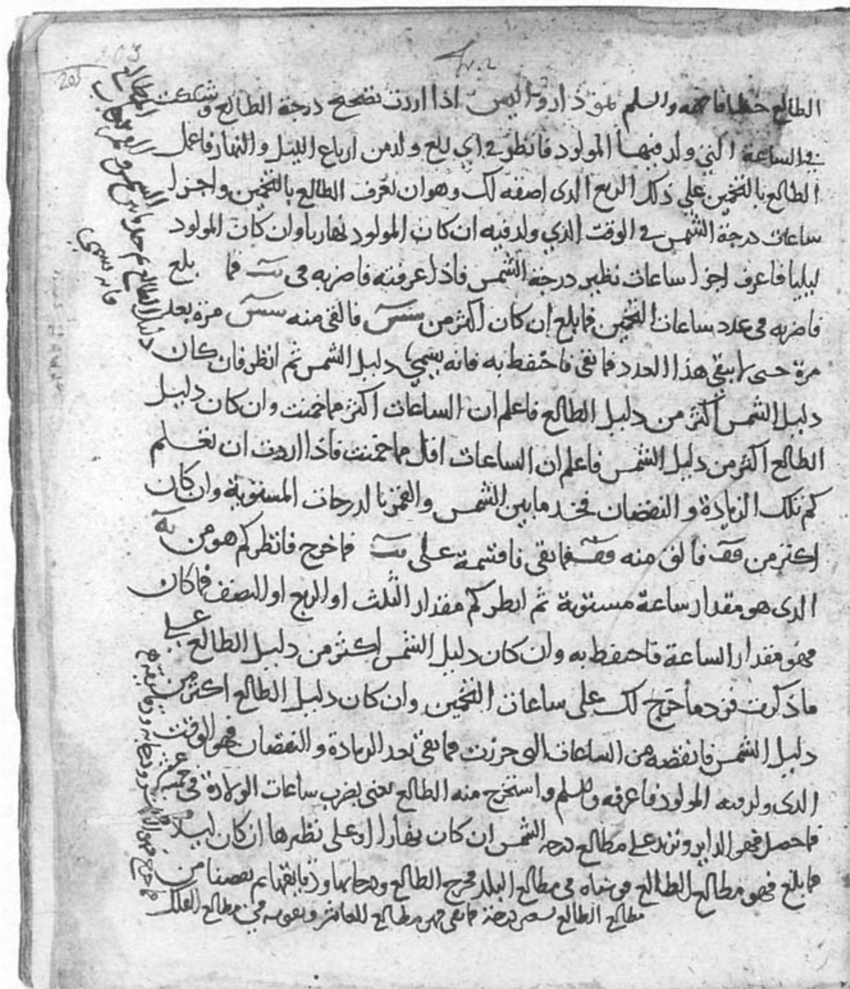


Figure 3f: The text on the *namūdār* of Wālīs in MS **L**. [Courtesy of the Deutsche Staatsbibliothek (Preussischer Kulturbesitz), Berlin.]

E: MS Cairo Dār al-Kutub *mīqāt* 157, 7 fols, copied ca. AH 850 [≈ AD 1450], contains a set of tables of the ascensions at the time of the *salām* (shortly before daybreak, when praises on the name of the Prophet are called out by the muezzin) by the fifteenth-century Cairo astronomer ʿIzz al-Dīn al-Wafāʾī.[50] On fol. 1r of this manuscript our table reappears copied upside down in a different, much later, hand, and with brief instructions on its use: see Figure 3c.

F: MS Cairo Dār al-Kutub *mīqāt* 637, 48 fols, copied 868 H [= 1463/64], consists mainly of a copy of the *Zīj* called *al-Lumʿa* compiled by the fifteenth-century Cairo astronomer Aḥmad al-Kawm al-Rīshī.[51] Appended to this in different hands are various magical tables and qibla diagrams and our table (fols 47v-48r) which now occurs together with another astrological table, illustrated in Figure 3d.

G: MS Cairo Dār al-Kutub *mīqāt* 1108, 59 fols, copied AH 1053 [= AD 1643/44] by ʿAlī ibn Muḥammad, a copyist of two important astronomical manuscripts in the Egyptian National Library,[52] contains a collection of treatises and tables relating to timekeeping and lunar crescent visibility.[53] Amongst these (fol. 20v) is the *jadwal al-ḥayā* (see Figure 3e) with some notes copied from the hand of Ibn Abi 'l-Fatḥ al-Ṣūfī (see MS D).

H: MS Cairo Dār al-Kutub *mīqāt* 1205, 215 fols, copied ca. AH 1250 [≈ AD 1850] in different hands, contains the *zīj* called *al-Zīj al-Mufīd* of the seventeenth-century Cairo astronomer Riḍwān Efendī.[54] At the end of this work there is appended (fol. 215v) the *jadwal al-ḥayā*.

J: MS Cairo Ṭal'at *mīqāt* 241, 43 fols, copied ca. AH 1250 [≈ AD 1850] in different hands, contains prayer-tables for Damietta by the seventeenth-century astronomer Quṭb al-Dīn al-Maḥallī

[50] Ibid., no. C61 (*ad* 3.1.24).
[51] Ibid., no. C41.
[52] *Cairo ENL Catalogue*, I, p. 704.
[53] Ibid., I, pp. 188-9, and *Cairo ENL Survey*, no. D226.
[54] Ibid., no. D58.

and other anonymous tables for Cairo.[55] Amidst these (fol. 34r) is our table, here entitled *jadwal al-tanwīr*, 'Table of illumination', a nice corruption of *jadwal al-tanawwur.*

K: MS Cairo Ṭal'at *mīqāt* 227, 163 fols, copied AH 1145 [= AD 1732/33] in legible *naskhī* script (provenance uncertain), contains seven astrological treatises, including Abbasid, Mamluk, and Rasulid Yemeni works,[56] amidst which (fols 87r-87v) there occurs the *jadwal al-ḥayā*, taken from a copy in the hand of ʿAlī ibn Ḥasan al-Baḥṭīṭī, a well-known copyist of astronomical works,[57] and a student of the Cairo astronomer Sibṭ al-Māridīnī ca. AH 900 [≈ AD 1500],[58] this in turn based on a copy by al-Dalāmī, perhaps our source B!

L: MS Berlin Ahlwardt 5751 (= Mq. 101), 221 pp., copied ca. AH 700 [≈ AD 1300],[59] is a fine copy of the astronomical handbook (*zīj*) of the early-eleventh-century Iranian astronomer Kūshyār ibn Labbān.[60] At the end of this copy occur various interesting spurious additions, including texts and tables, amongst which[61] we find a *namūdār* attributed to Wālīs—see Figure 3f and Section 4—along with others attributed to Hermes, unidentified 'early scholars', and the ninth-century philosopher and astrologer al-Kindī on the technique associated with the Indians.[62] In addition there are some tables for finding the period of gestation.[63]

M: MS Damascus Ẓāhiriyya *ʿāmm* 9354, 181 fols, copied AH 641 [= AD 1243/44], is a substantial fragment of the astrological compendium *Kitāb al-Mughnī* by Ibn Hibintā.[64] In this (fols 22v-

[55] Ibid., no. D37.

[56] On the manuscript see *Cairo ENL Catalogue*, I, *sub* ṬM 227.

[57] On al-Baḥṭīṭī see ibid., I, p. 703.

[58] On Sibṭ al-Māridīnī see Suter, *MAA*, no. 445, and *Cairo ENL Survey*, no. C97.

[59] On the manuscript see *Berlin Catalogue*, pp. 203-6, esp. p. 206.

[60] Kennedy, '*Zīj* Survey', nos. 7 and 9; Sezgin, *GAS*, VI, pp. 246-9; and King and Samsó, 'Islamic Astronomical Tables', p. 41.

[61] One example is a set of tables for timekeeping by the stars, computed for the latitude of Qandahar ca. 1000: see King, *SATMI*, I-3.2.1.

[62] No such treatise is listed in Sezgin, *GAS*, VII, pp. 259-61.

[63] See n. 85 below.

[64] See n. 10 above.

23r) is preserved the *namūdār* of Wālīs—see Section 4—along with others. See also MS N below.

N: MS Munich ar. 852, 240 fols, copied ca. AH 650 [≈ AD 1250], contains a second substantial fragment of the *Kitāb al-Mughnī*—see MS M above.

O: MS Dublin Chester Beatty 3640, 111 fols, copied ca. AH 670 [≈ AD 1270] is a copy of the astrological compendium *Safīnat al-aḥkām* of al-Ṭūsī.[65]

P: MS London British Library Add. 23400, 103 fols, copied in the tenth/sixteenth century, is another copy of the *Safīnat al-aḥkām*.[66]

Q: MS Escorial ar. 939,1, fols 1r-9v, copied ca. AH 900 (?) [≈AD 1500], contains Chs 10-15 of the astrological treatise *Mafātīḥ al-asrār* by Ibn al-Kammād—see MS A above.[67]

4 *Finding the horoscopus at birth*

The longitude λ_H of the horoscopus at birth can be found by measuring the time of the nativity and using tables of right and oblique ascensions $\alpha(\lambda)$ and $\alpha_\phi(\lambda)$ which were standard equipment of astronomers in late Antiquity and throughout the Middle Ages.[68] If the birth occurs at time T (in degrees) after sunrise on a day when the solar longitude is λ_S then:

$$\alpha_\phi(\lambda_H) = \alpha_\phi(\lambda_S) + T \text{..............} (2)$$

If it occurs when a star with known mediation (co-culminating ecliptic longitude) μ is observed to be culminating, then:

$$\alpha_\phi(\lambda_H) = \alpha(\mu) + 90° = \alpha'(\mu) \text{..................} (3)$$

where $\alpha'(\lambda) = \alpha(\lambda) + 90°$ are the normed right ascensions, in use since Antiquity for determining λ_H from μ. (Various Islamic

[65] Sezgin, *GAS*, VII, p. 24.
[66] Ibid. See also Pingree, 'Astronomy and Astrology in Iran', pp. 241-2.
[67] *Escorial Catalogue*, pp. 54-5.
[68] See the article 'Maṭāliʿ [= ascensions]' in *EI*$_2$.

tables are known in which λ_H is tabulated for each degree of altitude for various stars, and for each degree of both altitude and longitude for the sun.[69] One table is known where λ_H is tabulated for various significant times of day—significant, that is, for religious and/or astronomical reasons—throughout the year, the values having been determined using an astrolabe.[70])

Since the horoscopus cannot be directly observed and its computation involves these subtleties, various solutions were developed in Antiquity which purported to enable the user in possession of only vague details of a nativity to determine the horoscopus accurately: these are known as *namūdārs*.[71] Frequently cited in the Islamic sources are the *namūdārs* of the Indians, Ptolemy, Hermes, Zoroaster, and Māshā'allāh, to which we may now add that of Vettius Valens—see Appendix A for a translation of the text in MSS L, p. 203 (I have not used MS M, fols 22v-23r, or any other copies). The procedure outlined is by no means clear to me, and differs from what E.S. Kennedy has published recently from various other Arabic manuscripts.[72]

One is to guess the time of birth, say T' after sunrise, as best one can. Then, using the lunar longitude λ_M, the length of daylight $2D(\lambda_S)$ corresponding to the solar longitude λ_S, and the oblique ascensions, one determines the 'indicators' of the horoscopus and of the sun, denoted by i_H and i_S, thus:

$$i_H \equiv [(2D(\lambda_S)/12) \times 12 \times T'] \bmod 360 \qquad (4)$$
$$i_S = \alpha_\phi(\lambda_S) - \alpha_\phi(\lambda_M) \qquad (5)$$

Then one is to form the quantity:

$$\Delta\lambda \equiv (\lambda_S - \lambda_M) \bmod 180 \qquad (6a)$$

with which the time correction is:

$$\Delta T \equiv [\Delta\lambda/12]/15 \qquad (6b)$$

and the true time of the nativity T is:

$$T = T' \pm \Delta T \quad (\text{as } i_S \text{ is greater or less than } i_H) \qquad (7).$$

[69] King, *SATMI*, I-3.2.1-2 and 3.1.1.
[70] Ibid., II-14.1, and IV-5.3.
[71] See n. 37 above.
[72] Kennedy, 'Kāshī on the Ascendent', pp. 143-4.

The text of the *namūdār* concludes with statements equivalent to (2) and (3). These procedures apparently defy rational explanation. There is perhaps a need for further investigation of all the available sources. This material is of interest not least because it represents an Arabic rendering of a very obscure Greek text.[73]

5 *The table and its format*

The treatise accompanying the table in MS B and the derivative copy C (see Figure 3a and also Appendix B for a full translation) begins with the words *tafsīr al-hashtaq li-Wālīs* ... , 'an explanation of (the table called) *hashtaq* due to Vettius Valens ... ', and continues with the remark that he 'made a *namūdār* for rectify-

[73] Alexander Jones has kindly provided the following notes from the published Greek text. The 'indicators' of the sun and the ascendant are what Vettius Valens calls 'gnomons'. There are several passages in the *Anthology* that discuss these. The one used in the Arabic version is apparently that found in 8.3 (Pingree, *Vettius Valens*, pp. 285-6):

'A different way. We found also in another way the magnitude corresponding to the sun's degree and multiplied it by 12, or at night the diametrically opposite (degree). Again we will multiply the same by the given hours of the giving birth; and after casting out 360s, regard the remainder as the horoscopic gnomon. Then, taking the interval from the sun to the moon in ascension (*pros anaphoran*), compare with the first, horoscopic, gnomon. For if the solar gnomon is greater, add to the hour, and if it is less, subtract. One should add or subtract as much as the difference of the solar magnitude indicates. For the complete (what?) before the comparison will be obtained from the addition and subtraction at the hour or also fraction of an hour.' [Last sentence unclear, obviously corrupt.]

Inserted in the Arabic version of this is a method found in the fifth-century '*additamenta*', ch. 1 (Pingree, *Vettius Valens*, p. 350):

'Another way. Reckoning the interval from conjunction to the moon or the (interval) from full-moon to the moon, and if it is within 180 degrees, multiplying by 12, see what fraction results of 15 days, and if it is found more than 180 degrees, subtracting the 180 degrees, compare the remainder, what fraction it is of the moon's (daily) course, and deduct this from the hourly magnitude.' (Apparently Vettius Valens uses 15 as a round figure for the moon's daily motion here.)

Another version of this last rule is in the *Anthology* (8.3, Pingree, *Vettius Valens*, p. 285), but without using 15 explicitly for the moon's daily course:

'Another way. Reckoning the interval from conjunction to the moon or the (interval) from full moon to the moon—and if it is found inside 180 degrees, use it in the demonstrated way, but if over 180 degrees, subtracting the 180 degrees, compare the remainder, what fraction it is of the course, and subtract this from the hourly magnitude.'

ing the longitude of the horoscopus and the hour of the nativity so that there should be no doubt about either'—see Section 4. The anonymous author then states that 'we have mentioned and explained this in the chapter on *namūdārs* in this book of ours (presumably the *Kitāb al-Kāmil fi 'l-nujūm*?)' and that 'he (Vettius Valens) (made) a table which he called *al-hashtaq*'.

The author then describes in detail the structure of the table and mentions the significant entries therein, which correspond precisely to those in the various Arabic copies of the table:

> '(Vettius Valens) began in (the table) with Libra and put 2 for the first degree of it, 4 for the second, 6 for the third, 8 for the fourth, 10 for the fifth, 12 for the sixth, then he added to these twelve 14, and so there was 26 for the seventh, and then he again added 2 for the eighth making 28, and 30 for the ninth, and 2 for the tenth, etc.,'

The author explains that Vettius Valens made the entries for Leo and Pisces the same as those for Libra, and began the entries for Scorpio, as well as those for Aries and Virgo, with 14, and the entries for Sagittarius, and also Taurus, with 26, and the entries for Capricorn, as well as Gemini, with 8, and the entries for Aquarius, as well as Cancer, with 20. The rule is that one adds 2 for each degree and after each 5° adds 14, both operations being performed *modulo* 30, and that the 'degrees of increase at the beginnings of the signs are six (degrees), because the first degree has no contribution (*ishtirāk*) and the remaining (degrees) are contributing (*mushtarika*).' Here we should surely read 26 rather than 6 for the amounts added (again *modulo* 30) at the first degree of each sign to the value at the end of the previous sign. Vettius Valens is quoted as saying that there is a difference of 12° between the values for the first degree of consecutive signs, a remark confirmed by inspection of the table. Furthermore, the treatise indicates that the entries in the table begin from the left-hand side, as one would expect in a Greek astronomical table but not in an Arabic table, and also confirms the unusual distribution of the zodiacal signs at the head of the columns of the table (see Figure 4), which caused some confusion (see Figure 3c) amongst copyists used to transcribing tables of spherical astronomical functions symmetrical about the solstices, such as those in the main Cairo corpus of tables for timekeeping.[74]

[74] King, 'Cairo Corpus', pp. 351-3, and idem, *SATMI*, I-1.4.

THE TABLE OF LIFE					
	Sign				
	I VI VIII	II IX	III X	IV XI	V VII XII
Degree					
1	14	26	8	20	2
2	16	28	10	22	4
3	18	30	12	24	6
4	20	2	14	26	8
5	22	4	16	28	10
6	24	6	18	30	12
7	8	20	2	14	26
8	10	22	4	16	28
9	12	24	6	18	30
10	14	26	8	20	2
11	16	28	10	22	4
12	18	30	12	24	6
13	2	14	26	8	20
14	4	16	28	10	22
15	6	18	30	12	24
16	8	20	2	14	26
17	10	22	4	16	28
18	12	24	6	18	30
19	26	8	20	2	14
20	28	10	22	4	16
21	30	12	24	6	18
22	2	14	26	8	20
23	4	16	28	10	22
24	6	18	30	12	24
25	20	2	14	26	8
26	22	4	16	28	10
27	24	6	18	30	12
28	26	8	20	2	14
29	28	10	22	4	16
30	30	12	24	6	18

Figure 4: The format of the 'Table of Life' in the Arabic sources, the order of the columns now reversed.

The instructions on the use of the table indicate that one should first take the parts of the hours of the degree of the ecliptic which is instantaneously rising over the local horizon (i.e., the horoscopus) at the time of birth, and multiply them by 12. The 'parts of the hours of the horoscopus' are then $2D(\lambda_H, \phi)/12$, so that multiplication by 12 gives us back the quantity $2D(\lambda_H, \phi)$. (This part of the instructions is absurd in that the first quantity $2D/12$ would have had to have been computed from $2D$ anyway.) Then one takes the number in the table corresponding to the horoscopus and divides it by 60, and takes this fraction of the result of the first multiplication. (For the latitude of Cairo, say, which is 30°, $2D$ varies between about 151° and 209°; since ζ varies between 2 and 30, for the latitude of Cairo the length of life varies between a minimum of about 5 years and a maximum of about 105 years.) Next one takes the quantity $\zeta(\lambda_H)$, and determines the length of life $L(\lambda_H, \phi)$ in years using formula (1) above. A marginal note in MS D in hand of the Cairo astronomer Ibn Abi 'l-Fatḥ al-Ṣūfī[75] states that to find the life expectancy one should simply multiply the entry in the table by the 'degrees of daylight (of the horoscopus)', taking the entry as sexagesimal minutes; in MS G, the copyist records a note by al-Ṣūfī in which the operation is described for the entries taken as units.

6 *Some worked examples*

An example is given in the treatise in MSS B and C, namely, for the case where the horoscopus is Leo 7° and the locality is Baghdad. The 'degrees of hours of the horoscopus' are stated to be 17;8° which when multiplied by 12 yields 205;36°. The appropriate entry in the *hashtaq* table is 26, the ratio of which to 60 is given as $1/3 + 1/10$. The product of these two quantities is correctly calculated to be 89;5 (by truncation rather than rounding), that is, 89 years and 1 month. Another example is given in which the horoscopus is Cancer 6° and the degrees of its hours are 17;45°. The length of life is computed as 106 years and six months, a ripe old age which derives from the fact that the value of $\zeta(\lambda_H)$ in the table is 30. Values of $2D/12$ to two digits do not reflect small changes in latitude ϕ and obliquity of the ecliptic ε.

[75] See n. 49 above.

However, the value 17;8° corresponds exactly to the pair: $\phi = 33°$ and $\varepsilon = 23;51°$, which are elsewhere associated with the early-ninth-century astronomer al-Khwārizmī.[76] For the parameters of the astronomers of al-Ma'mūn,[77] namely: $\phi = 33;27°$ and $\varepsilon = 23;33°$, I compute 17;9°. For the second example both sets of parameters yield 17;46° instead of 17;45° in the text. So it is not possible to determine the underlying parameters uniquely.

In MSS D and E, a different example is given. The horoscopus is now Gemini 5°, and the 'degrees of the hours' are 17;44°. Since the appropriate entry in the table is 16, the life expectancy is computed as 56 years and about 9 months. The latitude underlying the value of the 'degrees of the hours' is neither that of Baghdad nor that of Cairo, where both MSS B and D were copied, but rather about 36°, serving Raqqa, Aleppo or Rayy, or more significantly perhaps, the middle of the fourth climate.[78]

In MS G, the copyist has added another example for a baby named Muḥammad, again not his own addition (*hākadhā wajadtuhu*). The date of birth is given (according to the astronomical reckoning for the Hijra calendar) as Wednesday, Shawwāl 29, 1006 Hijra [= June 3, 1598] and the solar longitude is given as 13;13° which must be in Gemini so that $\lambda_S = 73;13°$. At midday in Cairo on that day the longitude of the sun was, by modern computation (to the nearest 0.1°) in fact about 72;3°, which says a lot for the solar tables in use at the time.[79] The time since sunrise T is given as 63;13°. This could have been computed directly from an observation of the solar altitude or found from tables of $T(h, \lambda_S)$ for each degree of both arguments which formed part of the corpus of tables for timekeeping used in Cairo throughout the medieval period.[80] Then the oblique ascensions of the

[76] King, 'al-Khwārizmī', p. 2.

[77] These parameters were used in the table of oblique ascensions in MS Escorial ar. 927, fols 49b-50a, of a recension of the *Mumtaḥan Zīj*. The pair $\phi = 33;21°$ and $\varepsilon = 23;33°$ used in other spherical astronomical tables in this source is apparently due to the tenth-century astronomer Ibn al-Aʿlam. On early values for the latitude of Baghdad see further King, 'Earliest Muslim Geodetic Measurements', pp. 225-7.

[78] See King, 'Geography of Astrolabes', pp. 6-9.

[79] On the compilation of the solar ephemerides from which this value of the solar longitude was probably taken see Kennedy and King, 'Ibn al-Majdī's Auxiliary Tables'.

[80] King, 'Cairo Corpus', pp. 359-62; and idem, *SATMI*, I-2.1.1 and II-4-5.

horoscopus at sunrise (i.e., of the solar longitude) are stated to be 57;50°. A table of oblique ascensions for Cairo with values to degrees and minutes for each degree of ecliptic longitude was standard equipment of the Cairo astronomers.[81] Accurately for $\phi = 30;0°$ and $\varepsilon = 23;35°$, the parameters of the Cairo corpus and originally due to Ibn Yūnus, the value should be 57;56°. (For the same latitude and the more recent value of the obliquity found by Ulugh Beg, namely $\varepsilon = 23;30,17°$, which was popular amongst the Cairo astronomers from the late fifteenth century onwards, I compute 58;0°, which shows yet greater divergence.) Then the oblique ascensions of the horoscopus at birth are simply 57;50° + 63;13° = 121;3°, so that the longitude of the horoscopus is Leo 9;20° = 129;20°. Using accurate tables and an oblique ascension of 121;3° I obtain 129;24°. The corresponding entry in the 'Table of Life' is 30, and the arc of daylight is given as 201;44°. For the first set of parameters I compute that it should be 201;40°. Then the length of life is correctly given as 1,40;52 (= 30/60 × 201;44) years, which corresponds to 100 years, 10 months and 12 days, although this equivalent is not given. But our copyist writes that the age of the boy when he died (*ʿumruhu*) was reported to be (*ʿalā mā dhukira*) 6 years, 5 months and 6 days. It is not completely clear that the boy died at this early age, not least because the usual formulae for the deceased are missing but also because the fractional parts of the computed age and actual age are suspiciously related in the ratio 2:1. It would be tempting to restore the '6' (*wāw* in *abjad* notation) years to '50' (*nūn*), but that would mean that the now mature Muḥammad was still alive when MS G was copied (1053 H), some 47 lunar years after he was born.

7 *Finding the time of conception*

Two notes (*fā'ida*) are found in MS B (and hence also MS C), the second of which is also found in MS D. In the first is stated the well-known rule of thumb that '(the longitude of) the moon at birth is the horoscopus at conception (*masqaṭ al-nuṭfa*), and (the longitude of) the moon at conception is the horoscopus at (the time of) birth'. This is found already in the *Centiloquium* at-

[81] King, 'Cairo Corpus', p. 359. See also n. 68 above.

tributed to Ptolemy (and actually due to the tenth-century Egyptian scholar Aḥmad ibn Yūsuf) as well as in numerous later Arabic sources.[82] It is invariably cited in the *namūdār* for finding the time of conception attributed to Hermes.[83]

In the second note, the number of days that the newborn will remain in its mother's womb is related to the astrological house which the moon is in at the time of conception. If the moon is in the ascendant this time is 273 days (about 10 months). Otherwise the time of gestation is modified by the time taken by the moon to move from the position of the ascendant on the ecliptic to its instantaneous ecliptic position. The values given for the 12 houses starting with the ascendant are as follows:

273 275 278 280;30 283 285 288 260 263 265 268 270

The underlying scheme appears to be: take 273^d for the first house and add 2 $1/2^d$ for each house below the horizon and subtract 2 $1/2^d$ for each house above the horizon.[84] If we drop the resulting halves we obtain all of the values in the text except for the anomalous 280;30, which should perhaps be 280. As noted above, the value 273 is the number of days in ten months (actually 273^d 5^h). One could argue that one should reckon with 30 / 13;11 = 2^d 6.6^h per house, this being the time the moon moves through 30° of the ecliptic from the horoscopus, corresponding roughly (if we ignore effects of latitude and obliquity) to each house, but the text seems to use 12°/d for the lunar motion, corresponding to a month of 30^d and also roughly equal to the daily elongation of the moon from the sun. Several other Islamic tables for determining the length of gestation are available, but an investigation is beyond the scope of the present study.[85]

[82] See Lemay, 'Centiloquium', especially pp. 97-8. For some later sources see, for example, al-Bīrūnī, *Astrology*, pp. 329-31, and Kennedy, 'Kāshī on the Ascendent', pp. 140-1.

[83] Vernet, 'Tractat', p. 81.

[84] Ibid., pp. 91-4.

[85] See already the text to n. 39 above. In MS L, pp. 199-200, there are tables giving the number of days the fetus remains in the mother's womb (*ayyām makth al-janīn fī baṭn ummihi*). On pp. 204 and 216 of the same manuscript there is a 'table of the *namūdār* of conception' displaying the 'correction of the moon' (*taʿdīl al-qamar*) for the two situations where the moon is above and below the horizon. Future researchers should also investigate the tables

The connection between the various notes and the 'Table of Life' is immediately apparent. Firstly, in the more likely situation, we consider the case of a newborn for whom the longitude of the horoscopus λ_H and the longitude of the moon λ_M have been determined at the instant of birth. The position of the moon relative to the horizon enables us to find the length of gestation and hence the date of conception. The longitude of the moon at birth is the longitude of the horoscopus on the day of conception, and this enables us to compute the precise time of conception. Secondly, we consider the situation of a couple desirous of producing a child destined to enjoy long life. The second part of the rule in the first note means that the length of life is entirely determined by the lunar longitude at conception, for this can replace the longitude of the horoscopus at birth as argument in the 'Table of Life'. One should certainly be careful about computing the lunar longitude at that time, and rounding procedures are also all-important, because if λ_H is such that the entry in the table for ζ is 30, the entry for the next degree of longitude will be just 2. A popular Arab proverb states that *al-ʿajala mina 'l-Shayṭān—al-ta'annī mina 'l-Raḥmān*, 'Haste is from the Devil, procrastination is from (God) the Merciful'; in this particular case, *al-ta'annī* is not to be recommended. Or perhaps one would just need to get a better astrologer: a clever one could usually provide the kind of result his audience would want to hear.[86]

By considering the astrological house of the moon at the time of conception, the date of birth can be calculated using the information in the second note, and the precise time of birth is given by the first part of the rule in the first note: the lunar longitude at birth will be the same as the horoscopus at the time of conception. In either case, our material enables one to solve all of the astrological problems involved.

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associated with al-Sijzī (ca. 980) in MS Cairo ṬFF 18,3 (see *Cairo Survey*, no. B56-2.1.5), as well as those in the *Zījes* of al-Qāsim ibn Maḥfūẓ al-Baghdādī (1285), extant in MS Paris BNF ar. 2486, and Shams al-Dīn al-Wābkanawī al-Bukhārī (ca. 1320), extant in MS Istanbul Aya Sofya 2694.

[86] On the occasional necessity to fudge astronomical data in order to produce acceptable astrological predictions see North, *Horoscopes and History*, p. 52.

Appendix A: The namūdār *of Vettius Valens*

MS Berlin Ahlwardt 5751 (= L), p. 203—see Figure 3f

'The *namūdār* of Wālīs. If you want to rectify (the value of) the longitude of the horoscopus and you have doubts about the hour when the newborn was born, investigate in which quarter of the day and night he was born and determine [the time of day (!)] and (the approximate value of) the horoscopus by guessing (*al-takhmīn*) for that quadrant as I shall describe to you. You find the horoscopus and the degrees of the hours corresponding to the solar longitude for the time when he was born if the birth was during the day and corresponding to the opposite point of the ecliptic if the birth was during the night. When you have found it multiply by 12 and multiply the result by the number of hours you guessed. If the result is greater than 360 subtract 360s until the remainder is (less than 360): keep the result in mind; it is called the 'indicator of the horoscopus' (*dalīl al-ṭāliᶜ*). Then take the difference between the sun and moon in degrees of the ascensions and this is called the 'indicator of the sun' (*dalīl al-shams*). If the indicator of the sun is greater than the indicator of the horoscopus then you know that the hours are more than you guessed, and if the indicator of the horoscopus is greater than the indicator of the sun you know that they are less. If you want to know by how much they are greater or less then take the number of equal degrees between the sun and moon. If it is greater than 180 subtract 180 and divide the result by 12. Investigate what fraction the quotient is of 15, which is the amount of one equal hour (in degrees), and investigate whether it is one third, one quarter, one half (or whatever), and it will be the amount of the hour (*miqdār al-sāᶜa*); keep it in mind. If the indicator of the sun is greater than the indicator of the horoscopus, add your result to the hour you guessed, and if the indicator of the horoscopus is greater than the indicator of the sun, subtract it. The result after addition or subtraction will be the time when the newborn was born. Know this and peace (be upon you).

Find the horoscopus from (the time) [illegible word] by multiplying the hours at the birth by 15. The result will be the time since sunrise (in equatorial degrees). Add this to the (oblique) ascensions of the longitude of the sun by day or to the ascensions

of the opposite point of the ecliptic by night. The result will be the (oblique) ascensions of the horoscopus. We take the inverse oblique ascension, and the result in degrees and minutes will be the horoscopus. If we subtract 90° from the oblique ascensions of the horoscopus, the remainder will be the (right) ascensions of the tenth house (that is, upper midheaven): we take the inverse right ascensions and the result will be the longitude in degrees and minutes of the tenth house.'

Appendix B: Translation of the anonymous treatise on Vettius Valens' table

MSS Escorial ar. 961,5a (B)—see Figure 3a—and the derivative Cairo Ṭalʿat *mīqāt* 102,9 (C)

'An explanation of the *hashtaq* of Wālīs who compiled it without meanness and with no precociousness for finding the length of life. Wālīs, who is (also called) Wālinṭiyus (*W-'-l-x-ṭ-y-s*), devised a *namūdār* for correcting the longitude of the ascendant and the hour of birth so that no doubt remains (concerning their accuracy). We mentioned it and explained it in the chapter on *namūdārs* in this book of ours and we refrain from repeating this here.

(Wālīs also compiled) a table which he called the *hashtaq* starting with Libra, and he put 2 for the 1st degree of (this sign), 4 for the 2nd, 6 for the 3rd, 8 for the 4th, 10 for the 5th, and 12 for the 6th. Then he added 14 to these 12, and so the entry for the 7th degree was 26. Then he added 2 again for the 8th, and so for it the entry was 28, and then 30 for the 9th, 2 for the 10th, 4 for the 11th, and 6 for the 12th. Here he added 14 and so for the 13th the entry was 20, and then he started adding 2 degrees again so that the entry for the 14th degree was 22, for the 15th 24, for the 16th 26, for the 17th 28, and for the 18th 30. Here he added 14, and so for the 19th degree the entry was 14, for the 20th 16, for the 21st 18, for the 22nd 20, for the 23rd 22, and for the 24th 24. Here he added 14, and for the 25th the entry was 8, and then he started adding 2 degrees again so that for the 26th the entry was 10, for the 27th 12, for the 28th 14, for the 29th 16 , and for the 30th 18. He made Leo and Pisces the same. For the 1st degree of Scorpio (he put) 14 and added to this in

the same fashion, and he made Aries and Virgo the same. For the 1st degree of Sagittarius (he put) 26 and added to this in the same fashion, and he made Taurus the same. For the 1st degree of Capricorn (he put) 8 and added to this in the same fashion, and he made Gemini the same. For the 1st degree of Aquarius and also Cancer he put 20.

The way he increased was to add 2s, and if 5 degrees had passed, to add 14 and then to add 2s again until he had completed all of the signs [read: sign!]. The number of degrees of increase at the beginning of each sign is 6 [read: 26!], because the 1st degree has no contribution (*ishtirāk*) and the remaining (degrees) are contributing (*mushtarika*), and they are 5 degrees, (the value in the table) increasing by 14 between (each set) and the next (?*ukhtuhā*, lit., its sister). Wālīs also said that, if you add 12 to the entry for the 1st degree of each sign, you obtain the entry which must be for the 1st degree of the next sign. We checked this and it was correct. He did not mention the reason (*ʿilla*) for his procedures for obtaining these entries or for the increases which he applied.

Then (Wālīs described the following procedure) for determining the length of life of a newborn for whom 'the stars' have indicated that he will be healthy during the years of his early upbringing and that he will have a life (beyond childhood), if God Almighty wills. You take the degrees of the hours corresponding to the longitude of his ascendant in the locality where he was born, by day or night depending on the time of his birth, multiply these by 12 and obtain the product. Then investigate the entry in the *hashtaq* corresponding to an argument equal to the longitude of the ascendant, divide this by 60 and take this fraction of the product you obtained by multiplying the degrees of the hours by 12. The result will be the years and the fractional parts months, and this will be the length of his life, if God Almighty wills.

Example. The ascendant is Leo 7° and the corresponding degrees of the hours in Baghdad are 17;8°. When we multiply this by 12 the product is 205;36. In the *hashtaq* the entry for Leo 7° is 26, the ratio of which to 60 is as one third plus one tenth (to unity). If we take the share (of this fraction) of the 205 degrees and (the extra) minutes whose derivation we explained, the result is 89;5, which represents 89 years and one month.

Again suppose the ascendant is Cancer 6°. The degrees of its hours are 17;45 and if we multiply this by 12 the result is 213. (The entry) in the *hashtaq* is 30, the ratio of which to 60 is one half (to unity). If we take one half of the product we obtain 106 (years) and 6 months.

The procedure is the same as this in all other localities [reading *al-bilād* for *al-bāb*]: one (must) calculate with the (appropriate) degrees of the hours in each locality. We have compiled the *hashtaq* table for each degree of the signs (by the procedure) you know, without working out the years and months, because these change for (different) localities according to the variation in the degrees of the hours of daylight in each. In our opinion this chapter (or 'this method') should be helpful for finding the length of life. It should be compared with the indications of the *kadkhudā* and its supporters and the interruptions to life (*quṭūʿ*) necessitated by the *tasyīrāt* towards the bodies of the planets. (The truth ?) will become apparent by (comparing) one of these (methods) with the others, so that we can know through it what is correct of this noble science. God—may He be praised and exalted—knows best.'

Title of table: 'Table of life and the span of life for a newborn'.

Note to the left side of the table: '(The author) said that this table was found in the *Kitāb al-Kāmil fi 'l-nujūm*, 'The Perfect Book on Astronomy/Astrology', in the treasury of ʿAbdallāh ibn Ṭāhir ibn al-Ḥusayn, (sent to him) by al-Ma'mūn. (It was) written in gold.'

[On the other marginalia see Section 7.]

Appendix C: Shorter text to the 'Table of Life'

MS Cairo Dār al-Kutub *mīqāt* 1108,4b, fol. 20v (G), see Figure 3e.

'Praise be to God. This is the Table of life and the lifespan of a newborn as I found it in the *Kitāb al-Kāmil fi 'l-nujūm* in the treasury of ʿAbdallāh ibn Ṭāhir ibn al-Ḥusayn (sent to him) by al-Ma'mūn and written in gold. The way to use this table is (as follows). Take the degrees of the hours corresponding to the longitude of the ascendant, multiply them by 12, and keep (the product) in mind. Then look at the table of life and take

the entry opposite the argument equal to the longitude of the ascendant. Make what you find a ratio to 60 and take that ratio of the product you kept in mind. It will be the (length of) life of the newborn in years and months.

Example. A baby is born and has ascendant Gemini 5°. We take the corresponding entry in the table, which we find to be 16, which is one sixth plus one tenth of 60. The degrees of the hours corresponding to the longitude of the ascendant are 17;44. (When we) multiply (this) by 12 the result is 212;48°. One sixth plus one tenth of this is 56;45, which is 56 years and 9 months approximately. God knows best the Unseen.

(The following note) was found in the handwriting of Ibn Abi 'l-Fatḥ al-Ṣūfī. Take the diurnal arc corresponding to the ascendant and keep it in mind. Enter with the longitude of the ascendant in the table, multiply what you find by the quantity you kept in mind and divide the product by 60. The quotient will be years (of the individual's life). God knows best. This is as I found it, and as I found it I copied it.

Another example. The blessed child Muḥammad was born on Wednesday, 29 Shawwāl, 1006 (Hijra). The position (of the sun) was (Gemini) 13;13°, the time since sunrise 63;13°, the (oblique) ascensions (of the sun) 57;50°, the (oblique) ascensions of the ascendant 121;3°, and the ascendant Leo 9;20°. The (corresponding) value in the table is 30. The arc of daylight corresponding to the ascendant is 201;44°, and the product is 1,47;52, the first (digit) being 60s (*awwaluhu marfūʿ marra*) on the basis that the entries in the table are minutes. His lifespan according to what was related was six years, five months, and six days. God knows best. This is as I found it.'

Bibliography and bibliographical abbreviations

Berlin Catalogue: Wilhelm Ahlwardt, *Die Handschriften-Verzeichnisse der Koniglichen Bibliothek zu Berlin*, vol. 17: *Verzeichniss der arabischen Handschriften*, vol. 5, Berlin, 1893.

al-Bīrūnī, *Astrology*: R. Ramsay Wright, trans., *The Book of Instruction in the Elements of the Art of Astrology by ... al-Bīrūnī*, London, 1934.

——. *Chronology*: C. Edward Sachau, ed., *Chronologie orientalischer Volker von Alberuni*, Leipzig, 1923, repr. Baghdad: al-Muthannā, n.d., transl. as C. E. Sachau, *The Chronology of Ancient Nations—An English Version of the Arabic Text of the Athár-ul-bákiya of Albírúní, or 'Vestiges of the Past,' collected and reduced to writing by the author in AH 390-1, AD 1000*, London, 1879, repr. Frankfurt am Main, 1984.

——. see also Togan.

Bloom, 'The Blue Koran': Jonathan M. Bloom, 'The Blue Koran: An Early Fatimid Kufic Manuscript from the Maghrib', in *Les manuscripts du Moyen-Orient—Essais de codicologie et de paléographie. Actes du Colloque d'Istanbul (Istanbul, 26-29 mai 1986)*, ed. Francois Déroche, Istanbul and Paris, pp. 95-99 and pl. XIIIA.

A. Bouché-Leclercq, *L'astrologie grecque*, Paris, 1899.

Brockelmann, *GAL*: C. Brockelmann, *Geschichte der arabischen Litteratur*, 2nd edn., 2 vols, Leiden, 1943-49, and Supplementbände, 3 vols, Leiden, 1937-42.

Buck, *Dictionary*: C. D. Buck, *A Dictionary of Selected Synonyms in the Principal Indo-European Languages*, Chicago, Ill., 1949.

Cairo ENL Catalogue and Survey: David A. King, *A Catalogue of the Scientific Manuscripts in the Egyptian National Library* (in Arabic), 2 vols, Cairo, 1981 and 1986; and *idem, A Survey of the Scientific Manuscripts in the Egyptian National Library* (in English), Winona Lake, Ind., 1986.

Carmody, *Bibliography*: Francis Carmody, *Arabic Astronomical and Astrological Sciences in Latin Translation: A Critical Bibliography*, Berkeley and Los Angeles, 1956.

Centiloquium: see Lemay.

Chabás and Goldstein, '*Zīj* of Ibn al-Kammād': José Chabás and Bernard R. Goldstein, 'Andalusian Astronomy: *al-Zīj al-Muqtabis* of Ibn al-Kammād', *Archive for the History of Exact Science* 48 (1994), pp. 1-41.

——. *Zacut*: eidem, *Astronomy in the Iberian Peninsula: Abraham Zacut and the Transition from Manuscript to Print*, Transactions of the American Philosophical Society, 90.2, Philadelphia, 2000.

F. H. Cramer, *Astrology in Roman Law and Politics*, Memoirs of the American Philosophical Society, 37 (1954).

Cumont, *Astrology and Religion*: Franz Cumont, *Astrology and Religion among the Greeks and Romans*, first publ. 1912.

Dorotheos: see Pingree, *Dorotheus.*

EI$_2$: *The Encyclopedia of Islam*, new edn, 10 vols to date, Leiden, 1960- .

Elwell-Sutton, *Horoscope*: L. P. Elwell-Sutton, *The Horoscope of Asadullāh Mirzā—A Specimen of Nineteenth-Century Astrology*, Leiden, 1977.

Escorial Catalogue: H.-P.-J. Renaud, *Les manuscrits arabes de l'Escurial ...*, tome II, fasc. 3: *Sciences exactes et sciences occultes*, Paris, 1941.

Goldstein: see Chabás and Goldstein.

Ibn Hibintā, *al-Mughnī*: Ibn Hibintā, *al-Mughnī fī aḥkām al-nujūm*, 2 vols, Fuat Sezgin et al., eds, Frankfurt, 1987. [A facsimile of the Damascus and Munich manuscripts (MSS M and N)].

Ibn Nawbakht: see Labarta.

Jeffery, *Materials*: Arthur Jeffery, *Materials for the History of the Text of the Qur'ān—The old codices—The Kitāb al-Maṣāḥif of Ibn Abī Dāwūd together with a collection of variant readings*, Leiden, 1937.

Jones, *Astronomical Papyri from Oxyrhyncus*: Alexander Jones, *Astronomical Papyri from Oxyrhyncus*, 2 vols in 1, Philadelphia, Pa., 1999.

Kaufmann, 'R. Haja über die Lebensdauer': David Kaufmann, 'Ein Responsum des Go'ans R. Haja über Gottes Vorherwissen und über die Dauer des menschlichen Lebens', *Zeitschrift der Deutschen Morgenländischen Gesellschaft*, 49, 1895, pp. 73-84.

Kennedy, 'Kāshī on the Ascendent': Edward Stewart Kennedy, 'Treatise V of Kāshī's *Zīj*: Determination of the Ascendent', *ZGAIW*, 10, 1995-6, pp. 123-45.

——. 'Kāshī's Spherical Astronomy': idem, 'Spherical Astronomy in Kāshī's *Khāqānī Zīj*', *ZGAIW*, 2, 1990, pp. 1-46, reprinted in idem, *Studies*, VII.

——. *Studies*: idem, *Astronomy and Astrology in the Medieval Islamic World*, Variorum Collected Studies Series: CS600, Aldershot and Brookfield, Vt., 1998.

——. '*Zīj* Survey': idem, 'A Survey of Islamic Astronomical Tables', *Transactions of the American Philosophical Society*, n.s., 46.2, 1956, reprinted n.d. [ca. 1990].

Kennedy and Kennedy, *Islamic Geographical Coordinates*: idem and Mary Helen Kennedy, *Geographical Coordinates of Localities from Islamic Sources*, Frankfurt am Main, 1987.

Kennedy and King, 'Ibn al-Majdī's Auxiliary Tables': idem and David A. King, 'Ibn al-Majdī's Tables for Calculating Ephemerides', in *Journal for the History of Arabic Science* (Aleppo), 4, 1980, pp. 48-68, repr. in King, *Studies*, A-VI.

King, 'Astronomy in the Maghrib': David A. King, 'On the History of Astronomy in the Medieval Maghrib', in *Etudes Philosophiques et Sociologiques Dédiées à Jamal ed-Dine*

Alaoui, Université Sidi Mohamed Ben Abdallah, Publications de la Faculté des Lettres et des Sciences Humaines Dhar El Mahraz, Fès, N^0 Spécial 14 (Département de Philosophie, Sociologie et Psychologie), Fez, 1998 [published 1999], pp. 27-61.

——. 'Cairo Corpus': idem, 'Ibn Yūnus' *Very Useful Tables* for Reckoning Time by the Sun', *Archive for the History of Exact Science*, 10, 1973, pp. 342-94, repr. in idem, *Studies*, A-IX. [Superseded by idem, *SATMI*, II-4-5.]

——. 'Earliest Muslim Geodetic Measurements': idem, 'Too Many Cooks...—A Newly-Rediscovered Account of the First Islamic Geodetic Measurements', *Suhayl*, 1, 2000, pp. 71-99.

——. 'Geography of Astrolabes': idem, 'Bringing Astronomical Instruments Back to Earth: The Geographical Data on Medieval Astrolabes (to ca. 1100)', in *North Festschrift*, pp. 3-53.

——. 'al-Khwārizmī': idem, 'al-Khwārizmī and New Developments in Mathematical Astronomy in the Ninth Century', *Occasional Papers on the Near East* (New York University, Hagop Kevorkian Center for Near Eastern Studies) 2, 1983, 47 pp.

——. *SATMI*: idem, *Studies in Astronomical Timekeeping in Medieval Islam*, 12 parts, Leiden, in press, esp. I: *A Survey of Tables for Regulating Time by the Sun and Stars*, and II: *A Survey of Tables for Regulating the Times of Prayer.*

——. *Studies* A: idem, *Islamic Mathematical Astronomy*, London, 1986.

King and Samsó, 'Astronomical Handbooks and Tables': idem and Julio Samsó, with a contribution by Bernard R. Goldstein, 'Astronomical Handbooks and Tables from the Islamic World (750-1900): An Interim Report', *Suhayl*, 2, 2001, pp. 9-105. [A summary is in the article 'Zīdj' in EI_2.]

Kroll, *Vettius Valens*: Wilhelm Kroll ed., *Vettii Valentis anthologiarum libri*, Berlin, 1908.

Labarta, *Horóscopos históricos*: Ana Labarta, *Mūsā ibn Nawbajt: al-Kitāb al-Kāmil—Horóscopos históricos*, Madrid and Bellaterra, 1982.

Lane, *Lexicon*: Edward W. Lane, *An Arabic-English Lexicon*, 8 parts, London, 1863, reprinted Beirut, 1968.

Lemay, '*Centiloquium*': Richard Lemay, 'Origin and Success of the *Kitāb [al-]Thamara* of Abū Jaʿfar Aḥmad ibn Yūsuf ibn Ibrāhīm from the Tenth to the Seventeenth Century in the World of Islam and the Latin West', *Proceedings of the First International Symposium for the History of Arabic Science (Aleppo, 1976)*, 2 vols, Aleppo, 1978, II, pp. 91-107.

Levey, 'Medieval Arabic Bookmaking': Martin Levey, 'Mediaeval Arabic Bookmaking and its Relation to Early Chemistry and Pharmacology', *Transactions of the American Philosophical Society* (Philadelphia), n.s., 52:4, 1962.

Liddell and Scott, *Lexicon*: H. G. Liddell and R. Scott, *A Greek-English Lexicon*, 9th edn, Oxford, 1953.

Mestres, 'Hyderabad Manuscript': Angel Mestres, 'Maghribī Astronomy in the 13th Century: a Description of Manuscript Hyderabad Andra Pradesh State Library 298', in *Vernet Festschrift*, I, pp. 383-443.

——. Zīj *of Ibn Isḥāq*: idem, *Materials andalusins en el* Zīj *d'Ibn Isḥāq al-Tūnisī* (edited text and tables, with introduction and commentary in English), doctoral thesis, University of Barcelona, 2000.

Millás, *Estudios sobre Azarquiel*: José-María Millás Vallicrosa, *Estudios sobre Azarquiel*, Madrid and Granada, 1943-50.

Nallino, *ʿIlm al-falak*: Carlo Alfonso Nallino, *ʿIlm al-falak: ta'rīkhuhu ʿinda 'l-ʿArab fī 'l-qurūn al-wusṭā*, Rome, 1911, repr. Baghdad, ca. 1960.

——. *Scritti*, V-VI: idem, *Raccolta di scritti editi e inediti*, ed. Maria Nallino, vol. V: *Astrologia — Astronomia — Geografia*, and vol. VI: *Letteratura — Linguistica — Filosofia — Varia*, Rome, 1944 and 1948.

——. 'Tracce': idem, 'Tracce di opere greche giunte agli Arabi per trafila pehlevica', in *A Volume of Studies Presented to Professor E. G. Browne*, Cambridge, 1922, pp. 345-63, repr. in idem, *Scritti*, VI, pp. 285-303.

Neugebauer, *HAMA*: Otto Neugebauer, *A History of Ancient Mathematical Astronomy*, 3 vols, New York, etc., 1975.

Neugebauer and H. B. van Hoesen, *Greek Horoscopes*, Memoirs of the American Philosophical Society, 48, 1959.

Neugebauer and Saliba, 'Greek Numerology': idem and George Saliba, 'On Greek Numerology', *Centaurus*, 31, 1989, pp. 189-206.

North, *Horoscopes and History*: John D. North, *Horoscopes and History*, London, 1986.

North Festschrift: *Between Demonstration and Imagination: Essays in the History of Science and Philosophy Presented to John D. North*, eds. Arjo Vanderjagt and Lodi Nauta, Leiden, 1999.

Pauly-Wissowa, *Realencyclopädie*, 2-VIII,2: G. Wissowa et al., *Paulys Realencyclopädie der classischen Altertumswissenschaften*, 2. Reihe, Band VIII,2, Stuttgart, 1958.

Pingree, 'Astronomy and Astrology in Iran': David Pingree, 'Astronomy and Astrology in Iran', *Isis*, 54, 1963, pp. 229-46.

——. *Dorotheus*: idem, ed., *Dorotheus Sidonius carmen astrologicum*, Leipzig, 1976.

——. *Vettius Valens*: idem, ed., *Vettii Valentis Antiocheni anthologiarum libri novem*, Leipzig, 1986.

Renaud: see *Escorial Catalogue*.

Saliba: see Neugebauer and Saliba.

Samsó, 'Maghribī *Zīj*es': Julio Samsó, 'An Outline of the History of Maghribī *Zīj*es from the End of the Thirteenth Century', *Journal for the History of Astronomy*, 29, 1998, 93-102.

——. see also King and Samsó.

Sarton, *IHS*: George Sarton, *Introduction to the History of Science*, 5 vols in 3, Baltimore, 1927-48, reprinted 1962.

Sayılı, *The Observatory in Islam*: Aydin Sayılı, *The Observatory in Islam*, Publications of the Turkish Historical Society, Series VII, No. 38, Ankara, 1960, reprinted New York, 1981.

Sédillot-*fils*, *Oloug-Beg*: L. P. E. A. Sédillot, *Prolégomènes des tables astronomiques d'Oloug-Beg*, 2 vols, Paris, 1847-53.

Sezgin, *GAS*: Fuat Sezgin, *Geschichte des arabischen Schrifttums*, 9 vols to date, especially vol. 6: *Astronomie*, and vol. 7: *Astrologie, Meteorologie und Verwandtes*, Leiden, 1978 and 1979.

Steingass, *Dictionary*: F. Steingass, *A Comprehensive Persian-English Dictionary*, London, 1892, reprinted Beirut, 1975.

Suhayl = *Suhayl—Journal for the History of the Exact and Natural Sciences in Islamic Civilisation*, Barcelona, 2000-.

Suter, *MAA*: Heinrich Suter, 'Die Mathematiker und Astronomen der Araber und ihre Werke', *Abhandlungen zur Geschichte der mathematischen Wissenschaften*, 10, 1900, and 'Nachträge und Berichtigungen', ibid., 14, 1902, pp. 157-185, reprinted in idem, *Beiträge zur Geschichte der Mathematik und Astronomie im Islam*, 2 vols, Frankfurt, 1986, I, pp. 1-314.

Togan, 'al-Bīrūnī on the Length of Life': Zeki V. Togan, 'Muḥammad Ibn Aḥmad al-Bīrūnī: *Hikayat ṭariq ahl al-Hind fi istixrac al-'umr*', *Islām Tetkikleri Enstitüsü Dergisi* (Istanbul), 1, 1954, pp. 1-25 (appended with new pagination after p. 184 of the volume).

Ullmann, *NGWI*: Manfred Ullmann, *Die Natur- und Geheimwissenschaften im Islam*, Leiden, 1972.

Vernet, *Estudios*: Juan Vernet, *Estudios sobre historia de la ciencia medieval*, Barcelona, 1979.

———. 'Tractat': idem, 'Un tractat d'obstetricia astrologica', Boletín de la Real Academia de Buenas Letras de Barcelona, 22, 1949, pp. 69-96, reprinted in idem, *Estudios*, pp. 273-300.

Vernet Festschrift: *From Baghdad to Barcelona—Studies in the Islamic Exact Sciences in Honour of Prof. Juan Vernet*, eds Josep Casulleras and Julio Samsó, = *Anuari de Filologia*, 20, 1996, B-2, 2 vols, Barcelona, 1996.

Vettius Valens: see Kroll and Pingree.

ZGAIW: *Zeitschrift für Geschichte der arabisch-islamischen Wissenschaften* (Frankfurt am Main).

Weil, *Maimonides über die Lebensdauer*: Gotthold Weil, *Maimonides: Über die Lebensdauer – Ein unediertes Responsum*, Basel, 1953.

Magic Squares for Daily Life

JACQUES SESIANO

Introduction

The science of magic squares in Islamic civilization is in keeping with the history of Islamic mathematics. It appeared in the ninth century, developed over the tenth and eleventh, and began to decline in the thirteenth. From then on, magic squares (originally called *wafq al-aʿdād*, that is, 'harmonious disposition of the numbers') became increasingly put to magic purposes, due to the supposed capacities of some of them to create good or evil according to the wish of the user. Now this user did not have to be familiar with the general methods for constructing magic squares; all he needed was the knowledge of isolated squares and their properties. That is why the accompanying texts rarely if ever give the slightest hint as to how the squares depicted had been obtained. Europe came to know magic squares through such texts; this gave rise to their modern name and the first attempts to reconstitute their construction. In these magic texts translated into Latin, the magic squares of the first seven possible orders, from $n = 3$ to $n = 9$ (where n is the number of cells on each side), which are filled with the sequence of natural numbers, are associated with the seven planets then known. Therefore the magic attributes of these squares depended upon those of the planets and their efficacity upon the position, favourable or unfavourable, of the planets.

An example of such a text translated (or adapted) from an Arabic one of Moorish origin, occurs in MS Vienna, Österreichische Nationalbibliothek, cvp 5239, fols 147^{va}–149^{ra}, copied in the fourteenth century, an excellent reproduction of which is found in K. Nowotny's reprint of Cornelius Agrippa's *De occulta philosophia* (see his Appendix IX, pp. 665–6). This text, which we shall transcribe and translate here, has suffered from the carelessness of its copyist or his predecessors, so we have occasionally had to

modify the text, not only to correct some minor errors but also to remove repetitions or irrelevant glosses incorporated into the extant copy. (Angle brackets indicate the few additions of our own.) The correction was occasionally made easier by examining analogous passages occurring in later Latin writings, such as two works by the Jesuit Athanasius Kircher, who mentions such squares and their applications as examples of what he calls, depreciatingly, Saracenic and Hebrew cabbala (the amulets he found were often engraved with Hebrew characters). The translation of this Latin text will be followed by an explanation of the methods used to construct these squares.

Latin text

Incipiunt figure 7 planetarum.

Et scias quod in istis 7 figuris occultaverunt philosophi et sapientes antiqui 7 Dei nomina propter hoc quod nullus posset ea pronuntiare indigne, quia forte multi nescientes cum illis mala plurima perpetrarent. Unde videndum est de virtutibus earum.

Figura Saturni est quadrata, et sunt 3 multiplicata per 3, et sunt in quolibet latere 15 (Fig. 1). Cum volueris operari per eam ad mulierem laborantem in partu, scribe eam in panno novo de bombace in die Saturni et in hora eius cum Saturnus fuerit festinus cursu et auctus numero (148^{rb}) et lumine, et Luna sit velox, et sit Saturnus in bona dispositione cum Luna, et ligabis eam ad coxam mulieris dextram⟨; tunc parturiet felicissime⟩. Et si scripseris eam in lamina plumbi Saturno retrogrado, vel diminuto lumine et numero, et in mala dispositione ad Lunam, vel sub radiis vel sub stationario, et posueris eam in edificio novo vel plantatione nova, nunquam proficiet illud edificium et homines a plantatione fugient. Et si posueris eam in sede alicuius prelati, cito ab ea deponetur. Et si scripseris eam in lamina plumbi, Saturno in bono statu, et suffumigabis eam cum aluma, et portaveris tecum, non timebis regem neque personam aliquam que tibi nocere velit, et quod cuperis ab eis ferendum tibi observabis.

Figura Iovis quadrata est, et sunt 4^{or} multiplicata per 4⟨, et sunt in quolibet latere 34⟩ (Fig. 2). Cum volueris operari per eam, facies laminam argenteam in die et in hora Iovis, et sit Iupiter in bono statu; et in dicta lamina sculpes figuram et suffumigabis eam cum ligno aloes et cum ambra, et portabis tecum. Et diligent te

qui viderunt te et impetrabis ab eis quod quesieris. Et si posueris eam in repositorio mercatoris, mercatura eius augmentabitur. Et si posueris eam in columbario vel ubi sunt apes, congregabuntur ibi columbe et apes. Et si quis portaverit eam qui sit infortunatus, fortunabitur, de bono in melius proficiet. Et si posueris eam in sede prelati, prelatio eius durabitur et suos inimicos non timebit, sed proficiet apud eos.

Figura Martis infortunati significat bellum et graverias, et est figura quadrata, et sunt 5 multiplicata per 5, et sunt in quolibet latere 65 (Fig. 3). Cum volueris operari per eam, accipe laminam cupri in die et hora Martis cum Mars fuerit diminutus numero vel lumine, vel infortunatus et retrogradus (148^{ra}) vel in aliquo malo statu, et sculpes ipsam in lamina; et suffumigabis ipsam cum stercoribus murium vel murilegorum; et si posueris eam in edificio novo, non complebitur. Et si posueris eam in sede prelati, deteriorabitur cottidie et infortunabitur. Et si posueris eam in operatorio mercatoris, totum destruetur. Et si feceris predictam figuram ⟨cum⟩ nomine duorum mercatorum et subterraveris eam in domo unius illorum, cadet inter eos odium et inimicitia. Et si forte timorem de rege vel de aliquo magnato habueris, vel de inimicis, vel ad iudicium vel ante iudicem intrare volueris, sculpe hanc figuram ut supra dictum est, et sit Mars fortunatus, directus, auctus numero et lumine, et suffumiga eam cum dragma 1 de lapide corneola; et ponatur predicta lamina in panno serico rubeo, et portabis eam tecum; tunc vinces in iudicio et in bello adversarios tuos, et fugient a facie tua et timebunt te et reverebuntur. Et si posueris ⟨eam⟩ super crus mulieris, continuo fluxum sanguinis patietur. Et si scripseris eam in pergameno die et hora Martis et suffumigabis eam cum aristologia et eam posueris in loco apum, fugient omnes apes.

Figura Solis appropriata est regibus et principibus huius mundi. Et est figura quadrata, et sunt 6 multiplicata per 6, et ⟨est⟩ figura durationis totius regni, et sunt in quolibet latere 111 (Fig. 4). Cum volueris operari per eam, vide cum Sol fuerit in exaltatione sua, in 15 gradu Arietis, in mense Aprili; accipe 6 dragmas auri et fac inde laminam rotundam in die Solis et formabis in ea figuram Solis, et suffumiga eam cum croco, et lava eam cum aqua rosacea in qua fuerit muscus et ambra, et involves eam in panno serico croceo, et tenebis eam tecum continue; et eris fortunatus

in omnibus rebus et in petitionibus, et impetrabis ab omnibus quicquid quesieris, et ponet Deus benedictionem in manibus tuis ⟨et⟩ in omnibus rebus ⟨tuis⟩. In hoc est magnum secretum.

Figura Veneris est quadrata, et sunt 7 multiplicata per 7, et sunt in quolibet latere 175 (Fig. 5). Et est fortuna propria in facto mulierum et in venustate et de illis. Cum volueris operari per eam, vide quod sit Venus in Piscibus, quia ibi est exaltatio sua, vel in Thauro vel in Libra in quibus est domus eius, et quod sit fortunata, aucta lumine, velox cursu, directa; tunc accipe 7 dragmas argenti puri in die et hora Veneris, et fac laminam et sculpe in ea predictam figuram, et suffumiga eam cum ligno aloes et ambra et mastice, et involve eam in panno serico albo, et porta tecum; et omnes mulieres que viderunt ⟨te⟩ diligent te et videbis mirabilia. Et si vir vel mulier tardet se multum ad uxorandum, portet secum hanc figuram, et terminabitur eius negotium de maritatione et finem laudabilem consequetur. Et si uxor tua vel quelibet alia persona habet te odio, lava predictam laminam cum aqua pluviali, vel fontis vel roris, et da ei bibere, et diliget te et patietur quod quesieris. Et si coqueris camomillam cum aqua et laveris predictam figuram in aqua illa, et asperseris illam aquam in loco ubi est discordia vel in locis iudiciorum (148^{va}), cessabit omne malum et multiplicabitur omne bonum et sapientia. Et si asperseris aquam istam in locis ubi sunt mercature vel bestie, homines in illis mercaturis lucrabuntur, et multiplicabuntur et proficient illa que dicta sunt. Et si posueris figuram istam in columbario vel ⟨ubi⟩ sunt apes, multiplicabuntur ille res. Et si posueris eam in lecto, habundabis in coitu et diliget te uxor tua plurimum, et erit cito pregnans.

Figura Mercurii est quadrata, et sunt 8 multiplicata per 8, et sunt in quolibet latere 260 (Fig. 6). Mercurius est in motu velox, vel in mutatione velox, per vim et naturam habet inclinationem et complexionem aliarum planetarum, et habet complexionem similem signorum, et sibi data est anima naturalis et virtutes eius et sapientia et philosophia et totum quadrivium et scriptura. Cumque volueris operari per eam, accipe 8 dragmas puri argenti in die et hora Mercurii, et inde fac laminam, et figurabis in ea figuram Mercurii quando Mercurius fuerit fortunatus et directus, velox in cursu; suffumiga eam cum ligno aloes et gariofola et mastice. Et si non poteris habere argentum, scribe eam

in papiro citrino, et valebit tantum. Et porta eam tecum, et omnia que petieris impetrabis. Et si posueris eam in sede prelati, durabit illius prelatio, et multiplicabuntur bona illius. Et si imprimas et scribas istam figuram in annulo vel (148^{vb}) significaveris eam in scutella vitrea vel in vitro albo vel in pelvi ex latone in augmento lumine in die Mercurii mane secundum hoc in quinque primis diebus lunationis, vel septima die, et delaveris eam cum aqua fontis munda et bibes aquam istam per 3 dies, aufert a te omnem oblivionem et addisces de facili scientiam et omnia que volueris. Et si figuraveris eam in speculo calibeo et inspexit se in illo qui habet spasmum, curabitur, et ille qui est paraliticus, et ille qui amisit visum, vel intellectum per frequentiam, et e converso. Et si ieiunaveris per unum diem, vel melius per 3 dies continuos, quod ⟨est ut⟩ non commedas nisi panem cum melle vel cum uvis passis, et post 3 dies illos scripseris istam figuram in panno serico croceo vel citrino, et suffumiga eam cum ligno aloes sic dicendo 'Deus per virtutem istius figure ostende mihi talem rem', et ponas eam sub cussino vel sub capite tuo in nocte, tunc videbis in sompnis petitionem quam a Domino primitus postulasti.

Figura Lune est quadrata, et sunt 9 multiplicata per 9, et sunt in quolibet latere 369 (Fig. 7). Cum volueris per eam operari, accipe pergamenum virgineum et scribe in eo istam figuram in die et in hora Lune, et sit Luna in augmento, et scribe cum musco et croco distemperatis cum aqua rosacea, (149^{ra}) et suffumiga eam cum semine cucumerum et citrulorum et camphora; et postea duplica cartulam et mitte eam in canna facta de argento, et porta tecum. Et valet ad petendum res nobiles et pretiosas et honores, et aufert omne malum; et qui habuerit eam secum in via, non timebit latronem nec aliquam rem pessimam. Et quando volueris quod inimicus tuus vel alter exeat civitatem, scribe istam figuram in pergameno nigro ovis, et ex altera parte figuras figuram eius et scribas cum sanguine nigri galli decollati in die Lune quando Luna erit in diminutione; et cum illud feceris, mitte cartam illam in poculo novo parvulo et imple ipsum aqua, et dimitte ipsum per totam noctem ad stellas; sequenti die accipe urceolum illum, et dic 'Exeat etc., filius etc., ab ista civitate vel villa', et fiet quod volueris. Tunc effundas vel spergas aquam ex poculo contra 4 plagas mundi, et subterrabis poculum extra villam ubi volueris. Item si vis quod mulier nunquam nubat, scribe hanc

figuram in lamina plumbi in diminutione Lune in die Lune et ex altera parte figurabis figuram mulieris, et dic 'Virgo etc., filia etc., quod nunquam possit nubere', et subterrabis laminam in sepulcro ignoto. Et si mercimonia tua ⟨vel bestias⟩ vendere non possis, scribe ⟨eam⟩ in papiro Luna crescente et pone papirum plicatum in collo 1 bestie; ⟨vel⟩ scribe istam figuram in panno citrino in die et hora Lune cum croco distemperato cum aqua rosacea, et suffumigabis eam suffumigatione predicta, et duplicabis cartulam, et pone eam super res quas vendere velis vel in loco ubi sunt res; tunc cito ac prospere vendes eas.
Expliciunt figure 7 planetarum.

Translation.

The figures of the seven planets.

You are to know that in these seven figures the ancient philosophers and scholars have hidden the seven names of God, the reason being that nobody might pronounce them unworthily; for many ignorant persons may do much harm with them. So let us consider their qualities.

• The figure of Saturn is square, three by three, with 15 on each side (Fig. 1).

2	9	4
7	5	3
6	1	8

Figure 1

– If you wish to use it for a woman in labour during parturition: write it on a new piece of cotton on the day and the hour of Saturn when Saturn is moving rapidly and increasing in number and brightness, while the Moon is rapid and Saturn is in a favourable situation to the Moon; then you will bind this figure to the woman's right hip; this will grant a most successful delivery.
– If you write it on a lead plate while Saturn is retrograding, or decreasing in brightness and number, and is unfavourably situated to the Moon, or under the rays or stationary, and you place it in a new building or a new plantation, the building will never serve any purpose and people will flee the plantation.
– If you place it in the see of some prelate he will soon be removed from it.

– If you write it on a lead plate while Saturn is propitious and you fumigate it with alum and carry it with you, you will not fear the king nor any person who wishes to harm you, and you will see them bringing you anything you desire.

• The figure of Jupiter is square, four by four, with 34 on each side (Fig. 2).

– If you wish to operate with it: make a silver plate in the day and the hour of Jupiter, provided Jupiter is propitious, and engrave upon it the figure; you will fumigate it with aloes wood and amber. When you carry it with you, people who see you will love you and you will obtain from them whatever you request.

– If you place it in the store-house of a merchant, his trade will increase.

– If you place it in a dovecote or in a hive, a flock of birds or a swarm of bees will gather there.

– If someone unlucky carries it, he will prosper and be always more successful.

– If you place it in the see of a prelate, he will enjoy a long prelature and will not fear his enemies, but be successful among them.

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

Figure 2

14	10	1	22	18
20	11	7	3	24
21	17	13	9	5
2	23	19	15	6
8	4	25	16	12

Figure 3

• The figure of Mars when unfavourable means war and exactions. It is a square figure, five by five, with 65 on each side (Fig. 3).

– If you wish to operate with it, take a copper plate in the day and hour of Mars when Mars is decreasing in number and brightness, or malefic and retrograding, or in any way unfavourable, and engrave the plate with this figure; and you will fumigate it with the excrement of mice or cats; if you place it in a new building, it will never be completed.

– If you place it in the see of a prelate, he will suffer daily harm and misfortune.
– If you place it in the shop of a merchant, it will be wholly destroyed.
– If you make this plate with the names of two merchants and bury it in the house of one of them, hatred and hostility will come between them.
– If you happen to fear the king or some powerful person, or enemies, or have to appear before a judge or a court of justice, engrave this figure as said above when Mars is favourable, in direct motion, increasing in number and brightness; fumigate it with one drachma of carnelian stone; if you put this plate in a piece of red silk and carry it with you, you will win in court and against your enemies in war, for they will flee at the sight of you, fear you and treat you with deference.
– If you place it upon the leg of a woman, she will suffer from a continuous blood flow.
– If you write it on parchment on the day and the hour of Mars and fumigate it with birthwort and place it in a hive, the bees will all fly away.
• The figure of the Sun applies specifically to the kings and the princes of this world. It is a square figure, six by six, indicating the duration of a dynasty, with 111 on each side (Fig. 4).

1	32	34	3	35	6
30	8	27	28	11	7
20	24	15	16	13	23
19	17	21	22	18	14
10	26	12	9	29	25
31	4	2	33	5	36

Figure 4

– If you wish to operate with it, look for when the Sun is in its exaltation, in the 15th degree of Aries, during the month of April. Take six drachmas of gold, make with it a circular plate on the day of the Sun, and cast in it the figure of the Sun. Fumigate it

with saffron, and wash it with dew water containing musk and amber. Wrap it up in a piece of saffron-coloured silk, and keep it with you continuously. You will be lucky in everything and in your requests, and you will obtain from all whatever you ask. God will place his benediction in your hands and in everything belonging to you. In this figure lies a great secret.

• The figure of Venus is square, seven by seven, with 175 on each side (Fig. 5). It is particularly favourable in all matters concerning women, and brings luck in love.

4	35	10	41	16	47	22
29	11	42	17	48	23	5
12	36	18	49	24	6	30
37	19	43	25	7	31	13
20	44	26	1	32	14	38
45	27	2	33	8	39	21
28	3	34	9	40	15	46

Figure 5

– If you wish to operate with it, look for Venus to be in Pisces, because it has its exaltation there, or in Taurus or Libra, because its houses are there, and for Venus to be favourable, increasing in brightness, with a rapid and direct motion. Then take seven drachmas of pure silver on the day and the hour of Venus, make a plate and engrave in it the above figure. Fumigate it with aloes wood, amber and resin. Wrap it up in a piece of white silk, and carry it with you. All the women who see you will love you and you will see wonderful things.

– If a man or a woman is slow to marry, he should carry with him this plate; a marriage will be contracted and witness a happy end.

– If your wife or some other person hates you, wash the above plate with rain water, or water from a spring or the dew, and give it to them to drink; they will love you and let you have your own way.

– If you boil camomile with water and wash the above figure in this water, then sprinkle with it a place where there is discord, or a court of justice, any evil will cease, and good and wisdom increase.
– If you sprinkle this water in markets or where there are farm animals, men will make profit with their merchandise, they will increase and thrive.
– If you place this plate in a dovecote or a hive, they will multiply.
– If you place it in a bed, you will enjoy abundance of coitus, your wife will love you and become rapidly pregnant.
• The figure of Mercury is square, eight by eight, with 260 on each side (Fig. 6). Mercury has a rapid motion, changes rapidly, by its own power and nature has the tendency and the temperament of the other planets, and has a temperament similar to that of the signs; to it was given the natural soul and its virtues, and wisdom, philosophy, the whole quadrivium and writing.
– If you wish to operate with this figure, take eight drachmas of pure silver on the day and the hour of Mercury. Make a plate of it, where you will represent the figure of Mercury when Mercury is favourable and direct, rapid in motion. Fumigate it with aloes wood, clove and resin. But if you cannot have silver, write the figure on lemon-yellow paper; it will have the same effect. Carry it with you and you will obtain whatever you wish.

8	7	59	60	61	62	2	1
49	15	54	12	53	51	10	16
41	42	22	21	20	19	47	48
32	34	35	29	28	38	39	25
40	26	27	37	36	30	31	33
17	18	46	45	44	43	23	24
9	55	14	52	13	11	50	56
64	63	3	4	5	6	58	57

Figure 6

– If you place it in the see of a prelate, his prelature will last, and his goods will multiply.
– If you impress and mark this figure on a ring, or represent it on a glass dish, or on white glass, or in a brass cauldron, on the day of Mercury in the morning, when Mercury is increasing in brightness, and this during the first five days of the lunation, or the seventh day, and wash it off with clean spring water, and drink this water for three days, it will protect you from any forgetfulness, and you will learn with ease science or whatever you wish.
– If you represent it on a mirror of steel and someone having spasms looks at himself in it, he will be cured, so also whoever is paralysed, or whose sight or mind often fails, or conversely.
– If you fast for one day, or, better, for three consecutive days without eating anything but bread with honey or raisins, and write after these three days this figure on a piece of saffron or lemon-coloured silk, then fumigate it with aloes wood saying thus 'God, by the power of this figure, show me such a thing', and place it under your pillow or your head for the night, then you will see in your dreams what you had requested from God before.
• The figure of the Moon is square, nine by nine, with 369 on each side (Fig. 7).

37	78	29	70	21	62	13	54	5
6	38	79	30	71	22	63	14	46
47	7	39	80	31	72	23	55	15
16	48	8	40	81	32	64	24	56
57	17	49	9	41	73	33	65	25
26	58	18	50	1	42	74	34	66
67	27	59	10	51	2	43	75	35
36	68	19	60	11	52	3	44	76
77	28	69	20	61	12	53	4	45

Figure 7

– If you wish to operate with it, take a blank parchment, and write on it this figure on the day and the hour of the Moon while the Moon is increasing; you have to write with musk and saffron tempered with dew water. Fumigate it with cucumber and pumpkin seeds and camphor. Next, fold over this scrap and place it in a cane made of silver, and carry it with you. It will have the power to direct you towards noble or precious objects and honours, and it will remove any evil; whoever has it with him on his way will not fear any robber or misadventure.
– If you wish your enemy, or whoever, to leave the town, write this figure on a black sheep's skin and represent on the other side the person's picture, writing with the blood of a black cock beheaded on the day of the Moon when the Moon is decreasing; this being done, place it in a small new cup, fill this with water, and leave it for the whole night under the stars; on the next day, take this vessel and say: 'May X, son of Y, leave this town or village'. Then you must pour out or sprinkle the water from the cup towards the four cardinal directions, and you will bury the cup at any place outside the locality. What you wish will happen.
– Now if you wish a certain woman never to marry, write this figure on a lead plate when the Moon is decreasing on the day of the Moon, and represent on the other side the woman's picture, and say: 'May the virgin X, daughter of Y, never marry'. Bury the plate in an anonymous grave.
– If you cannot sell your merchandise or your farm animals, write this figure on paper when the Moon is increasing. Put the folded paper on the neck of one of the animals. Or write this figure on a piece of lemon-coloured cloth on the day and hour of the Moon, with saffron tempered with dew water, and fumigate it as indicated above. You will then fold this piece of paper and place it on the things you want to sell, or in the room where they are. You will sell them soon and with profit.

Construction of these squares

The general methods for constructing magic squares do not depend upon their size but upon the parity of their order: odd (3, 5, 7, ...), evenly-even (4, 8, 12, ...—thus even and divisible by 4), or oddly-even (6, 10, 14, ...—thus even but divisible by 2 only). A construction is possible for any order greater than 2. If the

squares are filled in with the sequence of natural numbers starting with 1, the magic sum, which is to be found in each row, each column and each of the two diagonals, is given by the formula

$$M_n = \frac{n(n^2+1)}{2}$$

where n represents the order of the considered square.

(a) Odd order squares.

Two general methods are seen to have been used in the construction of the squares presented.

First method (Fig. 8, here for $n = 7$). Put 1 in the cell below the central cell in the blank square. Then proceed with the next natural numbers from cell to cell diagonally downwards. When a side of the square is reached, continue by writing the next number on the opposite side, in the cell where we would be if the square were reproduced all around itself. After placing a quantity of numbers equal to the order n, we shall arrive at a cell already occupied. Move then two cells downwards vertically, remaining thus in the column where a number was last written. Resume diagonally from here: this will place the second set of n numbers. Do likewise for the following sets. It was by using such a method—the most common way to construct magic squares of odd order—that two squares appearing in our text were constructed (see Fig. 5 and 7; the diagonal movement is in the first case from right to left).

22	47	16	41	10	35	4
5	23	48	17	42	11	29
30	6	24	49	18	36	12
13	31	7	25	43	19	37
38	14	32	1	26	44	20
21	39	8	33	2	27	45
46	15	40	9	34	3	28

Figure 8

			1			
		15	9	3		
	29	23	17	11	5	
43	37	31	25	19	13	7
	45	39	33	27	21	
		47	41	35		
			49			

Figure 9

26	20	14	**1**	44	38	32
34	28	**15**	**9**	**3**	46	40
42	**29**	**23**	**17**	**11**	**5**	48
43	**37**	**31**	**25**	**19**	**13**	**7**
2	**45**	**39**	**33**	**27**	**21**	8
10	4	**47**	**41**	**35**	22	16
18	12	6	**49**	36	30	24

Figure 10

Second method. Imagine that the blank square contains a rhombus, the corners of which meet its sides in the middle. Fill the cells within this rhombus with the sequence of odd numbers taken in succession. The rhombus will then contain all the odd numbers to be placed in the square (Fig. 9). Write in the first even numbers opposite the first odd numbers, to form the hypotenuse of one of the two lower triangles (Fig. 10). Next, fill the hypotenuse of the other lower triangle by adding 1 to each of the odd numbers in the cells vertically above. Finish by adding successively to each of the two sets of even numbers already placed a quantity equal to the order of the square plus 1, and write the results in the sequence of cells below (moving to the top of the square when

the lower side of the square is reached). Such is the method used for the square of order 5 found in our Latin text (Fig. 3).

Both these methods were devised towards the end of the 10th century or at the beginning of the 11th in the Islamic world (Sesiano, *Traité médiéval*, pp. 32–40). The second received particular attention because of its separation of the numbers by parity. As a matter of fact, these two methods are closer than would seem at first sight. A relationship is already suggested by the presence of the same numbers in the rows and columns of two same-order squares thus constructed (comp. Fig. 8 & 10). An auxiliary device used for their construction in Arabic texts also points to such a relationship (Sesiano, HV I & III). Consider within the blank square a rhombus as before; but this time, make it a figure with the same number of cells as the square under consideration by connecting the opposite points of intersection of the rhombus's sides with the lines separating the rows and columns of the square (Fig. 11).

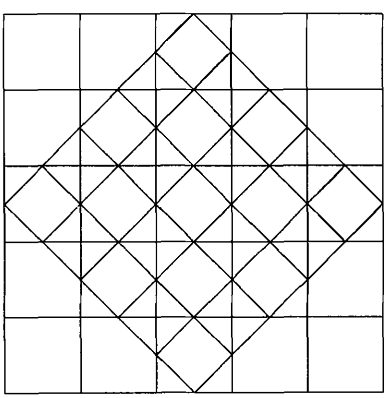


Figure 11

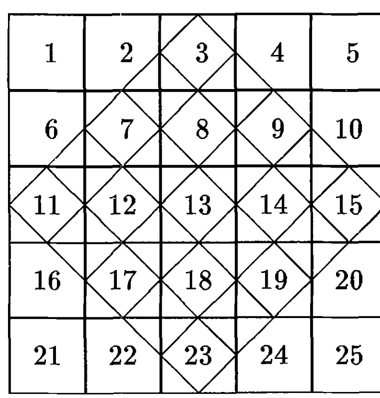


Figure 12

Then fill the square with the numbers in their natural sequence (Fig. 12). Move those in the four corner triangles towards the opposite sides of the rhombus (Fig. 13). They will thus find their place in the still empty cells of the rhombus, which will now be a magic square as if it had been constructed by the first method.

Suppose now you start with the natural sequence in the cells of the rhombus (Fig. 14). All the odd numbers then fall within cells of the square, leaving the even numbers on intersections. Considering then the rhombus to be divided in four triangles

meeting at its centre, move the even numbers in each into the empty corner triangles opposite (Fig. 15). This will make the proposed square magic, and arranged as if it had been constructed by the second method.

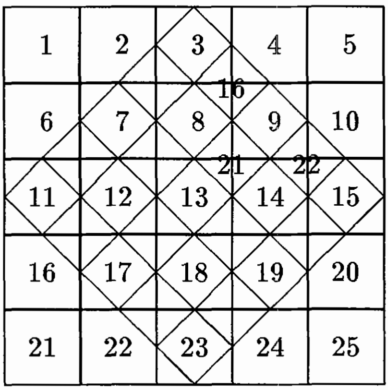


Figure 13

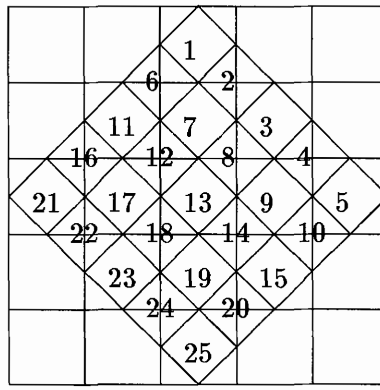


Figure 14

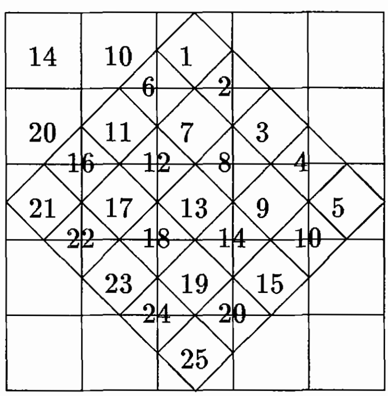


Figure 15

(*b*) Evenly-even order squares.

At this point, the reader may be somewhat disappointed to discover that the evenly-even order squares in our text (Fig. 2 and 6) were not constructed according to a general method — that is, a method applicable mechanically to any order of the same class — as was the case above. Nevertheless, the principles underlying their constructions *are* general. This may come as a surprise to anyone who already has some knowledge of magic squares, because the general methods of construction for evenly-even squares are so straightforward. But these methods, which are so easy to

apply, were not so easily invented; and did not appear until some time early in the 11th century, whereas the general principles for constructing squares of even order were already known around the mid 10th century: they are used by Abū'l-Wafā' for individual squares (Sesiano, 'Traité d'Abū'l-Wafā'', pp. 133-135) and are quite clearly stated by Ibn al-Haytham (Sesiano, HV I). This makes the squares in our text all the more interesting, for they are one of the few remnants surviving from this short interval between the discovery of the principles and the development of mechanically applicable methods.

8	7	6	5	4	3	2	1
16	15	14	13	12	11	10	9
24	23	22	21	20	19	18	17
32	31	30	29	28	27	26	25
40	39	38	37	36	35	34	33
48	47	46	45	44	43	42	41
56	55	54	53	52	51	50	49
64	63	62	61	60	59	58	57

Figure 16

Consider first, as we did with odd order squares, an even natural square, say of order 8 (Fig. 16, filled in from right to left, according to the Arabic way). Elementary mathematical considerations show that the excess in any lower row over the intended magic sum equals the deficit in the symmetrically located upper row, and the same holds for pairs of symmetrically located columns. Since the difference between corresponding elements in two such rows or columns is constant, we may obtain the magic sum in either case by merely interchanging half of their elements. Great care must be taken, however, in the process: it can only apply to elements in their original (natural) location; and elements in the main diagonals may not be moved as freely as the others, for our above consideration concerned the equalization of

rows and columns only. Ideally, the diagonal elements should not be touched at all: the diagonals of the natural square already give the magic sum. This is why the easy methods found later will leave them untouched. But this is not the case for one of the squares of our Latin text (Fig. 2). They have thus been constructed individually, but on the basis of the general principles.

Let us represent the interchanges with particular symbols: | will designate a vertical interchange between opposite rows in the natural square, – will designate a horizontal interchange between opposite columns, and X a diagonal interchange. To have the required number of interchanges, we must thus find, for a square of order n, bearing in mind that X performs the interchange for both row and column, altogether $\frac{n}{2}$ signs X and | in each row (the – being irrelevant here since the numbers remain in the same row); likewise, we must find altogether $\frac{n}{2}$ signs X and – in each column. In our cases, the arrangement is as shown in Fig. 17 and 18, the dots indicating where the corresponding numbers keep their original place.

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Figure 17

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Figure 18

Once this stage is reached, completing the magic square is an easy matter, for there is no need to perform the individual interchanges: it comes to the same thing if we count the cells from each corner and replace the relevant sign with the number reached. To that purpose, we choose a corner for the dots, count the cells and write in the numbers concerned. We do the same

from the opposite corner, but this time for the X. For the —, we start from the corner horizontally aligned with the corner of the dots, but for the | from the corner vertically aligned with that of the dots. Doing this with the above two figures produces the magic squares of the Latin manuscript (Fig. 2 and 6).

(*c*) The square of oddly-even order.

We are now left with the square of order 6 (Fig. 4). The comment will be brief: obviously, it has not been constructed according to the general principles of interchanges. On the one hand, this is not surprising since a general method for the oddly-even order had to await the end of the 11th century (Sesiano, HV III). On the other hand, constructions using the interchanges had, as for the evenly-even order, appeared earlier. Thus, a square of order 6 constructed by interchanges is already found in the treatise by Abū'l-Wafā'. Furthermore, the three types of interchange are seen in another square of order 6 found in Spain; it is found in a text, attributed to the astronomer al-Zarqālī, of a content similar to that of our Latin text (reproduction p. 660 of Nowotny's edition). This is the square described in our figures 19 and 20.

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Figure 19

6	32	3	34	35	1
7	11	27	28	8	30
19	14	16	15	23	24
18	20	22	21	17	13
25	29	10	9	26	12
36	5	33	4	2	31

Figure 20

Bibliography

Agrippa, Cornelius, *De occulta philosophia*, ed. K. A. Nowotny, Graz, 1967.

Kircher, Athanasius, *Arithmologia, sive de abditis numerorum mysteriis*, Rome, 1665.

——. *Œdipus ægyptiacus, hoc est universalis hieroglyphicæ doctrinæ instauratio*, Rome, 1652-4.

Sesiano, HV I: Jacques Sesiano, 'Herstellunsverfahren magischer Quadrate aus islamischer Zeit (I)', *Sudhoffs Archiv*, 64, 1980, pp. 187–96.

——. HV III: Jacques Sesiano, 'Herstellunsverfahren magischer Quadrate aus islamischer Zeit (III)', *Sudhoffs Archiv*, 79, 1995, pp. 193–226.

——. *Traité médiéval*: Jacques Sesiano, *Un traité médiéval sur les carrés magiques*, Lausanne, 1996.

——. 'Traité d'Abū'l-Wafā'': Jacques Sesiano, 'Le traité d'Abū'l-Wafā' sur les carrés magiques', *Zeitschrift für Geschichte der arabisch-islamischen Wissenschaften*, 12, 1998, pp. 121–244.

A Prognostication Based on the Conjunction of Saturn and Jupiter in 1166 [561 AH]

BERNARD R. GOLDSTEIN

1 *Introduction*

The theory of astrological history based on conjunctions of Saturn and Jupiter was already described by Māshā'allāh (d. ca. 815) and its roots lie in the Sasanian period.[1] This theory was applied by a number of Hebrew authors, including Abraham Bar Ḥiyya (d. ca. 1135) and Levi ben Gerson (d. 1344).[2] In the standard theory, a small conjunction takes place every 20 years indicating a change in the ruler; a middle conjunction takes place every 240 years (when the conjunction moves from one triplicity to another) indicating a change in dynasty; and a great conjunction takes place every 960 years when a cycle is completed, and the conjunction returns to Aries 0°. The conjunction of 1166 discussed in this prognostication is a small conjunction.

Abraham Ibn Ezra (d. 1167) considered the astrological theory of conjunctions in a number of places, notably in his *Sefer ha-ʿolam.*[3] It is therefore of some interest to see if he applied these rules to an actual conjunction. Indeed, a prognostication in Hebrew, based on the conjunction of Saturn and Jupiter in 1166 is ascribed to Ibn Ezra in the modern secondary literature. But it seems unlikely that he was the author, for the terminology in the prognostication is not consistent with Ibn Ezra's, and the planetary aspects and the astrological houses at the time of the conjunction are ignored, despite the rules given by Ibn Ezra (see

[1] See Kennedy and Pingree, *Astrological History*, p. vi; Yamamoto and Burnett, *Historical Astrology*, vol. I, pp. 582ff.

[2] Bar Ḥiyya, *Megillat ha-Megalleh*, eds. Poznanski and Guttmann; trans. Millàs, *Abraham Bar Hiia*; Goldstein and Pingree, *Levi ben Gerson's Prognostication.*

[3] Ibn Ezra, *Sefer ha-ʿolam*, ed. Fleischer. For a current bibliography, see Sela, 'Abraham Ibn Ezra's Scientific Corpus'.

Section 4, below). A personal horoscope for a native born in 1160 is also attributed to Ibn Ezra, but there are reasons to doubt that he was the author, some of which are mentioned by Fleischer.[4] These are the only horoscopes associated with Ibn Ezra.

It will be seen that the author's computations depend on the zij of al-Khwārizmī for the position of the conjunction in 1166 (see commentary *ad* § 3), and for the vernal equinox of 1166 (see commentary *ad* § 6). In the case of the eclipse of May 1, 1166, the planetary positions are given imprecisely, but they are consistent with computations according to the zij of al-Khwārizmī (see commentary *ad* § 7).

Jewish interest in astrological history was often associated with Messianic speculation; there are various hints of this in our text, although the term 'Messiah' does not occur in it (see §§ 5 and 9). However, the conjunction of 1166 is a small conjunction of Saturn and Jupiter and, according to the standard theory, should only mean a change in the ruler, not a major historical change such as the coming of a new prophet or the arrival of the Messiah.

The interpretation of this conjunction alludes to events associated with the Almohad invasion of Spain (1145-1148) that devastated the Jewish community in Muslim Spain (see Section 4, below), and it is likely that the author was a refugee from Muslim Spain residing in Aragon.

2 *Translation*

'Lift up your eyes and see: Who creates these? He who send out their host by count, Who calls them each by name: Because of His great might and vast power, Not one fails to appear' (Isa. 40.26). 'He reckoned the number of the stars; to each He gave its name' (Ps. 147.4).

§ 1. May the name of our God be blessed, the creator of the attendants (*mesharetim*) who serve Him, all standing in the high [place] of the universe and who make their voice heard in the glory of our God with the words of the living God, King of the universe. They are the attendants (*shammashim*) for the seven days of Creation — S[aturn], J[upiter], M[ars], S[un], V[enus], M[ercury], and the M[oon] — that move through the twelve houses [i.e.,

[4] Fleischer, 'Two Horoscopes', pp. 131ff.

the zodiacal signs] — A[ries], T[aurus], G[emini], C[ancer], L[eo], V[irgo], L[ibra], S[corpio], S[agittarius], C[apricorn], A[quarius], P[isces] — and, depending on them [i.e., their relative positions], there is good (*ṭov*) and evil (*muṭav*)[5] in every time and every moment for every event and every mishap; every dispute and every affliction; every pleasure and every honor; and the time of war and the time of peace.

§ 2. The two superior [planets], Saturn and Jupiter, are in conjunction (*mitḥabberim*) every 20 years at the same degree (*ḥeleq*) in one of the houses [i.e., zodiacal signs]; [at the end of] 20 years [they are] in the ninth zodiacal sign with respect to the [first] conjunction (*dibbuq*), and this is the rule until they complete 12 conjunctions in the 12 houses for each triplicity, and they are: A[ries], L[eo], S[agittarius] for the fiery [triplicity]; T[aurus], V[irgo], C[apricorn] for the earthy [triplicity]; G[emini], L[ibra], A[quarius] for the airy [triplicity]; and C[ancer], S[corpio], P[isces] for the watery [triplicity]. Then they return to their starting point at the head of Aries, and begin from the head of Aries until the end [of Pisces]; their influence is according to their [own] power and the [power due to] their conjunction (*hitḥabberam*) with the other five [planets, including the Sun and the Moon]. This is how they act, with the help of God, throughout time.

§ 3. The [following] prognostication (*ha-davar*) was [made] in the year [4]914 [AM = 1153-4] concerning a [forthcoming] conjunction:[6] on Monday, 1 Shawwāl of the months of the Muslims [*lit.* Ishmaʿʾel], in the year 561 [= Sunday, July 31, 1166] of the reign of the Madman [i.e., Muḥammad], may the name of the wicked rot (*shem reshaʿim yirqav*), the conjunction will take place in Capricorn 12° and a few minutes (*ḥalaqim*). They [i.e., Saturn and Jupiter] will be retrograde (*nezorim*), and Mars will enter this house near its end. Since, similarly, the conjunction will be retrograde, it is very strong and powerful. It is known that the house [i.e., zodiacal sign] at the time when the conjunction will take place is Capricorn which is the domicile of Saturn, and the exaltation (*kavod*) of Mars; hence, they are both strong in it. But this is not the case for Jupiter, for it is weak and infirm

[5] When *muṭav* occurs in an expression with *ṭov*, it is a euphemism for 'evil'; cf. Ben-Yehuda, *Hebrew Dictionary*, vol. III, pp. 1853f.

[6] For the syntax in this clause, cf. Jer. 25:1.

because it is located in the house of its dejection (*shiflut*); it has not strength like its two companions, for Saturn is in its domicile and Mars is in its exaltation.

§ 4. This [conjunction] indicates a consolidation (*tiqqun*) of the affairs of kings who have inherited their kingdom from their fathers and their fathers' fathers as a consequence of this conjunction. [By way of contrast,] every conspirator and traitor, not of royal seed, who rebels against them, will be weak and abased. This is a major indication concerning the Christian (*lit.* Edom) kingdom: their regency will be strengthened, whereas the strength of the conspirators of *al-Masmūd*[7] will diminish, their kingdom will fall and perish, and their alliance will be scattered. God will fall upon them with plague and sword, [with] panic [all] around, one man against another, one city against another, one kingdom against another; and they will [disappear] as if they had never existed. Also, strife (*qeṭaṭa*) will break out between *Kush* and *Qedar* — [in the land] which in Hebrew is called *Sefarad* [i.e., Spain] — [and will spread] in the east and in the west, and across the sea. And warfare (*ḥerum*)[8] will increase due to the influence (*ʿedut*)[9] of Mars in the house of the conjunction. And power, might, rule, and glory will be bestowed upon the knights (*parashim*), officials (*partemim*), warriors, and mighty men, by the kings and the rulers (*shaliṭim*) who will give them gifts to make them great and to exalt them everywhere. However, those who are educated in their [false] religion, who cling to their error, and who engage in hypocrisy with their smooth talk — they will be brought low, and will become a mockery and a disgrace.

§ 5. There is hope for our nation in these years of the conjunction: with the help of God, kings will honor them [our nation], elevate them, make them great, and join with them; perhaps, God will wish to give us hope for a destiny (*aḥarit*) better than this [i.e., our present situation]. For He knows future events and happenings as no one else, 'foretelling the end (*aḥarit*) from the

[7] For the term, al-Masmūd (or, more precisely, Maṣmūda), see Section 4, below.

[8] On this meaning of *ḥerum*, see Ben-Yehuda, *Hebrew Dictionary*, vol. III, p. 1749.

[9] The term *ʿedut* (evidence) is used by Bar Ḥiyya for astrological 'indication' or 'influence': cf. Bar Ḥiyya, *Megillat ha-megalleh*, eds. Poznanski and Guttmann, p. 153:29.

beginning' (Isa 46.10).

§ 6. As for what is seen of the ascendant in the first year of the conjunction: the Sun will enter the beginning of Aries in the middle of the 9th hour on Wednesday, the 20th day of Jumādā I, according to the reckoning of the Muslims [*lit.* Ishmaᶜ'el], in the year 568 [*sic*; read: 561],[10] or in the evening of March 23.[11] The ascendant then is Leo 18°, and the ruler of the house is the Sun, and the Sun is in the house of its exaltation.

§ 7. All this indicates the unfolding of the success of the king of Aragon[12] in that year, and the increase of his greatness. Since the Sun is eclipsed in Taurus, the domicile of Venus and the exaltation of the Moon, after noon when Saturn, Jupiter, and Mars are in the [same] triplicity as this house, the greatest among the princes of his kingdom will fall, and countries will make him tremble by means of a strong man who will rise by means of [the great man's] death and perishing, and by means of other men who will mock him both secretly and openly. Also, intense discord (*riḥuq ha-deᶜot*) will take place between one another, and conflicts in the Christian lands will become very numerous, as will warfare and the sword in many places. And events and happenings will come about in the world because of the strength of this conjunction and the eclipse in its triplicity, for plague will break out in the West and the people will feel a headache due to an increase in phlegm and black bile (*leḥa umerera sheḥora*), and many babies will die suddenly. Signs (*otot*) will be seen in the sky and comets in the air, and brightness will come forth from their paths and from behind shooting stars (*ziqot*). And there will be much trembling [i.e., earthquakes], winds and storms, and many houses will fall down. Some of the rulers will be lifted up, and some will be made low, according to the will of our God. And there will come about destruction of horses, cattle, and camels; birds, animals of the field, and some people will lose their progeny. Warfare and

[10] Both manuscripts read 568, but the astronomical data require that the year be emended to 561: see commentary ad § 6.

[11] This is the reading in both manuscripts: Q has 23 in alphabetic numerals; and Ox has twenty-three in words (despite Neubauer who transcribed 'twenty thirty' instead of 'twenty-three').

[12] Q: 'of the king of' (followed by a blank space); Ox: 'of the council of the king of Aragon' (but the syntax is faulty). Hence, I propose: 'of the king of Aragon.'

disruption of the roads will increase, and many ships will sink at sea. This king [will reappraise][13] his servants, banishing some of them, while relying [more heavily] on others. Even the beasts will not rest from fighting them and each other. Pregnant women will abort the fruit of their wombs.

§ 8. In the first years of the conjunction crops will increase. But in the fourth year there will be dearth and great famine in the lands of *Qedar* — they are the Berbers — and in the land of the West [i.e., the Maghrib]. Many will flee because of the sword, the famine, and the pestilence. Nevertheless, (re)construction (*ʿisqei*) of buildings will succeed, and the farmers will work the wildernesses and the desolate lands; they will rebuild the ruins, and will be very occupied with these matters.

§ 9. The people [of Israel?] will move from their place to other lands, and many will be occupied with other lands that they previously did not know. Many warriors will rest from war. The mighty ones will be made low, and the lowly will be raised up. Wonders will be seen in the East, and in the Land of Israel[14] new things will happen, as also in other lands. The summer will be made temperate, and the heat of the Sun will be milder. [On the other hand,] there will be intense cold in the winter. The king will muster his soldiers and marshal his armies to cross the sea to fight. This will bring about great sorrow for the people [of Israel?], and the tumult will increase; and the people will become mad[15] in many places for a limited time period. Thereafter, the hearts of the kings will return to the people, and the heart of the people to their lords. God will improve the lands, and the merchants will return with their merchandise; [holy] spirit will increase in the world, the faithful will return with deeds of righteousness for mankind, and there will be peace in the land.

§ 10. Blessed be God who knows future events, for there is none like Him. Blessed be the name of His glorious kingdom for

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[13] The translation is based on emending the text to read: *yimmalekh ha-melekh* (instead of: *ha-melekh*). For this meaning of *yimmalekh* (reconsider, reappraise), see Ben-Yehuda, *Hebrew Dictionary*, 4:3048f; cf. Neh. 5:7.

[14] The Land of Israel is often called 'the beautiful land' (*ereṣ ha-ṣevi*), as it is here: see Cohen, *Abraham Ibn Da'ud*, p. 134 n 4; cf. Dan. 11:16.

[15] Both manuscripts read *we-yitḥolelu*, but emending the text to *we-yitholelu* makes better sense; cf. Jer. 25:16 and 51:7. Kahana (*Qoveṣ Ḥokhmat ha-RAbE*, p. 118) introduced this emendation silently in his edition of the text.

ever and ever. Amen.

3 *Astronomical and Astrological Commentary*

Ad § 1. The prognostication begins with a general introduction on astrology, and alludes to Ps. 103:20-21 which has the term *mesharetim* (traditionally understood to refer to 'angels'). Ibn Ezra uses this term for 'planets' but, in the absence of any other occurrence in our text, it is not a sufficient reason to ascribe the text to Ibn Ezra.[16] The term *shammashim* for 'planets' is used by Bar Ḥiyya, as well as in the earlier treatises, *Baraita de-mazzalot*, chap. 7, and *Baraita di-Shmuel*, chap. 5.[17]

Ad § 2. In this section we are given some definitions of conjunctions of Saturn and Jupiter, and the association of the triplicities with the four elements.[18] The reference to conjunctions with the five other planets is probably to the 120 possible conjunctions of the seven planets (taken two, three, four, five, six, or seven at a time).[19] This number of possible planetary conjunctions goes back to the *Centiloquium*, § 50, attributed to Ptolemy in the Middle Ages.[20]

Ad § 3. Kahana[21] and, following him Barkai,[22] assumed that the year 4914 AM [1153-4] was intended to be the same as 561 AH [1165-6], and corrected the Hijra date to 548 AH to conform with the Jewish date, without paying attention to the fact that throughout 1153-4 Saturn and Jupiter were at least 100° apart! Since the date 561 AH for a conjunction of Saturn and Jupiter is secure astronomically (see below), a better solution is to take the date in the Jewish calendar as the time of composition of the

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[16] Cf. Ibn Ezra, *Ibn al-Muthannā's Commentary*, ed. B. R. Goldstein, p. 113 [Hebrew], MS Parma, 6a:14.

[17] Bar Ḥiyya, *Megillat ha-megalleh*, eds. Poznanski and Gutmann, pp. 11:22 and 23:29; Eisenstein, *Ozar Midrashim*, 2:280-285, 542-547; cf. Sarfatti, *'Barayta de-mazzalot'*, p. 73.

[18] For a list of the triplicities, see al-Bīrūnī, *Astrology*, ed. and trans. Wright, § 445.

[19] See, e.g., Ibn Ezra, *Sefer ha-ʿolam*, ed. Fleischer, p. 8; and Millás, *Las Tablas Astronómicas*, p. 87.

[20] See Sela, *Astrology in Ibn Ezra's Thought*, p. 81, for a discussion of the variants '119' and '120'.

[21] Kahana, *Qoveṣ Ḥokhmat ha-RAbE*, p. 139.

[22] Barkai 'L'astrologie juive médiévale', p. 341.

prognostication, in advance of the event, and this interpretation is consistent with the use of past and future tenses in the passage.[23] Calling the Hijra era 'years of the reign of the Madman' is highly polemical here, and not at all in the style of Ibn Ezra. But Muḥammad was often called the 'Madman' in medieval Hebrew literature.[24] On Sunday, July 31, 1166, the mean positions of Saturn and Jupiter, according to the zij of al-Khwārizmī, were Cap 12;51° and Cap 12;54°, respectively, and their true positions were Cap 4;46° and Cap 2;3°, respectively. So, there was a mean conjunction of Saturn and Jupiter at about Cap 12° (ignoring the minutes) on that day, and both planets were retrograde. The corresponding date in the Jewish calendar is Elul 1, 4926 AM and, in an anonymous Geniza fragment (C.U.L., T.-S. Ar. 29.45), there is a description of a conjunction of Saturn and Jupiter where the year is specified as 4926 AM and as 561 AH. But it is based on a different computation, for the date of the conjunction is given as Tuesday, Nisan 11, 4926, that corresponds to March 15, 1166. In another Geniza fragment in Arabic (C.U.L., T.-S. Ar. 51.55), ascribed to Abraham Ben Ezra on the first line of the text, a conjunction of Saturn and Jupiter is said to take place in 561 AH (with no further specification) in the last third of Capricorn (rather than in Cap 12°, as in our text). This location of the conjunction is based on a different computation, possibly that of al-Battānī whose zij yields a mean conjunction of these planets on August 23, 1166 at about Cap 24° which is in the last third of this zodiacal sign. Note that Ibn Ezra mentions the tables of al-Battānī in his *Sefer ha-ʿolam.*[25] To be sure, Ibn Ezra also knew the zij of al-Khwārizmī, for he translated a commentary on it.[26] One would expect a complete horoscope with the cusps of the twelve houses and the positions of all the planets, but they are not given in this prognostication. The domicile of Saturn is Capricorn, as stated in the text.[27] The term *kavod* corresponds to the Arabic *sharaf* ('exaltation'), and the exaltation of Mars

[23] I am grateful to G. Freudenthal for this suggestion.

[24] See, e.g., Maimonides, *Epistle to Yemen*, in Halkin and Hartman, *Epistles of Maimonides*, pp. 99 and 134 n. 48; cf. Steinschneider, *Polemische und apologetische Literatur*, pp. 302f.

[25] Ibn Ezra, *Sefer ha-ʿolam*, ed. Fleischer, p. 18.

[26] Ibn Ezra, *Ibn al-Muthannā's Commentary*, ed. B. R. Goldstein.

[27] See al-Bīrūnī, *Astrology*, ed. and trans. Wright, § 440.

is in the sign of Capricorn. The term *shiflut* corresponds to the Arabic *hubūṭ* (dejection), and the dejection of Jupiter is in the sign of Capricorn.[28]

Ad § 4. The strife between Kush and Qedar, mentioned in the text, is difficult to identify with a historical episode (see Section 4, below). Since Kush in the Bible is the name for Ethiopia and the text identifies Qedar with the Berbers (§ 8), it may be relevant to consider the following passage in *The Book of Religions and Dynasties* by Abū Maʿshar (d. 886): 'If [a conjunction of Saturn and Jupiter] happens in Capricorn, it indicates wars occurring between Ethiopia, Zanj, India, the Berbers, and the people of those regions. If the Sun and Mercury aspect them, this indicates illnesses destructive to kings, and that tumours, cutting, and burning <of the skin> come to them because of them, together with frequent winds, lightning, mists, fire, the lack of planets, and plenty of thieves.'[29]

Ad § 6. For the importance of the vernal equinox in this context, see, e.g., Ibn Ezra, *Sefer ha-ʿolam*: 'Doroneus [i.e., Dorotheus] the king said that he found in Enoch's book of secrets that he commanded: always consider the vernal equinox of the year (*tequfat ha-shana*) of the conjunction of Saturn and Jupiter — be it a great conjunction, a middle conjunction, or a small conjunction — and note the positions of the planets when the Sun enters Aries, ...'[30] According to modern tables, Jumādā I 20, 561 AH corresponds to Thursday, Mar. 24, 1166, whereas Jumādā I 20, 568 AH corresponds to Jan. 7, 1173. Hence, despite both manuscripts, the text should read 561 AH (The letters *aleph* and *ḥet* that are used for '1' and '8', respectively, are very similar in appearance in most medieval Hebrew scripts.) Since the two extant manuscripts come from places very distant from one another, it is likely that this corrupt reading entered the text at an early stage in its transmission. March 23, 1166 was a Wednesday and so Jumādā I 20, 561 AH corresponds to that date, as stated in the text. Modern tables of equivalences, based on the *mean* Hijra calendar, often give dates that differ by a day with respect to manuscript readings; in these cases the medieval manuscripts

[28] See al-Bīrūnī, *Astrology*, ed. and trans. Wright, § 443.

[29] Yamamoto and Burnett, *Historical Astrology*, vol. I, p. 135; cf. vol. II, p. 88.

[30] Ibn Ezra, *Sefer ha-ʿolam*, ed. Fleischer, p. 13.

are more reliable, particularly when the weekday is given. The middle (*lit.* half) of the 9th hour here is presumably counted from noon, for when the Sun is in Aries 0° and the ascendant is Leo 18°, the time is about 14;40h after sunrise: according to the table of right ascensions in the zij of al-Khwārizmī,[31] α(Ari 0°) = 0° and α(Leo 18°) = 140;32°; it follows that the time after sunrise is $360° - 140;32° = 219;28°$, corresponding to about 14;40h. On the day of the equinox the time from sunrise to noon is 6 hours, and so 14;40h after sunrise is equal to 8;40h after noon. Recomputation of the time of the equinox according to the zij of al-Khwārizmī yields March 23, 1166, about 7h after noon (in close agreement with our text); whereas recomputation with the zij of al-Battānī yields March 13, 1166 (close to the result using modern data: March 14, 1166). The Sun's domicile is Leo, and the sign of the Sun's exaltation is Aries.[32]

Ad § 7. The domicile of Venus is Taurus, and the sign of the Moon's exaltation is Taurus. There was a solar eclipse on May 1, 1166 (total in central Africa, but not noticeable in Spain), and true conjunction of the Sun and the Moon took place after noon (as stated in text): about 5h after noon according to al-Khwārizmī; about 3h after noon according to al-Battānī; and about 2h after noon according to recomputation with modern data. For the times of true conjunction, the computed true positions of the planets were approximately as follows:

	al-Khwārizmī	al-Battānī	Modern
Sun	Tau 7°	Tau 17°	Tau 17°
Saturn	Cap 10°	Cap 21°	Cap 23°
Jupiter	Cap 10°	Cap 20°	Cap 17°
Mars	Cap 12°	Cap 29°	Cap 24°

Capricorn and Taurus are both in the earthy triplicity; hence, the three outer planets were in the same triplicity as the Sun, as stated in the text. According to Ibn Ezra,[33] there is a difference of about 9° between 'tables of the scholars of India and the tables of the scholars of experience,' and in his introduction to Ibn al-Muthannā, he says that 'the positions of the planets

[31] Suter, *al-Khwārizmī*, pp. 171-3.
[32] See al-Bīrūnī, *Astrology*, ed. and trans. Wright, § 440 and § 443.
[33] Ibn Ezra, *Sefer ha-ʿolam*, ed. Fleischer, p. 9.

in the tables of al-Khwārizmī differ from the present positions by $9\frac{1}{2}^{\circ}$.'[34] It would seem that the tables of the scholars of India are represented by the zij of al-Khwārizmī, and the tables of the scholars of experience by the zij of al-Battānī (both zijes are cited by Ibn Ezra). The expression, 'scholars of experience,' is usually associated with astrologers,[35] but it is also possible that by 'the tables of the scholars of experience' Ibn Ezra intended to translate the expression, *al-zīj al-mumtaḥan* ('the zij tested by experience'), which was the title of a work by Yaḥyā ibn Abī Manṣūr (ninth century) who is mentioned by Ibn Ezra.[36] Saturn is associated with black bile and phlegm.[37] Moreover, Capricorn is associated with black bile (*ha-marra ha-sheḥora*).[38] The expression, *otot ba-shamayim* (signs in the sky, or heavens), is very close to the expression, *otot ha-shamayim* (signs of the heavens: cf. Jer. 10.2), in the title of Samuel Ibn Tibbon's Hebrew translation of Aristotle's *Meteorology* (completed in 1210). But I am not aware of this usage in Hebrew prior to Samuel Ibn Tibbon and, in his introduction, Samuel seems to emphasize his innovation.[39] For this reason, I assume that our author is referring to some unspecified, astrologically significant phenomena in the sky (cf. Babylonian Talmud, *Shabbat*, 156a). In the Middle Ages, 'meteors' (including comets) were taken to be sublunary. Note that the expression *ayāt samāwīya wa-zalāzil* (heavenly signs and earthquakes) occurs in the Arabic text of Abū Maʿshar's *Book of Religions and Dynasties*.[40]

4 *Authorship and Historical Context*

This prognostication was first published by A. Neubauer, based on the Oxford copy, and it was reprinted with a few notes by

[34] Ibn Ezra, *Ibn al-Muthannā's Commentary*, ed. B. R. Goldstein, p. 149.

[35] Sela, *Astrology in Ibn Ezra's Thought*, pp. 140-42.

[36] Ibn Ezra, *Ibn al-Muthannā's Commentary*, ed. B. R. Goldstein, p. 150.

[37] See al-Bīrūnī, *Astrology*, ed. and trans. Wright, § 420.

[38] See Ibn Ezra, *The Beginning of Wisdom*, trans. and ed. Levy and Cantera, p. xxx [Hebrew text].

[39] See Fontaine, *Otot ha-Shamayim*, pp. xiii f, 4-9.

[40] Yamamoto and Burnett, *Historical Astrology*, vol. I, p. 132 (line 802); cf. vol. II, p. 87 (line 803).

D. Kahana.[41] Since Kahana did not consult any manuscripts, I have not noted the divergences of his text from that of Neubauer. Both editors ascribe this prognostication to Abraham Ibn Ezra, mainly because it comes after Ibn Ezra's *Sefer ha-me'orot* in the Oxford manuscript. Moreover, in the Qafih manuscript — that has not been cited in the previous literature — it comes after Ibn Ezra's *Sefer mishpeṭei ʿolam* (= *Sefer ha-ʿolam*).[42] At the very least, this points to an association of our prognostication with Ibn Ezra by an early copyist. But the text itself does not identify its author and, as Fleischer noted, Ibn Ezra generally begins his treatises with 'Abraham said.'[43] Fleischer also offered a number of reasons to doubt the authorship of Ibn Ezra based on technical terminology and style. He noted that the terminology often differed from Ibn Ezra's usage elsewhere (e.g., *dibbuq* in our text is used for conjunction, but not by Ibn Ezra) and, in some cases, he argued that the grammatical usage was contrary to that of Ibn Ezra. In an earlier article, Fleischer recognized that the Almohad invasion was the subject of the prognostication, but he did not know the identity of al-Masmud (and at that time he accepted the claim that Ibn Ezra was the author).[44] Barkai correctly identified al-Masmud as a slightly corrupt form of Maṣmūda (see below); he also took the author to be Ibn Ezra, despite his allusion to Fleischer's doubts.[45] In fact, there is no proper study of Hebrew astronomical and astrological terminology comparable to the account of Hebrew mathematical terminology.[46] But a comparison of a few technical terms in this text with the corresponding terms used by Abraham Bar Ḥiyya and Abraham Ibn Ezra (the most prominent Hebrew authors who wrote on astrology in the twelfth century) casts doubt on the claim that Ibn Ezra is the author of our text. To be sure, Bar Ḥiyya was no longer alive when this prognostication was written, but we need to consider the possibility that the author was a disciple of his, particularly since the author seems to have resided in Aragon (Bar Ḥiyya lived

[41] Neubauer, 'Ibn Ezra-Literatur'; Kahana, *Qoveṣ Ḥokhmat ha-RAbE*, pp. 115-18 and 139.

[42] Cf. Sela, *Astrology in Ibn Ezra's Thought*, p. 376.

[43] Fleischer, 'Two Horoscopes', pp. 130-31.

[44] Fleischer, 'Ibn Ezra in France', pp. 291f.

[45] Barkai 'L'astrologie juive médiévale', pp. 338 and 341-2.

[46] Sarfatti, *Mathematical Terminology*.

in Barcelona). Ibn Ezra's expression for 'indicates' (astrologically) is *yoreh ʿal*;[47] Bar Ḥiyya's is *le-haʿid ʿal* or *ʿedut ʿal*;[48] but here we find *yokhiaḥ ʿal*. Moreover, for 'planet' Ibn Ezra uses *mesharet*[49] or *kokhav raṣ*,[50] whereas Bar Ḥiyya generally uses *kokhav navukh*, and occasionally *shammash* (see commentary *ad* § 1).[51] For 'eclipse' Ibn Ezra uses *qadrut*, whereas Bar Ḥiyya uses *laqut*; here we find *laqut*. Again, for 'conjunction' Ibn Ezra uses *maḥberet*, whereas Bar Ḥiyya uses *dibbuq*; here we find both.[52] It is surprising that the astrological term 'aspect' (*mabaṭ*) does not occur in our text, whereas it is frequently used by Ibn Ezra and would have been most appropriate here.[53] Indeed, aspects play an important role in the interpretation of a horoscope.[54] There is, however, one term that points specifically to Ibn Ezra's influence: *shalishut* (triplicity) occurs three times in MS Q (§§ 2 and 7) and twice in MS Ox (§ 7), whereas an older term, *ṭrigon*, taken from the Greek, is used once in MS Ox (§ 2). The term *ṭrigon* already occurs in the *Baraita de-mazzalot*,[55] whereas it seems that *shalishut* first appears in Ibn Ezra's *Sefer Mishpeṭei ha-mazzalot* in 1146,[56] and then frequently in his other astrological treatises. Since it seems unlikely that a copyist would change *ṭrigon* to *shalishut*, it is fair to conclude that the author of this prognostication was probably an early reader of Ibn Ezra's works. On balance, these considerations argue against Ibn Ezra's authorship, as Fleischer had already suggested. The terminology is also inconsistent with that of Abraham Bar Ḥiyya, and so it seems unlikely that the author was his direct disciple. Moreover, it is difficult to as-

[47] Ibn Ezra, *Sefer ha-ʿolam*, ed. Fleischer, p. 15:4.

[48] Bar Ḥiyya, *Megillat ha-megalleh*, eds. Poznanski and Guttmann, pp. 149:9 and 153:29.

[49] Ibn Ezra, *Ibn al-Muthannā's Commentary*, ed. B. R. Goldstein, MS Parma, 6a:14.

[50] Ibn Ezra, *Sefer Keli neḥoshet*, ed. Edelmann, p. 24:6.

[51] Bar Ḥiyya, *Libro del calculo*, ed. Millás, p. 98:1 [Hebrew Text].

[52] Cf. Sarfatti, *Mathematical Terminology*, p. 148.

[53] See, e.g., Ibn Ezra, *Keli neḥoshet*, ed. Edelmann, pp. 29-30.

[54] Cf. Ibn Ezra, *Sefer ha-ʿolam*, ed. Fleischer, p. 12. The astrological aspects are: conjunction, sextile, quartile, trine, and opposition, corresponding to distances between planets (including the Sun and the Moon) of 0°, 60°, 90°, 120°, and 180°, respectively.

[55] Sarfatti, '*Barayta de-mazzalot*', p. 71.

[56] Paris, Bibiotheque nationale de France, MS Heb. 1058, f. 15a: I am most grateful to Shlomo Sela for this reference.

sess the relationship between our text and the Geniza fragment ascribed to Ibn Ezra (see commentary *ad* § 3). Although it frequently happened that Ibn Ezra composed several versions of the same text, they are usually closer to each other than is the case here. Therefore, taking both texts at face value, the author of our text is not the same as the author of the text preserved in the Geniza fragment. The historical allusions in the text may help to locate its context. The term *al-masmūd* (§ 4) would seem to be the best clue, for it is a slightly corrupt form of Maṣmūda, one of the principal Berber ethnic groups in North Africa. Other emendations considered by Fleischer are not very convincing.[57] Similarly, the allusion to the Berbers (§§ 4 and 8) probably refers to the Almohads, and Barkai suggests that the term 'conspirators' applied to the Almohads means that the horoscope was written relatively soon after their invasion of Spain.[58] Ibn Tumart, the founder of the Almohads, died in 1130, and North Africa and Spain came under the control of this sect during the reign of his successor, ʿAbd al-Mu'min (d. 1163) whose army consisted of troops of the Maṣmūda and Zanāta tribes. A key victory took place in 1147 when the Almoravid capital, Marrakesh, fell to the Almohads. The conquest of al-Andalus (Muslim Spain) by the Almohads began in 1145 and culminated with the surrender of the Almoravid governor of Granada in 1156.[59] ʿAbd al-Mu'min unleashed a wave of persecutions against the Jews of al-Andalus in the first phase of his conquests, 1145-1148, offering them a choice of conversion or death. During the reign of his son, Abū Yaʿqūb Yūsuf (d. 1184), the Almohads continued to have a major military involvement in Spain.[60] A plausible conjecture is that *Kush* (§ 4) refers to the Almoravids, and *Qedar* to the Almohads, for the two groups did fight for control of al-Andalus (as alluded to in our text). The Almoravid sect began about 1040 in what is now southern Mauritania with predominantly Berber, as well as some Black African, adherents; later they ruled North Africa and al-Andalus until displaced by the Almohads.[61] Biblical names for tribes and countries were often given new meanings

[57] Fleischer, 'Two Horoscopes', p. 130, n. 8.
[58] Barkai 'L'astrologie juive médiévale', p. 342, n. 73.
[59] Huici Miranda, 'Gharnāṭa'.
[60] Shatzmiller, 'Al-Muwaḥḥidūn'.
[61] Norris, 'al-Murābiṭūn'; see also Lourie, 'Black Women Warriors', p. 199.

by medieval Hebrew authors, but the identification is not always obvious. For example, Judah Halevi uses the terms Philistines, Hittites, and descendants of Hagar in his poem, 'The Poet is urged to Remain in Spain'. Whereas it is argued that Philistines means the Berbers, and descendants of Hagar means the Arabs, it is not clear what is meant by Hittites.[62] The identity of *Kush* (Biblical Ethiopia) in our text is not specified, whereas *Qedar* (cf. Gen 25.13) is identified with the Berbers (see § 8). On the other hand, Maimonides, in his *Epistle to Yemen*, seems to identify *Qedar* with the Quraysh (the tribe of Muḥammad).[63] Moreover, the reference (§ 4) to 'a consolidation of the affairs of kings who have inherited their kingdom from their fathers and their fathers' fathers' seems to fit best with the Christians, for neither the Almoravids nor the Almohads had been in Spain very long. The next passage indicates that the author anticipated a defeat of the Almohads by the Christians, a plausible response to the relatively benign treatment of Jews by the Christians kings at the time: 'This is a major indication concerning the Christian (*lit.* Edom) kingdom: their regency will be strengthened, whereas the strength of the conspirators of *al-Masmūd* [i.e., the Almohads] will diminish, their kingdom will fall and perish, and their alliance will be scattered.' The reference to the King of Aragon (§ 7) suggests that the author probably lived in Aragon. Fleischer tried to find the name of this king in the text (allowing for an emendation), without success.[64] In sum, the author of this text was probably a Jew living in Aragon who may have been a refugee from al-Andalus as a result of the Almohad invasion.

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[62] D. Goldstein, *Hebrew Poems from Spain*, p. 166.

[63] See Halkin and Hartman, *Epistles of Maimonides*, p. 147, n. 250.

[64] Fleischer, 'Ibn Ezra in France', p. 292; and again in 'Two Horoscopes', p. 130, n. 8.

computations of planetary positions (using a program written by P. Huber for positions according to modern theory, and using a program written by E. S. Kennedy and H. Mielgo for positions according to the zijes of al-Khwārizmī and al-Battānī).

Bibliography

Barkai, R. 1987: 'L'astrologie juive médiévale: aspects théoriques et pratiques', *Le Moyen Age*, 93, pp. 323-48.

Ben-Yehuda, E. 1960: *A Complete Dictionary of Ancient and Modern Hebrew*, New York and London.

Cohen, G. D. 1967: *Abraham Ibn Da'ud: The Book of Tradition*, Philadelphia.

Edelmann, H., ed., 1845: Ṣ[evi] H[irsch] Ḥ[en] Ṭ[ov], *Abraham Ibn Ezra: Keli neḥoshet*. Konigsberg. Reprint edn.: Maqor (Jerusalem, 5730 [1970]).

Eisenstein, J. D. 1915: *Ozar Midrashim*, New York, reprinted Tel Aviv (?) 5729 [1969].

Fleischer, J. L. 5692 [1932]: 'R. Abraham Ibn Ezra in France', *Mizraḥ uMaʿarav*, 5, pp. 289-300 [in Hebrew].

——. ed., 1937: *Abraham Ibn Ezra: Sefer ha-ʿolam*, in Otzar ha-Ḥayyim, vol. 13, Homonna [Czechoslovakia], reprinted Maqor (Jerusalem), 5730 [1970].

——. 5710 [1950]: 'Two Horoscopes ascribed to R. Abraham Ibn Ezra', in *Minḥa li-Yhuda*, eds. S. Assaf et al., Jerusalem, pp. 129-40 [in Hebrew].

Fontaine, R. 1995: *Otot ha-Shamayim: Samuel Ibn Tibbon's Hebrew Version of Aristotle's Meteorology*, Leiden.

Goldstein, B. R. 1967: *Ibn al-Muthannā's Commentary on the Astronomical Tables of al-Khwārizmī. Two Hebrew versions*, edited and translated, with an astronomical commentary, New Haven and London.

Goldstein, B. R., and D. Pingree 1990: *Levi ben Gerson's Prognostication for the Conjunction of 1345*. Transactions of the American Philosophical Society, 80.6, Philadelphia.

Goldstein, D. 1966: *Hebrew Poems from Spain*, New York.

Halkin, A., and D. Hartmann 1985. *Crisis and Leadership: Epistles of Maimonides*, Philadelphia.

Huici Miranda, A.. 1960–: 'Gharnāṭa', in *Encyclopaedia of Islam*, 2nd edn, II, pp. 1012-14.

Kahana, D. 1894: *R. Abraham Ibn Ezra: Qoveṣ Ḥokhmat ha-RAbE*, vol. II, Warsaw.

Kennedy, E. S., and D. Pingree 1971: *The Astrological History of Māshā'allāh*, Cambridge (Mass.).

Levy, R., and F. Cantera, ed. and trans., 1939: *The Beginning of Wisdom, an astrological treatise by Abraham ibn Ezra*, Baltimore.

Lourie, E. 2000: 'Black Women Warriors in the Muslim Army Besieging Valencia and the Cid's Victory: A Problem of Interpretation', *Traditio*, 55, pp. 181-209.

Millàs, J. M., trans., 1929: Abraham Bar Hiia, *Meguillat Hamegallè*, Biblioteca Hebraico-Catalana, I, Barcelona.

——. ed. and trans., 1959: *La obra Séfer Ḥeshbón mahlekot ha-kokabim de R. Abraham Bar Ḥiyya ha-Bargeloni*, Barcelona.

——. 1962: *Las Tablas Astronómicas del Rey Don Pedro el Ceremonioso* Madrid–Barcelona.

Neubauer, A. 1881-2: 'Mittheilungen zur Ibn Ezra-Literatur', *Israelietische Letterbode*, 7, pp. 96-9.

Norris, H. T. 1960–: 'al-Murābiṭūn', in *Encyclopaedia of Islam*, 2nd edn, VII, pp. 583-9.

Poznanski, A., and J. Guttmann 1924: *Sefer Megillat ha-megalle von Abraham bar Chija*, Berlin.

Sarfatti, G. B. 1965: 'An Introduction to 'Barayta de-mazzalot'', *Annual of Bar Ilan University*, 3, pp. 56-82 [in Hebrew].

——. 1968: *Mathematical Terminology in Hebrew Scientific Literature of the Middle Ages*, Jerusalem [in Hebrew].

Sela, S. 1999: *Astrology and Biblical Exegesis in Abraham Ibn Ezra's Thought*, Ramat Gan. [in Hebrew]

——. 2001. 'Abraham Ibn Ezra's Scientific Corpus: Basic Constituents and General Characterization', *Arabic Sciences and Philosophy*, 11, pp. 91-149.

Shatzmiller, M. 1960–: 'Al-Muwaḥḥidūn', in *Encyclopaedia of Islam*, 2nd edn, VII, pp. 801-7.

Steinschneider, M. 1877: *Polemische und apologetische Literatur in arabischer Sprache*, Leipzig.

Suter, H. 1914: *Die astronomischen Tafeln des Muḥammad ibn Mūsā al-Khwārizmī*, Copenhagen.

Wright, R. R., ed. and trans., 1934: *The Book of Instruction in the Elements of the Art of Astrology by al-Bīrūnī*, London.

Yamamoto, K. and C. Burnett 2000: *Abū Maʿšar: On Historical Astrology*, 2 vols, Leiden.

Appendix: The Hebrew Text

There are two copies of this prognostication: Oxford, Bodleian Library, MS Opp. add. 4° 160, ff. 126b-128a [Ox]; and Jerusalem, Qafih, MS 36, ff. 164b-165b [Q], corresponding to films 22230 and 47427, respectively, in the Institute for Microfilmed Hebrew Manuscripts at the National Library in Jerusalem. I have depended on photocopies of the films in Jerusalem; the Oxford copy is not entirely clear and, in cases of doubt, I have accepted the readings in Neubauer's edition. The Qafih manuscript was copied in 5640 AM [= 1880] in Yemen from a manuscript of unstated age. Paragraphs and punctualtion have been added for ease in reading the text. Variants in spelling have not been noted.

Note added in proof: I have recently been informed by Renate Smithuis, who is completing her doctoral dissertation at the University of Manchester, that additional copies of this text may be found in MSS Cambridge, UL, Add. 1517, f. 47a-b, and Paris, BnF, Heb. 1057, ff. 94a-95a, and perhaps in other manuscripts as well.

א: כ״י אוקספורד; ק: כ״י קאפח.

‹א 126ב; ק 164ב› ״שאו מרום עיניכם וראו מי ברא אלה המוציא במספר צבאם לכלם בשם יקרא מרוב אונים ואמיץ כח איש לא נעדר״ (ישעי מ:כו). וכתוב ״מונה מספר לככבים לכלם שמות יקרא״ (תהלי קמז:ד).

‹1› יתברך שם אלהינו יוצר משרתים ואשר משרתיו כלם עומדים ברום עולם ומשמיעים בכבוד אלהינו קול בדברי אלהים[1] חיים ומלך העולם. הם שמשים לז׳ ימי בראשית שצ״ם חנכ״ל ההולכים בי״ב בתים בטש״ת סא״ב מע״ק גד״ד. ועל פיהם הטוב והמוטב בכל עת ובכל רגע וכל מקרה וכל פגע וכל ריב וכל נגע וכל ‹ק 165א› עונג וכל כבוד ועת מלחמה ועת שלום.

‹2› ושנים העליונים שהם שבתי וצדק מתחברים מכ׳ שנה לכ׳ שנה בחלק א׳ מן הבתים ולראש כ׳ שנה במזל התשיעי לדבוק ההוא וכן משפטם עד שישלמו הי״ב בתים לי״ב דבוקים בכל שלישות ושלישות[2] שהם טא״ק לאש שב״ג לעפר תמ״ד לרוח סע״ד למים. ואחרי כן חוזרים לתחלתם לראש מזל טלה ומתחילין מראש מזל טלה עד תכליתם וכפי כחם והתחברם עם האחרים החמשה. כן יהיה מעשיהם בע״ה כל הימים.

‹3› הדבר אשר היה על הדבוק בשנת תתקי״ד: בב׳ ימים ביום א׳[3] לירח שוא״ל לחדשי ישמעאל בשנת תקס״א למלכות[4] משוגע שר״י[5] ויהי הדבוק במזל גדי במע׳ י״ב בו ועוד מקצת חלקים. ויהיו נזורים ויבא מאדים בבית הזה בסופו. ויהיה הדבוק כמו

[1] א: אלים.

[2] א: טריגון וטריגון.

[3] בב׳ ימים ביום א׳] א: בשני ימים בו ביום ראשון.

[4] ק: למכות.

[5] א: שמו.

כן נזור ע״כ הוא חזק מאד ובעל כח. וידוע כי זה הבית בית גדי ובעת שיבא בו דבוק הוא בית שבתי וכבוד מאדים, ע״כ שניהם חזקים בו. ולא כן צדק כי הוא חלש ותשוש כח למען ‹א 127א› עמדו בבית שפלותו ואין לו כח כשני ריעיו כי שבתי בביתו ומאדים בכבודו.

‹4› על כן יוכיחו על תיקון ענין המלכים שתהיה להם מירושה ממלכות אבותם ואבות אבותם בדבוק הזה. וידל וישפל כל הקושר והבוגד עליהם והעומד מאליו שלא מזרע המלוכה. וזאת ראיה גדולה על מלכות אדום שתתחזק מלכותם וישפל כח אלמסמוד הקושרים ותפול[6] ותאבד מלכותם ותתפרד חברתם. ויפיל הקב״ה עליהם דבר וחרב ומגור מסביב איש באיש עיר בעיר ממלכה בממלכה והיו כלא היו. וגם תפול קטטה בין כוש וקדר ובעברי ספרד במזרח ובמערב[7] ועבר הים. וירבה החרום מפני עדות מאדים בבית הדבוק. ויגיע לפרשים ולפרתמים ואנשי הצבא והגבורים כח ועוז וממשלה וכבוד מאת המלכים והשליטים ולתת להם מתנות ולגדלם ולנשאם בכל מקום. והפקחים בדתם והאדוקים בטעותם ומחניפים בחלקלקותם ישפלו ויהיו לקלס ולבזיון.

‹5› ויש תקוה באלה שני הדבוק לאומתינו, בע״ה המלכים יכבדום וינשאום ויגדלום ויתחברו עמהם, ואולי ירצה השם[8] לתת לנו תקוה ואחרית טוב מזה. כי הוא היודע העתידות והחדשות מאין כמהו מגיד מראשית אחרית.

‹6› ואולם מה שנראה מן הצומח בשנה הראשונה לדבוק כי יכנס השמש במזל טלה בחלק א׳ בו בחצי השעה התשיעית יום ד׳ בכ׳ יום לחדש גמאדי[9] או״ל[10] לחשבון ישמעאל שנת תקס״ח[11]

[6] וישפל כח אלמסמוד הקושרים ותפול] א: וישפל כח אלמסמוד הקושרים; ק: ויפול כח למחמד הקוסרים [*sic*] ותפול.

[7] ק: ומערב.

[8] א: הקב״ה.

[9] ק: גמאד.

[10] ק: הראשון.

[11] כדאי לקרוא: תקס״א.

ובמוצאי חדש מארס כ״ג בו. והצומח אז אריה י״ח מע׳ ובעל הבית[12] השמש והוא בבית כבודו.

<7> וכל זה יוכיח <א 127ב> על הגלות הצלחת מלך[13] ארגון[14] בשנה הזאת ולהרבות גדולתו.[15] ולפי שלקה השמש במזל שור בית נגה וכבוד הלבנה לאחר חצות היום ושבתי וצדק ומאדים בשלישות זה הבית יפול הגדול משרי מלכותו וגם תרעשנה עליו מדינות ועל ידי אדם תקיף יתעלה במיתתו ופגירתו וע״י אנשים אחרים שישתבחו עליו בסתר ובגלוי. וכן יבוא רחוק הדעת[16] בין זה לזה ותרבה מאד המחלוקות בארצות אדום <ק 165ב> וחרום וחרב[17] במקומות רבים. ויתחדש בעולם מהמאורעות והקורות מכח זה הדבוק והלקות שבשלישותו כי יפול דבר בפאת מערב ויחושו העם בראשם מכאב[18] ראש ורבוי הלחה ומרירה[19] שחורה וימותו רבים מהתינוקים פתאם. ותראנה אותות בשמים וכוכבים באויר בעלי הזנבים ונגה להם ממסלותם ומאחרי[20] זיקות. ויהיה רעש הרבה ורוחות[21] וסערות ויפלו בתים רבים. ויעלו בעילוי מקצת מן המושלים ומקצתם יושפלו ברצון אלהינו. ויבא הפסד בסוסים בבקר ובגמלים[22] והעופות וחית[23] השדה תשכלם גם מבני אדם. וירבו החרום ושבוש הדרכים וישתקעו אניות רבות

[12] ק: ובעה״ב.

[13] ק: מלך; א: הועד מלך.

[14] א: ארגון; מקום חלק בכ״י ק.

[15] ק: גאולתו.

[16] ק: ריחוק הדעת יבא. כדאי לקרוא: יבוא ריחוק הדעות.

[17] ק: וחרוב.

[18] ק: מכוכב.

[19] ק: ממררה.

[20] א: ומאורי.

[21] ק: ברוחות.

[22] א: ובגמלים והחיות.

[23] ק: חית.

בים. מלך[24] זה על מקצת עבדיו להרחיק ממנו ולסמוך על אחרים. וגם החיות לא תשקוטנה מהלחם מהם ובהם. והמעוברות תשכלנה להפיל פרי בטניהן.[25]

‹8› וירבו התבואות בשני הדבוק הראשונים. ואולם בשנה הרביעית יהיה יוקר ורעב גדול בארצות קדר הם הברברים וארץ מערב. ויברחו רבים מפני החרב והרעב והדבר ובכל זה עסקי הבניינים ‹א 128א› יצליח ועובדי האדמה יעבדו המדברות והארצות הנשמות ויבנו החורבות ויתעסקו בענין זה מאד.

‹9› ויתעתקו העם ממקומם לארצות אחרות והרבה יתעסקו בארצות אחרות[26] מאשר לפנים לא ידעום. ורבים מבעלי המלחמה ינוחו ממלחמה. והגבורים יושפלו והשפלים יגבהו. ויראו פלאות בפאת מזרח ובארץ הצבי יתחדשו חדשות גם בארצות אחרות. וימזוג כעין[27] הקיץ ויקל חום השמש ויהיה קור עצום בחורף. ויעמיד המלך חיילים ויצביא צבאים לעבור את הים להלחם. ויהיה צער[28] גדול לעם על זה ותרבנה המהומות[29] ויתחוללו[30] העם בהרבה מקומות עד זמן ועידן מעט. ואח״כ ישובו לבות המלכים על העם ולב העם על אדוניהם. וייטיב הקב״ה את הארצות וישובו הסוחרים בסחורתם וירבה רוח בעולם וישובו בני אמונים בפעל יושר בבני[31] אדם ושלום בארץ.

‹10› וברוך האל היודע העתידות כי אין זולתו. ברוך שם כבוד מלכותו לעולם ועד. אמן.

[24] כדאי לקרוא: וימלך מלך.

[25] ק: פרי בטנם להפיל.

[26] ק: יתעסקו בארצות אחרות; א: יעסקו במלחמות.

[27] א: בהוגן ענין.

[28] ק: סער.

[29] ק: המקומות.

[30] כדאי לקרוא: ויתהוללו.

[31] א: בני.

Astrology and Prophecy: The Ikhwān al-Ṣafā' and the Legend of the Seven Sleepers

GODEFROID DE CALLATAŸ

1 *Introduction*

A glance at works such as Dom Michael Huber's *Die Wanderlegende von den Siebenschläfern*, a 600-page study written about a century ago,[1] would easily convince anyone of the immense popularity and of the complicated ramifications of the fable that came to be referred to as the Legend of the Seven Sleepers of Ephesus. It was Ernest Honigmann's merit to shed light on the Christian origin of the story by showing the crucial role played by the Bishop Stephen of Ephesus in the middle of the fifth century AD.[2] The Islamic developments of the story, whose basis lies in verses 9 to 26 of the *Sūrat al-Kahf* ('The Cave'),[3] have been best investigated by Louis Massignon, who devoted several studies to the legend between 1950 and 1962.[4]

Among those Muslim avatars, the very peculiar re-elaboration of the myth by the Ikhwān al-Ṣafā' (Brethren of Purity) at the end of their *Epistle on Resurrection* has been, it would seem, insufficiently analysed so far. Massignon himself referred to it only in passing, and made two serious confusions.[5] Yves Marquet

[1] M. Huber, Die Wanderlegende. An extensive, yet far from complete, bibliography on the legend may be found in F. Jourdan, *La Tradition.*

[2] E. Honigmann, 'Stephen of Ephesus', pp. 125-68.

[3] That is Sūrat 18 of the Qur'ān, which is generally assumed to belong the second Meccan period.

[4] See above all L. Massignon, 'Apocalypse', pp. 245-60 (repr. in Id., *Opera Minora*, III, pp. 104-118). See also his 'Les Sept Dormants'.

[5] See L. Massignon, 'Apocalypse', p. 109 of the reprint: 'Chez les Shi'ites, elle [i.e. the popular meditation on the Legend] a pris un aspect plus charnel, concernant les Imams Alides descendants du Prophète, ou plus spirituel, concernant les Clients Adoptés. (...) Aussi les Shi'ites identifient-ils les Sept Dormants avec les Sept Imams des Ismaëliens, cachés pendant 309 ans dans la "Caverne", qui fondèrent l'anticalifat des Fatimites en 309 à Mahdiya; leur

tackled the problem in several places and made many valuable suggestions on the occasion of a paraphrase he gave in one of his earliest articles.[6] As far as I know, the only more recent attempt at interpreting the Ikhwān's version is Geo Widengren's article entitled 'La Legénde des Sept Dormants dans les écrits des Frères Purs', but his is, similarly, a study based on a paraphrase which does not follow the Arabic text closely enough. In that case, the process led the author to make a considerable number of errors.[7]

Apart from Marquet's and Widengren's paraphrases,[8] two complete translations of our text have appeared, at approximately the same time, as part of broader undertakings. The former one by Jean Michot,[9] in French, was chiefly based on the old Cairo edition.[10] The other one, by Susanne Diwald,[11] in German, is established on more scientific grounds, for it was based on the Beirut edition[12] (which, in spite of its flaws, has remained ever since the standard edition of the *Rasā'il*) as well as on a certain number of manuscripts which the German translator listed in her *apparatus criticus.*

In order to appreciate better some points of the discussion, it has seemed worthwhile to quote here again the text of the legend in full, this time in English translation. The Arabic text I have followed is the one which appears on pp. 315-320 of Volume III of the Beirut edition, occasionally changed according to the *lectiones* of Diwald's work. Every such change is indicated in notes.

"Reveil" est la Rajʿa. C'est la doctrine des Ikhwān al-Ṣafā.' In the following, it will be shown that the Ikhwān do not connect the Seven Sleepers to the Seven Imāms of Ismailism and, moreover, that the figure of 309 years, which is the one of the Qur'ān, is replaced with that of '354 days of the days of the Sun according to the computation of the Moon.'

[6] For the paraphrase see Y. Marquet, 'Imāmat', pp. 75-9. The astrological background of the myth is best explained in Id., 'Les Cycles', pp. 51-54.

[7] G. Widengren, 'La Légende', pp. 509-26

[8] The old paraphrase in German by F. Dieterici is too defective to be of any use. A very brief summary of the Epistle is given in A. Bausani, *Enciclopedia.*

[9] J. R. Michot, 'L'épître de la résurrection', pp. 114-48 (pp. 139-43 for the myth).

[10] Ikhwañ al-Ṣafā', *Rasā'il*, Cairo.

[11] S. Diwald, *Arabische Philosophie.*

[12] Ikhwān al-Ṣafā', *Rasā'il*, Beirut. In this paper all references are made to the Beirut edition.

2 *Translation of the text*

[315,4] It has been told that there was a king of great importance, of strong authority, with a vast kingdom, [5] numerous soldiers and servants. A male child was born to him, who among all creatures most resembled him and his mother[13] in his nature and temper. As soon as he had grown up, developed and reached full maturity, his father entrusted him with a part of the kingdom and ordered his soldiers and servants to obey him. He advised him to govern them well and allowed him every favour. Yet he forbade him to <have access to> his own rank. The son remained <there> for a long time, the measure of half a day, surrounded by favours and pleasures, but he was inattentive[14] and absent-minded. One of his father's servants, of those who were leaders before him, [10] became envious of him (the son) and told him: 'Of course, you do not know any favour and you do not find any pleasure, since the peak of pleasures and favours has been forbidden to you, and the most delightful passion prohibited. If you hurry up and request the kingdom, you will have the precedence'. He was deceived by <the servant's> words, for he was inattentive and ignorant. He requested what it was not appropriate for him to obtain before his time or to request before the right moment. He fell down from his rank and was dismissed from his position in his father's court. His disgrace appeared to <his father> and his sins became evident to him. He ran away, in fear, from his father, [15] going about in his kingdom as in concealment. He experienced distress. Misery and adversity fell upon him. He endured pains and misfortune. He remembered the day in which he lived on his father's favours, felt sad about what he had lost and cried regretfully. Then he felt drowsy and fell asleep. He was brought to his father, who said: 'Let him sleep until Friday'.

Then on the second day he (the king) was granted another son, who resembled his brother more than anyone else. He grew up, developed, [20] reached full maturity and flourished. He was mild-tempered, dignified, grateful and patient. His father entrusted him with a part of his kingdom, [316,1] ordered <his subjects> to obey him and advised him to govern them. <The son> called

[13] Wālidati-hi D : wālidī-hi B ('his parents').

[14] Ghirran D : ghārran B ('vain', according to Michot's translation).

them together and gave them orders and proscriptions, but they did not listen to him nor did they obey his order, for he looked like Zuḥal (Saturn); rather, they molested him. He remained patient for a while, then he complained to his father, who for this reason became angry with them and threw most of them into the water. When <the son> saw what had fallen upon them, he became worried and sad. He felt drowsy and fell asleep. He was brought to his father, who said: 'Let him sleep until Friday'.

Then on the third day <the king> was granted another son, who resembled the two brothers we have just mentioned more than anyone else. He grew up, developed, reached full maturity and flourished. He was good, virtuous, very learned and keen on dispute. His father entrusted him with the position of his two brothers, ordered his subjects to obey him and advised him as he had advised his two brothers. He called them together and gave them orders and proscriptions, but they did not listen to him nor did they obey him, for he looked very much like al-Mushtarī (Jupiter). They frightened him with fire. He went to his father and built a temple for him. He offered up a sacrifice to him, performed rituals, and proclaimed among the people: 'Come on, come and see what you have not seen and hear what you have not heard'. Then he fell asleep and was brought to his father, who said: 'Let him sleep until Friday'. His proclamation remained in the ears of the souls. They transmit it as a legacy without hearing it. They go to his temple, see its exterior, but do not notice what its purpose is.[15] They perform the tradition of its rituals, but they do not understand the meaning of them, for they are 'deaf, dumb, blind and they do not realise'.[16] God preserve you, my brother, from being one of <those people>. Consider, with the light of your intellect, the *Epistle of the Acts of the Spiritual Beings*[17] and maybe you will know what we have said and understand what we have stated.

Then on the fourth day <the king> was granted another son, who grew up, developed, reached full maturity and flourished. He was strong, powerful, bold and brave. His father entrusted him with the position of his brothers, ordered the subjects to obey him

15 wa marmā-hu mimmā D : wa-marā-hu mā lā B (unintelligible)

16 See Qur'ān, 2, 171.

17 Ruḥāniyyāt D : ruḥāniyya B ('of spirituality'). The Risāla referred to here is *Epistle XLIX : On Modalities of the Status of Spiritual Beings.*

[20] and advised him as he had advised his brothers. He called them together and gave them orders and proscriptions, but they did not listen to him nor did they obey him, for he looked like al-Mirrīkh (Mars). They fought with him and he fought with them. They skirmished with him and he skirmished with them. He was supported with his father's power. He defeated them, scattered their union, broke up their gathering, dissolved their concord and cast them out on land and sea. Then he remained alone, like exile, calling without being answered, ordering without being respected. He became worried and sad, felt drowsy and fell asleep. He was brought to his father, who said: 'Let him [317,1] sleep until Friday'.

Then on the fifth day <the king> was granted another son, who resembled his first brother more than anyone else. He grew up, developed, reached full maturity and flourished. He was a leader and a guide, kind and friendly. His father entrusted him with the position of his brothers, ordered the subjects to obey him and advised him as he had advised his brothers. He called them together and gave them orders and proscriptions, [5] but they did not follow him or only a little, nor did they obey him or only in part, for he looked like al-Zuhara (Venus). Then they rushed on him and took away from him the tunic which his mother had sewn. He went to his father, who called out his soldiers for war and supported him with a spirit from him. <The spirit> flowed into their souls and held sway over their divinity as a compensation and retaliation for having held sway over his humanity. He wished to descend again, but his father said: 'Be patient[18] until Friday'.

[10] Then on the sixth day their father said to the celestial bodies: 'Choose for my son who looks like ʿUṭārid (Mercury) a day so that he may descend to the world of coming-to-be and passing-away, awake his sleeping brothers and call them to his truth, for I have been pleased with them. He will order them to prepare for praying. Indeed, tomorrow is the feast, Friday. The judges will appear, and he will decide among them about the things on which they disagreed'.

The masters of the celestial bodies and the leaders of the stars gathered together in the house of al-Mirrīkh (Mars) and

[18] Isbir D : isbirū B

took counsel together. The leader of the stars [15] and their king, the Sun, said: 'Of my power I choose for him, of my virtues I provide him with the greatness, the leadership and the reign, the strength and the elevation, the splendour and the magnificence, the laudation and the praise, the granting and the offering'.

The eldest of them, Kaywān (Saturn) said: 'Of my power I choose for him clemency and sobriety, patience and perseverance, insight and high-mindedness, preservation and faithfulness, reflection [20] and meditation'.

Birjīs (Jupiter), the rightful judge, said: 'Of my power I choose for him, I provide him with, religion [318,1] and piety, welfare and goodness, rightfulness and equity, truth and correctness, truthfulness and loyalty, integrity and valour'.

Bahrām (Mars), the master of the armies, said: 'Of my power I choose for him, of my virtues I provide him with, determination and rigour, intrepidity and bravery, ardour and courage, triumph [5] and victory, granting and munificence, and vigilance'.

Al-Nāhīd (Venus), the sister of the celestial bodies, said: 'Of my power I choose for him, of my virtues I provide him with, excellence and beauty, completion and perfection, compassion and mercy, ornamenting and neatness, love and affection, happiness and pleasure'.

The youngest brother, the most hidden of them in appearance, yet intrinsically the most illustrious of them—<the one> whose skill is [10] the most apparent, whose sciences are the most numerous, whose wonders are the most widely known and the most brilliant—said: 'Of my power I choose for him, of my virtues I provide him with, of my properties I confer on him, eloquence and speech, discernment and astuteness, insight and fineness, reading and intonation, sciences and wisdom'.

The mother of celestial bodies, namely the Moon, said: 'I will breast-feed him and make him grow. Of my power I choose for him, of my virtues I provide him with, light and brilliance, augmentation and increase, movement in the three [15] regions,[19] shifting in travels, fulfilment of hopes, ways of life and reports, and the science of appointed times'.

Then the spheres revolved and the powers of the spiritual beings were in labour, and the people of the heavens rejoiced.

[19] Al-thalātha B : - D

During the night of Destiny, before the rising dawn, the master of Resurrection came down to the world of coming-to-be, to blow the trumpet. This baby stayed in the womb for forty days [20] of the days of the Sun, and he stayed sucking for twenty days, until he had grown up, developed, reached full maturity and flourished. He resembled his third brother more than anyone else, for he looked like ʿUṭārid (Mercury), who is the brother of [319,1] al-Mushtarī (Jupiter), because of the opposition between them, their quadrature and the opposition of their spheres. Among his brothers this baby became the most complete with respect to body and the most perfect with respect to form. He was a very learned and wise man of letters, a strong king, a rightful *imām*, a messenger-prophet. His father entrusted him with his kingdom and the whole kingdom of his brothers. He appeared and subjugated whoever opposed him and made high and strong whoever was in agreement with him. He ruled over his kingdom for around [5] thirty days of the days of Sun. Then he thought high of himself and the evil eye fell upon him. He fell sick and remained on his bed for around a thousand days of the days of the Moon, ill in body and sick in soul. Then he left for another abode. He got up a little, walked and became stronger, became lively and happy. He drank from the love of this world, of its illusions and longings. He got drunk from the wine of his passions. He entered his father's cave, fell asleep with his brothers. They stayed <there> for a long time.

When the revolution of slumber [10] came to an end and the time was about to come, their father called them together: 'Is it not about time for you to awake from your sleep, to wake up from your negligence, to remember what you have forgotten regarding your principle, and to come back from your journeys to your place of return? Indeed, for every beginning there is an end, for every life an annihilation and for every death and sleeper an awakening. Hasten from your exile to your place of return. For the creation of the seven heavens was completed in six days, and tomorrow, Friday, your Lord will sit on the throne borne on this day by [15] eight <angels>'.

Therefore the brothers—of whom it is said that they were seven and that the eighth one was their dog—awoke after a torpor of 354 days[20] of the days of the Sun according to the computation of the Moon. They reminded each other of how long they had abided in their cave. Their father said to their brother: 'Do not enter, therefore, into controversies concerning them, except on a matter that is apparent, and do not take counsel with any one of them'.[21]

[20] They have hidden and concealed their secrets, since 'there is no confidential talk between three where He is not the fourth, nor between five where He is not the sixth, nor between fewer or more where He is not with them, wherever they might be. In the end, on the day of Rising, then He will notify them of what they have done'.[22]

Understand, my brother, these allusions and these warnings. Bring this into relation with what is similar to them and do not disclose the secrets. Perhaps you will awake from the sleep of negligence and from the torpor of ignorance before the trumpet [320,1] is blown, and before the herald calls for the prayer of Friday: 'Make haste to the remembrance of God and leave off this business of yours. This is best for you',[23] and before the criminals are mustered, in droves, in Gehenna ...

Take along provision from this world. Surely you are an emigrant and 'the best of provisions is piety, so fear Me, you that are sensible'.[24] 'Do not seek corruption on the earth'.[25] 'Successful be the one who will have purified <the soul>, and [5] unsuccessful be the one who will have depraved it'[26] . God give success to you, to us and to all our brothers on the path of Retribution. Surely He is merciful to the servants.

Here ends the *Epistle of Resurrection and Rising.* Next comes the *Epistle of the Quantity of the Kinds of Movements.*

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[20] Thalāthami'a D : + sana B ('years', i.e. '300 years and 54 days')
[21] Qur'ān, 18, 22.
[22] Qur'ān, 58, 7
[23] Qur'ān, 62, 9.
[24] Qur'ān, 2, 197.
[25] Qur'ān, 28, 77.
[26] Qur'ān, 91, 9-10.

3 *The cycle of prophetic history*

To a certain extent, as was noted by Marquet and Widengren, the allusions of the narrative are quite evident.[27] The seven-day week of the allegory must be identified with the seven-thousand year cycle which the Ikhwān, consider as one of the millennial revolutions.[28] In agreement with speculations well in favour with Ismaili movements,[29] 29 the Brethren connect each one of those seven millenniums with a prophet designed to bring on Earth a new law. In addition to the indications of our text, there are several passages in the *Rasā'il* which reveal to us that the Ikhwān adopt the following sequence: Adam, Noah, Abraham, Moses, Jesus, Muḥammad and the one they usually call the '*Qā'im* of Resurrection'.[30] In his paraphrase of the allegory, Marquet drew up a significant list of 'transparent allusions' to the life and deeds of each one of those prophets: Adam's banishment from Paradise, the deluge in the time of Noah, the sacrifice performed by Abraham, and so on.[31] It is not necessary to take up the matter again, at least for the five predecessors of Muḥammad.

For the Prophet of Islam, however, one may reasonably go into more detail than Marquet. The French scholar rightly identified the 40 days in the womb with the 40 years of life before the beginning of the Revelation, and the 20 days of suckling with the 20 years of that Revelation before Muḥammad's death. Yet he failed to notice that the next 30 years no longer refer to Muḥammad himself, but to the following phase in the history of Islam. Indeed, they correspond with the period of the first four caliphs of Islam, usually referred to as the *Rāshidūn*, that is, from the beginning of Abū Bakr's reign in AH 10 (AD 632) to the end of ʿAlī's caliphate

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[27] Y. Marquet, 'Les Cycles', p. 51: 'Le mythe de la caverne des sept dormants, dont les allusions sont transparentes...'; G. Widengren, 'La Légende', p. 513: 'Le contenu de cette allégorie est en partie parfaitement clair et l'analyse ne souffre point de difficulté'.

[28] On this, see for instance Ikhwān al-Ṣafā', *Rasā'il*, III, p. 250 (from *Epistle on Revolutions and Cycles*) and my notes on that passage in G. de Callataÿ, *Les Révolutions*, pp. 55-56. Elsewhere (*Rasā'il*, III, p. 219), the Ikhwān report the tradition according to which 'the world has a duration of 7000 years'.

[29] On this see H. Corbin, Temps Cyclique, especially pp. 39-69.

[30] See for instance Y. Marquet, 'La détermination', pp. 144-5.

[31] Y. Marquet, 'Imāmat', pp. 75-79. On Biblical prophets in the Qur'ān and in Arabic literature see the recent: R. Tottoli, *I profeti.*

in about AH 40 (AD 660), or more plausibly to the disaster of Ṣiffīn in AH 36 (AD 657). For the evil eye that struck the sixth son of the king is an obvious allusion to that great *Fitna* ('Trial' or 'Discord') by which the Islamic community was then divided;[32] it has, of course, nothing to do with Muḥammad himself, about whom the Ikhwān would never have used the expression 'to think high of oneself'. As for the '1000 days of the days of the Moon' during which the son remained sick on his bed, they would be in fair agreement with the 3 or 4 years between Ṣiffīn and the transfer of the Umayyad caliphate to Damascus, since the son is said to have 'left for another abode'. Then, after what seemed to be a period of pseudo-recovery, the man 'got drunk from the wine of his passions', and this is why, so the text goes on, he entered the cave and fell asleep with his brothers. The interpretation of all this causes no problems. The Ikhwān would have their readers believe that the reception of God's message entered a phase of concealment some time during the Umayyad period and that it has remained so ever since. The sleeping brothers, they say, stayed in the cave 'for a long time'.

We should not be misled by what follows in the story. 'When the revolution of slumber came to an end and the time was about to come' does not refer to the time of the Last Judgement, as Marquet assumed,[33] but only to the moment when God is about to wake up all his prophets 'from the sleep of negligence and from the torpor of ignorance', that is, in other words, when the reception of God's Word may come out of its concealment again. It is important to observe that at this stage we are still ahead of the transition to the seventh millennium of the cycle. Friday, the Day of Congregation and of the Last Judgement, is still to come, as the father tells his sons.

4 *The Time of the Awakening*

'Therefore the brothers—of whom it is said that they were seven and that the eighth one was their dog—awoke, after a torpor of 354 days of the days of the Sun according to the computation

[32] On the importance of the event for Islam see H. Djait, *La Grande Discorde*.

[33] Y. Marquet, 'Imāmat', p. 79: 'Enfin, lorsque fut terminé le cycle du sommeil et que fut proche l'Heure (du Jugement dernier), leur père les appela...'

of the Moon'. In this sentence there probably lies the key to what the Brethren intended to reveal to their partisans. The phrase is paradoxical in that it contains at the same time the most obvious reference to the Qur'ānic version of the story and the most patent contradiction of it. For if the first part is, as we noted, a literal reference to Qur'ān, 18, 22, the second part is thoroughly incompatible with Qur'ān, 18, 25, which reads: 'So they stayed in their cave 300 years, and 9 more'. How is this disagreement to be accounted for?

There is little need to take up again the discussion about the Qur'ānic value. Suffice to recall, after so many Muslim commentators of classical times, that those 309 years are lunar—i.e. Islamic years—and that they therefore perfectly equal the round figure of 300 solar years or three centuries.[34] From what has been stated above, the sleep of the sixth brother began a certain time after AH 40 (or AD 660). When the 309 lunar years of the Qur'ān are added, we obtain a *terminus post quem* of $40 + 309 =$ AH 349 (or about AD 960) for the date of the awakening. This, we may already note, nicely concides with the generally accepted *terminus post quem* for the definitive redaction of the *Epistles*.[35]

In another of their *Rasā'il* the Ikhwān do not hesitate to speak of their brotherhood as of a group of pure and noble friends who 'have been asleep for a certain period of time in the Cave of our father Adam, enduring the vicissitudes of time and the misfortunes of existence, until the appointed time, after a dispersal in the countries, in the kingdom of the Supreme Author of the Law'.[36] In the same *Risāla* the Ikhwān also speak of seven prophets, but the sequence and the names are not exactly the same. Thus,

[34] On this see for example F. Jourdan, *La Tradition*, pp. 97-8. The Graeco-Syriac tradition of the original legend generally hesitates between the figures of 372 years and of 'about 190 years'; see E. Honigmann, 'Stephen of Ephesus', pp. 136-137.

[35] See Y. Marquet, 'Ikhwān al-Ṣafā", p. 249: 'The epistles probably did not receive their definitive form until after the conquest of Egypt (969 AD)'. Elements of discussion for this controversial matter may found in: S. Stern, 'The Authorship', pp. 367-72 ; Id., 'New Information', pp. 405-28; A. Hamdāni, 'Abū Ḥayyān al-Tawḥīdī', pp. 345-353; Id., 'An Early Fātimid Source', pp. 62-75; Id., 'The Arrangement', pp. 97-110; Id., 'Brethren of Purity', pp. 73-82; Y. Marquet, '910', pp. 61-73; Id., 'Ibn al-Rumī', pp. 121-123.

[36] See Ikhwān al-Ṣafā', *Rasā'il*, IV, p. 18 (from Epistle XLIV: *Where one explains the belief of the Brethren of Purity and the doctrine of the men-of-the-Lord*).

Adam is followed by Noah, Abraham, Moses and Jesus, just as in the *Epistle of Resurrection*, but then we have al-Yazdān and Agathodaemon in the sixth place and Muḥammad in the seventh.[37] This time, it is no longer a question of a *Qā'im* at the end of the cycle, so that the comparison of our texts should not be pressed too hard. Yet it is interesting to observe, once again, the very peculiar—and decisively Hermetic—purport assigned to the sixth element of the sequence[38].

The Brethren chose to replace the Qur'ānic value by that of '354 days of the days of the Sun according to the computation of the Moon', which we may interpret with confidence as meaning 354 lunar—or Islamic—years.[39] There is, of course, absolutely no ground for suspecting that the Qur'ānic figure might have been altered in any way. As was already pointed out by Massignon, the *Sūrat al-Kahf* occupies an almost unrivalled position in Islamic liturgy, the proof being that it was customary, at least up to a recent past, to read it every Friday in every mosque.[40] Clearly the Ikhwān, who otherwise prove to be faithful transmitters of the Qur'ānic verses they quote, must have had a good reason for adding 45 lunar years to the figure of the Sacred Book.

I would venture to suggest that a certain opportunism could have been responsible for this chronological adaptation. If the moment of the final redaction of the *Rasā'il* is, as seems to be the case, slightly later than AD 960, then it would not have made any sense to refer to the Qur'ānic lapse of 309 years, for the long-awaited awakening of the brothers would have already occurred by then. In adding 45 years to the computation the Ikhwān could no doubt look forward to the future more serenely. The souls of

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[37] Ibid., IV, pp. 18-20. See also Y. Marquet, 'Imāmat', pp. 80-81; Id., *La philosophie*, pp. 375-6.

[38] In the same line we may wonder whether the figure of the dog in the story as told by the Ikhwān does not refer to Hermes's mission of leading the souls, since it is well-known that Thoth-Hermes was dog-headed. On the Hermeticism of the Ikhwān and the probable influence of the Sabeans from Ḥarrān, see: H. Corbin, 'Rituel'; Y. Marquet, 'Sabéens'.

[39] The Beirut edition has the rather unintelligible 'years' inserted between '300' and '54 days of the days of the Sun according to the computation of the Moon', an error which is probably due to the influence of the Qur'ānic expression of '300 years, and 9 more'. Diwald's apparatus criticus enables us to correct this aberration without difficulty.

[40] See L. Massignon, 'Apocalypse', p. 104 of the reprint.

their partisans could similarly be reassured. The fatal moment was near, but it was still ahead.

5 *Astrological determinism*

Now why did the Ikhwān elect the figure of 354 years rather than any other one? First, we note that this is a period in which the number of years corresponds to the number of days of the Muslim year cycle.[41] In other words, it could qualify as one of those 'Great Years' or 'Great Years of years' with no astronomical justification but with an obvious symbolic resonance. To give but one example in which the same process operates by using solar years, we may refer to the passage in which the Pseudo-Plutarch speaks of Diogenes's Great Year as having 365 times the length of that of Heraclitus.[42] The Ikhwān were very fond of such analogies, as is apparent in our text, *inter alia*, where they constantly alternate 'days of the Moon' (i.e. days in the proper sense) and 'days of the Sun' (i.e. in fact, years). Another of these analogies is, of course, the one they draw between the ordinary week (with the call for prayer on Friday) and that 7000-year Week of prophetic Revelation (with the Last Judgement at the end of the cycle), itself explicitly compared to the 6+1 Days for the creation of the world.

Yet in the case of people so lavishly imbued with astrological speculations we may reasonably doubt that this explanation is sufficient. The Ikhwān's re-interpretation of the *Sūrat al-Kahf*, which has no astrological implication in itself, is a very good example of that tendency. Here is a story in which prophets, or more exactly prophetic missions, are connected to planets,[43] a

[41] Geo Widengren observed this too, but then embarked upon a pretty desperate explanation so as to stress what he regarded as the 'religion ultra-shi'ite' of our authors; see G. Widengren, 'La Légende', p. 524: 'Essentiel est le fait que les 354 jours, l'année de Lune, sont divisibles par le nombre 7. Alors 4 jours constituent le reste. Si ces jours restants sont comptés à partir du jour du rassemblement, *yawm al-jum'a*, nous arrivons au signe du mois Muharram, le mois spécifiquement shi'ite. Mais si l'on veut ajouter le nombre 6 et compter à partir du dimanche, le résultat est le même [here Widengren refers to a passage from al-Bīrūnī's *Chronology of Ancient Nations*]. Le choix du nombre 354 s'explique ainsi sans aucune difficulté [sic]'.

[42] Pseudo-Plutarch, *Placita philosophorum*, II, 32. More on this in G. de Callataÿ, *Annus Platonicus*, p. 104.

[43] Of the seven prophets five are explicitly connected to a planet, namely:

story in which Islam is shown to be identified with Mercury,[44] a story in which the leaders of the stars all gather together 'in the house of Mars', each one of them being prepared to provide the future law with their specific qualities.

One of the two houses of Mars is Scorpio, traditionally recognised as the 'indicator' or 'significator' (*dalīl*) of the Arabs. In a famous passage of his *Muqaddima* Ibn Khaldūn reports a statement by Jirāsh ibn Aḥmad al-Ḥāsib in the time of the vizier Niẓām al-Mulk, according to which the birth of Muḥammad had been indicated by the conjunction of Jupiter and Saturn in the sign of Scorpio.[45] Even more interesting is, from the same source, the following assertion: 'Abū Maʿshar said in the *Book of Conjunctions*: "When the section reaches the twenty-seventh (degree) of Pisces, in which Venus has its exaltation, and when, at the same time, the conjunction occurs in Scorpio, which is the significator of the Arabs, then the Arab dynasty will make its appearance, and there will be a Prophet among them. The power and duration of his rule will correspond to the remaining degrees of the exaltation of Venus"'.[46]

This turns out to be a relatively faithful quotation; for the original text, recently edited by Keiji Yamamoto and Charles Burnett, states with regard to the appearance of the Arabs: 'Venus was in the sign of its exaltation, in the ninth place, indicating religion, and, being the indicator of the Arabs by nature, it gave the rulership to them, and transferred it to them and

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Noah with Saturn, Abraham with Jupiter, Moses with Mars, Jesus with Venus and Muḥammad with Mercury. From the analogy with the sequence given in what follows (the conferment of qualities by each one of the planets) we may infer that Adam should be associated with the Sun while the Qā'im of Resurrection should be related to the Moon, each one of the luminaries occupying an end of the chain.

[44] Whatever the sequence of the other planets may be, it is interesting to note that the Ikhwān differ from the majority of Muslim astrologers in connecting Venus with Christianity rather than to Islam. The reason why they identify Mercury with Islam, namely that it is the most secret of the planets, is strangely echoed in the Latin Middle Ages by someone like Roger Bacon (Opus maius, vol. I, pp. 255-267), who will identify Mercury to Christianity for exactly the same reason. More on this in: J. D. North, 'Astrology', pp. 181-201; G. de Callataÿ, *Annus Platonicus*, pp. 164-5; Y.J. Michot, 'Ibn Taymiyya', pp. 181-2.

[45] Ibn Khaldūn, *Muqaddima*, vol. II, p. 213.

[46] Id., vol. II, p. 214.

their land because of its predominance over the position indicating the Arabs in the Sign of Scorpio—that is, from its seventh to eleventh degree—because the extent of these degrees in the sign of Scorpio is the term of Venus. Because it (Venus) was in the ninth place, indicating religion, this indicates that their appearance was because of religion'.[47] On this basis Abū Maʿshar next calculates that the rulership is expected to last 693 (lunar) years from Hegira, which corresponds to AD 1294.[48] The same computation and result are given by al-Kindī in his letter 'On the Rule of the Arabs and its Duration', now re-edited in the same volume by Yamamoto and Burnett,[49] who convincingly demonstrate that we should speak of a common source rather than accuse one of the two scholars of plagiarism.

It is probable that the Ikhwān knew this tradition as well, just as they could have known this other assertion by Abū Maʿshar: 'in the period of year 333, they [Saturn and Mars] will conjoin in Cancer; they will indicate many riots and wars, and that Islam will triumph over most of the religions'.[50] When they wrote their *Epistle on Resurrection* and chose to speak of a 354-year period of sleep, the Brethren surely had many sources at their disposal to draw their inspiration from. Yet I think we should not expect too much of the comparison of their own astrological speculation with that of their predecessors, however famous they may be, like Abū Maʿshar, al-Kindī or Māshā'allāh.[51] The Ikhwān prove to be wholly original, as we have seen, in associating Mercury with Islam and Venus with Christianity. This alone should prevent us

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[47] Abū Maʿshar, *On Historical Astrology*, Part II, Chapter VIII. 5 (vol. I, pp. 126-7).

[48] Ibn Khaldūn (Muqaddima, vol. II, p. 214) records al-Kindī's calculation that 693 is also the sum of the numerical values of the isolated letters which one finds at the beginning of certain chapters of the Qur'ān. This is embarrassing, for there exists in Islam an arithmological tradition which interpret the '309 years' of the *Sūrat al-Kahf* in exactly the same way.

[49] Al-Kindī, *On the Rule* (as Appendix III of Abū Maʿshar, *On Historical Astrology*, vol. I, p. 533). Al-Kindī's text had been previously investigated in O. Loth, 'Al-Kindī', pp. 261-309.

[50] Abū Maʿshar, *On Historical Astrology*, Part II, Chapter VIII. 6 (vol. I, p. 131).

[51] On the importance of Māshā'allāh for Muslim astrology see: E. S. Kennedy-D. Pingree, The Astrological History, to which one should add: D. Pingree, 'Māshā'allāh: Some Sasanian and Syriac Sources' and more recently: Id., 'Māshā'allāh: Greek, Pahlavī, Arabic and Latin Astrology'.

from regarding them as mere transmitters of doctrines. After all, it is not so common to meet Muslim thinkers who deliberately alter a Qur'ānic figure for their own purposes.

Almost a century ago, Paul Casanova sought to demonstrate that in another *Risāla* the Ikhwān had alluded, by using the cryptic language of astrology, to the epoch in which their period of concealment would end.[52] The passage reads: 'Among the features of our Brethren is that they are connoisseurs in the field of religion, that they know the secrets of prophecies and that they are well-versed in philosophical computations. If you happen to meet one of them and seem to note integrity in him, let him know what will please him and remind him of the recommencement of the revolution of revealing and awakening, as well as of dissipation of worries for mankind, from the transfer of the conjunction, from the sign of fiery triplicities to the sign of vegetal and animal triplicities, in the tenth circle which corresponds to the house of power and the appearance of eminent people'.[53] Well aware of the importance of the conjunction of the two superior planets for Islamic astrology[54] and for the Ikhwān in particular,[55] the scholar arrived at the conclusion that the transfer alluded to here was the transfer of the conjunction of Jupiter and Saturn from the fiery triplicity (Sagittarius) to the earthy triplicity (Capricornus). On the basis of astronomical tables available to him, Casanova selected the date of 26 Jumāda I of the year AH 439 (i.e. 19 November AD 1047 according to the Julian Calendar).[56] The Ikhwān's prediction was excellent, he wrote, for he noted that a mere 11 years later, that is, on 13 Dhu l-Qa'da AH 450 (1 January 1059), the khutba was given in Baghdad in the name of the Fātimid Caliph al-Mustanṣir.

[52] See P. Casanova, 'Une Date', pp. 5-17.

[53] Ikhwān al-Ṣafā', *Rasā'il*, IV, pp. 146 (from *Epistle XLVIII: On Modalities of the Appeal to God*).

[54] On the Sasanian origin of this doctrine see for example: E. S. Kennedy, 'The Sasanian Astronomical Handbook', pp. 259-60; D. Pingree, 'Astronomy and Astrology', pp. 245-6; K. Yamamoto and Ch. Burnett in Abū Ma'shar, *On Historical Astrology*, vol. I, pp. 582-7.

[55] See Ikhwān al-Ṣafā', *Rasā'il*, III, p. 252 and 266, together with my notes on those passages in G. de Callataÿ, *Les Révolutions*, pp. 66-8 and 104-6.

[56] The date of 19 November 1047 corresponds in fact to the 28 Jumāda I of AH 439.

Later on, the question was reopened by Marquet,[57] who found in the *Jāmi'a*[58] yet another passage in which the expected event—provided it is the same —was a transfer of the same kind but this time from the watery to the fiery triplicity. But this obviously raises further problems, for this phenomenon, which had occurred in AH 193 (AD 809),[59] was not expected before the year AH 1175 (AD 1762), after the completion of another '960-year conjunction'. This prompted Marquet to hesitate between the years AD 909 (proclamation of al-Mahdī in Ifrīqiya) and AD 969 (conquest of Egypt)—neither of whom, it must be noted, correspond to a transfer of the conjunction in the triplicities—as the dates most possibly hinted at by the Ikhwān. Marquet's embarrassment is easily perceptible in the concluding lines of his article : 'Poursuivant ces déductions, on peut conclure que selon la théorie des Ikhwān, le Qā'im de la Résurrection aurait du apparaître vers 1531 [of the Christian era]; le jugement aurait dû intervenir vers 1571, et le nouveau cycle de 7000 ans devrait commencer vers l'an 2491.'[60] In a more recent article on 'La détermination astrale de l'évolution selon les Frères de la Pureté', Marquet still appears to set little value on astronomical data when he connects the cycle of prophethood, with its alternating phases of manifestation and clandestinity, to the four transfers of the conjunctions of Saturn and Jupiter : 'Le passage du groupe décadence-clandestinité au groupe renaissance-apogée est censé être déterminé par le prétendu passage de la conjonction de Saturne et de Jupiter des signes de feu aux signes de terre (par exemple en [AD] 928), ou des signes d'air aux signes d'eau (par exemple en 1404) ; l'inverse est lui déterminé par le prétendu passage de la conjonction des signes de terre aux signes d'air (par exemple en 1166) ou des signes d'eau aux signes de feu (par exemple en 690)'.[61]

Marquet's suggestion of connecting the prophetic and the astrological cycles is probably right, but the dates he indicates for the four transfers from one triplicity to the next are clearly wrong.

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[57] Y. Marquet, 'Les Cycles', pp. 62-9.

[58] Ikhwān al-Ṣafā', *al-Risālat-al-Jāmi'a*, I, pp. 323-4.

[59] For a table of conjunctions as computed by Māsha'allāh see E. S. Kennedy, 'Ramifications', p. 34 and E. S. Kennedy-D. Pingree, *The Astrological History*, pp. vi-vii.

[60] Y. Marquet, 'Les Cycles', p. 69.

[61] Y. Marquet, 'La détermination', p. 145.

The transfer from airy (Libra) to watery (Scorpio) signs took place in AD 571 and was considered by Muslim astrologers as indicating the birth of the Prophet.[62] The transfer from watery (Scorpio) to fiery (Sagittarius) signs occurred in AD 809 and was similarly associated with important events in the history of Islam.[63] In the same perspective of Muslim history, the next shift, from fiery (Sagittarius) to earthy (Capricorn) signs, could only be the transfer of the year AD 1047, i.e. the conjunction which Casanova assumed to be the one announced by the Ikhwān in *Epistle XLVIII.* The transfer from earthy (Capricorn) to airy (Aquarius) signs must be the one expected for the year AD 1286. From AD 571 to 1286 we have 715 (solar) years, that is only three quarters of the 'great conjunction' by which the transfer comes back to a sign of the same triplicity as the one in which it started. To complete this greater cycle, a new 'middle conjunction' of around 238 (solar) years is needed, which brings us to the year AD 1524, in which, let us note in passing, a famous and feared conjunction of nearly all planets took place in Pisces, accompanied by doomsday predictions.

If one is willing to keep close to the text, the only conclusion we can arrive at is that Casanova's interpretation was fundamentally right, even if he probably went too far in asserting that the proclamation of al-Mustanṣir in AD 1059 hinted at the remarkable sense of anticipation of the Ikhwān. In a brief note to the summary of the *Epistle* as given in his *Riassunto*, Alessandro Bausani similarly favoured, for the fateful transfer predicted by the Ikhwān, a date corresponding to 480 years—that is, twice the time of a 'middle conjunction'—after the one that announced the birth of the Prophet. Yet a regrettable confusion between solar and lunar years led him to choose the year AH 430 (AD 1038-1039) instead of AH 439 (AD 1047): 'Questo [the passage alluding to the transfer] mostra ancora una volta la estrema importanza anche politica che aveva per gli Ikhwān (e per gli ismailiti in genere) l'astrologia. La grande conjunzione (Saturno-Giove) in una triplicita di fuoco avvenne 240 anni dopo la nascita di Muḥammad. Nella triplicita seguente (di terra, cioè degli ani-

[62] See for instance Abū Ma'shar, *On Historical Astrology*, Part II, Chapter VIII. 32 (vol. I, p. 149) and D. Pingree, 'Historical Horoscopes', p. 488.

[63] Cf. D. Pingree, 'Historical Horoscopes', p. 489.

mali/vegetali) dovrebbe avvenire dopo altri 240 anni, quindi circa 480 anni dopo la nascita di Muḥammad ovvero, posta tale nascita a circa 50 anni avanti l'Egira, nel 430 H. = 1038-1039, cioè in epoca abbastanza vicina a quella degli Ikhwān, nel prossimo futuro'.[64]

Here is shown the horoscopic chart for the date of 28 Jumāda I AH 439 (19 November AD 1047) as obtained by modern calculations, using Raymond Mercier's Kairos-program. Saturn and Jupiter are, indeed, in conjunction in the sign of Capricorn, at a moment when Capricorn corresponds to the tenth house from the ascendant (Aries), i.e. 'the house of power and the appearance of eminent people'.[65]

II ♉ 36.7	I ♈ 5.2	XII ♓ 332.4	XI ♒ 301.7
III ♊ 65.9	1047 November 19 Latitude 31;0.0 Hour 14.0 Meridian 49;0.0		X ♑ 273.3 ♄ ♑ 275.2 ♃ ♑ 275.3
IV ♋ 93.3			IX ♐ 245.9 ♂ ♐ 254.4 ♀ ♐ 271.0
V ♌ 121.7	VI ♍ 152.4 ☊ 179.8	VII ♎ 185.2	VIII ♏ 216.7 ☉ ♐ 242.8 ☾ ♏ 232.4 ☿ ♏ 237.0

[64] A. Bausani, *Enciclopedia*, p. 271.

[65] On the theory of twelve houses see Ikhwān al-Ṣafā', *Rasā'il*, I, p. 138 (from *Epistle III: On Astronomy*).

6 *From concealment to manifestation*

Returning to the Legend of the Seven Sleepers, we may now wonder whether its allusions to historical facts concerning Islam fit with the computation they invited their readers to make. The surest way is to start from the year AD 1047 (or AH 439) and count backwards, assuming, as seems most likely to be the case, that the transfer of the conjunction coincides with the awakening of the Sleepers from their long period of sleep. By substracting 354 (lunar) years from AH 439, we get AH 85 for the time from which Islam as conceived by the Brethren would have been forced to enter a period of concealment.

The year AH 85 (or AD 704) does not appear to be recorded by Muslim astrologers as having been particularly rich in celestial omens. No Saturn-Jupiter or Saturn-Mars conjunction occurred in that year, for instance, but it would be only fair to admit that the Brethren could have considered many other kinds of celestial events, such as lunar or solar eclipses. In a passage from their *Epistle on Astronomy*, the Ikhwān regard the eclipses of both luminaries as signs indicating the trials that the prophets had to undergo as part of their mission, and on that occasion they cite examples for each one of the six last Legislators of the cycle, from Adam to Muḥammad.[66] This would surely make sense in such a theory of concealment as the one we are dealing with here. In the absence of more explicit data it would seem better, however, to leave the matter unsolved. After all, the requirement of such an astronomical phenomenon in the year AH 85 is only a conjecture which, to the best of my knowledge, the text itself does not invite us to make.

What can be asserted with confidence, on the other hand, is that the period around that year was one of crucial significance in the history of Islam, and in the history of Shīʿa Islam in particular. In that age the Muslim community was still experiencing, under the powerful government of the Umayyad ʿAbd al-Malik, the painful aftermath of the disaster of Karbalā'. With the multiplication of insurrections, martyrs of Islam accumulated here, there and everywhere. The crisis manifested by the murders of Ḥusayn in AH 61 (AD 680) and of the anti-caliph Ibn al-Zubayr

[66] Ikhwān al-Ṣafā', *Rasā'il*, I, pp. 139-40 (from *Epistle III: On Astronomy*).

a few years later[67] —to name but a few of the opponents of the regime—could easily be interpreted by future propagandists as the sign of the entry of the true Islam into a period of concealment.

And indeed we find in the *Epistle on Politics* the unambiguous assertion that the murder in Karbalā', the ultimate of the misfortunes which affected the People of the House, had been 'the cause of the concealment of the Brethren of Purity, and of the interruption of the empire of the Friends of Loyalty, until God allows the rising of the first, the second and the third of them at the moments at which it is appropriate for them to rise, when they come into view from their Cave and wake up from their long sleep.'[68] The sudden and drastic change seems to be confirmed by this curious fact, already noticed by Marquet,[69] that the *Rasā'il* do not mention any single name in the Umayyad caliphate after Yazīd I, whose rule Ḥusayn and the people of the House had 'refused to obey', as the Ikhwān tell us in yet another passage from *Epistle XLIV* specifically dedicated to the martyrs of Karbalā'.[70] Anyone reading through the entire corpus of the *Rasā'il* would probably arrive at the same conclusion. It is almost as though the authors had rejected as indiscriminately corrupt all the subsequent events in the political and religious history of Islam. *Mutatis mutandis*,

[67] In his book *On the Great Conjunctions* Abū Maʿshar refers to famous events such as the destruction of the Kaʿba and the killing of Ibn al-Zubayr as having been indicated by a Saturn-Jupiter conjunction which occurred in AH 71 (690 AD), around 120 years after the birth of Muḥammad and 60 years after his death; see Abū Maʿshar, *On Historical Astrology*, Part II, Chapter VIII. 32 (vol. I, p. 149): 'The indication of the shift of the conjunction from Libra to Scorpio and its triplicity was for the birth of the Prophet (Upon him be peace!). When the conjunction returned to Scorpio after 60 years, it indicated his death (God bless him and grant him salvation!), and his death was in the third year from the time of the return of the conjunction to the sign to which the shift was. Then the conjunction returned to Scorpio after 120 years, and in that year occurred the destruction of the Kaʿba and the killing of ʿAbd Allāh ibn az-Zubayr'.

[68] Ikhwān al-Ṣafā', *Rasā'il*, IV, p. 269 (from *Epistle L: On Modalities of the Species of Politics and of their Number*). The importance of the passage has been noticed by Y. Marquet, 'Les Cycles', p. 67, but he wants to connect it with dates like 969 AD or 909 AD, which fit no more with the figure of 354 years in our myth than with the theory of astrological transfers.

[69] Y. Marquet, 'La philosophie', p. 567.

[70] Ikhwān al-Ṣafā', *Rasā'il*, IV, p. 33 (from *Epistle XLIV: Where one explains the belief of the Brethren of Purity and the doctrine of the men-of-Lord*).

this is a way of reading the past history of Islam which calls to mind another text familiar with those interested in Islamic astrology. While casting the horoscopes of the most important dates of accession to power, Māshā'allāh, in his *Fī qiyām al-khulafā'* reserved a straightforward 'God curse and dishonour him' to Yazīd ibn Muᶜāwiya, whereas just a few lines before he had praised each one of the *Rāshidūn* with the more usual 'God is pleased with him'. In the year AH 85, the son and successor of Ḥusayn, ᶜAlī ibn Ḥusayn, better known as Zayn al-ᶜĀbidīn ('the Ornament of the Worshipers') in reference to his extreme piety, was clearly not in a position to defy the uncompromising rule of ᶜAbd al-Malik. Rather, classical sources agree in reporting that the fourth Imām—the great-grandson of the Prophet—abandoned to others the leadership of the resistance and retired from whatever could affect a life exclusively dedicated to prayer and mourning.[71] It is a time in which many Shīᶜa Muslims, especially among those who refused to resort to violence, may well have lost their last illusions of playing a role on the forefront. All these idealists could do was wait for better circumstances to show up again.

7 *Conclusion*

Does all this not make a good deal of sense? In order to conclude their *Epistle on Resurrection*, the Brethren of Purity refer to a well-known legend evoking the hopes of pure souls that were forced to hide from the exterior world for some time. But they transform the Qur'ānic version in two significant ways. First, they substitute a period of sleep of their own for the one mentioned in the Sacred Book. Then they superimpose on the original fable a truly fantastic framework of prophetic and astrological history. Taking up the classical Ismaili scheme of cycles made up of seven millenniums, they identify Islam with the last one of those millenniums before the great Resurrection and, as true followers of Hermeticism, choose to connect the 'Muḥammadian law' with the planet Mercury. Quite naturally for Muslim theorisers, they base their astrological history on the conjunctions of Saturn and

[71] See for instance: al-Ṭabarī, *The History*, vol. XXIII, pp. 132-3 ; al-Masᶜūdī, *The Golden Meadows*, V, p. 368. On Zayn al-ᶜĀbidīn see F. Sezgin, *Geschichte*, I, pp. 526-8.

Jupiter and, above all, on the transfer of the two planets in their triplicities. At the time they write, whether one adopts for this the shorter or the longer chronology, the next transfer in AH 439 appears as the most credible date for the return to full manifestation of their own belief and pretensions, the most suitable time for the re-establishment of genuine Islam. The entering into concealment occurred at a time which the Ikhwān in their narrative avoid to determine too strictly, but the figure of 354 (lunar) years combined with the famous prediction of *Epistle XLVIII*, would, I think, have enabled any of their partisans to fix it more precisely.

It would probably be presumptuous to draw from the above interpretation too many inferences on matters as controversial as the dating of composition of the *Rasā'il* or the doctrinal bias of their authors. The present study does not pretend to be more than a minor contribution to our knowledge in those fields. At the same time, it is hoped that it will help to cast some light on the originality of a work which is still too often described as a mere compilation, as well as on the inner coherence, more easily admitted, of this encyclopaedia as a whole. At the end of the myth, which is also the end of the *Epistle on Resurrection*, the Ikhwān tell their reader: 'Understand, my brother, these allusions and these warnings. Bring this into relation with what is similar to them and do not disclose the secrets.' In spite of the numerous issues that are better left undecided with thinkers using a language so deliberately allusive, I think the subject of the present investigation is another illustration of this outstanding coherence.

Bibliography

Abū Maʿshar, *On Historical Astrology. The Book of Religions and Dynasties (On the Great Conjunctions)*, ed. and transl. K. Yamamoto and Ch. Burnett, 2 vols. (i.: The Arabic Original; ii: The Latin Versions), Leiden, 2000. [*On Historical Astrology*]

Bausani, A., *L'Enciclopedia dei Fratelli della Purità. Riassunto, con Introduzione e breve commento, dei 52 Trattati o Epistole degli Ikhwān aṣ-Ṣafā'*, Naples, 1978. [*Enciclopedia*]

Casanova P., 'Une Date astronomique dans les épîtres des Ikhwān aṣ-Ṣafā", in *Journal Asiatique*, series XI, 5, 1915, pp. 5-17. ['Une date astronomique']

Corbin H., *Temps Cyclique et Gnose Ismaélienne*, Paris, 1982. [*Temps Cyclique*]

—, 'Rituel sabéen et exégèse ismailienne du rituel', in *Eranos Jahrbuch*, 19, 1950, pp. 181-246. ['Rituel']

de Callataÿ G., *Annus Platonicus. A Study of World Cycles in Greek, Latin and Arabic Sources*, Louvain-la-Neuve, 1996. [*Annus Platonicus*]

—, *Ikhwān al-Ṣafā'. Les Révolutions et les Cycles (Épîtres des Frères de la Pureté*, XXXVI), Beirut—Louvain-la-Neuve, 1996. [*Les Révolutions*]

Diwald S., *Arabische Philosophie und Wissenschaft in der Enzyklopadie. Kitāb Iḫwān aṣ-Ṣafā' (III) Die Lehre von Seele und Intellekt*, Wiesbaden, 1975. [*Arabische Philosophie*]

Djait H.,*La Grande Discorde. Religion et politique dans l'Islam des origines*, Paris, 1989. [*La Grande Discorde*]

Hamdāni A., 'Abū Ḥayyān al-Tawḥīdī and the Brethren of Purity', in *International Journal of Middle East Studies*, 9, 1978, pp. 345-53. ['Abu Hayyān al-Tawḥīdī']

—, 'An Early Fātimid Source on the Time and Authorship of the *Rasā'il* Ikhwān al-Ṣafā", in *Arabica*, 26, 1979, pp. 62-75. ['An Early Fātimid Source']

—, 'The Arrangement of the *Rasā'il Ikhwān al-Ṣafā'* and the Problem of Interpolations', in *Journal of Semitic Studies*, 29, 1984, pp. 97-110. ['The Arrangement']

—, 'Brethren of Purity, a Secret Society for the Establishment of the Fātimid Caliphate: New Evidence for the Early Dating of their Encyclopaedia', in M. Barrucand (ed.), *L'Égypte Fatimide: son art et son histoire. Actes du colloque organisé à Paris les 28, 29 et 30 mai 1998*, Paris, 1999, pp. 73-82 ['Brethren of Purity']

Honigmann E., 'Stephen of Ephesus (April 15, 448—Oct. 29, 451) and the Legend of the Seven Sleepers', in *Patristic Studies* (Studi e Testi, 173), Città del Vaticano, 1953, pp. 125-68. ['Stephen of Ephesus']

Huber M., *Die Wanderlegende von den Siebenschläfern. Eine literargeschichtliche Untersuchung*, Leipzig, 1910. [*Die Wanderlegende*]

Ibn Khaldūn, *Muqaddima*, transl. F. Rosenthal, *Ibn Khaldūn. The Muqaddimah. An Introduction to History*, London, 1958. [Muqaddima]

Ikhwān al-Ṣafā' wa Khillān al-Wafā', *Rasā'il*, ed. Khayr al-Dīn Zarkalī, Cairo, 1928, in four volumes. [*Rasā'il*, Cairo]

—, *Rasā'il*, ed. B. al-Bustānī, Beirut (Dār Ṣādir), 1957, in four volumes. [*Rasā'il*, Beirut]

—, *al-Risālat al-Jāmiʿa*, Damascus, 1949. [*al-Risālat al-Jāmiʿa*]

Kennedy E. S.-Pingree D., *The Astrological History of Māshā'allāh*, Cambridge Massachusetts, 1971. [*The Astrological History*]

Kennedy E. S., 'The Sasanian Astronomical Handbook Zīj-i Shāh and the Astrological Doctrine of "Transit" (*mamarr*)', in *Journal of the American Oriental Society*, 78, 1958, pp. 246-62. ['The Sasanian Astronomical Handbook']

—, 'Ramifications of the World-Year Concept in Islamic Astrology', in *Proceedings of the Tenth International Congress of History of Sciences* (Ithaca, 26 Oct. 1962—2 Nov. 1962), Paris, 1964, pp. 23-45. ['Ramifications']

Al-Kindī, *On the Rule of the Arabs and its Duration.* (see Appendix III of Abū Maʿshar, *On Historical Astrology*, ed. and transl. K. Yamamoto and Ch. Burnett). [*On the Rule*]

Jourdan F., *La Tradition des Sept Dormants: une rencontre entre chrétiens et musulmans*, Paris, 1983. [*La Tradition*]

Loth O., 'Al-Kindī als Astrolog', in *Morgenländische Forschungen. Festschrift Herrn Professor Dr H. L. Fleischer*, Leipzig, 1875, pp. 261-309. ['Al-Kindī']

Marquet Y., *La philosophie des Ikhwān al-Ṣafā'. Thèse présentée devant l'Université de Paris IV*, le 12 juin 1971, Algiers, 1973. [*La philosophie*]

—, 'Imāmat, résurrection et hiérarchie selon les Ikhwān al-Ṣafā'', in *Revue des Études Islamiques*, 30, 1962, pp. 49-142. ['Imāmat']

—, 'Sabéens et Ikhwān al-Ṣafā'', in *Studia Islamica*, 24, 1966, pp. 35-80 and 25, 1966, pp. 77-109. ['Sabéens']

—, 'Les Cycles de la Souveraineté selon les Épîtres des Ikhwān al-Ṣafā'', in *Studia Islamica*, 36, 1972, pp. 47-69. ['Les Cycles']

—, '910 en Ifrīqiya: une épître des Ikhwān al-Ṣafā'', in *Bulletin d'Études Orientales*, 30, 1978, pp. 61-73. ['910']

—, 'Ikhwān al-Ṣafā'', in *Dictionary of Scientific Biography*, Vol. XV, Suppl. I, New York, 1978, pp. 249-51.

—, 'La détermination astrale de l'évolution selon les Frères de la Pureté', in *Bulletin d'Études Orientales*, 44, 1992, pp. 127-46. ['La détermination']

—, 'Ibn al-Rumī et les Ikhwān al-Ṣafā'', in *Arabica*, 47.1, 2000, pp. 121-3. ['Ibn al-Rumī']

Massignon L., *Opera Minora*, III, Beirut, 1963. [*Opera Minora*, III]

—, 'Les Sept Dormants d'Éphèse (Ahl al-Kahf) en Islam et en Chrétienté: Recueil documentaire et iconographique réuni avec le concours d'Émile Dermenghem, Lounis Mahfoud, Dr. Suhail Unver, Nicolas de Witt et Mathias Zender', published in several parts in the *Revue des Études Islamiques* from 1954 to 1962. ['Les Sept Dormants']

—, 'Les "Sept Dormants": Apocalypse de l'Islam', in *Analecta Bollandiana*, 58, 1950, pp. 245-60 (repr. in Id., *Opera Minora*, III, Beirut, 1963, pp. 104-118). ['Apocalypse']

al-Masʿūdī, *The Golden Meadows (Murūj al-dhahab)*, ed. and French translation by C. Barbier de Meynard, *Les prairies d'or*, Paris, 1869, in nine volumes. [*The Golden Meadows*]

Māshā'allāh, *On the Accession of the Caliphs and the Knowledge of the Accession of Every King (Fī qiyām al-khulafā' wa maʿrifa qiyām kull malik)*, see under E. S. Kennedy —D. Pingree, *The Astrological History.*

Michot J. R., 'L'épître de la résurrection des Ikhwān al-Ṣafā'', in *Bulletin de Philosophie Médiévale*, 16-17, 1974-5, pp. 114-48. ['L'épître de la résurrection']

—, (alias Y. J. Michot), 'Ibn Taymiyya on Astrology. Annotated Translation of Three Fatwas', in *Journal of Islamic Studies*, 11.2, 2000, pp. 147-208. ['Ibn Taymiyya']

North J.D., 'Astrology and the Fortune of Churches', in *Centaurus*, 24, 1980, pp. 181-201. ['Astrology']

Pingree D., *The Astrological History of Māshā'allāh*, Cambridge Massachusetts, 1971 (see under E. S. Kennedy). [*The Astrological History*]

—, 'Historical Horoscopes', in *Journal of the American Oriental Society*, 82, 1962, pp. 487-502. ['Historical Horoscopes']

—, 'Astronomy and Astrology in India and Iran', in *Isis*, 54, 1963, pp. 229-246. ['Astronomy and Astrology']

—, 'Māshā'allāh: Some Sasanian and Syriac Sources', in Id., *Essays on Islamic Philosophy and Science*, Albany, 1975, pp. 5-14. ['Māshā'allāh: Some Sasanian and Syriac Sources']

—, 'Māshā'allāh: Greek, Pahlavī, Arabic and Latin Astrology', in A. Hasnawi—A. Elamrani-Jamal—M. Aouad (eds), *Perspectives arabes et médiévales sur la tradition scientifique et philosophique grecque. Actes du colloque de la SIHS-PAI (Société internationale d'histoire des sciences et de la*

philosophie arabes et islamiques) Paris, 31 mars—3 avril 1993, Paris—Leuven, 1997, pp. 123-36. ['Māshā'allāh: Greek Pahlavī, Arabic and Latin Astrology']

Plutarch (Pseudo-), *Placita philosophorum*, ed. G. Lachenaud, Plutarque. *Œvres morales. Opinions des philosophes*, Paris, 1993. [*Placita philosophorum*]

Qur'ān, *The Holy Qur'ān. English Translation of the Meanings and Commentary. Revised and edited by the Presidency of Islamic Researches, Iftā', Call and Guidance*, Madina: King Fahd Holy Qur'ān Printing Complex, 1410/1989. [Qur'ān]

Roger Bacon, *Opus maius*, ed. J.H. Bridges, Oxford, 1897. [Opus maius]

Sezgin F., *Geschichte des arabischen Schrifttums*, I, Leiden, 1967. [Geschichte, I]

Stern S., 'The Authorship of the Epistles of the Ikhwān aṣ-Ṣafā", in *Islamic Culture* (Hyderabad), 20, 1946, pp. 367-72. ['The Authorship']

—, 'New Information about the Authors of the "Epistles of the Sincere Brethren"', in *Islamic Studies* (Karachi), 3, 1964, pp. 405-28. ['New Information']

al-Ṭabarī, *The History of al-Ṭabarī (Ta'rīkh al-rusul wa'l-mulūk), Vol. XXIII: The Zenith of the Marwānid House*, translated and annotated by M. Hinds (Bibliotheca Persica), New York, 1990. [*The History*]

Tottoli, *I profeti biblici nella tradizione islamica*, Brescia, 1999. [*I profeti*]

Widengren G., 'La Légende des Sept Dormants dans les écrits des Frères Purs', in E. Castelli (ed.), *Démythisation et Idéologie. Actes du colloque organisé par le Centre International d'Etudes Humanistes et par l'Institut d'Études Philosophiques de Rome (Rome, 4-9 janvier 1973)*, Paris, 1973, pp. 509-26. ['La Légende']

Ibn al-Haytham and Eudoxus: The Revival of Homocentric Modeling in Islam

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David Pingree has written the book (actually many) on the transmission of science between cultures. So it seems appropriate to present him with a Persian text for his seventieth birthday, a text describing an astronomical model whose roots go back to fourth-century B.C. Athens, one that managed to get criticized in second-century A.D. Alexandria, that somehow influenced an eleventh-century Cairene transplanted from Basra, that was picked up in thirteenth-century Iran, and then made its way—through channels unknown—to Europe. A convoluted and still puzzling tale to warm the heart of the master!

1 *Introduction*

Ibn al-Haytham (965–ca. 1040) wrote a number of works on astronomy.[1] Historically he was best known for his *Maqāla fī hay'at al-ʿālam* (Treatise on the Configuration of the World), which gained for him considerable renown in both Islamdom and in Europe.[2] In particular, Islamic sources often accorded him the role of having taken Ptolemy's imaginary circles and turning them into solid, spherical bodies. Though this is, at best, dubious history, it did provide a nice story for Islamic astronomers that con-

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[1] Sabra [1972], 197–9, 205–8; Sezgin [1978], 251–61.

[2] Langermann [1990] provides an edition and translation along with a discussion of the influence of the work.

trasted their work with that of their Alexandrian predecessor.[3] It also provided a backdrop for another of Ibn al-Haytham's works, namely his *al-Shukūk ʿalā Baṭlamyūs* (Doubts About Ptolemy).[4] In it, he criticized Ptolemy for certain models (for example those employing the equant) that allowed irregular motions to occur in the heavens.

It is often stated that Ibn al-Haytham (as well as other early critics of Ptolemy) 'did not propose new models to replace those to which they were objecting...' (Saliba [1994], 21) but this turns out not to be the case. A number of years ago, A. I. Sabra [1979] published Ibn al-Haytham's reply to an anonymous critic of a work of his called *Maqāla fī ḥarakat al-iltifāf*, in which Ibn al-Haytham proposed a model to account for the latitudinal motions of the planetary epicycles. This work was evidently written before the *Doubts About Ptolemy*, which is promised at the end of his reply (Sabra [1979], 398 = 207 (Arabic numeration)). Unfortunately, the *Maqāla fī ḥarakat al-iltifāf* itself seems to be irretrievably lost. Its contents, however, can be reconstructed, thanks to a number of later accounts, the most extensive of which turns out to be in Persian. Naṣīr al-Dīn al-Ṭūsī (1201–1274) wrote this exposition in a supplement that he appended to his *Risālah-i Muʿīniyya*, written in 1235 while he was part of the court of the Ismāʿīlī governor of Qā'in in eastern Iran. This Appendix, variously entitled *Ḥall-i mushkilāt-i Muʿīniyya, Sharḥ-i Muʿīniyya*, or *Dhayl-i Muʿīniyya* ('The Solution of Difficulties', 'A Commentary' or 'An Appendix' for the *Muʿīniyya*), is important for a number of reasons, not the least of which is that it is the first instance in which Ṭūsī presents the rectilinear version of his famous 'couple' by which he used an oscillation on a straight line to deal with a number of difficulties in Ptolemaic planetary theory. But it is clear that he had not yet figured out how to produce a curvilinear oscillation on a great circle arc (which he later used in the *Tadhkira* to deal with such problems as Ptolemy's latitude theory), so in the Appendix he instead presented Ibn al-Haytham's model from the *Maqāla fī ḥarakat al-iltifāf* as one possible way to

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[3] See, for example, Ibn al-Akfānī (d. 1348), *Irshād al-Qāṣid*, in Witkam [1989], 57–8 (= 408–407), who is probably following astronomers such as al-Khiraqī (d. 1138–9) ; cf. Ragep [1993], 1: 30–3.

[4] The work has been edited by A. I. Sabra and N. Shehaby [1971]. An English translation is in D. L. Voss [1985].

deal with the problem (Ragep [1993], 1: 65-70 and Ragep [2000]). It is this chapter of the Appendix that is presented below in a Persian edition and English translation.

That Ṭūsī was not satisfied with Ibn al-Haytham's proposal was made abundantly clear in his *al-Tadhkira fī ʿilm al-hay'a*, where he notes that such a model, by retaining Ptolemy's small circles, will produce motion in longitude as well as latitude, thus altering the correct positions of the apex and perigee. He also was dissatisfied with the fact that motion on the small circles is uniform about a point other than the center of the circle, thus making the motion analogous to the irregular motion of the epicycle center on the deferent due to its uniformity with respect to the equant (Ragep [1993], 1: 214-7, 2: 450-2). It is interesting that Ṭūsī indicates similar dissatisfaction with Ibn al-Haytham's model in the *Risālah-i Muʿīniyya*, where he says that 'even with this postulation the irregularity is not ordered, and in addition several other corruptions come into being. But this is not the place to explain them' (Ragep [2000], 125). Nevertheless, he presents Ibn al-Haytham's model without criticism of any sort in the Appendix, thus clearly signaling that, despite having devised and introduced the rectilinear couple in the Appendix, he has yet to produce a model of his own (i.e. the curvilinear couple of the *Tadhkira*) to account for Ptolemy's latitudinal motions.

The model itself is Ibn al-Haytham's attempt to give a physical representation to Ptolemy's 'small circles', which the latter introduced in *Almagest*, XIII.2 to account for latitudinal variations in the epicycles, the so-called deviation and slant (Toomer [1984], 599-600). What is of great interest is that this model turns out to be essentially the same as one that is attributed to Eudoxus of Cnidus (fourth century B.C.). In particular, Ibn al-Haytham proposes to produce the small circles by means of two homocentric spheres that have different axes of rotation, each rotating at the same speed but with opposite angular rotations (see Figure C1). One rotating sphere (viz. KL), of course, could cause a given point to describe the needed circle, but in such a case the entire epicycle would also rotate, thus seriously disrupting the position of the planet. The second sphere (MN), which is contained inside the first and has an axis that always goes through the apex and perigee of the epicycle, would return the epicycle to its correct

position. At least this is the theory. In fact, these two orbs of Ibn al-Haytham correspond to orbs three and four of Eudoxus's planetary models, i.e. the ones that are meant to account for the retrograde motion by means of the 'hippopede' (Figure C2).[5]

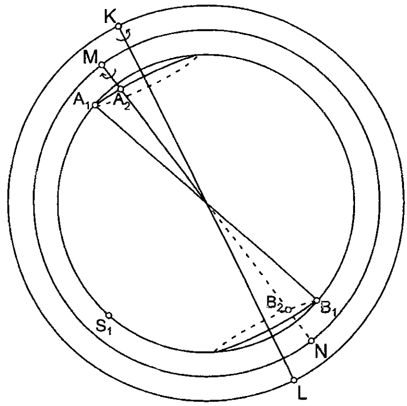


Figure C1

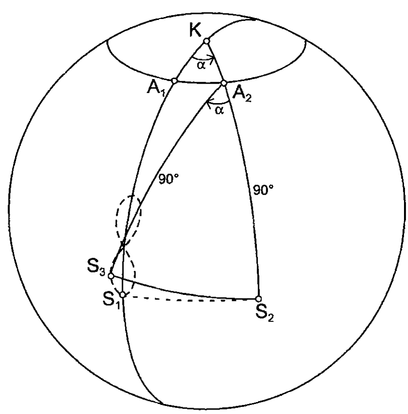


Figure C2

But this consequence of the Eudoxan model plays no role in Ibn al-Haytham's theory and indeed the hippopedal motion of the mean distance S between the epicyclic apex A and perigee B, if acknowledged, would have been an unwelcome complication of an already complex theory, since Ibn al-Haytham's goal is to have sphere MN return all points—other than the apex and perigee—of the epicycle, including S, to the eccentric plane.

[5] For a lucid account of the Eudoxan system, see Heath [1913], 190–211.

Has Ibn al-Haytham been somehow influenced by Eudoxus, most likely through the intermediation of Aristotle who presented Eudoxan planetary theory in Book XII, Chap. 8 of his *Metaphysics*? Ibn al-Haytham himself in his reply to a critic of his original *Iltifāf* treatise states that there is a generic affiliation between Aristotle's *iltifāf* and the *iltifāf* motion that he discusses in his own treatise. But the two are distinctly different, according to Ibn al-Haytham, since 'the latter is not the former, it resembles it only. The proof of this is that the mathematicians do not use it nor mention it, i.e. that which is referred to by Aristotle, since they do not need it and Aristotle does not employ the motion of the epicycle orb, nor does he [even] have a word for it' (Sabra [1979], 401=204 (Arabic numeration)). It is interesting that Ibn al-Haytham does not cite the obvious connection between his model and that of Eudoxus's hippopedal model but instead merely refers to 'Aristotle's' solar model that employs orbs with different axes for the daily motion, ecliptic motion and latitudinal motion (Sabra [1979], 403–402=202–203 (Arabic numeration)). Thus the 'generic' affiliation seems to be that homocentric orbs with different axes are employed both by himself and Aristotle to achieve an *iltifāf* motion. But for Aristotle, this is an 'accidental' result and the end product is a spiral motion with endpoints at the solstices whereas Ibn al-Haytham's *iltifāf* is based on 'specific bodies' that are designed to produce a circular motion on the epicycle orb (Sabra [1979], 401=204 (Arabic numeration)). From this discussion, it is clear that Ibn al-Haytham has not explicitly connected what he is doing with Eudoxus's planetary orbs three and four; there remains, though, the indirect influence from Aristotle of using connected homocentric spheres with different axes to produce a desired result. In the *Doubts* Ibn al-Haytham criticizes Ptolemy's abandonment of the 'small circle' latitude model in the *Almagest* in favor of a two-sphere arrangement in the *Planetary Hypotheses*.[6] But it is curious that he refrains from referring to his own treatise on *Iltifāf*, which undoubtedly was written earlier; this tends to underscore that the *Doubts* is a work of criticism rather than constructive engage-

[6] For an account of the latitude model of the *Planetary Hypotheses*, see Neugebauer [1975], 2: 908–9, 924–5. For the Arabic text of Ibn al-Haytham's criticisms, see Sabra and Shehaby [1971], 43–58; Voss [1985] provides an English translation, 61–78 and discussion, 147–71.

ment.

What is the significance of Ibn al-Haytham's *Iltifāf*? Ultimately, it was a minor work that nevertheless possesses a certain historical significance. For one thing, it is one of several works by early (pre-thirteenth century) Islamic astronomers that proposed new models to deal with a variety of ills they felt they had inherited from the Ancients.[7] It also evidently played an important role in the thinking of Naṣīr al-Dīn al-Ṭūsī , who not only summarized it for his Persian readers but also would later give it a certain prominence by transforming it in the *Tadhkira* into his own curvilinear version of the Ṭūsī couple.[8]

Finally Ibn al-Haytham's text marks an important step in the revival of homocentric modeling in the Middle Ages. The latter is best known in Islam through the work of the twelfth-century Andalusian Nūr al-Dīn al-Biṭrūjī, who sought to abandon the eccentrics and epicycles of Ptolemaic astronomy in favor of a purer, homocentric system in which all the orbs were geocentric. Whether this was inspired by the Eudoxan system has been a matter of some controversy.[9] Perhaps it would help to distinguish between a homocentric model and a homocentric system. An astronomical model (*aṣl* in Arabic; *hypóthesis* in Greek) is a device that can be used for a particular purpose within a larger system (Arabic: *hay'a*). Thus one may speak of an eccentric or epicyclic model but not of an eccentric or epicyclic system. On the other hand, one may refer to either a homocentric model or a homocentric system. The latter would include the systems of Eudoxus and al-Biṭrūjī. One might say that Biṭrūjī's *system* is doubtless inspired by an Aristotelian view that all motion in the heavens must be about a single center. Since Biṭrūjī also knows the Eudoxan system through Aristotle, it does not take much of a leap to conclude, as has E. S. Kennedy, that Biṭrūjī's system owes much to fourth-century B.C. Greek astronomy. On the other

[7] Examples include the various early theories of trepidation and Jūzjānī's eleventh-century attempt to deal with the equant; cf. Ragep [1993], 2: 452.

[8] It is of great interest that Ṭūsī presents his model as a transformation of Ibn al-Haytham's; see Ragep [1993], 1: 216–17, 2: 452–5.

[9] See, for example, Kennedy [1973] and Goldstein [1964] and [1971]. The former held that Eudoxus was the ultimate source of Biṭrūjī's astronomical system, whereas the latter insisted upon an origin in the various Islamic trepidation theories.

hand, this does not necessarily imply that Biṭrūjī knew or understood all the details of Eudoxan astronomy per se; as was the case with Ibn al-Haytham, he could have simply understood that the systems described in Aristotle's *Metaphysics* were homocentric and based upon a series of embedded, interconnected orbs with different axes of rotation. As B. Goldstein has noted, there could have been other sources that Biṭrūjī drew upon for the details of his *models*, and he pointed to several theories of trepidation that employ homocentric models. But as we now know, there are other astronomers before Biṭrūjī, such as Ibn al-Haytham, who were using homocentric modeling for purposes other than trepidation. And we know that this aspect of Ibn al-Haytham's astronomical corpus was influential both in Islam and the Latin West.[10]

2 *Edition and Translation of Chapter 5 of Ṭūsī's Ḥall-i mushkilāt-i Muʿīniyya.*

The edition is based upon 3 manuscripts:

1) F Istanbul, Fatih 5302/4, ff. 204a-205a (722 H./1322 A.D.)
2) K Oxford Or. 208 (Bodl.), ff. 145b-148a (n.d.)
3) M Tehran, Malik 3503, pp. 14–17 (658 H./1260 A.D.); published in facsimile in al-Ṭūsī 1956

Abbreviations used in Apparatus:

F_{ab} = above in MS F
F_{cr} = crossed out in MS F
F_{mr} = in margin of MS F
F_{rp} = repeated in MS F

The Persian text is part of a collaborative effort by Professor Wheeler Thackston, Ms. Sally Ragep and myself to establish the entire *Risālah-i Muʿīniyya* and its *Ḥall.* Thackston has established a preliminary edition of both works based upon MSS F and M, and S. Ragep has been collating this preliminary edition with another nine manuscripts. It has been my responsibility to then establish a 'final' apparatus, text and commentary. In the case of this particular chapter, only the three listed manuscripts

[10] For evidence of the influence of his homocentric models in the Latin West, see Mancha [1990].

of the eleven we have been using contained all of Chapter 5 of the *Ḥall.*

The English translation generally follows the procedures and vocabulary used in my translation of Ṭūsī's *Tadhkira* (Ragep [1993]). This represents a revised version of an earlier translation done a number of years ago by Prof. Thackston and myself. But, as with the edition, all remaining shortcomings are the responsibility of the present author alone.

Glosses are keyed to the translation and are given as footnotes. Only problematic readings or interpretations (such as the mix-up in the order of the orbs) are discussed; a more detailed commentary on Ibn al-Haytham's model and its relation to that of Ṭūsī can be found in Ragep [1993], 2: 450-5.

Figures 1 and 2 have been established from the three manuscripts, which exhibit minor variations. Figure 2 follows the medieval convention of rendering a 3-dimensional effect by 'flipping' the circles horizontally onto the page. (See Ragep [1993], 1: 218-9 for another example.) Figure 2c is an attempt to present the diagram using a more modern perspective.

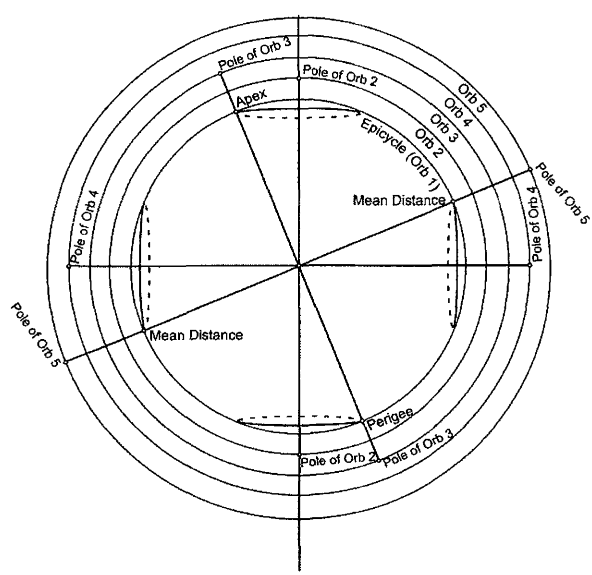


Figure 2C

حلّ مشکلات معینیه در هیئه لنصیر الدین طوسی

فصل ۵

در هیأت افلاك تداویر سیارگان[1]
بر مذهب ابو علی بن الهیثم[2]

این ابو علی[3] از مبرزان علم ریاضی بوذه است و هیأت افلاك بر وجه تجسیم بیشتر مستفاد از سخن اوست . و اورا رسالهٔ است در بیان افلاك تداویر کواکب بر وجهی که این حرکات مختلف از آن صادر شوذ . می‌گویذ هر یکی از[4] کواکب علوی سه فلك تدویر دارند بیکدیگر محیط .

فلك اول که در میان دو فلك دیگر باشد ، فلکی بوذ مصمت که کوکب[5] بر یك طرف او باشد . و آن کره متحرك بوذ بحرکت خاصهٔ کوکب . و منطقهٔ او توهم کنیم که از سطح منطقهٔ خارج مرکز خارج بوذ و با او مقاطع[6] در دو[7] بعد اوسط . پس قطری که بدو بعد اوسط بگذرذ در سطح منطقهٔ خارج بوذ ، و قطری که بذروه و حضیض[8] بگذرذ یك نیمه در جهتی و دیگر نیمه در جهتی دیگر[9]. و چون خطی از مرکز مایل[10] بمرکز تدویر کشند و اخراج کنند تا بر تدویر بگذرذ ، هر آینه با این قطر[11] که بذروه و حضیض گذشته است بر مرکز تدویر تقاطع کنذ . و چون این خط در سطح فلك خارج باشد ، بعد میان این خط و قطر تدویر در ذروه و حضیض بقدر میل ذروه و[12] حضیض باشد برین صورت :

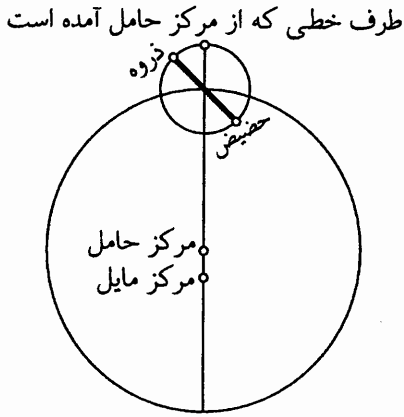


Figure 1A

پس فلك دوم توهم کنیم که بذین فلك محیط باشد و مرکز هر دو فلك[13] یکی بوذ . و این فلك[13] بر دو قطب ازین خط که[14] از مرکز مایل آمذه است ، متحرك باشد بحرکتی شبیه بحرکت مرکز تدویر بر[15] محیط مرکز معدل مسیر . و لامحالة چون این فلك حرکت کند ، و فلك اول را با خوذ ببرذ ، ذروه و حضیض را دو[16] مدار حادث شوذ که مرکز هر یکی از آن دو مدار بر خطی بوذ که از مرکز مایل آمذه است . و آن دو دایرهٔ خرد[17] بوذ که سطح هر یکی با سطح فلك خارج متقاطع بوذ بر زوایای[18] قایمه ، مانند هدفی که قطرش بر محیط سپری نهند . پس دو موضع ازین دایره[19] در سطح فلك حامل بود . و چون ذروه و حضیض بر محیط این دایره[19] حرکت کنند ، هرگاه که بذین دو نقطه رسند در سطح فلك خارج باشند . و چون بر منتصف هر دو نقطه باشند در غایت میل باشند از سطح خارج .

لیکن[20] ازین حرکت فسادی لازم آیذ . و آن چنان بوذ که چون همه تدویر بذان حرکت متحرك بوذ ، قطری که بدو بعد اوسط گذشته باشد از سطح حامل خارج شوذ و دوری[21] بکند که در اثنای آن نصف شرقی از تدویر غربی شوذ و نصف غربی شرقی . پس از جهت دفع این فساد ، فلکی[22] دیگر توهم کنیم که آن فلك سوم باشد محیط باین هر دو فلك چنانکه مرکزش مرکز هر دو فلك بوذ[23] ، و دو قطب او بر دو طرف قطر فلك تدویر بوذ که مارّ باشد بذروه[24] و حضیض ، و حرکت او در خلاف جهت حرکت فلك دوم بوذ ، و هم بمقدار آن حرکت تا آن قدر که منطقهٔ فلك تدویر بحرکت فلك دوم از وضع خوذ زایل شوذ ، این فلك اورا با وضع خوذ برد[25] و قطر بعد اوسط همیشه در سطح فلك خارج بماند ، اما دور[26] ذروه و حضیض بر مدارهای[27] مذکور بر قرار بماند از جهت آن که قطبهای[28] این فلك بر دو طرف قطر فلك تدویر است و قطبهای[29] فلك دوم غیر این[30] دو قطب است ، و میان هر دو قطبی[31] ازین چهار قطب[32] بقدر نصف قطر مدار ذروه یا حضیض[33] . پس ازین حرکات لازم آیذ که نصف ذروه دایما در جهتی بوذ و نصف حضیض در جهتی دیگر مخالف جهت اول ، و در هر[34] دوری از ادوار دو نوبت[35] منطقهٔ تدویر در سطح منطقهٔ خارج آیذ و بگذرذ بر[36] وجهی که جهات متبادل شوذ .

و چنانکه مرکز تدویر محیط مایل را[37] بحرکتی قطع می کنذ[38] بنسبت با مرکز مایل غیر متشابه است بنسبت[39] با

مرکز معدل مسیر متشابه تا در دو ربع که در نصف اوج افتد بطئ باشد[40] و در دو ربع دیگر سریع . همچنان[41] ذروه و حضیض این دو مداررا قطع کنند[42] بحرکتی که بنسبت با مرکز مدار[43] غیر متشابه بوذ و بنسبت با نقطهٔ دیگر غیر مرکز[44] مدار و داخل مدار که بجای مرکز معدل مسیر باشد متشابه بوذ[45] تا سیر ذروه بر محیط این مدار در دو ربع بطئ بوذ و در دو ربع سریع . و هم چنین سیر حضیض نا[46] متشابهست تا سیر مرکز محفوظ بوذ .

و این دو دایرهٔ خرد آنست[47] که صاحب منتهی الادراك در اثنای[48] اثبات اجسامی که مبادی حرکات باشند ایراد[49] کرده است و بر آن اقتصار[50] نموذه . و هر چند باول اثبات این دو دایره بطلمیوس کرده است در مجسطی ، اما چون بطلمیوس در همه احوال بر دوایر اقتصار[51] کرده است ، این باب مناسب دیگر ابواب باشذ . و کسی که در دیگر مواضع اثبات اجسام کند و اینجا بر[52] ایراد دوایر اقتصار کنذ ، شرط مناسبت رعایت نکرده باشد .

و اما در دو کوکب[53] سفلی همین دو فلك زاید[54] بر فلك تدویر جهت میل ذروه و حضیض اثبات می کند . [55] و جهت حرکت انحراف دو فلك دیگر اثبات می‌کند . فلك اول ، که فلك چهارم بوذ ، از افلاك تداویر ایشان محیط[55] بوذ[56] بذان سه فلك . و دو قطب این فلك[57] دو نقطه بوذ از خطی که بمرکز تدویر بگذرذ در سطح فلك حامل و با خطی که از مرکز مایل آمذه باشد بر زوایای[58] قایمه متقاطع بوذ . و چون[59] فلك حرکت کند قطری‌را[60]

که مارّ بوذ بدو بعد اوسط لامحالة بر حوالی این دو قطب حرکت بایذ کرد . پس حرکت انحراف حادث شوذ ، الا آنکه چون همه منطقهٔ تدویر[61] حرکت کند ، ذروه و حضیض از وضع خوذ زایل شود و ذروه بجای[62] حضیض آیذ و حضیض بجای[63] ذروه . پس فلکی پنجم برین چهار فلك محیط بوذ که دو قطبش دو طرف خطی بوذ که بدو بعد اوسط گذشته باشد و حرکت او مخالف حرکت فلك چهارم و مساوی او تا آنچه از موضع خود[64] زایل شوذ با وضع اول آیذ . و دو بعد اوسطرا از[65] حرکت[66] فلك چهارم دو مدار لازم آیذ ، و آن دو دایرهٔ خرد بوذ که با سطح[67] فلك حامل بر زوایای[68] قایمه متقاطع باشذ ، مانند هدفی که بر سپری نهند ، چنانکه[69] هر دو محیط بر نقطهٔ مماس باشند و سطح با سطح متقاطع بر زوایای قایمه[70] . و حرکت این دو بعد بر محیط این دو دایره مختلف در یك نیمه سریع و دیگر نیمه بطئ[71] ، مانند[72] سیر مرکز بر فلك[73] مایل . و چون ذروه در مدار خوذ در غایت میل بوذ از سطح مایل ، بعد اوسط در سطح مایل بوذ . و چون بعد اوسط از[74] سطح مایل در غایت میل بوذ ، ذروه در سطح مایل بوذ تا این دو عرض بر سبیل تبادل لازم آیذ .

و صورت افلاك تداویر این دو کوکب[75] بر حسب[76] آنچه بر سطح توان کشید اینست . و صورت افلاك کواکب علوی هم از اینجا معلوم شوذ چون بر سه فلك[77] اقتصار[78] کنند .

اینست بیان این مقالت ، و الله اعلم بالصواب[79] .

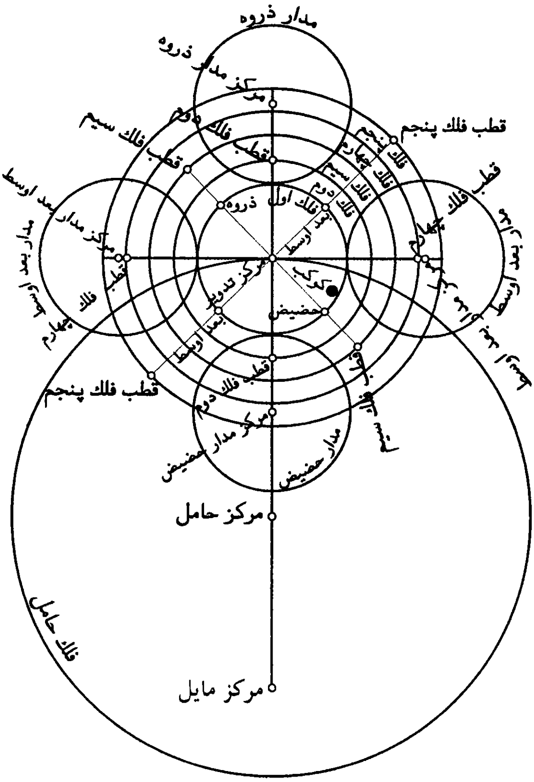

مدار ذروه
مركز مدار ذروه
قطب فلك سيم
قطب فلك پنجم
فلك اول
ذروه
بعد اوسط
مدار بعد اوسط
مركز مدار بعد اوسط
كوكب
حضيض
مركز تدوير
قطب فلك پنجم
مركز مدار حضيض
مدار حضيض
مركز حامل
فلك حامل
مركز مايل

Figure 2a

Text Variants

[1] سیارگان] ستارکان : F . [2] الهیثم] +رحمه الله : F . [3] ابو علی]
مرد : F . [4] از] ازین : K . [5] که کوکب] که ~~فلك~~ کوکب : K .
[6] مقاطع] K_{mr} ، K_{ab} = مطالع : K_{cr} .. [7] در دو] + بروج :
M_{cr} . [8] و حضیض] حضیض: K . [9] دیگر] + افتد : K . [10]
مایل] حامل : K . [11] قطر] قدر : F . [12] این] آن : K . [13] یکی بوذ
و این فلك] –F . [14] که] –M . [15] بر] –K . [16] دو] هر دو : M .
[17] خرد] F_{mr} . [18] زوایای] زوایا : M . [19] در سطح ... محیط این
دایره] –M . [20] لیکن] لکن : M . [21] و دوری] دوری : K .
[22] فلکی] فلك : K, M . [23] بوذ] –F . [24] باشد] باشند : M . [25] برد]
–M = اورد : K . [26] دور] در : F . [27] مدارهای] مدارها : M . [28]
قطبهای] قطبها : M . [29] قطبهای] قطبها : M . [30] این] ان : K . [31]
قطبی] قطب : K . [32] قطب] قطر : M . [33] یا حضیض] تا
حضیض : F = نا حضیض : M = + باشد : K . [34] در هر] در هم
: M . [35] دو نوبت] و نوبت : M . [36] بر] و بر : M .
[37] مایل‌را] حامل‌را : K . [38] کنذ] + که : F, K . [39] بنسبت] و
بنسبت : F, K . [40] بطئ باشد] بطیست : M . [41] همچنان] و
همچنان : K . [42] کنند] کند : K . [43] مدار] K_{rp} . [44] مرکز]
مر~~و~~کز : M . [45] بوذ] باشد : F . [46] نا] تا : F . [47] آنست]
است : F . [48] اثنای] اثنا : F . [49] ایراد] این او : M . [50] اقتصار]
اقتضا : M . [51] اقتصار] اختصار : M . [52] بر] –K . [53] دو
کوکب] کوکب : K . [54] زاید] M_{rp} . [55] و جهت ...ایشان
محیط] و بجهت میل طرف قطر بعد اوسط همجنین فلکی چهارم
توهم کنند که محیط : K . [56] بوذ] –F . [57] این فلك] این

~~دو~~ فلك : K . 58 زوایای] زوایا : K, M . 59 و چون] جون : K
= + این : F . 60 قطری‌را] قطری : F, K . 61 تدویر] + نیز : F .
62 بجای] بحالی : M . 63 بجای] بحالی : M . 64 خود] –M . 65 را از]
راز : M . 66 حرکت] M_{ab} . 67 با سطح] با ~~دو~~ سطح : F .
68 زوایای] زوایا : M . 69 چنانکه] F_{rp} . 70 زوایای قایمه] زوایا :
M . 71 و دیگر] و در دیکر : K = دیکر : M . 72 مانند] باشد :
F . 73 بر فلك] –K . 74 از] در : K, M . 75 دو کوکب]
کواکب : F . 76 حسب] سبیل : F . 77 فلك] + تدویر : F .
78 اقتصار] اختصار : K . 79 بالصواب] –F, –K .

The Solution of Difficulties in the Muʿīniyya [Treatise] on Astronomy by Naṣīr al-Dīn al-Ṭūsī

Chapter Five: 'On the Configuration of the Epicycle Orbs of the Wandering Planets According to the Theory of Abū ʿAlī b. al-Haytham'

This Abū ʿAlī was a prominent mathematician, and the configuration of the orbs as solid bodies is mostly taken from his work.[11] He has a treatise[12] explaining the orbs of the planets' epicycles in such a way that the various motions result from them. He states that each of the upper planets has three epicycle orbs, one enclosing another.

The first orb, which is inside the two other orbs, is a complete solid orb on one side of which is the planet.[13] That sphere moves with the proper motion of the planet. Let us conceive that its equator is in a plane different from that of the eccentric equator, intersecting the latter at the two mean distances. Now the diameter passing through the two mean distances is in the plane of the eccentric equator and the diameter passing through the epicyclic apex and perigee will be in one half in one direction and in the other half in the other direction. If a line is drawn from the center of the inclined [orb] to the center of the epicycle and extended until it reaches the epicycle, then it necessarily intersects the diameter passing through the apex and the perigee at the center of the epicycle. Since this line is in the plane of the eccentric orb, the distance between this line and the diameter of the epicycle [passing] through the apex and perigee is equal to the inclination of the apex and the perigee according to this illustration.[14]

---

[11] As is common among Islamic astronomers and encyclopedists starting at least as early as al-Khiraqī (d. 1138-9), Ṭūsī reiterates the view that Ibn al-Haytham is responsible for putting forward the basic solid configuration (*hay'a*) of the celestial orbs. No doubt this is due to Ibn al-Haytham's *Maqāla fī hay'at al-ʿālam* (Treatise on the Configuration of the World). That Ptolemy had attempted this in the *Planetary Hypotheses*, a work known to Ibn al-Haytham and Ṭūsī, seems not to have made much of an impression (cf. Ragep [1993], 1: 30-3 and the introduction above).

[12] This is Ibn al-Haytham's work whose title was most likely *Maqāla fī ḥarakat al-iltifāf*; see Sezgin [1978], 260 (no. 25) and Sabra [1979], 390.

[13] This first orb, of course, is the 'original' epicycle.

[14] Apparently the apex and perigee in Fig. 1 are meant to be above and below the plane of the paper at the maximal inclination of the epicyclic

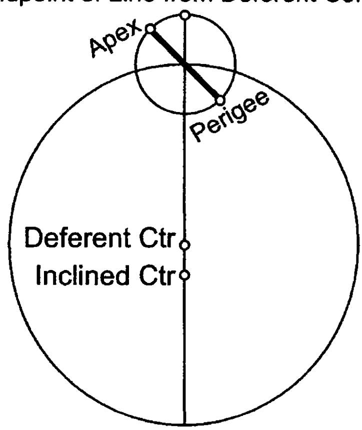


Figure 1B

Let us now conceive the second orb as enclosing this [first] orb, and each of these two orbs shares the same center. This orb, which is on two poles on the line coming from the center of the inclined orb, moves with a motion like that of the epicycle center on the circumference of the equant center [circle].[15] There is no doubt that when this orb moves and carries the first orb with it, then [both] the apex and the perigee describe two circuits each of whose centers is on the line which comes from the center of the inclined [orb]. And these two small circles are such that each of their planes is perpendicular to the plane of the eccentric orb, like a stud whose diameter is on the circumference of a shield. Now two places on [each] circle are in the plane of the deferent orb. When the apex and perigee move along the circumference of [each] circle, they will be in the plane of the eccentric orb whenever they reach these two points. At the mid-point between these two points, they are at the maximum inclination from the plane of the eccentric.

There follows, however, from this motion a distortion, namely that since the entire epicycle moves with this motion, the diameter passing through the two mean distances goes out of the plane

'deviation' (cf. Ragep [1993], 1: 190–3).

[15] This is somewhat misleading. In fact Orb 2 moves the apex and perigee with a motion correlated with the actual irregular motion of the epicycle center on the deferent, not with the idealized uniform motion on the imaginary equant circle. Cf. note 17 below.

of the deferent. As it moves around, the eastern half of the epicycle becomes western and the western half becomes eastern. Then in order to rectify this distortion, let us conceive another orb, which is the third orb enclosing these [previous] two orbs, in such a way that its center is the center of both orbs. Its two poles are at the two end-points of the diameter of the epicycle orb that passes through the apex and perigee. Its motion is in the opposite direction of the second orb's motion but equal to it, so that by however much the equator of the epicyclic orb is displaced from its proper place by the motion of the second orb, this orb brings it back to its proper place, and the diameter of the mean distance always remains in the plane of the eccentric orb. However the revolutions of the apex and the perigee on the above-mentioned circuits remain fixed inasmuch as the poles of this orb are at the two end-points of the diameter of the epicycle orb. The poles of the second orb are different from these two poles. The distance between each two of these four poles is equal to the radius of the circuit of the apex or of the perigee. Therefore from these motions it follows that the apex half is always in one direction and the perigee half is in another direction opposite that of the first. In every revolution the equator of the epicycle passes twice through the plane of the eccentric equator in such a way that the directions interchange.[16]

As the epicycle center traverses the circumference of the inclined [orb] with a motion that is nonuniform with respect to the inclined center, it is uniform with respect to the equant center, so that in the two quadrants that fall in the apogee half it is slower while in the other two quadrants it is faster. Similarly, the apex and the perigee traverse [their] two circuits with a motion that is nonuniform with respect to the circuit's center but uniform with respect to a point other than the circuit's center that is within the circuit in a position [corresponding to that of] the

[16] There is a serious error in the order of the orbs. For the model to actually work, Orb 3 should be contained inside Orb 2 and not the reverse, as is presented here and in Figure 2; otherwise the apex and perigee will not remain on the small circle and the diameter connecting them will not stay aligned with the poles of Orb 3. Whether this was a careless error due to Ibn al-Haytham or Ṭūsī is not clear. Ṭūsī does correct the mistake, without comment, in the *Tadhkira* where he places Orb 3 between Orb 2 and the epicycle (Ragep [1993], 1: 214–5 and 358, Fig. C23).

equant center, so that the movement of the apex on the circumference of this circuit is slower in two quadrants and faster in [the other] two quadrants. And likewise the movement of the perigee is nonuniform so that the motion of center is preserved.[17]

These two small circles are those that the author of *Muntahā al-idrāk* introduced when positing the bodies that are the principles of motion, and he limited himself to that. Even though these two circles were first posited by Ptolemy in the *Almagest*, this chapter is consistent with other chapters inasmuch as Ptolemy limited himself in all cases to circles. [However] someone who in other places posits bodies but here limits himself to circles is not observing the condition of consistency.[18]

Moreover for the two lower planets, he [Ibn al-Haytham] likewise posits two orbs in addition to the epicycle orb to account for the inclination of the apex and the perigee; to account for the motion of the slant, he posits two other orbs. The first orb, which is the fourth orb of their [i.e. Ibn al-Haytham's] orbs for the epicycles, encloses the other three orbs. The two poles of this orb are two points on a line passing through the epicycle center in the plane of the deferent orb and intersecting at right angles the line that comes from the center of the inclined [orb]. When

[17] Though Ptolemy claims that the small circle 'revolves with uniform motion, with a period equal to that of the motion in longitude' (Toomer [1984], 599), it is actually, as Ṭūsī notes here, nonuniform since the motion must be correlated with the motion of the epicycle center on the deferent, which is uniform with respect to the equant center but nonuniform with respect to the actual deferent center. Ṭūsī himself presented this in the *Tadhkira* as part of his criticism of Ptolemy's model (and by implication Ibn al-Haytham's) since this results in the apex and perigee moving nonuniformly on the small circle (Ragep [1993], 1: 212–7). One might well interpret what he says in the *Muʿīniyya* itself as also being a criticism of this aspect of Ibn al-Haytham's model: 'Yet even with this postulation the irregularity is not ordered, and in addition several other corruptions come into being' (Ragep [2000], 125). But here he presents Ibn al-Haytham's model without explicit criticism.

[18] The author in question is Shams al-Dīn abū Bakr Muḥammad b. Aḥmad al-Khiraqī (d. 533 H./1138–9), who, in addition to the *Muntahā al-idrāk fī taqāsīm al-aflāk*, wrote *al-Tabṣira fī ʿilm al-hay'a*, both of which were very influential in the development of the *hay'a* (mathematical cosmography) tradition in Islam; see Ragep [1993], 1: 33, 36. Note the criticism of Khiraqī's inconsistency in presenting circles in some places but physical bodies in others; at least, according to Ṭūsī, Ptolemy was consistent in always using circles in the *Almagest*. (This represents a rather rare apology for the much maligned Alexandrian astronomer.)

the orb moves, the diameter, which passes through the two mean distances, must necessarily move around these two poles. Thus there results the motion of the slant except that since the entire epicyclic equator moves, the apex and the perigee will become displaced from their proper places; the apex goes to the perigee's place and the perigee to the place of the apex. Therefore a fifth orb encloses these four orbs; its two poles are the two end-points of the line that passes through the two mean distances. Its motion is opposite and equal to the motion of the fourth orb so that whatever is displaced from its proper place will return to its original position.[19] Now two circuits for the two mean distances result from the motion of the fourth orb, and these are the two small circles that intersect the plane of the deferent at right angles, like a stud on a shield, such that each of the two circumferences are tangent at a point and one plane intersects the other at right angles. The motion of these two [mean] distances on the circumference of these two circles varies in each half, being faster in one half, slower in the other, corresponding to the motion of center on the inclined orb. When the apex on its own circuit is at the maximum inclination from the inclined plane, the mean distance is in the inclined plane. And when the mean distance is at the maximum inclination from the inclined plane, the apex is in the inclined plane. Thus it follows that these two latitudes are inverses of one another.

The illustration of the orbs of the epicycles of these two planets, to the extent they can be drawn in a plane, is as follows. The illustration of the orbs of the upper planets may also be known from [the following] since they are limited to three orbs.

This is the exposition of this treatise, and God is the Knower of Truth.

[19] The same criticism regarding the ordering of Orbs 2 and 3 also applies here to Orbs 4 and 5, which should be reversed. See note 16 above.

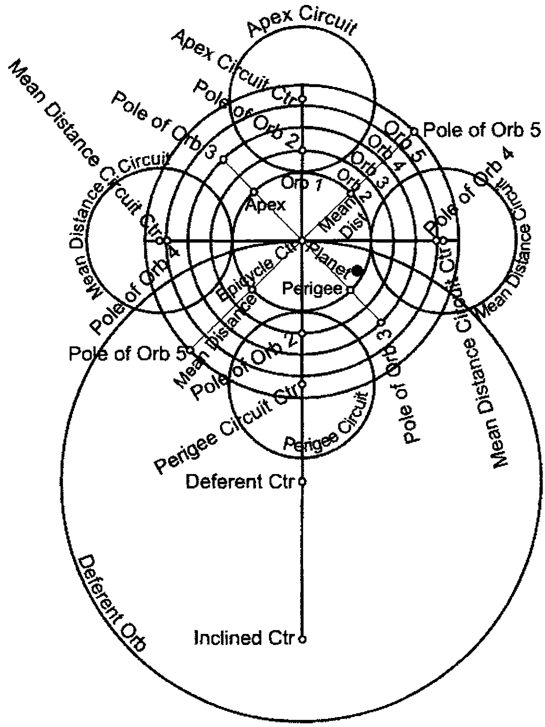


Figure 2B

Bibliography

Goldstein [1964]: Bernard R. Goldstein, 'On the Theory of Trepidation according to Thābit b. Qurra and al-Zarqāllu and its Implications for Homocentric Planetary Theory', *Centaurus*, 10, pp. 232–47.

——. [1971]: *Al-Biṭrūjī: On the Principles of Astronomy*, ed., trans., comm. by Bernard R. Goldstein, 2 vols., New Haven.

Heath [1913]: Thomas L. Heath, *Aristarchus of Samos, the Ancient Copernicus: A History of Greek Astronomy to Aristarchus Together with Aristarchus's Treatise 'On the Sizes and Distances of the Sun and Moon', a New Greek Text with Translation and Notes*, Oxford. Reprinted 1959.

Kennedy [1973]: E. S. Kennedy, 'Alpetragius's Astronomy', *Journal for the History of Astronomy*, 4, pp. 134–6.

Langermann [1990]: *Ibn al-Haytham's 'On the Configuration of the World'*, ed. and trans. Y. Tzvi Langermann, New York.

Mancha [1990]: José Luis Mancha, 'Ibn al-Haytham's Homocentric Epicycles in Latin Astronomical Texts of the XIVth and XVth Centuries', *Centaurus*, 33, pp. 70–89.

Neugebauer [1975]: Otto Neugebauer, *A History of Ancient Mathematical Astronomy*, 3 parts, New York.

Ragep [1993]: F. Jamil Ragep, *Naṣīr al-Dīn al-Ṭūsī's Memoir on Astronomy (al-Tadhkira fī ʿilm al-hay'a)*, 2 vols., New York.

——. [2000]: F. Jamil Ragep, 'The Persian Context of the Ṭūsī Couple', in *Naṣīr al-Dīn al-Ṭūsī: Philosophe et Savant du XIIIe Siècle*, eds. N. Pourjavady and Ž. Vesel, Tehran, pp. 113–30.

Sabra [1972]: A. I. Sabra, 'Ibn al-Haytham', in *Dictionary of Scientific Biography*, VI, New York, pp. 189–210.

——. [1979]: A. I. Sabra, 'Ibn al-Haytham's Treatise: Solutions of Difficulties Concerning the Movement of *Iltifāf*', *Journal for the History of Arabic Science*, 3, pp. 422–388 (=183-217).

Sabra and Shehaby [1971]: Ibn al-Haytham, *Al-Shukūk ʿalā Baṭlamyūs*, eds. A. I. Sabra and N. Shehaby, Cairo.

Saliba [1994]: George Saliba, *A History of Arabic Astronomy*, New York.

Sezgin [1978]: Fuat Sezgin, *Geschichte des arabischen Schrifttums*, VI: Astronomie, Leiden.

Toomer [1984]: Ptolemy, *Almagest*, trans. and comm. Gerald J. Toomer, New York.

al-Ṭūsī [1956]: Naṣīr al-Dīn al-Ṭūsī, *Ḥall-i mushkilāt-i Muʿīniyya*, facsimile of Tehran, Malik 3503, introduction by Muḥammad Taqī Dānish-Pizhūh, Tehran, 1335 H. Sh.

Voss [1985]: Don L. Voss, 'Ibn al-Haytham's "Doubts Concerning Ptolemy": A Translation and Commentary', Ph.D. dissertation, University of Chicago.

Witkam [1989]: Jan Just Witkam, *De Egyptische Arts Ibn al-Akfānī*, Leiden.

Reform of Ptolemaic Astronomy at the Court of Ulugh Beg

GEORGE SALIBA

1 *Introduction*

In the third issue of the journal *Arabic Sciences and Philosophy*, I have published a critical edition, with a translation and commentary, of a treatise written by ʿAlāʾ al-Dīn al-Qushjī sometime between the years 1420 and 1449, in which Qushjī developed a critique of the Ptolemaic model for the planet Mercury and gave his own detailed alternative to that model.[1] In that article, I also explained all the technical details connected with that model. I demonstrated, for example, that as far as the observational qualities of the Ptolemaic model were concerned, i.e. by considering the observational points where the Ptolemaic model was tested, Qushjī's model could account for those observations in exactly the same manner the Ptolemaic model did. At all other points in between, where there were no Ptolemaic observations to compare the two models, Qushjī's model was nevertheless shown to differ from the projected positions of the Ptolemaic model by a maximum value of five minutes of arc for an observer placed on the earth. In the same context, I have asserted that as far as medieval astronomers were concerned, a variation of five minutes of arc was well within their tolerable limits. Those limits would have even allowed for a variation of as much as ten minutes of arc, if not more. But most importantly I used the evidence of that treatise to assert that the discussion of alternatives to Ptolemaic astronomy did not cease with the astronomers of the Marāgha school, but continued well into the fifteenth and sixteenth centuries with Qushjī as a brilliant example of that continuing tradition.

[1] 'Al-Qushjī's Reform of the Ptolemaic Model for Mercury', *Arabic Sciences and Philosophy*, 3, 1993, pp. 161-203.

In this paper, I will review very quickly the results already established in the above-mentioned article, introducing some new evidence to support the claim of continued interest in non-Ptolemaic astronomy during the fifteenth century and thereafter, and will then devote the rest of the paper to a discussion of the importance of this evidence, and the image it paints of the intellectual conditions prevalent at the court of Ulugh Beg in particular. In other words, I would like to contextualize Qushjī's treatise and reconsider its ramifications for our understanding of the history of Arabic astronomy in the fifteenth century, as well as for our current periodization of the more general intellectual Islamic history, and in particular the tradition that dealt with the reform of Ptolemaic astronomy. I hasten to say at this point that I use the term 'Arabic astronomy' in this context only to designate the language of Qushjī's treatise, as well as the language of other treatises like it which were written during this century and the centuries following it, all dealing with theoretical planetary theories, especially of the non-Ptolemaic type. The reasons for this linguistic phenomenon have already been discussed in a separate context, and I need not repeat them here.[2]

2 *The Model for the Planet Mercury*

Without repeating the technical details of Qushjī's reform of the Ptolemaic model for Mercury—for that was already done in the article cited above—I should note here that in his new model, Qushjī made use twice of a theorem that was first developed by Mu'ayyad al-Dīn al-ʿUrḍī (d. 1266) of Damascus, but in a fashion not anticipated by al-ʿUrḍī or anyone else before. Qushjī did not, for example, change the direction of motions, as was done by al-ʿUrḍī in his own model for Mercury, and only added two small epicyclets functioning just like the small epicyclet introduced by al-ʿUrḍī in his own model for the upper planets.

As was already stated in my previous article, Qushjī's re-

[2] See, for example, my discussion of the phenomenon of theoretical astronomical texts dealing with planetary theories being written in Arabic, even when the authors themselves would have spoken Persian at home, in 'Persian Scientists in the Islamic World: Astronomy from Maragha to Samarqand', in *The Persian Presence in the Islamic World*, eds Richard G. Hovannisian and Georges Sabagh, Cambridge, 1998, pp. 126–46.

form achieved a remarkable success by its ability to preserve uniform motion for all the spheres concerned, while requiring that all spheres moved around an axis that passed through their centers and still managing to achieve a perfect match with the same Ptolemaic observations.

In general, then, one can safely assert that Qushjī's model did indeed satisfy the axiomatic requirement of uniform motion, and at the same time, also accounted for all the observations which were recorded by Ptolemy without any variation at all.

3 *The Importance of Qushjī's Model*

Now that the technical success of Qushjī's model has been demonstrated, let us turn to the assessment of this model in the light of what we already know about the development of planetary theories in Arabic.

First, it is not new to say that the period when Qushjī and his colleagues were working is generally considered to have been a period of general decline in Islamic intellectual history, and a decline in Islamic science in particular. People who work on the Ottoman or Safavid sciences, for example, know very well of the negative effects such a characterization has had on their fields of inquiry. With the new scientific evidence now preserved in the work of such people as Qushjī and others who preceded him and followed him, going well into the sixteenth century, such a periodization will have to be drastically revised, if not abandoned altogether.[3] In the face of this new evidence, and there is a lot of it that cannot all be recounted here, one can no longer say, as it is often said, that the rise of Islamic orthodoxy, ushered in by the attack of the twelfth century Ashʿarite Abū Ḥāmid al-Ghazālī (d. 1111) against the philosophers was indeed responsible for the age of decline in Islamic science where no such new and original material would have been expected. I will return shortly to this relationship between Islam as a religion and the signs of originality in planetary theories.

But before I do so, let me try to describe the conditions at

[3] I have argued this point in more detail in *A History of Arabic Astronomy: Planetary Theories During the Golden Age of Islam*, New York, 1994, pp. 13–19.

the court of Ulugh Beg under which this work of Qushjī was produced, in order to gain a better understanding of the social and intellectual conditions at that court. From Qushjī's biography, aptly recorded by Ṭasköprülü-Zāde (d. 1561) ,[4] we know that his father Muḥammad was the falconer of Ulugh Beg, thus his name Qushjī/u. We also know that Qushjī the younger studied the mathematical sciences with both Ulugh Beg himself, and with Mūsā b. Qāḍī Maḥmūd, known as Qāḍīzādeh al-Rūmī (d. ca. 1440) . Qushjī admitted that much in the introduction to his own treatise on the model for Mercury, where he acknowledged Ulugh Beg's favors to him by saying that Ulugh Beg had taught him the mathematical sciences when he was young. Considering the relative ages of Ulugh Beg, who was born in 1394, and Qushjī who was probably born around 1400, for he died in 1474, their relationship must have been more like companions, one of them being about 6 years older than the other. This despite the fact that Ulugh Beg refers to Qushjī in the introduction of his own *Zīj-i Jadīd Sulṭānī* as 'the young lad'. In that *zīj* he was probably using the royal prerogative to refer to everyone else as the young lad, despite the proximity of age. What is certain however is that Qushjī grew up at the court of Ulugh Beg, and that Ulugh Beg himself was interested in instructing him in the mathematical sciences, and was apparently interested as well in the non-Ptolemaic direction that astronomy was taking, as we shall soon see. I shall also point out that this relationship between Ulugh Beg and Qushjī was known to all who knew them at the time.

Qushjī's biography goes on to say that at some point he had to leave the court and seek teachers in Kirman. From the introduction to the treatise on the model for Mercury Qushjī insinuates that his departure was instigated more by court intrigues than by the need to seek new teachers, which is not really surprising, since we know that Ulugh Beg himself was eventually killed by his own son in 1449 as a result of such intrigues. Ṭasköprülü-Zāde continues to say that on his return from Kirman, Qushjī brought to Ulugh Beg a treatise on the solution of the equant

[4] Ṭasköprülü-Zāde, 'Iṣāmu d-dīn Ebu l'Khayr Ahmed Efendi, *Es-Seḳā'iḳun-Nu'mānīye fī 'Ulemā'i d-Devleti l-'Os̱mānīye*, ed. by Ahmed Subhi Furat, Istanbul, 1985 (hereafter *TZ*), pp. 159–62.

problem of the Moon (*sic*)—a problem very similar to the one just described with respect to the planet Mercury. Ulugh Beg was supposed to have asked Qushjī to read the treatise to him, right there and then, to which request Qushjī complied and so earned the admiration of Ulugh Beg.

In the same article which contained the edition of Qushjī's reform of the Mercury model, I ventured to say that the information in Ṭasköprülü-Zāde regarding the reform of the lunar model is actually erroneous, and the treatise mentioned by the biographers was in all likelihood the very same one dealing with the reform of the Mercury model. I came to this conclusion because (1) Ṭasköprülü-Zāde did not mention the well-attested treatise on the reform of the Mercury model, which formed the subject of the study mentioned above, and (2) because the two models are so close that they could be legitimately confused by someone like Ṭasköprülü-Zāde, who did not have the technical abilities to tell the difference.

There is no doubt, therefore, that the court of Ulugh Beg entertained such discussions as the reform of Ptolemaic astronomy, at least as far as the reform of the planet Mercury is concerned and for which we have this detailed documentation. This fact was until recently unknown from any other source. We had originally thought that with the production of the two famous Persian *zīj*es in that court, namely, the *Zīj-i Khāqānī* of Jamshīd b. Masʿūd Ghiyāth al-Dīn al-Kashī (d. 1436) and the just mentioned *Zīj-i Jadīd Sulṭānī* of Ulugh Beg himself, the interest in that court was more in observational astronomy than in the theoretical planetary theories dealt with in Qushjī's treatise on Mercury. One should also not underestimate the importance of the evidence Qushjī's treatise could bring to bear on Marāgha school studies and the impact of that school on later astronomers, now documented well into the fifteenth and sixteenth centuries if not beyond.

Qushjī's treaties raises yet another important question. Of all the treatises on non-Ptolemaic astronomy that have so far been subjected to serious study, we find that most of them were produced in circles slightly removed from the direct centers of political power, unless one would want to raise the debatable point that the studies that were produced at the Marāgha observatory were indeed produced at a center of political power. With Qushjī's

treatise we have no ambiguity: here we have the patron himself, in this case Ulugh Beg, encouraging such study and expressing his eagerness to hear of the results of Qushjī's research as soon as he learned of those results.

The fact that a political court, at its highest level, would be interested in such kind of studies is a phenomenon worthy of much more attention than the attention we could devote to it in this limited space. And the fact that we can find additional supporting documentation for that interest is even more interesting and some of that documentation will be reviewed in the sequel.

The only similar cases that come to mind, namely, of texts on theoretical planetary theories dedicated to patrons in political power, are those of the Persian *Risāleh-i Muʿīnīye*, of Naṣīr al-Dīn al-Ṭūṣī (d. 1274) which was apparently composed for Muʿīn al-Dīn (c. 1235), the son of the Ismāʿīlī governor of Qūhistān, and the Arabic *al-Tuḥfa al-Shāhīya* of Quṭb al-Dīn al-Shīrāzī, which was dedicated to Amīr Shāh (d. 1285), the governor of Shīrāz. But in these two latter instances we have no information whether Muʿīn al-Dīn or Amīr Shāh ever understood the subject of the two works dedicated to them or were capable of appreciating them in the same fashion as Ulugh Beg did. From that perspective, we can only say that Qushjī's treatise was probably the only treatise on some aspect of planetary theories that is known to us and that was recited for a political patron, and that the patron was knowledgeable enough to understand it.

This result is very significant for, as I had argued at a different occasion, political patrons were usually interested in that genre of astronomy which would serve astrology, and thus restricted themselves to patronizing *zīj*es and the like which were essential for astrological computations.[5] At least this was the case for the production of the *Ilkhānī zīj* at Marāgha, for which the observatory was built in the first place. All the other non-Ptolemaic astronomical texts produced at that observatory came as an additional bonus, and the political patron had probably never heard of them. I have also argued that the distinction drawn between astrology and astronomy which must have taken place sometime during the eleventh century, if not before, can also explain why most of the *zīj*es written in the Iranian domain were written in

[5] Saliba, *A History of Arabic Astronomy*, pp. 32–9.

Persian, to be used by the less educated astrologers at the court, while the theoretical astronomical works dealing with planetary theories, written in the same domain and in the same period, were written around the school system and thus continued to be written in Arabic. The population of such schools could easily understand Arabic, for that language was used as part of their religious curriculum. I am told that this situation still prevails in Iran till this day.

With the example of Qushjī's treatise, it becomes obvious then that such distinctions can not be so clear cut, and that an intelligent patron such as Ulugh Beg could indeed find himself interested in planetary theories, as well as in the *zīj*-type literature. His interest in planetary theories also demonstrates that he must have had an excellent command of Arabic, a fact not very well known either, but which can be easily documented, as we shall soon see.

4 *New Evidence of Interest in Non-Ptolemaic Astronomy at the Court of Ulugh Beg*

Now, all of a sudden, it seems that we know much more about Ulugh Beg's intellectual interests than we used to think before. For, in addition to this treatise of Qushjī, other evidence has more recently come to light that documents this interest of Ulugh Beg on a larger scale still. In a rarely used text written by the Mulla Fatḥ Allāh al-Shirwānī (d. ca. 1486)[6] as a commentary on Naṣīr al-Dīn al-Ṭūsī's *al-Tadhkira fī al-Hay'a*,[7] we are told that Ulugh Beg used to visit the school he had built at Samarqand on a regular basis, two to three times a week, and would attend the classes of Qāḍīzāde al-Rūmī, together with the other teachers of the school. The classes were obviously conducted in Arabic, for the sole text that was read in astronomy was apparently the commentary of Niẓām al-Dīn al-Nīsābūrī (c. 1311) on the same *Tadhkira* of Ṭūsī, which was also written in Arabic. During those visits Ulugh Beg used to interrupt the students and seek commentaries from them on the astronomical work that they were read-

[6] See *TZ*, pp. 107-8.

[7] See J. Ragep, *Ṭūsī's Memoir on Astronomy*, New York, 1993, pp. 59 and 62–3. According to *TZ*, p. 108, Shirwānī died in the early years of the rule of Mehmed the Conqueror, i.e. around 1453 A.D.

ing. We are also told by Shirwānī that the teacher of that class was none other than the famous Qāḍīzāde al-Rūmī who was also involved in the observations for the *Zīj-i Jadīd Sulṭānī*. Because of the importance of this information, I shall quote Shirwānī's report in some detail, in order to give the reader the flavor of the original, as well as to allow him/her to appreciate its full nuances. More importantly, this report sheds direct light on the type of evidence we now have about the intellectual environment created by Ulugh Beg, about Ulugh Beg's knowledge of Arabic, about the extended life of Marāgha research, and finally the type and manner of education that was conducted in the celebrated Ulugh Beg school of Samarqand. Needless to say this education was obviously conducted in Arabic, as the only evidence for texts used in those classes were texts that were written in Arabic and were not known to have been translated into Persian, or any other language.

At one point in his commentary on Ṭūsī's *Tadhkira*, namely, when Ṭūsī makes a statement regarding parallel straight lines in the same plane versus the non-straight lines that could also be called parallel, al-Shirwānī says the following:

وكنت من السامعين لشرح النظام على شيخي وأستاذي قاضي زاده الرومي، ذلك الإمام الذي لم تسمح بمثله الأدوار ما دار الفلك الدوار بسمرقند عند قراءة بعض أعالي مجلسه الشريف معروفًا بالذكاء والعلاء في مدرسة واليها السلطان ابن السلطان الغ بيك گورگان بعد أن كنت قرأت شرح السيد [الشريف الجرجاني؟] شهاب الثاقب على السيد الأيّد أبي طالب بمشهد الإمام علي الرضا رضي الله عنهم أجمعين فقال السلطان ذالك الفيلسوف يومًا وقد حضر مجلس الدرس وكان يحضره في أسبوع يومًا أو يومين ووصل الدرس إلى هذا المحل : لِمَ قال " الكاينة في سطح " وما الفائدة في التقييد به؟ فبادر أحد المدرسين في مدرسته وكانوا يحضرونه مع حضوره إلى الجواب وقال لأن أقليدس برهن في كتابه، الخ . .

And I was among the listeners to the commentary of al-Niẓām [al-Nīsābūrī] under the tutorship of my teacher and professor

> *Qāḍīzāde al-Rūmī, that master whose likes the [celestial] revolutions have not yet produced as long as the orbs have been revolving, in Samarqand, at the reading of one of the most distinguished in his noble assembly, known for his intelligence and excellence at the school of the Sultan son of the Sultan Ulugh Beg Gurgān. And after hearing the explanation of al-Sayyid [al-Sharīf al-Jurjānī?], the brilliant spark, under the tutorship of the mighty Sayyid Abu Ṭālib at the shrine of al-Imām ʿAlī al-Riḍā [at Meshhed], may God be pleased with all of them, that Sultan philosopher said one day, when he had visited the class, which he used to visit one or two days a week, and the class had reached this point [i.e. the parallel lines]: Why did he say 'occurring in a plane', and what is the use of that restriction. One of the teachers of the school—for they all used to attend [the class] when he [i.e. the Sultan] did—responded and said: Euclid had proved in his book, etc.*

Shirwānī went on to say that the answer given by the teacher was not accurate. And he knew that because he used to read the *Elements* of Euclid during his vacation days (*fī ayyām al-ʿuṭla*), and he knew that the Sultan Ulugh Beg himself was 'well-versed in that book' (*mustaḥḍiran li-dhālika al-kitāb*). Shirwānī said that he did not interrupt the speaker at that time in order that everyone would hear his erroneous answer, knowing very well that the Sultan would return to this point time and again, for 'it was the Sultan's habit to return to the quest twice and three times' (*idh kāna min daydab al-sulṭān wa-da'bihi an yaʿūda ilā al-taftīsh thāniyan wa-thālithan*), as if he himself was examining those who were in his school. This despite the fact that he had already appointed a specific teacher to perform such exams, as we are told by Shirwānī. The report went on to say that when the Sultan returned to the same point, and the teacher repeated the same answer, it was then that Shirwānī had to declare that the answer was erroneous, a gesture that caused the Sultan to lift his head on account of the fact that the reader was blocking the Sultan from his view. At this juncture Shirwānī specifically mentioned that the reader, the Sultan and the teacher were all seated near one another in the shape of a triangle whose side was about one cubit. A remarkable statement worth a thousand paintings, as it gives us a clear image of the intimacy with which such discussions were taking place.

Shirwānī concluded by saying that, when he gave the correct answer to the question, the Sultan confirmed it and said so to

all those who were standing in his presence. Most of those who were standing were among the distinguished scholars who used to accompany him. At that point the Sultan departed, and Shirwānī was appointed as the class reader from then on. When Qāḍīzāde returned from the company of the Sultan, as he saw him out, he said to Shirwānī with a smile: 'have you read the part of Euclid that deals with solid figures'? To which Shirwānī responded by saying that he read what he read thanks to the teacher in that assembly, but did not read the last three treatises for he did not own a copy of that book. Qāḍīzāde then ordered the first of his servants that he saw to bring the book to Shirwānī, who in turn copied it fully and thus finished his reading of Euclid's *Elements*. Shirwānī concluded the story by saying that he continued to read Niẓām's commentary with Qāḍīzāde for five more years, when his father finally came to take him to Shirwān at the request of the potentate of that district. At that time Qāḍīzāde gave Shirwānī a certificate to that effect (*ajāzanī*), the text of which is fully preserved in the same commentary of Shirwānī.[8]

This long story describes very clearly what was going on at the school and court of Ulugh Beg. First, we learn from it that Qāḍīzāde was indeed teaching theoretical astronomy at that school. For a text, he was using Nīsābūrī's commentary on Ṭūsī's *Tadhkira*, and that text was in Arabic. The best of the students would apparently read it aloud, and answer questions, while the others listened. We also learn that Ulugh Beg was almost a regular attendant at that school and obviously must have known Arabic in order to participate in such discussions. Furthermore, he was also apparently concerned at least with that kind of education, and would interfere in the class discussion, and ask his own questions to solicit commentaries from the students or anyone present. In this specific instance, as reported by Shirwānī, the answer was volunteered by one of the other teachers. We learn as well that all the teachers would attend the class when Ulugh Beg was present at a class discussion. And finally, we learn that such discussions and texts were apparently studied elsewhere under other teachers, as in the case of the text of al-Sharīf al-Jurjānī (c.

[8] I intend to publish the full text of the report and the certificate which is still preserved in the commentary of Shirwānī, e.g., Süleimaniye Library, Damat Ibrahim 847, fols 14v–16r (forthcoming).

1413) being studied at the shrine of Imām ʿAlī, presumably at Meshhed, where Shirwānī had apparently read it, as he explicitly told us.

On another occasion, Shirwānī shared with his readers the account of another visit to the same school by Ulugh Beg. I will also quote this account in some detail:

وبعد أن قرأت هذا المحل من شرحه بيومين إذ أخبر الأستاذ أن السلطان المذكور رحمهما الله توجّه إلى مجلس الدرس فقال لي اقرأ اليوم درس أمس الأمس إذ لا إشكال فيما بعده من الدرسين · وكان السلطان ينقبض من درس لا إشكال فيه · وكنت قد تعمّقت فيه بعد القراءة حتى سنح لي فيه شيء على الشارح · فلمّا وصلت القراءة إلى قوله فإن لم يكن فيه أوجب تحريكه التقاطعين بقدر تحريك الأوج ، وهو محل السنوح ، [و١٠٩و] صدّقه فيه من تلامذة السلطان من كان يعرف بعلي القوشجي وكان يتكلم قائمًا · قلت : بل فيه بحث ·

Two days after I had read this part of his commentary, the teacher reported that the Sultan, may the lord have mercy on both of them, was on his way to the class and thus asked me to read the lesson of the day before yesterday, for he said that there was no problem in the two lessons that followed it. The Sultan used to feel unhappy when there were no problems (ishkāl) in the lesson. After reading it, I had already studied the problem very well until something new had occurred to me against the commentator. When the reading had reached his statement 'if it is not in it, then it must move the two nodes by the same amount as the apogee', that being the point where I had the new idea, one of the students of the Sultan, one known by the name of ʿAlī al-Qushjī, who spoke while standing, confirmed it. I then said: This bears discussing.

The story went on to report that the teacher Qāḍīzāde had asked him to think about that point some more, obviously to avoid embarrassment on account of the conflict with Qushjī's opinion on the subject, and ended up by demonstrating how once more Shirwānī was proven right when the same discussion was brought up again at the house of the teacher on a vacation day. All those who used to blame and envy him (presumably for speaking so rashly), turned to blame themselves, when the teacher himself finally admitted that the same point was indeed taken against Nīsābūrī. But the importance of the story lies in the fact that it

demonstrates the vitality of the discussion and the disagreements that would inevitably arise in what must have looked like a very advanced seminar on theoretical astronomy.

Here again, we are told, in no uncertain terms, that Ulugh Beg was very much involved in the discussions of these theoretical matters in his school, that he was fully cognizant of the contents of each class, and that he would sometimes bring along his very own student Qushjī. From the way Shirwānī refers to Qushjī we can surmise that Qushjī was not then a regular student at the school. At the time when Shirwānī was reporting Qushjī was probably already in his forties, or very close to that, and in all likelihood did not need any more schooling. He may have been one of the dignitaries mentioned in the first report, who usually accompanied Ulugh Beg on his school visits. But the importance of his presence is to assert once more the general interest in non-Ptolemaic astronomy among the members of the court where Qushjī was functioning, as well as the members of the school that was being visited by those courtiers almost on regular basis.

When taken together, these two reports, that of Qushjī's treatise and of Shirwānī's report in his own treatise on non-Ptolemaic astronomy, give us a fair description of the circumstances of education at the school of Ulugh Beg, and inform us very clearly about the subject matter of the classes. When Shirwānī was still a student at that school, we are told that the course on theoretical astronomy lasted for at least five years. The most exciting topics in that field were those that touched on novel ideas emerging from critical readings of the texts. Knowing that all the commentaries occasioned by Ṭūsī's *Tadhkira* indeed represented a continuous tradition in which Greek theoretical astronomy was severely criticized, one can assume that the spirit of criticism and re-evaluation was alive and well at the court of Ulugh Beg and in his school, a fact not at all reflected in the two *zīj*es composed under his patronage. In the second report Shirwānī rendered the excitement of the new research by speaking at one point about 'the class sessions burning with research and discussion' (*al-majlis al-mushtaʿil bi-nār al-baḥth wa-l-bayān*).

5 *Concluding Remarks: The Age of Decline and Science versus Religion*

In conclusion, I would like to look at these reports from a different perspective, in order to draw some more general, but brief, conclusions regarding the status of science in those centuries usually described as centuries of decline, and the status of religion versus science in those same centuries.

As we have already seen, the school system was obviously involved in the instruction of theoretical astronomy at the most advanced level, as I have already stipulated elsewhere as well.[9] Now we see that a book such as that of Nīsābūrī used to take at least five years to finish. This can best be described as a very advanced on-going seminar on theoretical astronomy. Ulugh Beg's special leanings towards such matters, and their inclusion in his school, was apparently not so unique, for we now know, from Shirwānī's new evidence, that Shirwānī himself had already studied similar material at the shrine of Imām ʿAlī. More recently, I published a report of someone else who was also reading theoretical astronomy at Meshhed as late as the seventeenth century.[10] When we put these reports together with the similar ones from the previous century, one should begin to conclude that our image of the school curriculum should probably be also modified. The schools (*madrasas*) were certainly intended to be schools of law in the first place, but the discussion of theoretical astronomy in those schools was apparently considered as a complement to such legal studies, and the phenomenon was evidently widely spread. On the other hand, the study of theoretical astronomy apparently took place at religious institutions as well, thus allowing for the integration of theoretical *hay'a*-type astronomy and religious studies.

One need not think that, because we can now document very well the study of theoretical astronomy in the schools of Iran and Central Asia, this phenomenon had in any way something to do with the later adoption of Shīʿī Islam by the Safavids of Iran. At least in the case of Shirwānī there is no doubt about his allegiance to Sunnī Islam, for he expressly paid homage to the first

[9] Saliba, *A History of Arabic Astronomy*, pp. 32–9.

[10] Saliba, G., 'A Sixteenth-Century Arabic Critique of Ptolemaic Astronomy: The Work of Shams al-Dīn al-Khafrī', *Journal for the History of Astronomy*, 25, 1994, pp. 15–38, especially pp. 35–6.

four caliphs in the introduction of the same book we have been quoting. One presumes that Qāḍīzāde was of the same persuasion, as were probably Jurjānī and Nīsābūrī. The question of the inclusion of such theoretical discussion into the school curriculum has to be studied, therefore, in isolation of the kind of Islam one professed or practiced, and other factors, probably social, political and economic, have to be investigated before one can reach more reliable results regarding this aspect of the relationship between astronomy and religion at this period.

All we can say now is that there is enough evidence to consider the relationship between religion and theoretical astronomy as being a harmonious relationship. The general implication of this result is that we can no longer apply the conflict model usually applied to Renaissance Europe when speaking of the antagonistic relationship between science and religion. In the context of the *madrasas* too the problem can now be focused on which sciences were accepted into the curricula of those *madrasas* and which sciences were rejected. With the new evidence at our disposal, we can now say, with some certainty, that theoretical astronomy was accepted, and very critically pursued. Moreover, the research conducted in this environment, which encouraged the pursuit of theoretical astronomy, can now be demonstrated also to have produced theoretical results of the highest order. The examples of Qushjī and Shirwānī are the only two we know so far. But I am sure others will be found when more manuscripts are investigated, especially those produced in what is today Iran and Central Asia. Finally, we may say that the tradition of theoretical astronomy was obviously quite alive well into those centuries commonly considered as centuries of decline.

To return to Qushjī once more, we can now also see that he was in no way the very private student of Ulugh Beg, pampered by the Sultan, allowed to study what he pleased. Rather, he was only participating in an on-going research which intrigued Ulugh Beg himself, as his treatise on the reform of the Mercury model, and now Shirwānī's evidence, clearly demonstrate. The difficulty of the subject, and the ability of Qushjī to solve one of its major problems successfully, is what distinguished him and gave him the prestige that he so richly deserved in the eyes of his intellectual

successors such as Khafrī,[11] and the future Ottoman rulers such as Mehmed the Conqueror. But the one who should command our admiration and interest is the Sultan Ulugh Beg himself, who not only tolerated such studies, but rather encouraged them and participated personally in their propagation.

[11] On the tribute paid by Khafrī to Qushjī, see Saliba, 'Al-Qushjī's Reform', pp. 165, 177–9 and 201–3.

The *Zīj-i Nāṣirī* by Maḥmūd ibn ʿUmar
The Earliest Indian-Islamic Astronomical Handbook with Tables and its Relation to the *ʿAlāʾī Zīj*

BENNO VAN DALEN

1 *Introduction*

Until recently, virtually nothing was known about the *Zīj-i Nāṣirī* by Maḥmūd ibn ʿUmar, the earliest known Islamic astronomical handbook with tables that was written in India. Storey was the first western scholar to mention this work in the astronomical section of his *Persian Literature* [Storey 1958, p. 52]. He quoted an entry in [Āghā Buzurg Ṭihrānī 1936–78, vol. 8, p. 215] stating that the *Nāṣirī Zīj* was dedicated to Naṣīr al-Dīn Maḥmūd ibn Shams al-Dīn Iltutmish, sultan of Delhi from 1246 to 1265, and that a copy of the work was located in the important manuscript collection of Ḥusayn Āghā Nakhjawānī in Tabriz. Storey also mentioned a reference [*Oriens* 5 (1952), p. 193] to a letter of a certain Muḥammad Qazwīnī published in *Nashra-yi Dānishgada-yi Adabīyāt-i Tabrīz* (*Revue de la Faculté des Lettres de Tabriz*) 2 (1328 H.S./A.D. 1949–50), pp. 119–126, which confirms the dedication of the zīj. Recent attempts to locate the manuscript of Nakhjawānī have been in vain. Furthermore, two small fragments of the *Nāṣirī Zīj* listed in the catalogues of the Mulla Firuz Library in Bombay turned out to be of little interest.

Our knowledge of the *Nāṣirī Zīj* has drastically improved with the appearance in 1994 of volume 23 of the catalogue of the Marʿashī Library in Qum [Ḥusaynī & Marʿashī 1994, p. 293]. This volume provides a one-page description of the Persian manuscript 9176 (165 folios) which contains a complete copy of the *Nāṣirī Zīj*. It was due to the efforts of Mr. Mohammad Bagheri (Encyclopaedia Islamica Foundation, Tehran), to whom we would like to express our deepest gratitude, and Prof. S. M. Razaullah Ansari (Aligarh Muslim University) that a photocopy of the whole manuscript was obtained. In the summer of 2002, Prof. Ansari

and the present author had a chance to study various aspects of the *Nāṣirī Zīj* in Frankfurt. In the future, Prof. Ansari intends to publish a detailed account of the whole work. Below follow some preliminary results, concentrating on the tables for calculating planetary longitudes. It will be shown that nearly all of these tables derive directly or indirectly from the *ʿAlāʾī Zīj*, the latest of the six zījes written by the Caucasian astronomer al-Fahhād (ca. 1180), which is lost in its original form but influenced various later astronomical works. It partially survives in a reworking for the Yemen by al-Fārisī (ca. 1270) and in a Byzantine recension by Gregory Chioniades (Constantinople, ca. 1305), which was in turn based on a Persian translation by Shams al-Dīn al-Bukhārī (Tabriz, 1295/96). In the course of our investigation it will be made plausible that Chioniades' version of the *ʿAlāʾī Zīj* contains the original planetary tables of al-Fahhād.

The *Nāṣirī Zīj* consists of two divisions (*rukn*), the first on "details" (*juzʾiyāt)* in 66 chapters (*bāb*, 121 folios), the second on "general principles" (*kulliyāt*) in 60 chapters (44 folios). The introduction (folios 1v–2v) states that Maḥmūd ibn ʿUmar studied the zījes of earlier "astronomers who made observations" (*aṣḥāb-i arṣād*) during thirty years and calculated from these works planetary conjunctions and solar and lunar eclipses for the purpose of comparison. The following astronomers are mentioned explicitly: Hipparchus, Ptolemy, Yaḥyā ibn Abī Manṣūr, Khālid al-Marwarrūdhī, Muḥammad ibn ʿAlī al-Makkī, Muḥammad ibn Mūsā ibn Shākir, al-Battānī, Sulaymān ibn ʿIṣma al-Samarqandī, al-Ṣūfī, Abu 'l-Wafāʾ, al-Bīrūnī, Ibn al-Aʿlam, Ḥabash al-Ḥāsib (these two are the only deviations from the chronological order), al-Khāzinī, and finally ʿAbd al-Karīm al-Shirwānī, i.e., al-Fahhād, the author of the *ʿAlāʾī Zīj*.

From the table of contents on folios 3r–v it becomes clear that the first division of the zīj deals with practical calculations, such as the conversion of dates in various calendars, spherical astronomical functions, planetary longitudes and latitudes, solar and lunar eclipses, and astrological quantities. The range of topics treated is quite extensive and includes, for example, the Jewish calendar, the latitude of the visible climate, tables of mean transfers (*wasaṭ al-taḥwīl*), equalization of the houses, projection of the rays according to the equator and the horizon, and *tasyīr* (*aphesis* or *directio*). The second division of the *Nāṣirī Zīj*, at which

we have not yet looked in detail, presents explanations and proofs of the methods applied in the first division.

In this article we will limit ourselves to an investigation of the tables for calculating planetary longitudes. In order to make an attempt to place the *Nāṣirī Zīj* within the history of Islamic astronomy, we have used some preliminary results of the present author's project for the compilation of an updated survey of Islamic zījes. In 1956, Prof. E.S. Kennedy published *A Survey of Islamic Astronomical Tables*, in which he listed approximately 125 zījes, abstracted twelve of the most important of these works, and summarized the available historical information. During the last forty-five years, nearly one hundred additional works have come to light [cf. King & Samsó 2001]. The purpose of the author's project is first to compile a short list of all zījes now known with basic information on author, title, date, geographical origin, and available manuscripts, as well as references to the most important bio-bibliographical works and specific literature. Secondly, extensive studies on the treatment in zījes of topics such as the calculation of planetary positions, the prediction of solar and lunar eclipses, and mathematical astrology are envisaged on the basis of between 25 and 50 extant, mostly unpublished works. These studies will make it possible to describe the historical development of Islamic astronomy in more detail, whereas indexes of the mathematical characteristics of tables will facilitate the determination of the origin of unattributed materials. A preliminary database of the planetary parameters in approximately 25 zījes, relying heavily on a handwritten file by Prof. Kennedy, and an overview of the characteristics of planetary tables in around fifteen extant eastern-Islamic works predating the *Nāṣirī Zīj* have already been prepared and have been used for the present article.

Reference will be made to the following astronomers and zījes:

- The *Mumtaḥan Zīj* by Yaḥyā ibn Abī Manṣūr (Baghdad, 830), based on the observations carried out under the caliph al-Maʾmūn. Extant in a thirteenth-century revision in Escorial árabe 927 and published in facsimile in [Yaḥyā ibn Abī Manṣūr].
- The *Zīj* by Ḥabash al-Ḥāsib (Samarra, ca. 870), strongly related to the *Mumtaḥan Zīj* and extant in the manuscripts Istanbul Yeni Cami 784/1 (close to the original) and Berlin Ahlwardt 5750 (later recension). The introduction was edited

and translated in [Sayılı 1955], and a detailed summary presented in [Debarnot 1987].

- The *Ṣābi' Zīj* by al-Battānī (Raqqa, ca. 920). Extant as Escorial árabe 908 and published in [Nallino 1899–1907].
- The *ʿAḍūdī Zīj* by Ibn al-Aʿlam (Baghdad, d. 983). The author was well-known for his planetary observations, but his zīj is lost. Its parameters were reconstructed in [Kennedy 1977] and [Mercier 1985].
- The *Jāmiʿ Zīj* by Kūshyār ibn Labbān (Iran, ca. 970). Highly popular and extant in more than twenty manuscripts of which Istanbul Fatih 3418 is the oldest complete one.
- Two zījes are attributed to Abu 'l-Wafā' (Baghdad, ca. 980): the *Wāḍiḥ Zīj* is lost, whereas part of *al-Majisṭī* is extant in Paris BNF arabe 2494 (with the tables left out).
- The *Ḥākimī Zīj* by Ibn Yūnus (Cairo, ca. 1000). Extant in Leiden Or. 143 (chapters 1–20) and Oxford Bodleian Hunt. 331 (chapters 21–44). The introduction and observation accounts were translated into French in [Caussin de Perceval 1804].
- *al-Qānūn al-Masʿūdī* by al-Bīrūnī (Ghazna, ca. 1030), a very extensive and learned work with complete explanations of the Ptolemaic planetary models and the determination of parameters. Extant in numerous copies and edited in [al-Bīrūnī].
- The *Sanjarī Zīj* by al-Khāzinī (Marw, ca. 1120). Extant in the manuscripts Vatican arabo 761 and London BL Or. 6669, both defective. The same author's *Wajīz* ("Summary") contains similar planetary tables. Editions and translations of various versions are being prepared by Prof. Pingree and his students. See already [Pingree 1999].
- The *ʿAlā'ī Zīj* by al-Fahhād (Shirwan, ca. 1180), the last of his six zījes. Highly influential but lost in its original version. All following works in this list are helpful in reconstructing al-Fahhād's tables and parameters.
- The *Shāmil Zīj* (anonymous, ca. 1240), said to be based on the planetary parameters of Abu 'l-Wafā' but also related to the *ʿAlā'ī Zīj*. Extant in more than ten manuscripts, of which Florence Laurenziana Or. 95 appears to be the oldest. The *Athīrī Zīj* by al-Abharī (Mardin, ca. 1240) can be considered to be a variant of the *Shāmil*, whereas the so-called *Utrecht Zīj* and the *Durr al-muntakhib* by the priest Cyriacus (Mardin, ca. 1480) were based on it.

- The Persian *Nāṣirī Zīj* by Maḥmūd ibn ʿUmar (Delhi, 1250), the earliest zīj written in India. Extant in the manuscript Qum Marʿashī 9176. It is studied for the first time in this article.
- The *Muẓaffarī Zīj* by al-Fārisī (Yemen, ca. 1270), a reworking of the *ʿAlāʾī Zīj* with adjustments for the author's geographical location and time. Extant in Cambridge Gg. 3.27/2.
- The *Zīj* by Jamāl al-Dīn Abu l-Qāsim ibn Maḥfūz al-Munajjim al-Baghdādī (1285), with materials going back to Ḥabash al-Ḥāsib and Abu 'l-Wafāʾ. Extant in Paris BNF arabe 2486.
- The Byzantine recension of the *ʿAlāʾī Zīj* by Gregory Chioniades (Constantinople, ca. 1305), based on a Persian version by Shams al-Dīn al-Bukhārī (Tabriz, 1295/96). Extant in Greek manuscripts in Florence and the Vatican, edited and translated in [Pingree 1985–86].
- The *Zīj* of al-Sanjufīnī (Tibet, 1366). Based on observations made by Muslim astronomers in Yuan China [see, for instance, van Dalen 2002], happens to contain a complete list of *ʿAlāʾī* parameters.

More information on these zījes can be found in [Kennedy 1956], [King & Samsó 2001], and, for sources related to the *ʿAlāʾī Zīj*, in [Pingree 1985–86, vol. 1, pp. 7–9 and 16–18].

The reader is assumed to be familiar with the general characteristics of Ptolemaic planetary theory and the setup and use of the tables for planetary mean motions and equations as found in the *Handy Tables* and Islamic zījes. Full explanations of theory and tables can be found in, for instance, [Neugebauer 1975, vol. 1, pp. 53–190] and [Pedersen 1974, Chapters 5, 6, 9, and 10], whereas Appendix 1 in [Neugebauer 1957] provides a brief but clear overview of the theories of eccentres and epicycles. The use of the *Handy Tables* is expounded in [Van der Waerden 1958]. An example of a full analysis of the mean motion tables in a particular zīj is [van Dalen 2000]. Information on the calendars involved can be found in the article TAʾRĪKH in the *Encyclopaedia of Islam, new edition.* Standard notation is used for sexagesimal numbers, e.g., 33;1,2 stands for $33 + \frac{1}{60} + \frac{2}{60^2}$. A superscript "s" denotes a zodiacal sign, or also a general unit of 30° on the ecliptic, deferent, or epicycle. For example, $3^s12;25$ may denote 12;25° Cancer, but also an anomalistic planetary position of 102;25° measured from the apogee of the epicycle. By "the first sign" I will refer to the arguments 0 to 30° of a table for a planetary equation, by

"the second sign" to arguments 30 to 60°, etc. Superscripts "iv", "v" and "vi" stand for sexagesimal fourths, fifths and sixths.

2 *The Daily Solar, Lunar, and Planetary Mean Motions*

Folios 53r–58r of the *Nāṣirī Zīj* contain a set of tables for planetary mean motions. These include, for each of the periods of time listed below, the solar anomaly (*wasaṭ*, lit. "centrum"), the lunar longitude and anomaly, the elongation, the lunar node, and the centrum (*wasaṭ*) and anomaly of each of the five planets. All mean motions are tabulated to seconds for the following periods of time: 1, 2, ..., 10, 20, ..., 100, 200, ..., 1000 completed Persian years of 365 days (folios 53v–54r); current[1] Persian months Farwardīn, Urdībihisht, ..., Isfandār[mudh] plus a value for *tamām al-sana*, a "complete year" (folio 55r); current days 1, 2, 3, ..., 30 (folios 55v–56r); and 1 to 24 hours (folios 56v–57r). A table of the "difference [in mean motion] between the two longitudes" (*faḍl mā bayn al-ṭūlayn*) allows the adjustment of the mean positions found from the tables to geographical longitudes differing by 1, 2, ..., 10, 20, ..., 100 degrees from the base longitude of the zīj (folios 57v–58r). A separate table is provided for the motion of the solar and planetary apogees (folio 53r), which also lists the apogee positions at the epoch of the zīj (see below and Table 2). On folio 54v we find the motion of the pseudo-comet Kaid in the above-mentioned periods of years (which may have been inadvertently omitted from the general table of years) as well as the epoch positions at Delhi for all mean motions (also reproduced in Table 2).

Note that, unlike most Islamic zījes, the *Nāṣirī Zīj* does not include tables with actual mean *positions* for the beginnings of a certain collection of years; all tabulated values are mean *motions* in the given periods. For the actual positions we have to rely completely on the epoch positions listed on folios 53r (apogees)

[1] Sub-tables for months may be for "current" (*nāqiṣa*) or for "completed" (*tāmma*) months. In the former case, the tabulated motion for a given month is the motion that has taken place between New Year and the beginning of that month, which implies that the value for the first month is equal to zero. In the latter case, the tabulated motion is that between New Year and the end of the month, so that, for instance, to calculate a mean position during the month Ṣafar, the motion listed for Muḥarram must be used. Also the sub-tables for days may be for current or for completed days.

and 54v (all others). The only ways to correct scribal errors in these values are a check by means of the elementary relations between mean longitudes, centrums, and anomalies as described in the Appendix to this article and a comparison with the positions for Delhi at the Yazdigird, Hijra and Alexander epochs listed on folio 50v. As will be shown below, an attempt to reconstruct the epoch positions from those in other zījes will not prove succesful. In general, none of the zījes listed in the introduction to this article contain mean motion tables similar to the *Nāṣirī Zīj*, neither with respect to their setup nor their tabular values.

The parameters underlying the mean motion tables in the *Nāṣirī Zīj* were estimated by means of the Least Number of Errors Criterion (LNEC, introduced in [van Dalen 1993, Section 2.5]) which determines the range of parameter values for which the largest number of values in a given sub-table is correctly recomputed. A more extensive discussion of the method is presented in [Van Dalen 2000], whereas a slightly different approach is explained in [Mielgo 1996]. The application of the LNEC showed that the majority of the mean motion tables in the *Nāṣirī Zīj* were computed with a very high accuracy. As a matter of fact, most of the sub-tables do not contain any errors at all, others at most one or, very incidentally, two, which can partially be explained as scribal mistakes. Note that, due to the use of the Persian calendar with its constant year-length of 365 days, every sub-table is completely linear, consisting simply of multiples of the first value.[2] The ranges of the daily mean motions found by means of the LNEC are given in the second column of Table 1, where the notation $\mu \pm \epsilon$ indicates that all daily mean motions in the range $[\mu - \epsilon, \mu + \epsilon]$ produce the smallest possible number of

[2] In estimating the underlying daily mean motions, I have treated the values for 1, 2, ..., 10 years, those for 10, 20, ..., 100 years, and those for 100, 200, ..., 1000 years as three separate sub-tables, since it is possible that the value calculated for 10 or 100 years was first rounded to a lesser precision before being used as a constant multiplier for the next higher range of years. Since the later variant of the Persian calendar is used, in which the five epagomenal days are placed at the end of the year as opposed to after the eighth month Ābān, the sub-table for months displays the mean motions in 0, 30, 60, ..., 360 days, and is therefore also completely linear. Note that the last value, in spite of the indication *tamām al-sana*, is not in fact the motion during a complete year, i.e., 365 days, but that between New Year and the beginning of the epagomenal days.

motion	LNEC estimates	*ʿAlāʾī Zīj*
Apogee Longitude	0; 0, 0, 8,57,37,29 $\pm$ 30^{vi}	0; 0, 0, 8,57,46 *
Solar Longitude		0;59, 8,20,35,25
Solar Anomaly	0;59, 8,11,37,46,56 $\pm$ 5^{vi}	0;59, 8,11,37,39 *
Lunar Longitude	13;10,35, 1,55,32, 1 $\pm 2\frac{1}{2}^{vi}$	13;10,35, 1,55,32
Lunar Anomaly	13; 3,53,56,17,52, 3 $\pm$ 4^{vi}	13; 3,53,56,17,51,59
Lunar Elongation	12;11,26,41,20, 7, 2 $\pm$ 3^{vi}	12;11,26,41,20, 7 *
Lunar Nodes	−0; 3,10,37,35,29,15 $\pm 4\frac{1}{2}^{vi}$	−0; 3,10,37,35,29,19
Saturn Longitude		0; 2, 0,36, 4,33,33
Saturn Centrum	0; 2, 0,27, 6,55,32 $\pm$ 11^{vi}	0; 2, 0,27, 6,47,33 *
Saturn Anomaly	0;57, 7,44,30,51,26 $\pm$ 9^{vi}	0;57, 7,44,30,51,27 *
Jupiter Longitude		0; 4,59,15,39,41
Jupiter Centrum	0; 4,59, 6,42, 2,57 $\pm$ 7^{vi}	0; 4,59, 6,41,55 *
Jupiter Anomaly	0;54, 9, 4,55,44, 4 $\pm$ 11^{vi}	0;54, 9, 4,55,44 *
Mars Longitude		0;31,26,39,51,21
Mars Centrum	0;31,26,30,53,43, 2 $\pm$ 3^{vi}	0;31,26,30,53,35 *
Mars Anomaly	0;27,41,40,44, 3,57 $\pm 4\frac{1}{2}^{vi}$	0;27,41,40,44, 4 *
Venus Longitude	*same as solar longitude*	
Venus Centrum	*same as solar anomaly*	
Venus Anomaly	0;36,59,28,43, 1,44 $\pm$ 5^{vi}	0;36,59,28,43, 1,38
Mercury Longitude	*same as solar longitude*	
Mercury Centrum	*same as solar anomaly*	
Mercury Anomaly	3; 6,24,22, 7,59, 5 $\pm$ 4^{vi}	3; 6,24,22, 7,59
Kaid	−2°30′ / Persian year	

Table 1. Second column: *LNEC-estimates of the daily mean motions underlying the tables in the Nāṣirī Zīj.* Third column: *The mean motion parameters of the ʿAlāʾī Zīj. Values with an asterisk are not listed by Chioniades but were recovered from his tables, the list in the Sanjufīnī Zīj, and other sources. Note that in the Nāṣirī Zīj the daily mean motions in centrum are all* 8^{v} *larger than those in the ʿAlāʾī Zīj due to the use of a different daily motion of the apogee.*

errors in the sub-tables. In this procedure, the tabular values for the range 100, 200, ..., 1000 years have been given the largest weight, since they are most significant and allow the most accurate determination of the underlying mean motion parameters.

From Table 1 we first note that, even though the parameter intervals typically have a width of approximately 10^{vi}, nine out of fourteen (not counting the centrums of Venus and Mercury because they are identical to the solar anomaly and the motion of Kaid because it is based on a retrograde motion of precisely 2°30′ per year) include a round value to sexagesimal fifths. In the case of the mean anomaly of Mercury, such a round value is only narrowly missed. It is thus likely that most of the mean motion

tables in the *Nāṣirī Zīj* were computed on the basis of daily mean motions to a precision of fifths.

A comparison of the estimated daily mean motions with values in Prof. Kennedy's parameter database of Islamic astronomy shows an obvious agreement with one particular set of daily mean motions, namely that listed in the Byzantine version of the *ʿAlāʾī Zīj* by Gregory Chioniades, in the Tibetan *Sanjufīnī Zīj*, and in a group of strongly related zījes including the anonymous *Shāmil Zīj*, the *Athīrī Zīj* by al-Abharī, the *Utrecht Zīj*, and the *Durr al-muntakhib* by the Priest Cyriacus. There is little doubt that this set of mean motions stems from the non-extant *ʿAlāʾī Zīj*, the latest of the six zījes by al-Fahhād (ca. 1180). In fact, the mean motion tables in Chioniades' reworking of this zīj, which was drawn upon a Persian version by Shams al-Dīn al-Bukhārī (Tabriz, 1295/96), are based on the daily mean motions concerned, as are those in the Yemeni *Muẓaffarī Zīj* by Muḥammad ibn Abū Bakr al-Fārisī (ca. 1270), who acknowledges the use of al-Fahhād's observations. The introduction to the *Shāmil Zīj* maintains that the author of the *ʿAlāʾī Zīj* had presented Abu 'l-Wafāʾ's mean motion parameters as the results of his own observations; the tables in the *Shāmil Zīj* are said to be based on Abu 'l-Wafāʾ's parameters, which is particularly interesting because this astronomer's planetary tables are completely lost. The mean motion tables in the *Shāmil Zīj* have a rather different structure from those in the Byzantine version of the *ʿAlāʾī Zīj* and the *Muẓaffarī Zīj* but the underlying daily mean motions never deviate by more than one sexagesimal fourth from the parameters in those works. That the list of *ʿAlāʾī* parameters also occurs in the *Sanjufīnī Zīj* appears to be a historical coincidence: the planetary mean motion tables in this work were not computed on the basis of the *ʿAlāʾī* parameters but from the tables in the *Huihuili*, a Persian zīj compiled in Mongol China in the 1270s and translated into Chinese in 1383.

A comparison of the estimates found above with the lists of *ʿAlāʾī* parameters allows us to determine exactly which daily mean motions were used to compute the tables in the *Nāṣirī Zīj* and which were the original parameters of the *ʿAlāʾī Zīj*. This comparison is complicated by the fact that some of the sources involved tabulate or list the solar and planetary *longitudes* and others the *centrums*, which differ by amounts of the daily apogee motion

that are not the same for each source. Moreover, the lists in the works from the *Shāmil* group contain many scribal mistakes.

The clearest picture of the situation arises if we compare the daily mean motions from the Byzantine version of the *ʿAlāʾī Zīj*, which are listed without an asterisk in the third column of Table 1, with: 1) the actual mean motion tables in that same work; 2) the estimates derived from the *Nāṣirī Zīj*; and 3) the complete list of parameters in the *Sanjufīnī Zīj*. It turns out that each of the latter three sets of data can be derived from the basic set of *ʿAlāʾī* parameters listed in the Byzantine version. Both Chioniades (or possibly al-Fahhād himself) and Maḥmūd ibn ʿUmar needed to convert the daily mean motions in *longitude* from the basic set into daily mean motions in *centrum* in order to compute the tables that we find in their works. This conversion is carried out by subtracting the daily motion of the apogee from the daily mean motion in longitude. However, the daily motion of the apogee is not listed to the required precision of five or six sexagesimal fractional digits in Chioniades' version of the *ʿAlāʾī Zīj*, and hence possibly in the original work by al-Fahhād. This may have been the reason why Chioniades (or al-Fahhād) and Maḥmūd ibn ʿUmar used slightly different rates for the conversion. In fact, Chioniades consistently used the value 0;0,0,8,57,46 °/day underlying his table for the "equation of the solar apogee" [Pingree 1985–86, vol. 2, pp. 35–36], whereas Maḥmūd used a value within the range 0;0,0,8,57,37,40–38,0 °/day which we have not yet found in other sources but is quite close to a motion of 1° in 66 Julian years. Finally, the author of the list in the *Sanjufīnī Zīj* picked still another value for the apogee motion, namely, the very common 0;0,0,8,57,58 °/day, close to 1° in 66 Persian years, to calculate his daily mean motions in centrum. That the value used by Chioniades is the original parameter of the *ʿAlāʾī Zīj* is made plausible by the fact that the table for apogee motion in the *Muẓaffarī Zīj* by al-Fārisī is basically identical to that presented by Chioniades, the only difference being a change of the epoch for which the apogee positions are given from the year 541 Yazdigird to the year 631 (A.D. 1262 instead of 1172).

The parameter lists in Chioniades' work and in the *Sanjufīnī Zīj* contain a number of daily mean motions expressed to a precision of sexagesimal sixths (rather than fifths). These correspond very well to the intervals of estimates obtained from the *Nāṣirī*

Zīj that do *not* contain a round value to fifths. For instance, the motion of the lunar node listed by Chioniades falls within the interval of estimates, as do the motions of Saturn in centrum and in anomaly that can be derived from the listed motion in longitude. The listed anomaly of Venus misses the interval of estimates by only a sixth, whereas the lunar anomaly, equal to Ptolemy's value, falls within the interval together with a round value to fifths. Thus we can conclude that, for the computation of his mean motion tables, Maḥmūd ibn ʿUmar based himself on exactly the values listed by Chioniades, but used a slightly different apogee motion to convert longitudes to centrums.

The works of the *Shāmil* group contain complete lists of daily mean motions in longitude, centrum and anomaly, but with values to fifths only. In general, the values are identical with, or extremely close to, those of the *ʿAlāʾī Zīj* in the third column of Table 1, except that the motions in centrum were determined from those in longitude by subtracting a daily apogee motion of 0;0,0,8,57,58°, as in the *Sanjufīnī Zīj*. In the *Shāmil* group, Saturn's daily mean motion in longitude is 0;2,0,36,4,35° (instead of $33^{v}33^{vi}$), in centrum 0;2,0,27,6,37 (instead of $35^{v}33^{vi}$), and in anomaly 0;57,7,44,30,21 (instead of $51^{v}27^{vi}$; since this is inconsistent with the longitudes of the Sun and Saturn, it could be a scribal error). If the introduction to the *Shāmil Zīj* is correct, we may thus conclude that both al-Fahhād and the author of the *Shāmil Zīj* used the mean motions of Abu 'l-Wafāʾ; otherwise, the *Shāmil* group must have copied the parameters of the *ʿAlāʾī Zīj* with a minor change in the motion of the apogee.

The only data we have to verify the statement in the *Shāmil Zīj* that the *ʿAlāʾī* parameters stem from Abu 'l-Wafāʾ are a number of incredibly precise values in the margins of the mean motion tables of Ḥabash al-Ḥāsib in the manuscript Berlin Ahlwardt 5750. It is not impossible that these are Abu 'l-Wafāʾ's, since the present author has noticed that from this thirteenth-century manuscript some of Ḥabash's original tables were scraped away with a knife or similar object (in many cases producing small holes in the paper) and replaced by that of Abu 'l-Wafāʾ, which has a precision of thirds instead of seconds (unpublished result).[3] The

[3] It may be noted, though, that at first sight the hands in which the marginal notes and the substituted table were written are not the same.

values attributed to Abu 'l-Wafā' in the margins are as follows (these were partially published in [Kennedy 1956, p. 169]):

Solar Longitude	0;59, 8,20,43,17,38,41,42,20, 5
Lunar Longitude	13;10,35, 1,55,37,39, 6,16,45,43
Lunar Anomaly	13; 3,53,56,17,50,25, 7,59,17,31
Lunar Elongation	12;11,26,41,12,20, 0,54,24,25, ?
Lunar Node	0; 3,10,37,35,10, 1,51,42,13,28
Saturn Longitude	0; 2, 0,36, 4,27,58,33,41,41,42
Jupiter Longitude	0; 4,59,16,58,50,44,30,49,53,17
Mars Longitude	*not included*
Venus Anomaly	0;36,59,29, 7,49, 1,36, 9,21,59
Mercury Anomaly	3; 6,24, 6,59,45,22, 0,37,26,24

It will be clear that, even taking into account the possibility of multiple scribal errors, there cannot be a simple relationship between the parameters of the *ʿAlāʾī Zīj* in Table 1 and those attributed to Abu 'l-Wafā'.

As far as the origin of the mean motion parameters in the *ʿAlāʾī Zīj* is concerned, it appears that not all of them were based on new observations. In the introduction of the *Muẓaffarī Zīj*, al-Fārisī presents some interesting statements concerning observations made by al-Fahhād [Lee 1822, pp. 257–259]. For instance, he writes that al-Fahhād found the mean motions of the Sun and the Moon to be in agreement with the observations of the astronomers working under al-Maʾmūn, namely Yaḥyā ibn Abī Manṣūr, Khālid al-Marwarrūdhī, al-ʿAbbās al-Jawharī, and Ḥabash al-Ḥāsib. In fact, it can be checked that the solar and lunar mean longitudes and the lunar mean anomaly found from the *ʿAlāʾī Zīj* differ by less than a minute of arc from those of Yaḥyā and Ḥabash. Al-Fahhād also found that the mean positions of Mars and Venus were in agreement with Ibn al-Aʿlam, whose *ʿAḍūdī Zīj* is likewise lost. From data in later works it can be seen that the *ʿAlāʾī* mean positions of these planets are indeed quite close to Ibn al-Aʿlam's (cf. [Mercier 1985]), although the agreement is not quite as good as in the case of the solar and lunar mean motions. Two more interesting statements by al-Fārisī to the extent that, unlike most astronomers, al-Fahhād found the apogee of Venus to be unequal to that of the Sun, and that the most accurate true positions of Mercury were produced by Ptolemy's tables, deserve further investigation.

3 *The Solar, Lunar, and Planetary Epoch Positions*

As mentioned above, the mean motion tables in the *Nāṣirī Zīj* display only motions, not actual mean positions. The motions obtained from the tables need to be added to corresponding mean positions listed on folios 53r (apogees) and 54v (all others). These positions have been reproduced in the second column of Table 2. From the description of the use of the mean motion tables (as well as from the values themselves) it becomes clear that the given positions are in fact for New Year (1 Farwardīn) of the Persian year 1 *before* Yazdigird. In this way, a mean position for, for example, the year 1372 Yazdigird can be obtained by simply adding the mean motions found for arguments 1000, 300, 70, and 2 in the table for years to the epoch position concerned.

The epoch positions to be used with the mean motion tables can be compared with a table on folio 50v that lists mean positions at Delhi for the Yazdigird, Hijra, and Byzantine (i.e., Dhu 'l-qarnayn or Alexander, 1 October 312 B.C.) epochs. Even though this list includes mean longitudes rather than centrums (although both are called *wasaṭ*) and does not include the anomalies of the superior planets,[4] it is an easy matter to verify that the two sets of positions are fully compatible. In nearly all cases the epoch positions of the mean motion tables can be obtained by subtracting the motion in a single Persian year from the positions for the Yazdigird epoch found in the table on folio 50v or derived from it by means of the elementary rules explained in the Appendix. It turns out that the positions for the Hijra epoch (Maḥmūd ibn ʿUmar used the civil variant, i.e., Friday, 16 July, A.D. 622) and for the Byzantine epoch were likewise accurately computed using the mean motion tables in the *Nāṣirī Zīj*.[5] Thus we can reliably reconstruct a complete set of mean positions for Delhi at the Yazdigird epoch, which is found in the third column of Table 2. Values that were not taken directly from the table on folio 50v are marked with an asterisk. Scribal errors in the manuscript have been indicated in notes to the table.

[4] For the Yazdigird epoch the lunar elongation, the anomaly of the superior planets, and the centrum of the inferior planets are written in the margin.

[5] As a matter of fact, due to the large time span between the Byzantine and the Yazdigird epochs, it is even possible to conclude that not the original rate of apogee motion from the *ʿAlāʾī Zīj*, 0;0,0,8,57,46 °/day, was involved, but that from the *Nāṣirī Zīj*, 0;0,0,8,57,38 °/day.

motion	1 before Yazdigird	1 Yazdigird
Solar Longitude		$2^s26;57,24$
Solar Anomaly	0^s 2;32,45	0^s 7;17,36 *
Solar Apogee	$2^s19;38,53$	$2^s19;39,48$
Lunar Longitude	$7^s23,31,36$	0^s 2,54,43
Lunar Anomaly	7^s 7; 0, 0	10^s 5;43, 7 [1]
Elongation	$4^s26;19,58$	9^s 5;57,19 †
Lunar Node	$10^s13;36,39$	2^s 5;43,20 [2]
Saturn Longitude		7^s29; 7,58
Saturn Centrum	11^s17; 4,25	$11^s29;17,10$ *
Saturn Anomaly	$7^s10;17,20$	$6^s27;49,26$ †
Saturn Apogee	$4^s29;49,53$	$7^s29;50,48$
Jupiter Longitude		9^s 2;47,10
Jupiter Centrum	$2^s12;54,46$	$3^s13;14,22$ *
Jupiter Anomaly	$7^s24;44,59$	$5^s24;10,14$ †
Jupiter Apogee	$5^s19;31,53$	$5^s19;32,48$
Mars Longitude		$10^s10;35,56$ [3]
Mars Centrum	$11^s18;45,50$	6^s 0; 2, 8 *
Mars Anomaly	$10^s27;52,55$	$4^s16;20,48$ †
Mars Apogee	$3^s10;32,53$	$4^s10;33,48$
Venus Longitude		$2^s26;57,24$ † [4]
Venus Centrum	$0^s19;28,45$	$0^s19;13,36$ *
Venus Anomaly	$8^s13;57$, 6	$3^s28;58,56$
Venus Apogee	2^s 2;42,53	2^s 7;43,48 [5]
Mercury Longitude		$2^s26;57,24$ † [6]
Mercury Centrum	8^s 6; 3,45	8^s 6;48,28 *
Mercury Anomaly	3^s 9; 9,51	6^s 3; 8, 6 [7]
Mercury Apogee	6^s21; 7,53	6^s21; 8,48
Kaid	$7^s17;30$, 0	4^s15; 0, 0 [8]

Table 2. *Solar, lunar, and planetary epoch positions for Delhi as found in the Nāṣirī Zīj.* Second column: *Positions for New Year of the year 1 before Yazdigird (folios 53r and 54v), to be used with the mean motion tables.* Third column: *Positions for New Year of the year 1 Yazdigird as found on folio 50v. Values indicated by an asterisk were reconstructed, values indicated by a dagger are written in the margin.*

Notes to the table: [1] The positions for the Hijra and Alexander epochs are in agreement with a value of $10^s5;44,7$. [2] This is 360° minus the actual position of the ascending node, possibly pointing to a dependence on mean motion tables that tabulate the supplement of the nodal position rather than its actual position. [3] In the manuscript this value is corrected to $10^s10;36,36$, which is in fact consistent with the tabulated anomaly and the solar longitude. [4] The other epoch positions for Venus and Mercury are in agreement with a mean longitude of $2^s27;11,38$, differing from the tabulated longitude by the motion in nearly a quarter of a day. [5] The manuscript makes the apogee of Venus equal to that of the Sun for all three epochs. [6] See note 5. [7] The positions for the Hijra and Alexander epochs are in agreement with a value of $6^s0;8,6$. [8] This is 360° minus the actual position of Kaid (cf. note 2).

It turns out that the epoch positions of the *Nāṣirī Zīj* cannot be easily derived from the mean positions in Chioniades' version of the *ʿAlāʾī Zīj* or in al-Fārisī's *Muẓaffarī Zīj*. Chioniades systematically maintains al-Fahhād's original epoch 541 Yazdigird (A.D. 1172) and his geographical longitude 84° (measured from the Fortunate Isles) for the region Shirwan in Azarbaijan. Thus he lists the longitudes of the apogees for the year 541 (besides for the Yazdigird and Hijra epochs) [Pingree 1985–86, vol. 2, p. 33], and his table for the "equation of the solar apogee" (see above) assumes its zero for that year. Al-Fārisī, on the other hand, changes the epoch to his own time, namely, to 631 Yazdigird (A.D. 1262), and consequently shifts the zero in his "equation of the solar apogee". Furthermore, al-Fārisī adjusts the original tables of the *ʿAlāʾī Zīj* for use in the Yemen (longitude 63°30′ from the Fortunate Isles, rounded to 64°). As a result, the mean positions given by al-Fārisī all differ from those in Chioniades by exactly the motion in $1^h 20^m$, corresponding to a longitude difference of 20°.

As was mentioned above, the base meridian of the *Nāṣirī Zīj* is that of Delhi, to which Maḥmūd ibn ʿUmar attaches a longitude of 103°35′ (measured from the Western Shore of Africa) in an example in Division 1, Chapter 25 (folios 50v–51r) on the adjustment of planetary mean positions to different locations.[6] This corresponds to 113°35′ from the Fortunate Isles and hence to a longitude difference from Shirwan equal to 29°35′. This implies that Maḥmūd ibn ʿUmar would have needed to subtract the mean motion in nearly two hours from the epoch positions of the *ʿAlāʾī Zīj* in order to obtain the corresponding epoch positions for Delhi. An attempt to reconstruct Maḥmūd's epoch positions along these lines clearly showed that he did *not* use the epoch positions from the *ʿAlāʾī Zīj*. Whereas his solar longitude is in exact

[6] The longitude value 103°35′ for Delhi occurs in some more chapters of the *Nāṣirī Zīj*. It has not been found in other zījes but on various astrolabes (cf. [Kennedy & Kennedy 1987, p. 105]). The *Nāṣirī Zīj* also includes a geographical table (folios 32v–34r), which presents longitudes (likewise measured from the Western Shore of Africa) and latitudes for 109 localities distributed over eight climates. This table gives the longitude of Delhi as 104°29′, a value not yet known from other sources. The listed longitudes of Shirwan (57°30′, possibly a mistake for 67°30′) and Tabriz (73°10′) stem from al-Bīrūnī and are not compatible with al-Fahhād's longitude of 84° (measured from the Fortunate Isles) for Shirwan.

agreement with Chioniades if we assume a correction for a longitude difference of 30°15′, most of the other longitudes, centrums and anomalies are not even close to values thus reconstructed. The longitudes of the solar and planetary apogees, however, correspond with the values to minutes given by Chioniades [Pingree 1985–86, vol. 2, p. 33] with the exception of that of Mars, for which Chioniades has $4^s6;34$ instead of Maḥmūd's $4^s10;33,48$. In Chioniades, as well as in the *Nāṣirī Zīj*, the longitude of the apogee of Venus is different from that for the Sun, again pointing to a relation to al-Fahhād.

I have further compared the planetary positions in the *Nāṣirī Zīj* with the *Sanjarī Zīj* of al-Khāzinī (Marw, ca. 1120), the next earlier astronomer who is mentioned in the introduction, but likewise with a negative result. The epoch positions listed in the *Shāmil Zīj* and its relatives, which are said to be for a geographical longitude of 84°, turn out to be in full agreement with the *ʿAlāʾī Zīj* and hence not with the *Nāṣirī Zīj*. We conclude that Maḥmūd ibn ʿUmar either used positions based on still another zīj, or adjusted the epoch positions on the basis of his own observations.

4 *The Solar, Lunar, and Planetary Equations*

We will now turn our attention towards the tables for the solar, lunar, and planetary equations in the *Nāṣirī Zīj* and compare them with Chioniades' Byzantine version of the *ʿAlāʾī Zīj*, al-Fārisī's *Muẓaffarī Zīj* and the group of zījes related to the *Shāmil Zīj*. As will become apparent, the equations in all these works are to a smaller or larger extent related to each other, and in many cases show a dependency on the equation tables in the zījes of al-Khāzinī. Before comparing actual tabular values, we will look at more general characteristics of the tables, in particular the "displacements". Displaced equations have been described in [Salam & Kennedy 1967, for the lunar tables of Ḥabash], [Saliba 1976, for Cyriacus], [Kennedy 1977, in connection with Ibn al-Aʿlam], [Saliba 1977, for ʿAbd al-Raḥīm al-Qazwīnī], [Saliba 1978, again for Cyriacus], [van Dalen 1996, for Kūshyār's solar equation], [Van Brummelen 1998, for Kūshyār's planetary tables], and others. Here we will only present a brief general description of displacements.

In Ptolemy's *Almagest* and many Islamic zījes, the planetary equations need to be either added to, or subtracted from, specific quantities depending on whether their arguments fall in certain ranges. Displaced equations eliminate the conditional addition or subtraction by making the equations additive (or, sometimes, subtractive) throughout. This is done by increasing every equation value by a positive constant c equal to, or larger than, the maximum equation. Thus, instead of a positive function $q(x)$ that has a maximum value $q_{\max}$ and is sometimes additive and sometimes subtractive, we obtain a positive, always additive function $c \pm q(x)$ with values between $c - q_{\max} \geq 0$ and $c + q_{\max}$. Since the equation now exceeds its actual value by the amount c, a mean motion μ to which the equation is added must be decreased by c if the addition is to yield the same result; thus the mean motion is tabulated as $\mu - c$ instead of μ. If μ is itself the argument of an equation, the arguments of this equation must be shifted by c, so that the equation for the original argument μ appears next to $\mu - c$. It is in particular in cases where this shift is necessary that c is chosen as an integer number. In general, we define the displacement of a given planetary equation by a constant $c > 0$ as the operation in which every additive value of the equation is increased by c and every subtractive value is subtracted from c, after which multiples of 12^s ($360°$) are discarded.

The planetary equations in the *Nāṣirī Zīj* are basically of standard type, except that they are displaced by 12 zodiacal signs. In accordance with the above definition, this means that subtractive values q of the equation are represented as $12^s - q$, and that all tabular values can be *added* to the mean position concerned in order to obtain a true position. In the case of (originally) subtractive values, the result of this addition will almost always be larger than 12^s, requiring 12^s to be subtracted again in order to obtain a number between 0 and $360°$. As far as I know, the *Nāṣirī Zīj* is the earliest extant zīj in which displacements of 12 zodiacal signs occur, although displacements by the maximum equation (or next larger integer) were already used by Ḥabash al-Ḥāsib (ca. 850, only for the moon) and Kūshyār ibn Labbān (ca. 970).[7]

[7] [Kennedy 1977] and [Mercier 1989] disagree about the question whether the non-extant *ʿAḍūdī Zīj* by Ibn al-Aʿlam (ca. 970) utilized displaced equations. I have re-inspected the main primary source for information on Ibn al-Aʿlam's planetary tables, namely, the *Ashrafī Zīj* by Muḥammad ibn Abī

Since some of the tables in Chioniades' version of the *ʿAlāʾī Zīj* and in al-Fārisī's *Muẓaffarī Zīj* are also displaced by 12 signs, the *ʿAlāʾī Zīj* by al-Fahhād may have been the original source for this type of displacement.

The tables for the planetary equations in the Byzantine version of the *ʿAlāʾī Zīj* and in the *Muẓaffarī Zīj* are partially very different from those in the *Nāṣirī Zīj* but very similar to each other. The tables of the solar equation and of the planetary equations of anomaly are displaced by the longitude of the apogees concerned, so that the equation must always be added to the tabulated mean centrum (rather than to the mean longitude) in order to obtain the true longitude. Furthermore, the tables for the lunar and planetary equations of anomaly are not only displaced but also of what I will call "mixed type", which means that the first halves of the tables display values of the equation for one position of the epicycle on the deferent, the second half for another. The mixed equations eliminate the conditional addition or

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ʿAbd Allāh Sanjar al-Kamālī of Yazd, known as Sayf-i Munajjim (1302). The only surviving manuscript of this work is Paris, Bibliothèque Nationale de France, supplement persane 1488. Al-Kamālī first presents his own planetary equations, which are essentially displaced copies of those of Ḥabash al-Ḥāsib, and then enables the reader to calculate planetary positions according to twelve well-known zījes by including all planetary equations from those works that are different from his own. For Ibn al-Aʿlam, al-Kamālī tabulates the solar equation (without displacement), the equations of centrum for Saturn and Jupiter (with displacements in concordance with al-Kamālī's own equations of anomaly for these planets), and both equations for Mercury (without displacements). By comparing all alternative equations presented in the *Ashrafī Zīj* with the zījes from which they originate (in so far as these are extant), I found that al-Kamālī in general correctly reproduces the actual equation values, but that in various cases displacements and shifts were omitted, modified, or introduced. Since every single alternative equation has a maximum value different from that of al-Kamālī himself, it is thus clear that the author's purpose was to accurately represent the magnitude of the equations but not necessarily their displacements. If we make the plausible assumption that Ibn al-Aʿlam's planetary equations were all either of the displaced type or of the standard type, it follows that al-Kamālī removed the displacements from the Mercury equations (and possibly the solar equation), or that he introduced the displacements of the equations of centrum for Saturn and Jupiter. In my opinion, the latter possibility is more likely, since it made it possible to use Ibn al-Aʿlam's equations of centrum for Saturn and Jupiter in combination with al-Kamālī's own displaced equations of anomaly for these planets. I will therefore for the time being assume that Ibn al-Aʿlam did not use displaced equations.

subtraction that occurs in most cases in the course of the Ptolemaic interpolation involved in the calculation of the equation of anomaly (see below). As a result, the only operations left in the calculation are one multiplication and two additions. However, this requires additional tables for the equation itself and for the interpolation function. An extensive explanation of the "mixed equation of anomaly" will be found in a forthcoming publication by the present author on the characteristics of tables for calculating planetary longitudes in Islamic zījes. This publication will also discuss characteristics not treated here such as terminology, exact layout of the tables, etc. As far as I know, Chioniades' version of the *ʿAlāʾī Zīj* and the *Muẓaffarī Zīj* are the earliest extant works that utilize equations of anomaly of mixed type. Given that the equation tables in the two works share such a highly peculiar characteristic, whereas we have already seen that al-Fārisī's tables for planetary mean motions can be derived from those of Chioniades, we conclude with reasonable certainty that Chioniades' tables, which are set up for the longitude of al-Fahhād's location Shirwan and his epoch A.D. 1172, are the tables from the original *ʿAlāʾī Zīj*.

Solar Equation

The solar equation in the *Nāṣirī Zīj* is tabulated to seconds of a degree as a function of the mean anomaly and is displaced by 12 zodiacal signs (see above). It assumes a maximum value of 1°59′0″, which corresponds to a solar eccentricity of 2;4,35,30 units and originally goes back to the Mumtaḥan observations. The solar equation tables for this parameter by Yaḥyā ibn Abī Manṣūr and Ḥabash al-Ḥāsib were still highly inaccurate, whereas al-Battānī and Kūshyār used the minimally different eccentricity value 2;4,45. However, Abu 'l-Wafāʾ (ca. 980) provided a table, extant in the Berlin manuscript of Ḥabash's *Zīj* and in the *Zīj* by Jamāl al-Dīn ibn Maḥfūẓ al-Baghdādī (1286), with maximum 1°59′, values to sexagesimal thirds, and errors of at most 2 thirds. Other early Muslim astronomers, such as Ibn al-Aʿlam and al-Bīrūnī, observed maximum equations slightly different from the Mumtaḥan value, whereas al-Khāzinī used the clearly larger 2°12′23″.

Instead of the plain solar equation, Chioniades' version of the

ʿAlāʾī Zīj and the *Muẓaffarī Zīj* tabulate the true solar longitude to seconds for every degree of the mean anomaly. Thus the tabulated function is $\lambda_A + \bar{a} \pm q(\bar{a})$, where λ_A is the longitude of the solar apogee, $\bar{a}$ the mean anomaly, and $q(\bar{a})$ the solar equation. Here Chioniades uses a longitude of the solar apogee for the year 541 Yazdigird, namely $2^s27°50'43''$, which is presumably al-Fahhād's original value. Al-Fārisī adjusts this longitude to his own epoch 631 Yazdigird to obtain $2^s29°12'30''$. In fact, Chioniades' and al-Fārisī's tables differ throughout by exactly the apogee motion in 90 Persian years, $1°21'47''$.[8] In both sources, a correction for the motion of the apogee is necessary for years other than the epoch year. This correction is carried out by means of the "equation of the apogee" (see above), which needs to be added to, or subtracted from, the true solar longitude depending on whether the desired year follows or precedes the respective epochs.

It is a simple matter to reconstruct the actual solar equation values from the tables of Chioniades and al-Fārisī, so that they can be compared with other sources. (The same holds for lunar and planetary equations that are displaced by the apogee longitude or involve a mixed equation.) In the remainder of this article, whenever I compare tables with different displacements or formats, I will tacitly assume that they have been reduced to a standard form. It turns out that Chioniades' solar equation contains only eight errors (in 180 tabular values) of at most $1''$. Five of these errors match with errors in al-Fārisī and Maḥmūd ibn ʿUmar, whose tables have a total of nine and eleven errors respectively. It is therefore probable that the three tables come from a common source, for which al-Fahhād is the most likely candidate. Chioniades would then have copied the original form of the table directly from the *ʿAlāʾī Zīj* without adjusting the apogee longitude, whereas al-Fārisī did carry out such an adjustment. Furthermore, Maḥmūd ibn ʿUmar would have extracted the standard form of the solar equation from al-Fahhād's table of the true solar longitude. Because of the small number of errors, it is not possible to decide whether al-Fahhād used the highly accurate table of Abu 'l-Wafāʾ or performed an inde-

[8] There are only eight deviations (out of 360 tabular values) from this constant difference, five of which can easily be explained as scribal errors.

pendent computation. It is quite certain, though, that the undisplaced solar equation table in the *Shāmil Zīj* [see van Dalen 1989, pp. 106–113], which is tabulated for every 12′ of the mean anomaly and has 17 errors for integer arguments but only one in common with the *ʿAlāʾī Zīj*, constitutes an independent computation.

Lunar Equation of Centrum

In the *Nāṣirī Zīj*, the lunar equation of centrum is tabulated as a function of the elongation, which is somewhat less common than the double elongation. The table has values to minutes for every degree of the argument and is displaced by 12 zodiacal signs. The maximum equation is Ptolemy's standard value 13°8′, corresponding to his lunar eccentricity of 10;19 units.

In Chioniades' version of the *ʿAlāʾī Zīj* and in the *Muẓaffarī Zīj*, the lunar equation of centrum is displaced by the maximum equation, 13°8′, and shifted upwards by 5°, in agreement with the displacement of the equation of anomaly (see below). A comparison of tabular values does not yield much information in this case, since all three tables differ in at most five or six places from Ptolemy's table in the *Handy Tables* (which also has the elongation as its argument). The lunar equation of centrum in the *Shāmil Zīj*, on the other hand, has the double elongation as its argument and a clearly larger number of deviations from Ptolemy.

Lunar Equation of Anomaly

In Ptolemy's *Almagest* and *Handy Tables*, as in practically all Islamic zījes, the lunar and planetary equations of anomaly are calculated by performing so-called "Ptolemaic interpolation" between values of the equation for two or three particular positions of the epicycle on the deferent. This interpolation is carried out by means of a non-trivial function of the position of the epicycle, the "interpolation function". In the case of the moon, the tables provided for the calculation of the equation of anomaly are usually the equation at the apogee of the deferent (often called the "second equation"), the differences in the equation between apogee and perigee, and the interpolation function.

In the *Nāṣirī Zīj*, the three tables to be used in the calculation of the lunar equation of anomaly display values to minutes for

every degree of the respective arguments. They are of standard type, except that the second equation is displaced by 12 zodiacal signs. The equation at apogee reaches a Ptolemaic maximum of $5°0'$, the *ikhtilāf* ("difference [in the equation]") of $2°39'$.

The structure and use of the tables for the lunar equation of anomaly in the *ʿAlāʾī Zīj* and the *Muẓaffarī Zīj* are completely different from the *Nāṣirī*, since they are not only displaced but also of "mixed type". Thus the table of the second lunar equation contains (originally) subtractive values for the equation of anomaly at the perigee of the deferent in its first half (arguments 0 to 180°), and (originally) additive values for the equation of anomaly at the apogee in its second half (arguments 180 to 360°). All values are displaced by 5°, and, since the subtractive values in the first half of the table assume a maximum of $7°39'$, another 12^s is added to those subtractive values which have an absolute value larger than 5° (we can thus say that the table is displaced by $12^s5°$). It seems highly probable that the insufficient displacement of 5° (instead of 8°, the next larger integer of the maximum equation) provides a historical clue as to the origin of this table. Earlier tables for the lunar equation of anomaly with a displacement of 5° include those of Ḥabash al-Ḥāsib and al-Bīrūnī, whereas Kūshyār used a displacement of 8°. As will be shown below, the underlying values of the equation in Chioniades' version of the *ʿAlāʾī Zīj* and in the *Muẓaffarī Zīj* point to a dependence on Ḥabash, since al-Bīrūnī used a different parameter and tabulated the equation to seconds instead of to minutes.

Of course, the Ptolemaic interpolation on values in the first half of the table of the second lunar equation now needs to be carried out differently from the second half. In fact, the table of differences in the equation of anomaly is accompanied by two interpolation functions that are supplementary, i.e., whose values sum up to one. The standard interpolation function as found in most zījes is used with the additive values of the equation, i.e., in the second half of the table of the second equation, whereas the supplementary function is used when the equation is taken from the first half of the table. The result is that also the Ptolemaic interpolation is *always* carried out by means of an addition and never requires a subtraction. That this procedure in fact yields the correct equation of anomaly, will be shown in my forthcoming publication on the characteristics of tables for calculating plane-

tary longitudes in Islamic zījes.

Even though the structure of the tables for the lunar equation of anomaly in Chioniades and al-Fārisī is so different, it turns out that the underlying values are basically the same as those in the *Nāṣirī Zīj*. A comparison with the tables in earlier extant zījes shows that this is much more significant than in the case of the equation of centrum, since many Muslim astronomers appear to have calculated the lunar equation of anomaly anew. The only tables to which those in the *Nāṣirī Zīj* and the *ʿAlāʾī Zīj* are really close are those of Yaḥyā ibn Abī Manṣūr and Ḥabash al-Ḥāsib, which, in turn, seem to have been independent of Ptolemy's. Since we have noted above that also the insufficient displacement of 5° may point to a dependence on Ḥabash (whereas we had already seen in the section on the daily mean motions that al-Fahhād found the lunar parameters of the Mumtaḥan astronomers to be the most correct), we may conclude that the table for the second lunar equation in the *ʿAlāʾī Zīj* most probably derives from that of Ḥabash.

The situation is more complicated with the differences in the lunar equation of anomaly between the apogee and perigee of the deferent. The tables for this function in the *Nāṣirī*, *ʿAlāʾī*, and *Muẓaffarī* Zījes are all three basically identical with the table in the *Handy Tables* or in al-Battānī. However, the differences that can be reconstructed from the two halves of the mixed table of the equation of anomaly in Chioniades and al-Fārisī show more than 40 deviations from the explicitly given differences and exhibit a shift in the tabular values between arguments 60 and 90° which also occurs in the *Mumtaḥan Zīj* and with Ḥabash al-Ḥāsib. Thus again a dependence on Ḥabash is plausible.

A comparison of the tables for the lunar interpolation function in various sources is more cumbersome because these usually consist of numerous repetitive stretches of the same numbers between 0 and 60 minutes, and are therefore extremely sensitive to mistakes in copying. Furthermore, their argument may be the elongation or the double elongation and will be shifted in the case of displaced lunar equation tables. We therefore only note that again the interpolation functions in the *Nāṣirī Zīj*, Chioniades' version of the *ʿAlāʾī Zīj*, and the *Muẓaffarī Zīj* are basically identical, and that they are close to that of Ḥabash al-Ḥāsib.

planet	maximum equation	eccentricity
Saturn	6;31	3;25
Jupiter	5;15	2;45
Mars	11;25	6; 0
Venus	1;59	1; 2
Mercury	3; 2	3; 0

Table 3. *Parameters of the planetary equations of centrum in the Nāṣirī Zīj, the Byzantine version of the ʿAlāʾī Zīj, and the Muẓaffarī Zīj.*

Planetary Equation of Centrum

In the *Nāṣirī Zīj*, the planetary equations of centrum are tabulated to minutes for each degree of the mean centrum and are displaced by 12 zodiacal signs. Except for Venus, the maximum equations, and hence the underlying eccentricities, are equal to Ptolemy's values in the *Almagest* and the *Handy Tables* (cf. Table 3). The new Islamic maximum equation for Venus, 1°59′, like that of the Sun, stems from the observations made in Baghdad under the caliph al-Maʾmūn, and occurs in the zījes of Yaḥyā ibn Abī Manṣūr, Ḥabash al-Ḥāsib, al-Battānī, and others. The only other zījes that include the displacement of 12 signs are Chioniades' version of the *ʿAlāʾī Zīj* and the *Muẓaffarī Zīj*; Kūshyār uses displacements equal to the maximum equation or next higher integer degree.

As far as the tabular values for the equations of centrum are concerned, it appears that most Muslim astronomers up to the thirteenth century simply copied those from the *Handy Tables* (except, of course, for Venus). Exceptions to this rule are found, in particular, with Ibn al-Aʿlam, who observed new maximum equations for Saturn, Jupiter, and Mercury; Kūshyār, who adjusted the maximum equation for Mars to 11°30′; Ibn Yūnus, who modified the maximum equation for Mercury to 4°2′; and al-Bīrūnī, who introduced an error in the computation for Mercury [cf. Yano 2002] and in general carried out a different type of interpolation between values from the *Almagest* rather than using the *Handy Tables* directly. For Venus, both al-Bīrūnī and al-Khāzinī reverted to a Ptolemaic maximum equation of 2°23′, whereas most other astronomers stayed with the Mumtaḥan value, 1°59′.

The values for the equations of centrum in the *Nāṣirī Zīj* are generally close to those in the *Handy Tables*, and show hardly any differences from the tables in Chioniades' version of the *ʿAlāʾī Zīj*

and the *Muẓaffarī Zīj*. In a few cases, certain deviations from the *Handy Tables* allow us to draw more detailed conclusions about the origin of the tables in the *ʿAlāʾī*, *Muẓaffarī*, and *Nāṣirī* Zījes. For instance, the tables for Saturn in these three works contain a peculiar interpolation pattern between arguments 0 and 18°, which is further only found with al-Khāzinī. Also the tables for Mars in the *ʿAlāʾī Zīj* and the *Muẓaffarī Zīj* are clearly closer to the table of al-Khāzinī than to the *Handy Tables*; however, in this case the *Nāṣirī Zīj* contains a shift of the tabular values in the fourth sign (arguments 90–120°) which is furthermore only found in the *Shāmil Zīj*.

Planetary Equation of Anomaly

As was explained above, the lunar and planetary equations of anomaly are usually calculated by means of Ptolemaic interpolation between two or three fixed positions of the epicycle on the deferent. In the case of the planets, these positions are the apogee, the perigee, and a "central position" at which the distance of the epicycle centre from the earth is precisely 60 units, and hence the equation is independent of the planetary eccentricity.[9] Tables are usually provided for: the central equation of anomaly (the "second equation"); the differences in the equation between the central position and the apogee, which I will call "decrements" since they need to be subtracted from the central equation; the differences in the equation between the central position and the perigee, the "increments"; and an interpolation function. We will now discuss the tables in the *Nāṣirī Zīj* of each of these types and their relations to tables in other works.

The tables in the *Nāṣirī Zīj* for calculating the planetary equation of anomaly are of standard type, except that the central equation is displaced by 12 zodiacal signs. All functions are tabulated to minutes for every degree of the respective arguments. Table 4 displays for each planet the maximum values of the decrements, the central equation of anomaly, and the increments, and

[9] Even though the Mercury model is somewhat different from that for the other planets, the setup and use of the tables for its equations are basically the same. A setup of the tables for the planetary equation of anomaly in which only two reference equations (namely, for the apogee and the perigee) are used, is found in some later zījes, for example, al-Kāshī and Ulugh Beg.

planet	maximum values			epicycle radius	apogee longitude
	decrements	central eq.	increments		
Saturn	0;21	6;13	0;25	6;30	8^s 8; 2
Jupiter	0;30	11; 3	0;34	11;30	5^s27;44
Mars	5;38	41; 9	8; 3	39;30	4^s14;45
Venus	1;42	45;59	1;52	43;10	2^s15;55
Mercury	3;12	22; 2	2; 1	22;30	6^s29;20

Table 4. *Parameters of the planetary equation of anomaly in the Nāṣirī Zīj, the Byzantine version of the ʿAlāʾī Zīj, and the Muẓaffarī Zīj (the apogee longitudes are as found in the ʿAlāʾī Zīj for al-Fahhād's epoch 541 Yazdigird). Note that the decrements and increments for Mercury in the ʿAlāʾī Zīj and the Muẓaffarī Zīj (maximum values 2;50 and 2;13 respectively) are not in agreement with the parameters that underlie the equation of centrum and the central equation of anomaly.*

the underlying radius of the epicycle (the eccentricities are already listed in Table 3; the last column of Table 4 shows the apogee longitudes for 541 Yazdigird that will be seen below to be involved in the equation of anomaly tables in Chioniades' version of the *ʿAlāʾī Zīj*). All maximum values, and hence the underlying eccentricities and epicycle radii, are Ptolemaic, which implies that the Maʾmūnic value of the eccentrity of Venus (cf. above) was *not* taken into account in the decrements and increments.

The planetary equation of anomaly in the Byzantine version of the *ʿAlāʾī Zīj* and in the *Muẓaffarī Zīj* is again implemented by means of a "mixed equation" (cf. the description of the lunar tables above). In this case, two mixed tables are needed, one displaying the equation of anomaly at the apogee of the deferent in its first half (arguments 0 to 180°) and the central equation in its second half (arguments 180 to 360°), and one displaying the central equation in its first half and the equation at perigee in its second half. Both tables are accompanied by a table with respectively decrements and increments of the central equation of anomaly and two interpolation functions whose values add up to one. The mixed tables are displaced by the longitude of the planetary apogee, so that the equation found from them can simply be added to the true centrum of the planet in order to obtain its true longitude. Again, Chioniades maintains the original apogee longitudes of al-Fahhād, whereas al-Fārisī adjusts them to his own time. The maximum values of the decrements, central equation, and increments in the *ʿAlāʾī Zīj* and in the *Muẓaffarī Zīj* are

the same as those in the *Nāṣirī*, except for the decrements and increments of Mercury (respectively 2;50 instead of 3;12 and 2;13 instead of 2;1; cf. below).

In general, also the tables for the central equation of anomaly in Islamic zījes up to the thirteenth century were simply copied from the *Handy Tables*. Ibn Yūnus reproduced the original tables particularly accurately; the typically 20 deviations found in many other zījes can mostly be explained as scribal errors or from some small adjustments of the interpolation pattern, in particular around the maximum equation. Also for the equation of anomaly, al-Bīrūnī performed entirely new interpolations between the values from the *Almagest*. Clear evidence for the fact that the tables in the *Nāṣirī Zīj*, the *ʿAlāʾī Zīj*, and the *Muẓaffarī Zīj* are strongly related to each other, and all depend on al-Khāzinī, is provided by the tables for Jupiter and Mars. Where these differ in all four sources in around 20 (Jupiter) or even 50 values (Mars, due to large systematic differences in the fifth and sixth signs) from the *Handy Tables*, they do not differ in more than five values among each other.

The tables of decrements and increments of the central equation of anomaly show much less computational variation. This is partly because they were calculated by means of "distributed linear interpolation" [cf. Van Brummelen 1998, p. 278] and are thus monotonically increasing before their maximum and monotonically decreasing thereafter.[10] Furthermore, since the decrements and increments are tabulated to only minutes, whereas, except for Mars, their maximum values are at most a little more than one degree, the tables concerned consist for the most part of repetitions of the same values.

Similar to the equation of centrum and the central equation of anomaly, most of the tables of planetary decrements and increments in Islamic zījes up to the thirteenth century are based on the *Handy Tables*. The variations that we find are mostly of two types: 1) adjustments of the interpolation pattern, for instance in order to evenly spread out tabular differences in a linear part of the table, or to smoothen the section surrounding a maximum; 2) shifts of tabular values by one (sometimes two) row(s) upwards

[10] It can be verified that if the decrements and increments were computed as the actual differences of accurately computed equation of anomaly values at two different epicycle positions they would not generally be monotone.

or downwards. These shifts are often scribal mistakes, in which case they may well extend to the bottom of the column concerned, where the error would finally be noticed by the copyist. Shifts are particularly common in sections of the tables where values are repeated at least four times in a row. It seems that many of the deviations from the *Handy Tables* in tables of decrements and increments of the central equation of anomaly arise from this kind of scribal errors, the number of deviations gradually increasing in the course of the centuries, sometimes to even more than half of the tabular values.

It is precisely the shifts in large parts of columns that allow us to draw conclusions about the relations between tables of decrements and increments of the planetary equation of anomaly in Islamic zījes. For instance, the decrements and increments for Saturn, the increments for Jupiter, and the decrements and increments for Venus in the *ʿAlāʾī*, *Muẓaffarī*, and *Nāṣirī* Zījes have long shifts in common of up to a total of 63 deviations from the *Handy Tables*, which are further only found with al-Khāzinī. Also most of the other decrement and increment tables in the *Nāṣirī Zīj* are very close to the *ʿAlāʾī* and *Muẓaffarī* Zījes, but here the evidence for a dependence on al-Khāzinī is less conclusive. The increments for Mars in the *Nāṣirī Zīj* display an upward shift of one row through arguments 158 to 177° (due to the omission of the value for 158°), but besides there are only two differences from the *Muẓaffarī Zīj* and seven from the *ʿAlāʾī*. A special case is Mercury, for which al-Khāzinī follows the *Handy Tables*, whereas the *ʿAlāʾī Zīj* and the *Muẓaffarī Zīj* tabulate a completely different function which is in agreement with the equations of anomaly at the apogee, central position and perigee as found in the mixed tables (as a matter of fact, as opposed to the lunar equation of anomaly, the decrements and increments of the planetary equations of anomaly are in each case in full agreement with the equations given in the mixed tables). I do not currently have an explanation for these deviating decrements and increments. In any case, the author of the *Nāṣirī Zīj* apparently restored the original functions from the *Handy Tables* or one of its direct descendants (the number of deviations from al-Khāzinī's tables is here clearly larger than from the *Handy Tables*). In this process he introduced a shift of the complete first sign of the decrements by inserting an extra zero for argument 1°.

It is remarkable that the new value for the eccentricity of Venus that occurred for the first time in the *Mumtaḥan Zīj* and was used in the tables for the central equation of anomaly in various later zījes, was not incorporated into the equation decrements and increments in those zījes except in the *Mumtaḥan Zīj* itself and, independently, in the *Ḥākimī Zīj*. All other zījes that used the new value simply copied the decrements and increments from the *Handy Tables*, thus introducing errors of up to 20′ in the final equation of anomaly.[11] Since Ibn Yūnus observed a new epicycle radius for Venus besides a new eccentricity, and also adjusted the parameters of the Mercury model, he had to compute the equations for both planets completely anew. Kūshyār experimented with a variant of Ptolemaic interpolation in which the roles of the strong and weak variables were interchanged [cf. Van Brummelen 1998]; his tables for decrements, increments, and the interpolation function therefore cannot be compared with other zījes.

The tables of the interpolation function for the planetary equation of anomaly, whose values lie exclusively between 0′ and 60′, share the property with the tables for decrements and increments that they consist largely of repetitions of the same values and are therefore extremely susceptible to accidental shifts of parts of columns.[12] Most interpolation tables appear to be variants of those in the *Handy Tables* with a smaller or larger number of shifts, presumably introduced by careless copyists. Some of these shifts are so peculiar that they allow us to establish dependences between tables of the interpolation function. As a matter of fact, a comparison of the deviations from the *Handy Tables* in the interpolation tables in the two zījes of al-Khāzinī and in the *ʿAlāʾī Zīj*, the *Muẓaffarī Zīj*, and the *Nāṣirī Zīj* shows very

[11] In his *Ḥākimī Zīj*, Ibn Yūnus notes the inconsistency in various zījes between the new Mumtaḥan value of the eccentricity of Venus and the Ptolemaic decrements and increments of the equation of anomaly, and states that this first occured with Yaḥyā ibn Abī Manṣūr [Caussin de Perceval 1804, p. 74 (58 in separatum)]; the eccentricity value 2;3,35 that Ibn Yūnus associates with Yaḥyā is undoubtedly a scribal mistake for 2;4,35. Since the decrements and increments for Venus in the extant recension of the *Mumtaḥan Zīj* were correctly computed for the new eccentricity value, they may not stem from the original zīj.

[12] The only exceptions to this rule are tables with values to seconds as are found in Ptolemy's *Almagest* and in al-Bīrūnī's *Masʿūdic Canon*.

clear relations between all these works, especially in the cases of Jupiter and Venus. That the exact dependences may be complicated is shown by the Jupiter table. The *ʿAlāʾī*, *Muẓaffarī*, and *Nāṣirī* Zījes copy four longer shifts from the *Sanjarī Zīj*, whereas the *Wajīz* omits one of these and adds still another. The *ʿAlāʾī* and *Muẓaffarī* add three more shifts of which one, however, is not found in the *Nāṣirī Zīj*. In the case of Venus, Chioniades' version of the *ʿAlāʾī Zīj* provides an additional shift of the complete fourth sign (arguments 90–120°) and some more smaller ones, none of which is found in the *Muẓaffarī* and *Nāṣirī* Zījes.

Al-Fahhād presumably introduced one peculiar type of shift in the interpolation functions that was not yet found with al-Khāzinī. For all planets except Mercury, the central position of the epicycle center corresponds to an angular distance of approximately 88° from the apogee of the deferent. This means that the interpolation coefficients for values of the true centrum from 1 to 87° are to be used with the equation of anomaly decrements, and those for values from 88 to 180° with the increments. Al-Khāzinī included the two ranges of coefficients in separate tables. He thus needed four columns for the coefficients that are used with the increments, of which the first contains only values for arguments 88 and 89°. Apparently al-Fahhād wanted to avoid this waste of space and squeezed the values for 88 and 89° into the column for the fourth sign (arguments 90–120°). At the same time he spread out the values in the last section of the table that was used with the decrements in order to fill up the free space for arguments 88 and 89°. The result is that al-Fahhād's interpolation functions for all planets except Mercury have a clear distortion with respect to the *Handy Tables* in the neighborhood of 90 degrees. This distortion can be clearly recognized in the Byzantine version of the *ʿAlāʾī Zīj*, the *Muẓaffarī Zīj*, and the *Nāṣirī Zīj*, as well as in the *Shāmil Zīj*.

5 *Summary and Conclusions*

In our study of the *Nāṣirī Zīj* by Maḥmūd ibn ʿUmar we have clearly established the dependence of this work on the *ʿAlāʾī Zīj* by al-Fahhād. We have seen that Maḥmūd computed accurate mean motion tables on the basis of the daily mean motions in longitude and in anomaly listed in the Byzantine version of the

ʿAlāʾī Zīj by Gregory Chioniades and in the further unrelated *Sanjufīnī Zīj*. Since these parameters also underlie the mean motion tables in Chioniades and in al-Fārisī's *Muẓaffarī Zīj*, which is said to be based on al-Fahhād's observations, we may conclude that they stem from the original *ʿAlāʾī Zīj*. For calculating the mean motions in centrum, Maḥmūd utilized a slightly different value of the daily apogee motion. There is no direct relation between the epoch positions in the *Nāṣirī Zīj*, which are for the longitude of Delhi (103°35′ from the Western Shore of Africa) and the first day of the year 1 before Yazdigird, and those in the *ʿAlāʾī Zīj*; thus it is still unclear whether Maḥmūd found his epoch positions from new observations or from still another zīj.

The solar, lunar and planetary equations in the *Nāṣirī Zīj* are all displaced by 12 zodiacal signs, which means that subtractive values q are tabulated as $12^s - q$ and thus become additive. We have seen that this type of displacement stems from al-Fahhād; the more complicated type of displacement by the maximum equation or next higher integer is already found with Ḥabash al-Ḥāsib and Kūshyār ibn Labbān. The values of the equations in the *Nāṣirī Zīj* agree very well with those in Chioniades' version of the *ʿAlāʾī Zīj* and in al-Fārisī's *Muẓaffarī Zīj*. It is thus very probable that Maḥmūd reconstructed his solar equation from the table for the true solar longitude in the *ʿAlāʾī Zīj* and his plain equations of anomaly from the mixed and displaced tables in that work. In the equation of centrum and the equation of anomaly increments for Mars, Maḥmūd or a later scribe introduced shifts in a whole column by omitting one particular value at the beginning. For Mercury, Maḥmūd restored the decrements and increments of the central equation of anomaly from the *Handy Tables*, again accidentally shifting a whole column by one row; for reasons not yet known to us, the decrements and increments in the *ʿAlāʾī Zīj* are completely different functions (see also below).

In the course of our investigation, we have been able to establish to a large extent the original form of the planetary tables in the *ʿAlāʾī Zīj*. Because of the close similarity of the tables of Chioniades with those of al-Fārisī, and since Chioniades uses what is very probably al-Fahhād's epoch year A.D. 1172 and his base longitude of 84° for Shirwan, we may conclude that the Byzantine version of the *ʿAlāʾī Zīj* faithfully reproduces al-Fahhād's plane-

tary tables. In the *Muẓaffarī Zīj*, al-Fārisī adjusted the planetary equations to his own epoch year A.D. 1262 and the mean positions to the longitude of Yemen (64° or 63°30′), but further left the structure of the tables unchanged. We have seen that the solar and lunar mean motions in the *ʿAlāʾī Zīj* were in agreement with those of the Mumtaḥan astronomers, as stated by al-Fārisī in the introduction to his zīj. Furthermore, we have verified the statement that the mean motions of Mars and Venus are quite close to those of Ibn al-Aʿlam.

None of the planetary equations in the original *ʿAlāʾī Zīj* were given in the standard form. Instead of the solar equation, al-Fahhād tabulated the true solar longitude, which, for years other than the epoch year, needed to be corrected for the motion of the apogee. The lunar equation of centrum was displaced by the maximum equation, 13°8′. The lunar equation of anomaly was displaced by the maximum equation at apogee, 5°0′, but was also of "mixed type", which means that one half of the table displayed the equation at apogee, the other at perigee. In this way, the conditional addition or subtraction that occurs in the calculation of the general equation of anomaly by means of Ptolemaic interpolation, is eliminated at the cost of some extra tables. The planetary equations of centrum were all displaced by 12 zodiacal signs and hence of the same form as found in the *Nāṣirī Zīj*. The planetary equations of anomaly, finally, were of mixed type and displaced by the longitude of the apogees concerned. Also here, a correction for the motion of the apogee was necessary.

As far as the origin of the tables for the planetary equations in the original *ʿAlāʾī Zīj* is concerned, we have seen that the solar equation, based on the Mumtaḥan maximum of 1°59′0″, was independently computed or possibly rounded from Abu 'l-Wafāʾ's table. The lunar equations have their displacements in common with the *Zīj* of Ḥabash al-Ḥāsib, with which they also show the highest coincidence of the tabular values. Most of the planetary equations ultimately derive from the *Handy Tables*, but numerous peculiar error patterns make clear that al-Fahhād's direct sources for the tables for Saturn, Jupiter, Mars and Venus must have been the *Sanjarī Zīj* and the *Wajīz* of al-Khāzinī. The one exception to this rule is the equation of centrum for Venus, for which the *ʿAlāʾī Zīj* seems to have used the table of Kūshyār with the Mumtaḥan maximum of 1°59′, whereas al-Khāzinī per-

formed an independent computation for the Ptolemaic maximum 2°23′. Although the decrements and increments of the equation of anomaly also depend on the maximum equation of centrum, al-Fahhād here sticked to al-Khāzinī's tables for Ptolemy's parameters, thus introducing an inconsistency that leads to errors in the longitude of Venus of up to 20′. Contrary to common usage, al-Fahhād took the longitude of the apogee of Venus to be different from that of the Sun. In the interpolation functions of all planets except Mercury, al-Fahhād introduced a peculiar distortion in the neighborhood of the central position of the epicycle on the deferent. In agreement with al-Fārisī's statement that al-Fahhād found his observations of Mercury to be in best agreement with Ptolemy, the *ʿAlāʾī Zīj* includes the tables of the equation of centrum, the central equation of anomaly, and the interpolation function for this planet from the *Handy Tables.* However, the decrements and increments of the central equation of anomaly are represented by a completely different function that we have not been able to explain.

We have also presented some scattered information on the planetary tables in the *Shāmil Zīj.* The statement in its introduction that the *ʿAlāʾī Zīj* and the *Shāmil Zīj* use the mean motions of Abu 'l-Wafāʾ could not be confirmed. The daily mean motions listed in the *Shāmil Zīj* are very close to those in the the *ʿAlāʾī Zīj*, and the epoch positions are in full agreement. The solar equation also has a maximum value 1°59′0″, but was independently computed for every 12′ of the mean anomaly. The lunar equation of centrum is tabulated as a function of the double elongation and with clearly more deviations from the *Handy Tables* than most other zījes. The lunar equation of anomaly at apogee was undoubtedly copied from Kūshyār's *Jāmiʿ Zīj.* The tables for the planetary equations are all of standard type without displacements. The tabular values exhibit some similarities to other zījes, such as Kūshyār's *Jāmiʿ Zīj*, the *ʿAlāʾī Zīj*, and the *Nāṣirī Zīj.* In particular, the equations of Venus were taken from Kūshyār, whereas the interpolation functions include the peculiar distortion around the central position on the deferent that was introduced by al-Fahhād.

Appendix: Elementary Relations between Mean Motions

Because of the way in which the Ptolemaic solar, lunar and planetary models are set up, certain elementary relations exist between the mean motions and positions. These have been used in this article to verify the consistency of the epoch positions given in the *Nāṣirī Zīj* and to correct scribal errors in them. Note that the relations hold for actual mean positions as well as for mean motions in any given period. They are the following:

1. The difference of the solar mean longitude and the solar mean anomaly is the longitude of the solar apogee; the difference of the planetary mean longitude and the planetary mean centrum is the longitude of the apogee of the planet concerned.
2. The lunar elongation is the difference of the lunar and solar mean longitudes, the double elongation is twice that difference.
3. The sum of the mean longitude and mean anomaly of the superior planets equals the solar mean longitude; the sum of the mean centrum and mean anomaly of the superior planets equals the solar mean centrum.
4. The mean longitude of the inferior planets is equal to the solar mean longitude; the mean centrum of the inferior planets is equal to the solar mean centrum.

Bibliography

Āghā Buzurg Ṭihrānī, Muḥammad Muḥsin 1936–78. *al-Dharīʿat ilā taṣānīf al-Shīʿa* (*Bibliography of Shiʿit'es Literary Works*, in Arabic), 25 vols., Najaf / Tehran.

al-Bīrūnī, Abū Rayḥān Muḥammad. *al-Qānūnu'l-Masʿūdī (Canon Masudicus). An Encyclopaedia of Astronomical Sciences*, 3 vols., Hyderabad (Osmania Oriental Publications Bureau), 1954–56.

Caussin de Perceval, Armand Pierre 1804 (an XII de la République). Le livre de la grande table hakémite, *Notices et extraits des manuscrits de la Bibliothèque Nationale* 7, Paris

(Bibliothèque Nationale), pp. 16–240 (pp. 1–224 in separatum).

Dalen, Benno van 1989. A Statistical Method for Recovering Unknown Parameters from Medieval Astronomical Tables, *Centaurus* 32, pp. 85–145.

——. 1993. *Ancient and Mediaeval Astronomical tables: mathematical structure and parameter values* (doctoral dissertation), Utrecht (Utrecht University, Mathematical Institute).

——. 1996. al-Khwārizmī's Astronomical Tables Revisited: Analysis of the Equation of Time, in *From Baghdad to Barcelona. Studies in the Islamic Exact Sciences in Honour of Prof. Juan Vernet* (Josep Casulleras & Julio Samsó, eds.), Barcelona (Instituto Millás Vallicrosa de Historia de la Ciencia Arabe), vol. 1, pp. 195–252.

——. 2000. Origin of the Mean Motion Tables of Jai Singh, *Indian Journal of History of Science* 35, pp. 41–66.

——. 2002. Islamic Astronomical Tables in China: The Sources for the *Huihui li*, in *History of Oriental Astronomy. Proceedings of the Joint Discussion-17 at the 23rd General Assembly of the International Astronomical Union, organised by the Commission 41 (History of Astronomy), held in Kyoto, August 25–26, 1997* (S. M. Razaullah Ansari, ed.), Dordrecht (Kluwer), pp. 19–31.

Debarnot, Marie-Thérèse 1987. The Zīj of Ḥabash al-Ḥāsib: A Survey of MS Istanbul Yeni Cami 784/2, in *From Deferent to Equant. A Volume of Studies in the History of Science in the Ancient and Medieval Near East in Honor of E.S. Kennedy* (David A. King & George A. Saliba, eds.), New York (The New York Academy of Sciences), pp. 35–69.

Ḥusaynī, Aḥmad & Marʿashī, Maḥmūd 1994. *Fihrist-i nuskhahā-yi khaṭṭī-yi Kitābkhānah-yi ʿumūmī-yi Ḥaḍrat-i Āyatallāh al-ʿUẓmā Marʿashī Najafī* (Catalogue of the manuscripts of the Public Library Marʿashī, in Persian), vol. 23, Qum (Chāpkhānah-yi Mihr-i Ustuwār).

Kennedy, Edward S. 1956. A Survey of Islamic Astronomical Tables, *Transactions of the American Philosophical Society, New Series* 46-2, pp. 123–177. Reprinted in 1989 with page numbers 1–55.

——. 1977. The Astronomical Tables of Ibn al-Aʿlam, *Journal for the History of Arabic Science* 1, pp. 13–23.

Kennedy, Edward S. & Kennedy, Mary Helen 1987. *Geographical Coordinates of Localities from Islamic Sources*, Frankfurt am Main (Institute for the History of Arabic-Islamic Science).

King, David & Samsó, Julio 2001. Astronomical Handbooks and Tables from the Islamic World (750-1900): an Interim Report, *Suhayl* 2, pp. 9–105.

Lee, Samuel 1822. Notice of the Astronomical Tables of Mohammed Abibekr al-Farsi, *Transactions of the Cambridge Philosophical Society* 1, pp. 249–265. Reprinted in Fuat Sezgin (ed.), *Islamic Mathematics and Astronomy*, volume 77, Frankfurt (Institute for the History of Arabic-Islamic Science), 1998, pp. 315–331.

Mercier, Raymond P. 1989. The Parameters of the *Zīj* of Ibn al-Aʿlam, *Archives Internationales d'Histoire des Sciences* 39, pp. 22–50.

Mielgo, Honorino 1996. A Method of Analysis for Mean Motion Astronomical Tables, in *From Baghdad to Barcelona. Studies in the Islamic Exact Sciences in Honour of Prof. Juan Vernet* (Josep Casulleras & Julio Samsó, eds.), Barcelona (Instituto Millás Vallicrosa de Historia de la Ciencia Arabe), vol. 1, pp. 159–179.

Nallino, Carlo Alfonso 1899–1907. *al-Battani sive Albatenii opus astronomicum (al-Zīj al-Ṣābiʾ)*, 3 vols, Milan (Ulrich Hoepli).

Neugebauer, Otto 1957. *The Exact Sciences in Antiquity*, second edition, Providence (Brown University Press).

——. 1975. *A History of Ancient Mathematical Astronomy*, 3 vols., Berlin (Springer).

Pedersen, Olaf 1974. *A Survey of the Almagest*, Odense (Odense University Press).

Pingree, David 1985–86. *The Astronomical Works of Gregory Chioniades. Part I: The Zīj al-ʿAlāʾī*, 2 vols., Amsterdam (Gieben).

——. 1999. A Preliminary Assesment of the Problems of Editing the Zīj al-Sanjarī of al-Khazini, in *Editing Islamic Manuscripts on Science. Proceedings of the Fourth Conference of Al-Furqān Islamic Heritage Foundation. 29th–30th November 1997* (Yusuf Ibish, ed.), London (Al-Furqān Islamic Heritage Foundation).

Salam, Hala & Kennedy, Edward S. 1967. Solar and Lunar Tables in Early Islamic Astronomy, *Journal of the American Oriental Society* 87, pp. 492–497.

Saliba, George A. 1976. The Double-Argument Lunar Tables of Cyriacus, *Journal for the History of Astronomy* 7, pp. 41–46.

——. 1977. Computational Techniques in a Set of Late Medieval Astronomical Tables, *Journal for the History of Arabic Science* 1, pp. 24–32.

——. 1978. The Planetary Tables of Cyriacus, *Journal for the History of Arabic Science* 2, pp. 53–65.

Sayılı, Aydin 1955. The Introductory Section of Ḥabash's Astronomical Tables Known as the "Damascene" Zīj (in Turkish, English and Arabic), *Ankara Üniversitesi Dil ve Tarih-Coğrafya Fakültesi dergisi* 13, pp. 132–151.

Storey, Charles Ambrose 1958. *Persian Literature: a Bio-Bibliographical Survey*, vol. 2, London (Luzac & Co.).

Mathematical Methods in the Tables of Planetary Motion in Kūshyār ibn Labbān's *Jāmiʿ Zīj*, *Historia Mathematica* 25, pp. 265–280.

Waerden, Bartel L. van der 1958. Die Handlichen Tafeln des Ptolemaios, *Osiris* 13, pp. 54–78.

Yaḥyā ibn Abī Manṣūr. *The Verified Astronomical Tables for the Caliph al-Maʾmūn (al-zīj al-maʾmūnī al-mumtaḥan). Fac-*

simile of Escorial Library MS árabe 927, with an introduction by E.S. Kennedy, Frankfurt am Main (Institute for the History of Arabic-Islamic Science), 1986.

Yano, Michio 2002. The First Equation Table for Mercury in the *Huihui li*, in *History of Oriental Astronomy. Proceedings of the Joint Discussion-17 at the 23rd General Assembly of the International Astronomical Union, organised by the Commission 41 (History of Astronomy), held in Kyoto, August 25–26, 1997* (S.M. Razaullah Ansari, ed.), Dordrecht (Kluwer), pp. 33–43.

Current Bibliography of David Pingree
(as of July 2003)

Abbreviations

AIHS	*Archives internationales d'histoire des sciences*
BO	*Bibliotheca Orientalis*
DOP	*Dumbarton Oaks Papers*
HM	*Historia Mathematica*
IJCT	*International Journal of the Classical Tradition*
JAOS	*Journal of the American Oriental Society*
JHAS	*Journal for the History of Arabic Science*
JHA	*Journal for the History of Astronomy*
JNES	*Journal of Near Eastern Studies*
JOI Baroda	*Journal of the Oriental Institute, Baroda*
JOR Madras	*Journal of Oriental Research, Madras*
JRAS	*Journal of the Royal Asiatic Society*
JWCI	*Journal of the Warburg and Courtauld Institutes*
PAPS	*Proceedings of the American Philosophical Society*

1 *Books and Monographs* (in chronological order)

The Thousands of Abū Ma'shar, London, 1968.
Albumasaris De revolutionibus nativitatum, Leipzig, 1968.
Sanskrit Astronomical Tables in the United States, Philadelphia, 1968.
The Vidvajjanavallabha of Bhojarāja, Baroda, 1970.
Census of the Exact Sciences in Sanskrit, Series A, vols 1–5, Philadelphia, 1970–94.
The Pañcasiddhāntikā of Varāhamihira (with O. Neugebauer), 2 vols, Copenhagen, 1970–1.

The Astrological History of Māshā'allāh (with E. S. Kennedy), Cambridge MA, 1971.
The Bṛhadyātrā of Varāhamihira, Madras, 1972.
Hephaestionis Thebani Apotelesmaticorum libri tres, 2 vols, Leipzig, 1973–4.
Sanskrit Astronomical Tables in England, Madras, 1973.
Babylonian Planetary Omens (with E. Reiner), Malibu CA, 1975–81 (2 fascs).
The Laghukhecarasiddhi of Śrīdhara, Baroda, 1975.
Dorothei Sidonii Carmen astrologicum, Leipzig, 1976.
The Vṛddhayavanajātaka of Mīnarāja, 2 vols, Baroda, 1976.
The Yavanajātaka of Sphujidhvaja, 2 vols, Cambridge MA, 1978.
The Book of the Reasons behind Astronomical Tables (with E. S. Kennedy), Delmar NY, 1981.
Jyotiḥśāstra, Wiesbaden, 1981.
A Catalogue of the Chandra Shum Shere Collection in the Bodleian Library, Part I. Jyotiḥśāstra, Oxford, 1984.
The Astronomical Works of Gregory Chioniades, Part 1. The Zīj al-'Alā'ī, 2 vols, Amsterdam, 1985–6.
Vettii Valentis Anthologiarum libri novem, Leipzig, 1986.
The Latin Picatrix, London, 1986.
The Rājamṛgāṅka of Bhojarāja, Aligarh, 1987.
MUL.APIN. An Astronomical Compendium in Cuneiform (with H. Hunger), Horn, 1989.
The Astronomical Works of Daśabala, Aligarh, 1990.
Levi ben Gerson's Prognostication for the Conjunction of 1345, (with B. R. Goldstein), Philadelphia, 1990.
The Grahajñāna of Āśādhara together with the Gaṇitacūḍāmaṇi of Harihara, Aligarh, 1989 (but actually published *ca.* 1993).
The Liber Aristotilis of Hugo of Santalla (with C. Burnett), London, 1997.
From Astral Omens to Astrology, From Babylon to Bīkāner, Rome, 1997.
Preceptum Canonis Ptolomei, Louvain-la-Neuve, 1997.
Babylonian Planetary Omens. Part Three (with E. Reiner), Groningen, 1998.
Astral Sciences in Mesopotamia (with H. Hunger), Leiden, 1999.
Arabic Astronomy in Sanskrit: Al-Birjandī on Tadhkira II, Chapter 11 and its Sanskrit Translation (with T. Kusuba), Leiden, 2002.

2 *Chapters in Books* (in chronological order)

'The *Apologia* of Alphonsus Lyncurius', in *Italian Reformation Studies in Honor of Laelius Socinus*, ed. J. Tedeschi, Florence, 1965, pp. 199–214.

'Astrology', in *Dictionary of the History of Ideas*, ed. Philip P. Wiener, 4 vols, New York, 1973-4, I, pp. 118–26.

'The *Karaṇavaiṣṇava* of Śaṅkara', in *Charudeva Shastri Felicitation Volume*, Delhi, 1974, pp. 588–600.

'Māsha'allāh: Some Sasanian and Syriac Sources', in *Essays in Islamic Philosophy and Science*, ed. George F. Hourani, Albany NY, 1975, pp. 5–14.

'Al-Bīrūnī's Knowledge of Sanskrit Astronomical Texts', in *The Scholar and the Saint: Studies in Commemoration of Abu'l-Rayhan al-Biruni and Jalal al-Din al-Rumi*, ed. Peter J. Chelkowski, New York, 1975, pp. 67–81.

'Thessalus Astrologus', in *Catalogus Translationum et Commentariorum*, vol. 3, ed. F. Edward Cranz, Washington DC, 1976, pp. 83–6.

'The Library of George, Count of Corinth', in *Studia Codicologica*, ed. K. Treu, Berlin, 1977, pp. 351–62.

'The Horoscope of Constantinople', in Πρίσματα: *Festschrift für Willy Hartner*, ed. Y. Maeyama and W. Saltzer, Wiesbaden, 1977, pp. 305–15.

'History of Mathematical Astronomy in India', in *Dictionary of Scientific Biography*, vol. 15, ed. Charles C. Gillespie, New York, 1978, pp. 533–633.

'The *Gaṇitapañcaviṃśī* of Śrīdhara', in *Ludwik Sternbach Felicitation Volume*, ed. J. P. Sinha, Lucknow, 1979, pp. 887–909.

'The *Kheṭamuktāvalī* of Nṛsiṃha', in *Sanskrit and Indian Studies: Essays in Honor of Daniel H. H. Ingalls*, ed. M. Nagatomi et al., Dordrecht–Boston, 1980, pp. 143–57.

'Mesopotamian Astronomy and Astral Omens in Other Civilizations', in *Mesopotamien und Seine Nachbarn*, ed. Hans J. Nissen and Johannes Renger, Berlin, 1982, pp. 613–31.

'The Zodiac Ceilings of Petosiris and Petubastis' (with O. Neugebauer and R. Parker), in *Denkmaler der Oase Dachla*, Mainz, 1982, pp. 96–101.

'The Diffusion of Arabic Magical Texts in Western Europe', in *La diffusione delle scienze islamiche nel medio evo europeo*, ed. B.-M. Scarcia Amoretti, Rome, 1987, pp. 57–102.

'Venus Omens in India and Babylon', in *Language, Literature, and History: Philological and Historical Studies Presented to Erica Reiner*, ed. Francesca Rochberg-Halton, New Haven CT, 1987, pp. 293–315.

'Sumatiharṣa Gaṇi and Some Other Jaina Jyotiṣīs', in *Astha aura cintana*, Delhi, 1987, pp. 99–104.

'Indian and Islamic Astronomy at Jayasiṃha's Court', in *From Deferent to Equant: a Volume of Studies in the History of Science in the Ancient and Medieval Near East in Honor of E. S. Kennedy*, ed. David A. King and George Saliba, New York, 1987, pp. 313–28.

'Babylonian Planetary Theory in Sanskrit Omen Texts', in *From Ancient Omens to Statistical Mechanics: Essays on the Exact Sciences Presented to Asger Aaboe*, ed. J. L. Berggren and B. R. Goldstein, Copenhagen, 1987, pp. 91–9.

'MUL.APIN and Vedic Astronomy', in *DUBU-E_2-DUB-BA-A. Studies in Honor of Ake W. Sjöberg*, ed. Hermann Behrens et al., Philadelphia, 1989, pp. 439–45.

'A Babylonian Star Catalogue: BM 78161' (with C. Walker), in *A Scientific Humanist: Studies in Memory of Abraham Sachs*, ed. Erle Leichty et al., Philadelphia, 1988, pp. 313–21.

'Astrology', in *The Cambridge History of Arabic Literature: Religion, Learning and Science in the 'Abbāsid Period*, ed. M. J. L. Young, J. D. Latham and R. B. Serjeant, Cambridge, 1990, pp. 290-300.

'The Preceptum canonis Ptolomei', in *Rencontres de culture dans la philosophie médiévale*, ed. M. Fattori and J. Hamesse, Louvain-la-Neuve-Cassino, 1990, pp. 355–75.

'Mesopotamian Omens in Sanskrit', in *La circulation des biens, des personnes et des idées dans la Proche-Orient ancien*, ed. Dominique Charpin and Francis Joannes, Paris, 1992, pp. 375–9.

'Innovation and Stagnation in Medieval Indian Astronomy', in *17° Congreso Internacional de ciencias historicas*, Madrid, 1992, pp. 519–26.

'Thessalus Astrologus Addenda', in *Catalogus Translationum et Commentariorum*, vol. 7, ed. Paul Oscar Kristeller, Washington DC, 1992, pp. 330–2.

'Venus Phenomena in *Enūma Anu Enlil*', in *Die Rolle der Astronomie in den Kulturen Mesopotamiens*, ed. Hannes Galter and Bernhard Scholz, Graz, 1993, pp. 259–73.

'Plato's Hermetic Book of the Cow', in *Il Neoplatonismo nel Rinascimento*, ed. Pietro Prini, Rome, 1993, pp. 133–45.

'La magia dotta', in *Federico II e le scienze*, 3 vols, ed. P. Toubert and A. Paravicini Bagliani, Palermo, 1994, II, pp. 354–70.

'Astronomy in India', in *Astronomy before the Telescope*, ed. Christopher Walker, London, 1996, pp. 123–42.

'Indian Astronomy in Medieval Spain', in *From Baghdad to Barcelona: Studies in the Islamic Exact Sciences in Honour of Prof. Juan Vernet*, 2 vols, ed. Josep Casulleras and Julio Samsó, Barcelona, 1996, pp. 39–48.

'Indian Reception of Muslim Versions of Ptolemaic Astronomy', in *Tradition, Transmission, Transformation: Proceedings of Two Conferences on Premodern Science Held at the University of Oklahoma*, ed. F. Jamil Ragep and Sally P. Ragep, Leiden, 1996, pp. 471–85.

'Māshā'allāh: Greek, Pahlavī, Arabic, and Latin Astrology', in *Perspectives arabes et médiévales sur la tradition scientifique et philosophique grecque*, ed. A. Hasnawi et al., Leuven–Paris, 1997, pp. 123–36.

'Legacies in Astronomy and Celestial Omens', in *The Legacy of Mesopotamia*, ed. Stephanie Dalley, Oxford, 1998, pp. 125–37.

'Mathematics and Mathematical Astronomy', in *India's Worlds and U. S. Scholars. 1947–1997*, ed. Joseph W. Elder et al., New Delhi, 1998, pp. 355–61.

'Preliminary Assessment of the Problems of Editing the Zīj al-Sanjarī of al-Khāzinī', in *Editing Islamic Manuscripts on Science: Proceedings of the Fourth Conference of Al-Furqān Islamic Heritage Foundation, 29th–30th November 1997*, ed. Yusuf Ibish, London, 1999, pp. 105–13.

'Avranches 235 dans la tradition manuscrite du *Preceptum Canonis Ptolomei*', in *Science antique, science médiévale: Actes du colloque international (Mont-Saint-Michel, 4–7 septembre 1998)*, ed. Louis Callebat and Olivier Desbordes, Hildesheim, 2000, pp. 163–9.

'A Greek List of Astrolabe Stars', in *Sic itur ad astra: Studien zur Geschichte der Mathematik und Naturwissenschaften. Festschrift für den Arabisten Paul Kunitzsch zum 70. Geburtstag*, ed. Menso Folkerts and Richard Lorch, Wiesbaden, 2000, pp. 474–7.

'The Coining of New Words to Express New Concepts in Sanskrit Astronomy', in *Harānandalaharī*, Reinbek, 2000, pp. 217–20.

'Ravikās in Indian Astronomy and the Kālacakra', in *Le parole e i marmi: studi in onore di Raniero Gnoli nel suo 70° compleanno*, ed. Raffaele Torella, Rome, 2001, pp. 655–64.

'I professionisti della scienza e la loro formazione', in *Storia della scienza*, gen. ed. S. Petruccioli, vol. II, Rome, 2001, pp. 690–707.

'Cosmologia vedica, cosmologia puranica', in *Storia della scienza*, gen. ed. S. Petruccioli, vol. II, Rome, 2001, pp. 715–28.

'Stelle e costellazioni', 'Il Sole e la Luna', 'I calendari', in *Storia della scienza*, gen. ed. S. Petruccioli, vol. II, Rome, 2001, pp. 729–33.

'La calendaristica vedica (Jyotisa)', in *Storia della scienza*, gen. ed. S. Petruccioli, vol. II, Rome, 2001, pp. 769–71.

'Astronomia', in *Storia della scienza*, gen. ed. S. Petruccioli, vol. II, Rome, 2001, pp. 790–813.

'Divinazione e astrologia', in *Storia della scienza*, gen. ed. S. Petruccioli, vol. II, Rome, 2001, pp. 813–20.

'Philippe de La Hire at the Court of Jayasiṃha', in *History of Oriental Astronomy: Proceedings of the Joint Discussion-17 at the 23rd General Assembly of the International Astronomical Union, organised by the Commission 41 (History of Astronomy), held in Kyoto, August 25–26, 1997*, ed. S. M. Razaullah Ansari, Dordrecht, 2002, pp. 123–31.

'The Sarvasiddhāntarāja of Nityānanda', in *The Enterprise of Science in Islam: New Perspectives*, ed. Jan P. Hogendijk and Abdelhamid I. Sabra, Cambridge MA, 2003, pp. 269–84.

'Zero and the Symbol for Zero in Early Sexagesimal and Decimal Place-Value Systems', in *The Concept of śūnya*, ed. A. K. Bag and S. R. Sarma, New Delhi, 2003.

3 *Journal Articles* (alphabetically by journal title, then chronologically)

'Some Little Known Commentators on Bhāskara's *Karaṇakutūhala*', *Aligarh Journal of Oriental Studies*, 2, 1985, pp. 158–68.

'The Teaching of the *Almagest* in Late Antiquity', *Apeiron*, 27, 1994, pp. 75–98.

'A Neo-Babylonian Report on Seasonal Hours' (with E. Reiner), *Archiv für Orientforschung*, 25, 1974–7, pp. 50–5.

'Concentric with Equant', *AIHS*, 24, 1974, pp. 26–9.

'Al-Khwārizmī in Samaria', *AIHS*, 33, 1983, pp. 15–21.

'In Defence of Gregory Chioniades', *AIHS*, 35, 1985, pp. 114–5.

'The *Paitāmahasiddhānta* of the *Viṣṇudharmottarapurāṇa*', *Brahmavidyā*, 31–2, 1967–8, pp. 472–510.

'Frammento astrologico (PL II(27))' (with R. Pintaudi), *Bulletin of the American Society of Papyrologists*, 18, 1981, pp. 83–7.

Δάρας τὸ νῦν καλούμενον Ταυρές, *Bulletin de la Classe des Sciences de l'Academie Royale de Belgique*, CI, lettres 5, 48, 1962, pp. 323–6.

'Al-Ṭabarī on the Prayers to the Planets', *Bulletin d'Études Orientales*, 44, 1992, pp. 105–17.

'The Later Pauliśasiddhānta', *Centaurus*, 14, 1969, pp. 172–241.

'Vasiṣṭha's Theory of Venus: the Misinterpretation of an Emendation', *Centaurus*, 19, 1975, pp. 36–9.

'Astronomical Computations for 1299 from the Cairo Geniza' (with B. R. Goldstein), *Centaurus*, 25, 1982, pp. 303–18.

'Antiochus and Rhetorius', *Classical Philology*, 72, 1977, pp. 203–23.

'Gregory Chioniades and Palaeologan Astronomy', *DOP*, 18, 1964, pp. 133–60.

'The Astrological School at John Abramius', *DOP*, 25, 1971, pp. 191–215.

'The Horoscope of Constantine VII Porphyrogenitus', *DOP*, 27, 1973, pp. 219–31.

'The Byzantine Version of the Toledan Tables: the Work of George Lapithes?', *DOP*, 30, 1977, pp. 85–132.

'Political Horoscopes from the Reign of Zeno', *DOP*, 30, 1977, pp. 133–50.

'Classical and Byzantine Astrology in Sassanian Persia', *DOP*, 43, 1989, pp. 227–88.

'On the Date of the *Mahāsiddhānta* of the Second Āryabhaṭa', *Gaṇita Bhāratī*, 14, 1992, pp. 55–6.

'The Byzantine Tradition of Vettius Valens' *Anthologies*', *Harvard Ukrainian Studies*, 7, 1983, pp. 532–41.

'A Hitherto Unknown Sanskrit Work Concerning Mādhava's Derivation of the Power Series for Sine and Cosine' (with D. Gold), *Historia Scientiarum*, 42, 1991, pp. 49–65.

'Sanskrit Geographical Tables', *Indian Journal of History of Science*, 31, 1996, pp. 173–220.

'Representation of the Planets in Indian Astrology', *Indo-Iranian Journal*, 8, 1965, pp. 249–67.

'From Alexandria to Baghdād to Byzantium: The Transmission of Astrology', *IJCT*, 8, 2001, pp. 3–21.

'Astronomy and Astrology in India and Iran', *Isis*, 54, 1963, pp. 229–46.

'Hellenophilia versus the History of Science', *Isis*, 83, 1992, pp. 554–63.

'The Empires of Rudradāman and Yaśodharman: Evidence from Two Astrological Geographies', *JAOS*, 79, 1959, pp. 267–70.

'A Greek Linear Planetary Text in India', *JAOS*, 79, 1959, pp. 282–4.

'Historical Horoscopes', *JAOS*, 82, 1962, pp. 487–502.

'Rejoinder to Wayman's 'The Buddha's Birthdate'', *JAOS*, 84, 1964, pp. 174–5.

'The Greek Influence on Early Islamic Mathematical Astronomy', *JAOS*, 93, 1973, pp. 32–43.

'The Beginning of Utpala's Commentary on the *Khaṇḍakhādyaka*', *JAOS*, 93, 1973, pp. 469–81.

'The Astronomical Tables of al-Khwārizmī in a Nineteenth Century Egyptian Text' (with B. R. Goldstein), *JAOS*, 98, 1978, pp. 96–9.

'A Note on the Calendars Used in Early Indian Inscriptions', *JAOS*, 102, 1982, pp. 355–9.

'Brahmagupta, Balabhadra, Pṛthūdaka, and al-Bīrūnī', *JAOS*, 103, 1983, pp. 353–60.
'Additional Astrological Almanacs from the Cairo Geniza' (with B. R. Goldstein), *JAOS*, 103, 1983, pp. 673–90.
'The Purāṇas and Jyotiḥśāstra: Astronomy', *JAOS*, 110, 1990, pp. 274–80.
'Two *Karmavipāka* Texts on Curing Diseases and Other Misfortunes', *Journal of the European Ayurvedic Society*, 5, 1997, pp. 46–52.
'The *Liber universus* of 'Umar al-Farrukhān al-Ṭabarī', *JHAS*, 1, 1977, pp. 8–12.
'Islamic Astronomy in Sanskrit', *JHAS*, 2, 1978, pp. 315–30.
'On the Classification of Indian Planetary Tables', *JHA*, 1, 1970, pp. 95–108.
'On the Greek Origin of the Indian Planetary Model Employing a Double Epicycle', *JHA*, 2, 1971, pp. 80–5.
'Precession and Trepidation in Indian Astronomy before A.D. 1200', *JHA*, 3, 1972, pp. 27–35.
'The Mesopotamian Origin of Early Indian Mathematical Astronomy', *JHA*, 4, 1973, pp. 1–12.
'The Recovery of Early Greek Astronomy from India', *JHA*, 7, 1976, pp. 109–23.
'Reply to B. L. van der Waerden', *JHA*, 11, 1980, pp. 58–62.
'On the Identification of the *Yogatārās* of the Indian *Nakṣatras*' (with P. Morrissey), *JHA*, 20, 1989, pp. 99–119.
'Bīja Corrections in Indian Astronomy', *JHA*, 27, 1996, pp. 161–72.
'Some Fourteenth-century Byzantine Astronomical Texts', *JHA*, 29, 1998, pp. 103–8.
'Nīlakaṇṭha's Planetary Models', *Journal of Indian Philosophy*, 29, 2001, pp. 187–95.
'The Persian 'Observation' of the Solar Apogee in *ca.* A.D. 450', *JNES*, 24, 1965, pp. 334–6.
'The Fragments of the Works of Ya'qūb ibn Ṭāriq', *JNES*, 27, 1968, pp. 97–125.
'The Fragments of the Works of al-Fazārī', *JNES*, 29, 1970, pp. 103–23.
'Horoscopes from the Cairo Geniza' (with B. R. Goldstein), *JNES*, 36, 1977, pp. 113–44.

'Political Horoscopes Relating to Late Ninth Century 'Alids' (with W. Madelung), *JNES*, 36, 1977, pp. 247–75.
'Astrological Almanacs from the Cairo Geniza' (with B. R. Goldstein), *JNES*, 38, 1979, pp. 153–75 and 231–56.
'Sanskrit Evidence for the Presence of Arabs, Jews, and Persians in Western India *ca.* 700–1300', *JOI Baroda*, 31, 1981, pp. 172–82.
'The *Śīghrasiddhi* of Lakṣmīdhara', *JOI Baroda*, 37, 1987–8, pp. 65–81.
'The Yavanajātaka of Sphujidhvaja', *JOR Madras*, 31, 1961–2, pp. 16–31.
'Indian Influence on Early Sasanian and Arabic Astronomy', *JOR Madras*, 33, 1964, pp. 1–8.
'Indian Influence on Sasanian and Early Islamic Astronomy and Astrology', *JOR Madras*, 34–5, 1964–6 (1973), pp. 118–26.

'The Indian Iconography of the Decans and Horās', *JWCI*, 26, 1963, pp. 223–54.
'Some of the Sources of the *Ghāyat al-ḥakīm*', *JWCI*, 43, 1980, pp. 1–15.
'A New Look at *Melancolia I*', *JWCI*, 43, 1980, pp. 257–8.
'Between the *Ghāya* and *Picatrix* I: the Spanish Version', *JWCI*, 44, 1981, pp. 27–56.
'An Illustrated Greek Astronomical Manuscript', *JWCI*, 45, 1982, pp. 185–92.
'Ibn al-Ḥātim on the Talismans of the Lunar Mansions' (with Kristen Lippincott), *JWCI*, 50, 1987, pp. 57–81.
'Indian Planetary Images and the Tradition of Astral Magic', *JWCI*, 52, 1989, pp. 1–13.
'Learned Magic in the Time of Frederick II', *Micrologus*, 2, 1994, pp. 39–56.
'The Astronomical Tables of Mahādeva' (with O. Neugebauer), *PAPS*, 111, 1967, pp. 69–92.
'Indian Astronomy', *PAPS*, 122, 1978, pp. 361–4.
'More Horoscopes from the Cairo Geniza' (with B. R. Goldstein), *PAPS*, 125, 1981, pp. 155–89.
'An Astronomer's Progress', *PAPS*, 143, 1999, pp. 73–85.
'Māshā'allāh's(?) Arabic Translation of Dorotheus', *Res Orientales*, 12, 1999, pp. 191–209.

'Artificial Demons and Miracles', *Res Orientales*, 13, 2001, pp. 109–22.

'Observational Texts Concerning the Planet Mercury' (with E. Reiner), *Revue d'Assyriologie*, 69, 1975, pp. 175–80.

'Amṛtalaharī of Nityānanda', *SCIAMVS*, 1, 2000, pp. 209–17.

'Āryabhaṭa, the Paitāmahasiddhānta, and Greek Astronomy', *Studies in History of Medicine and Science*, New Series 12, 1993, pp. 69–79.

'Sanskrit Translations of Arabic and Persian Astronomical Texts at the Court of Jayasiṃha of Jayapura', *Suhayl*, 1, 2000, pp. 101–6.

'The Indian and Pseudo-Indian Passages in Greek and Latin Astronomical and Astrological Texts', *Viator*, 7, 1976, pp. 141–95.

'Al-Bīrūnī's Treatise on Astrological Lots' (with F. I. Haddad and E. S. Kennedy), *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften*, 1, 1984, pp. 9–54.

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Subject
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5 *Book Reviews* (alphabetically by journal title, then chronologically)

J. Mogenet, *Le Grand Commentaire de Théon d'Alexandrie aux Tables Faciles de Ptolémée (reviewed and completed by A. Tihon with a commentary by the same)*, *AIHS*, 37, 1987, pp. 370–1.

Paul Kunitzsch, *Der Almagest: Die Syntaxis Mathematica des Claudius Ptolemäus in arabisch-lateinischer Überlieferung*, *BO*, 33, 1976, p. 246.

Paul Kunitzsch, *Ibn aṣ-Ṣalāḥ Zur Kritik der Koordinatenüberlieferung im Sternkatalog des Almagest*, *BO*, 34, 1977, p. 233.

Otto Neugebauer, *A History of Ancient Mathematical Astronomy*, *BO*, 34, 1977, pp. 303–4.

Ernst Hammerschmidt, *Äthiopische Kalendertafeln*, *BO*, 36, 1979, p. 262.

Sylvia Powels, *Der Kalender der Samaritaner anhand des Kitāb Ḥisāb as-Sinīn und anderer Handschriften*, *BO*, 38, 1981, pp. 563–4.

B. L. van der Waerden, *Das Heliozentrische System in der Griechischen, Persischen, und Indischen Astronomie*, *Centaurus*, 20, 1976, pp. 258–60.

Paul Kunitzsch, *Claudius Ptolemäus. Der Sternkatalog des Almagest. Die arabischmittelalterliche Tradition. II. Die lateinische Übersetzung Gerhards von Cremona*, *Centaurus*, 35, 1992, pp. 67–8.

Paul Kunitzsch, *Claudius Ptolemäus. Der Sternkatalog des Almagest. Die arabischmittelalterliche Tradition. III. Gesamtkonkordanz der Sternkoordinaten*, *Centaurus*, 36, 1993, p. 167.

Germaine Aujac, *Géminos. Introduction aux Phénomènes*, *The Classical World*, 70, 1977, pp. 148–9.

V. De Falco and M. Krause, *Hypsikles: Die Aufgangszeiten der Gestirne*, *Gnomon*, 40, 1968, pp. 13–17.

Wilhelm Gundel and Hans Georg Gundel, *Astrologoumena*, *Gnomon*, 40, 1968, pp. 276–80.

Manfred Erren, *Die Phainomena des Aratos von Soloi* (reviewed with Walther Ludwig), *Gnomon*, 43, 1971, pp. 346–54.

B. L. van der Waerden, *Science Awakening II: the Birth of Astronomy*, *HM*, 3, 1976, pp. 90–91.

A. S. Saidan, *The Arithmetic of Al-Uqlidisi*, *HM*, 7, 1980, pp. 97–8.

A. Y. al-Hassan, G. Karmi and N. Namnum, *Proceedings of the First International Symposium for the History of Arabic Science*, *HM*, 8, 1981, pp. 95–7.

S. N. Sen and A. K. Bag, *The Śulbasūtras of Baudhāyana, Āpastamba, Kātyāyana and Mānava with Text, English Translation and Commentary*, *HM*, 15, 1988, pp. 183–5.

W. Hübner, *Grade und Gradbezirke der Tierkreiszeichen*, *IJCT*, 6, 2000, pp. 473–6.

A. Jones, *Astronomical Papyri from Oxyrhynchus*, *IJCT*, 7, 2001, pp. 610–15.

O. Neugebauer and R. Parker, *Egyptian Astronomical Texts II. The Ramesside Star Clocks*, *Isis*, 57, 1966, pp. 136–7.

O. Jaggi, *History of Science and Technology in India, Vol. I: Dawn of Indian Technology (Pre- and Proto-Historic Period)*, and *vol. II: Dawn of Indian Science (Vedic and Upanishadic Period)*, *Isis*, 61, 1970, pp. 407–8.

Glenn R. Morrow (tr.), *Proclus: A Commentary on the First Book of Euclid's Elements*, *Isis*, 62, 1971, pp. 252–3.

B. Rama Rao, *A Check-List of Sanskrit Medical Manuscripts in India*, *Isis*, 65, 1974, pp. 115–16.

Alex Michaels, *Beweisverfahren in der vedischen Sakralgeometrie: Ein Beitrag zur Entstehungsgeschichte von Wissenschaft*, *Isis*, 72, 1981, pp. 140–41.

Priyadaranjan Ray; Hirendranath Gupta; Mira Roy, *Suśruta Saṃhitā: A Scientific Synopsis*, *Isis*, 73, 1982, p. 600.

Bīna Chatterjee, *Śiṣyadhīvṛddhida Tantra of Lalla, with the Commentary of Mallikārjuna Suri. Volumes I–II*, *Isis*, 74, 1983, pp. 284–5.

J. Mogenet, A. Tihon, R. Royez, and A. Berg (ed.), *Nicéphore Grégoras: Calcul de l'Éclipse de Soleil du 16 Juillet 1330*, *Isis*, 76, 1985, p. 433.

David A. King, *Islamic Mathematical Astronomy*, *Isis*, 80, 1989, pp. 310–11.

George Gheverghese Joseph, *The Crest of the Peacock: Non-European Roots of Mathematics*, *Isis*, 84, 1993, pp. 548–9.

William R. Newman, *The* Summa Perfectionis *of Pseudo-Geber: A Critical Edition, Translation, and Study*, *Isis*, 84, 1993, pp. 789–90.

André Allard, Roshdi Rashed (preface), Muḥammad ibn Mūsā al-Khwārizmī (original), *Le Calcul Indien (Algorismus). Histoire des textes, édition critique, traduction et commentaire des plus anciennes versions latines remaniées du XIIe siècle*, *Isis*, 85, 1994, pp. 307–8.

F. J. Ragep, *Naṣīr al-Dīn al-Ṭūsī's Memoir on Astronomy (al-Tadhkira fī 'ilm al-hay'a)*, *Isis*, 86, 1995, pp. 313–14.

S. N. Sen with A. K. Bag and S. Rajeswar Sarma, *A Bibliography of Sanskrit Works on Astronomy and Mathematics. Part I: Manuscripts, Texts, Translations and Studies*, *JAOS*, 87, 1967, p. 196.

K. V. Sharma, *Grahaṇamaṇḍana of Parameśvara*, *JAOS*, 87, 1967, pp. 337–9.

Hans Georg Gundel, *Weltbild und Astrologie in die Griechischen Zauberpapyri*, *JAOS*, 92, 1972, pp. 183–4.

Ajay Mitra Shastri, *India as Seen in the Bṛhatsaṃhitā of Varāhamihira*, *JAOS*, 94, 1974, pp. 487–8.

Fuat Sezgin, *Geschichte des Arabischen Schrifttums. Band VII: Astrologie Meteorologie und Verwandes bis ca 430H*, *JAOS*, 102, 1982, pp. 559–61.

Frits Staal, *The Fidelity of Oral Tradition and the Origins of Science*, *JAOS*, 108, 1988, pp. 637–8.

Francis Zimmerman, *The Jungle and the Aroma of Meats: An Ecological Theme in Hindu Medicine*, *JAOS*, 109, 1989, pp. 664–5.

Otto Neugebauer, *Abu Shaker's 'Chronography': A Treatise of the 13th Century on Chronological, Calendrical, and Astronomical Matters, Written by a Christian Arab, Preserved in Ethiopic Chronography in Ethiopic Sources*, *JAOS*, 111, 1991, pp. 166–7.

Hans-Georg Türstig, *Yantracintāmaṇiḥ of Dāmodara*, *JAOS*, 111, 1991, pp. 416–17.

Francis Zimmerman, *Le Discours des Remèdes au Pays des Épices*. *JAOS*, 111, 1991, pp. 839–40.

R. S. Webster, P. R. MacAlister, and F. Etting, *Astrolabe Kit*, and MacAlister and Etting, *A Trilogy of Time Instruments*, *JHAS*, 1, 1977, pp. 325–6.

Bīna Chatterjee, *The Khaṇḍakhādyaka (an Astronomical Treatise) of Brahmagupta with the Commentary of Bhaṭṭotpala*, *JHA*, 2, 1971, pp. 121–2.

Jack Lindsay, *The Origins of Astrology*, *JHA* 4, 1973, p. 59.

J. Eric S. Thompson, *A Commentary on the Dresden Codex: A Maya Hieroglyphic Book*, *JHA*, 5, 1974, pp. 137–8.

Anne Tihon, *Le 'Petit Commentaire' de Théon d'Alexandrie aux Tables Faciles de Ptolémée*, *JHA*, 11, 1980, pp. 137–8.

G. Aujac (with J. P. Brunet and R. Nadal), *Le Sphere en Mouvement. Levers et Couchers Héliaques, Testimonia*, *JHA*, 12, 1981, pp. 148–9.

Fitzedward Hall (ed.), *The Sūrya Siddhānta, or an Ancient System of Hindu Astronomy. With the Exposition of Raṅganātha, the Gūḍhārtha-prakāśaka*, *JHA*, 15, 1984, pp. 47–8.

Pundit Babu Deva Sastri and Lancelot Wilkinson, *The Sūrya Siddhānta, or an Ancient System of Hindu Astronomy. Followed by the Siddhānta Śiromaṇi*, *JHA*, 15, 1984, pp. 47–8.

Paul Kunitzsch and Tim Smart, *Short Guide to Modern Star Names and Their Derivatives*, *JHA*, 19, 1988, p. 59.

Nikolaus Gross, *Senecas Naturales Quaestiones: Komposition, Naturphilosophische Aussagen und Ihre Quellen*, *JHA*, 23, 1992, pp. 146–7.

'Ptolemy's Geographical Guide Explored', essay review of J. L. Berggren and A. Jones, *Ptolemy's Geography*, *JHA*, 34, 2003, pp. 235–7.

K. V. Sarma, *Līlāvatī of Bhāskarācārya with Kriyākramakarī of Śaṅkara and Nārāyaṇa*, *Journal of the Oriental Institute*, 25, 1975, pp. 104–5.

R. E. Emmerick, *Siddhasāra of Ravigupta, vol. I*, *JRAS*, 1982, pp. 70–71; *vol. 2*, *JRAS*, 1984, pp. 157–8.

Hans-Georg Türstig, *Jyotisa; das System der indischen Astrologie*, *JRAS*, 1982, pp. 72–3.

Arvind Sharma, *Studies in 'Alberuni's India'*, *JRAS*, 1984, pp. 295–6.

D. Wujastyk, *A Handlist of the Sanskrit and Prakrit Manuscripts in the Library of the Wellcome Institute for the History of Medicine*, *JRAS*, 1987, pp. 347–8.

Asger Aaboe and Norman T. Hamilton, *Contributions to the Study of Babylonian Lunar Theory*, *Orientalia*, 51, 1982, pp. 141–2.

6 *Others*

'Otto Neugebauer, 26 May 1899 – 19 February 1990', *Isis*, 82, 1991, pp. 87–8.

'Letter to the Editor', *Isis*, 85, 1994, pp. 668–9.

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